Investigation of $K^{bar}NN$ resonances with a coupled-channel Complex Scaling Method + Feshbach projection

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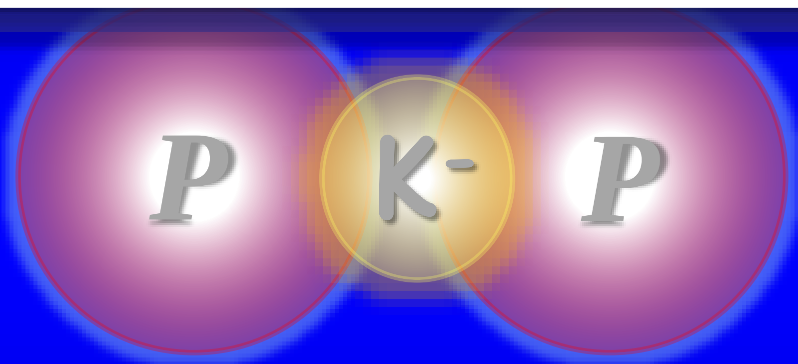
1. Introduction
2. Effective single-channel potential by "Feshbach method with coupled-channel Complex Scaling Method"
3. Result of "$K^{-}pp$" calculated with ccCSM+Feshbach method
4. Further investigation
   - Other quantum number case – $J^{\pi}=1^{-}$ … $S_{NN}=1$
   - $K^{-}pp$ with SIDDHARA data
5. Summary and future plan

"Hadrons and Hadron Interactions in QCD 2015 – Effective Theories and Lattice –"
03. Mar. '15 @ YITP, Kyoto univ., Kyoto, Japan

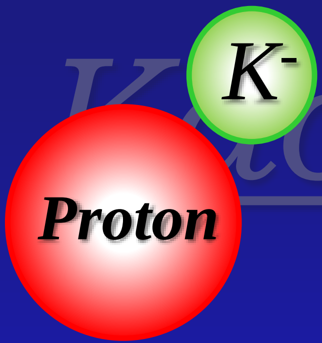
Kaonic nuclei

1. Introduction



Prototype system = $K^{-}pp$

Kaonic nuclei

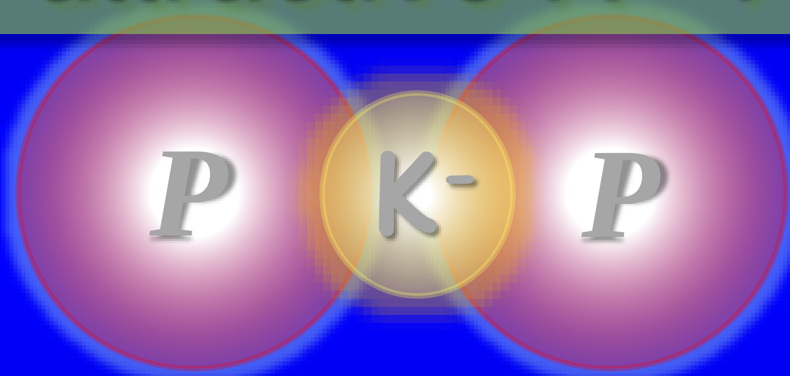


$K^{bar}N$ two-body system

Low energy scattering data, 1s level shift of kaonic hydrogen atom

*"Excited hyperon **Λ(1405)** = K^- proton quasi-bound state"*

Strongly attractive $K^{bar}N$ potential

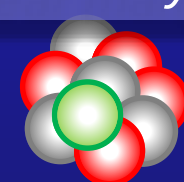


Prototype system = $K^- pp$

- *Doorway to **dense matter**†*
  → Chiral symmetry restoration in dense matter
- *Interesting structure†*
- *Neutron star*

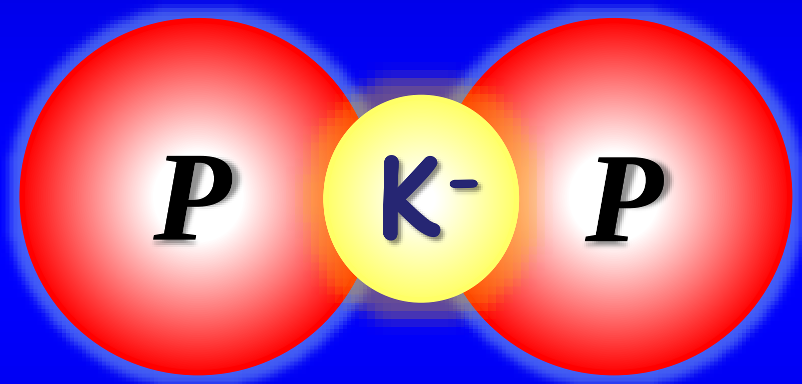
† A. D., H. Horiuchi, Y. Akaishi and T. Yamazaki, PRC70, 044313 (2004)

Nuclear many-body system with K^-



$^3HeK^-$, $pppK^-$,
$^4HeK^-$, $pppnK^-$,
…, $^8BeK^-$, …

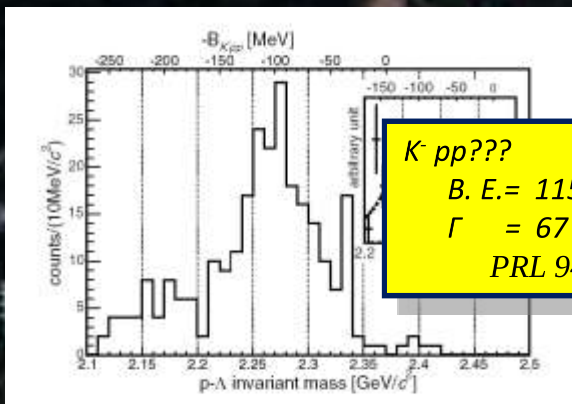
Kaonic nuclei



Prototype system = K^-pp

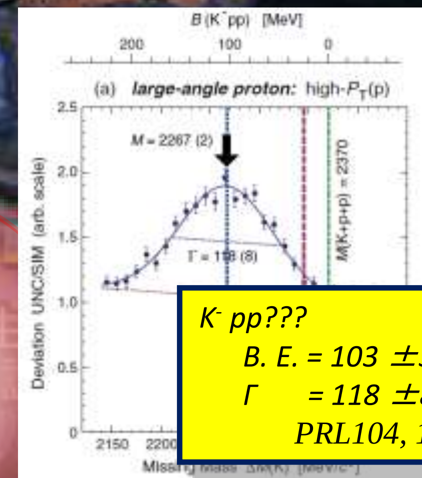
Experiments of K⁻pp search

FINUDA



K⁻pp???
B. E.= 115 MeV
Γ = 67 MeV
PRL 94, 212303 (2005)

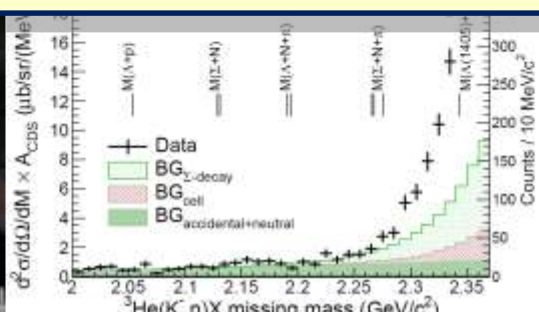
DISTO



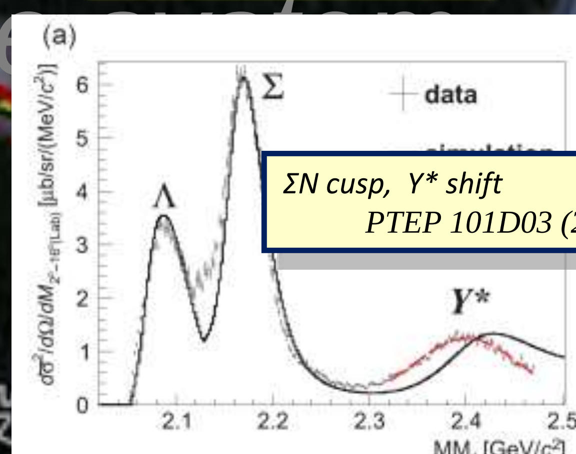
K⁻pp???
B. E. = 103 ±3 ±5 MeV
Γ = 118 ±8 ±10 MeV
PRL104, 132502 (2010)

J-PARC E15

Attraction in K⁻pp subthreshold region
arXiv:1408.5637 [nucl-ex]

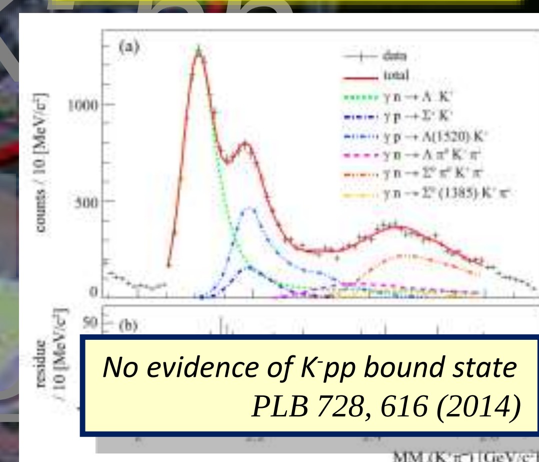


J-PARC E27



ΣN cusp, Y shift*
PTEP 101D03 (2014)

SPring8/LEPS



No evidence of K⁻pp bound state
PLB 728, 616 (2014)

Theoretical studies of $K^{-}pp$

	Dote-Hyodo-Weise	Barnea-Gal-Liverts	Akaishi-Yamazaki	Ikeda-Kamano-Sato	Shevchenko-Gal-Mares
	PRC79, 014003 (2009)	PLB712, 132 (2012)	PRC76, 045201 (2007)	PTP124, 533 (2010)	PRC76, 044004 (2007)
$B(K^{-}pp)$	**20±3**	**16**	**47**	**9 ～ 16**	**50 ～ 70**
Γ	40 ～ 70	41	61	34 ～ 46	90 ～ 110
Method	Variational (Gauss)	Variational (H. H.)	Variational (Gauss)	Faddeev-AGS	Faddeev-AGS
Potential	*Chiral (E-dep.)*	*Chiral (E-dep.)*	*Pheno.*	*Chiral (E-dep.)*	*Pheno.*

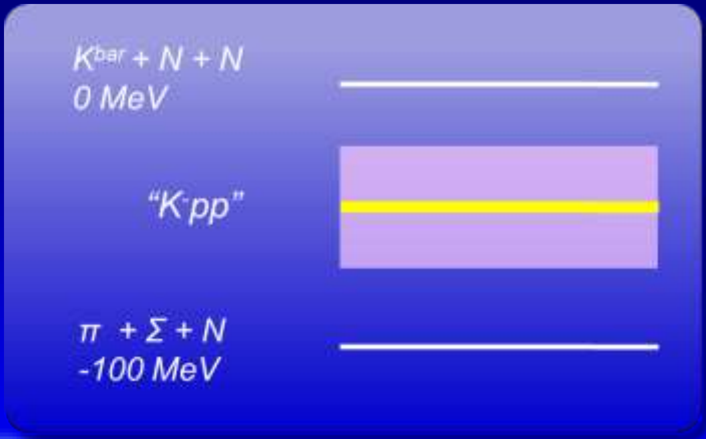
- ***Chiral pot. (E-dep.) → Small binding***
- ***Phenomenological pot. (E-indep.) → Large binding***

$B(K^{-}pp)$ < 100 MeV

$K^{-}pp$ should be bound, but exist as a resonance between $K^{bar}NN$ and $\pi\Sigma N$ thresholds.

Kaonic nuclei

- **Resonant state**
- **Coupled-channel system**



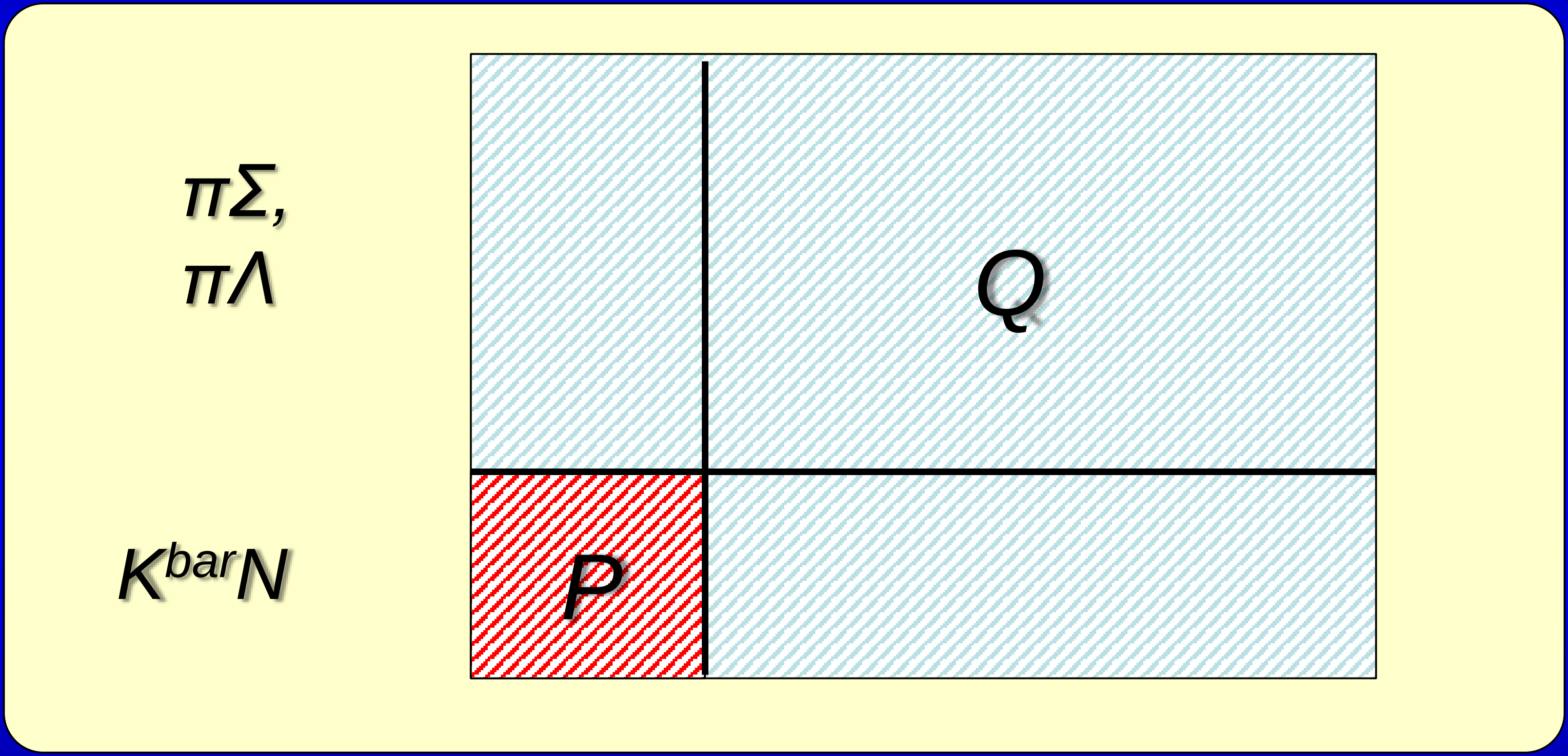
⇒ coupled-channel Complex Scaling Method

Prototype system = $K^{-}pp$

- **Anti-kaon = Nambu-Goldstone boson**

⇒ Chiral SU(3)-based $K^{bar}N$ potential

2. Effective single-channel potential by "Feshbach projection with ccCSM"



Reduce the coupled-channel problem to a single channel problem

Complex Scaling Method for Resonance

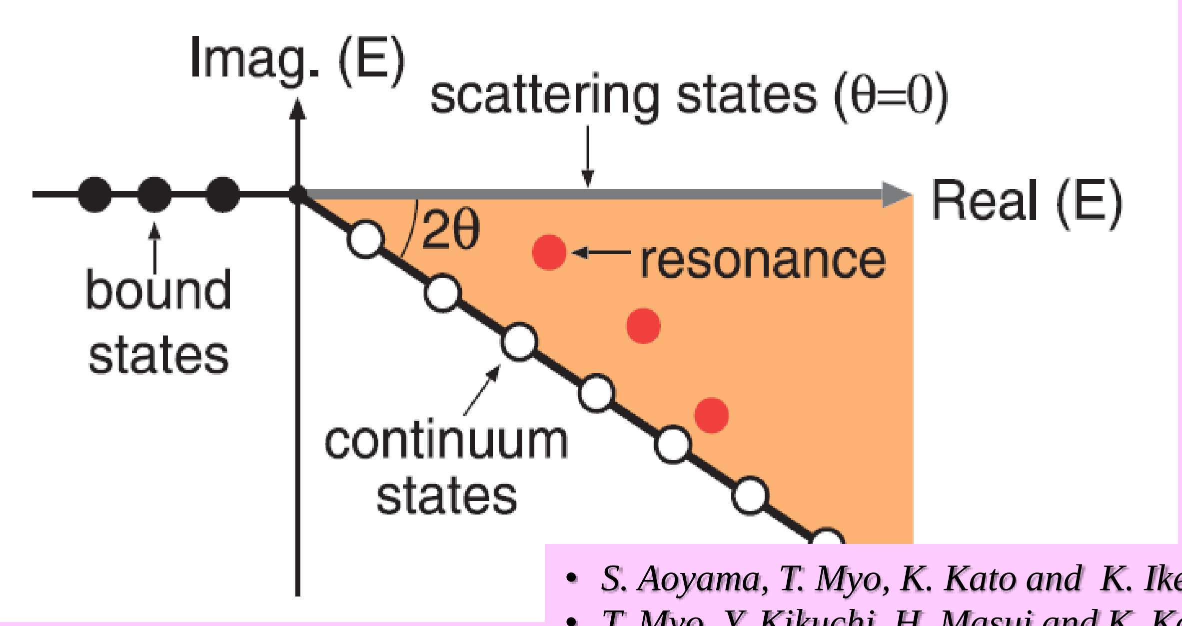
Complex rotation of coordinate (Complex scaling)

$$U(\theta): \quad \mathbf{r} \to \mathbf{r}\, e^{i\theta}, \quad \mathbf{k} \to \mathbf{k}\, e^{-i\theta}$$

$$H_\theta \equiv U(\theta) H U^{-1}(\theta), \quad |\Phi_\theta\rangle \equiv U(\theta)|\Phi\rangle$$

Diagonalize H_θ with Gaussian base,

we can obtain resonant states, in the same way as bound states!



- *S. Aoyama, T. Myo, K. Kato and K. Ikeda, PTP 116, 1 (2006)*
- *T. Myo, Y. Kikuchi, H. Masui and K. Kato, PPNP 79, 1 (2014)*

† J. Aguilar and J. M. Combes, Commun. Math. Phys. 22 (1971),269.
E. Balslev and J. M. Combes, Commun. Math. Phys. 22 (1971),280

- *Continuum state appears on 2ϑ line.*
- *Resonance pole is off from 2ϑ line, and independent of ϑ. (ABC theorem†)*

Formalism of ccCSM + Feshbach method

Elimination of channels by Feshbash method

Schrödinger eq.
in model space "P" and out of model space "Q"

$$\begin{pmatrix} T_P + v_P & V_{PQ} \\ V_{QP} & T_Q + v_Q \end{pmatrix}\begin{pmatrix} \Phi_P \\ \Phi_Q \end{pmatrix} = E\begin{pmatrix} \Phi_P \\ \Phi_Q \end{pmatrix}$$

Schrödinger eq. in P-space : $\left(T_P + U_P^{Eff}(E) \right)\Phi_P = E\,\Phi_P$

Effective potential for P-space

$$U_P^{Eff}(E) = v_P + V_{PQ}\,G_Q(E)V_{QP}$$

Q-space Green function: $G_Q(E) = \dfrac{1}{E - H_{QQ}}$

Extended Closure Relation in Complex Scaling Method

$$H_{QQ}^{\theta}\left|\chi_n^{\theta}\right\rangle = \varepsilon_n^{\theta}\left|\chi_n^{\theta}\right\rangle$$

$$H_{QQ}^{\theta} = U(\theta)H_{QQ}U^{-1}(\theta)$$

$$\int_C \sum_{R+B} \left|\chi_n^{\theta}\right\rangle\left\langle\chi_n^{\theta}\right| = 1$$

Diagonalize H^{θ}_{QQ} with Gaussian base,

$$\sum_n \left|\chi_n^{\theta}\right\rangle\left\langle\chi_n^{\theta}\right| \approx 1$$

Well approximated

T. Myo, A. Ohnishi and K. Kato, PTP 99, 801 (1998)
R. Suzuki, T. Myo and K. Kato, PTP 113, 1273 (2005)

Express the $G_Q(E)$ with Gaussian base using ECR

$$G_Q^{\theta}(E) = \frac{1}{E - H_{QQ}^{\theta}} \approx \sum_n \left|\chi_n^{\theta}\right\rangle \frac{1}{E - \varepsilon_n^{\theta}}\left\langle\chi_n^{\theta}\right|$$

$$U_P^{Eff}(E) = v_P + V_{PQ}\,\underbrace{U^{-1}(\theta)G_Q^{\theta}(E)U(\theta)}_{G_Q(E)}\,V_{QP}$$

$\left\{ \left|\chi_n^{\theta}\right\rangle \right\}$: expanded with Gaussian base.

3. Result of K^-pp calculated with ccCSM + Feshbach method

Apply ccCSM + Feshbach method to K^-pp

"K^-pp" … $K^{bar}NN$ - $\pi\Sigma N$ - $\pi\Lambda N$ ($J^\pi=0^-$, $T=1/2$)

For the two-body system, $P = K^{bar}N$, $Q = \pi Y$

$$V\left(K^{bar}N-\pi Y; I=0,1\right)$$
$$V\left(\pi Y-\pi Y' \quad ; I=0,1\right)$$

Feshbach + ccCSM →

$$U^{Eff}_{K^{bar}N(I=0,1)}\left(E\right)$$

- Schrödinger eq. for $K^{bar}NN$ channel :

$$\left(T_{K^{bar}NN}+V_{NN}+\sum_{i=1,2}U^{Eff}_{K^{bar}N_i(I)}\left(E_{K^{bar}N}\right)\right)\Phi_{K^{bar}NN} = E\,\Phi_{K^{bar}NN}$$

- Trial wave function

$$\left|"K^-pp"\right\rangle = \sum_a C_a^{(KNN,1)}\left\{G_a^{(KNN,1)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)+G_a^{(KNN,1)}\left(-\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)\right\}\left|S_{NN}=0\right\rangle\left|\left[K\left[NN\right]_1\right]_{T=1/2}\right\rangle$$

Ch. 1: $K^{bar}NN$, NN: ^1E

$$+\sum_a C_a^{(KNN,2)}\left\{G_a^{(KNN,2)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)-G_a^{(KNN,2)}\left(-\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)\right\}\left|S_{NN}=0\right\rangle\left|\left[K\left[NN\right]_0\right]_{T=1/2}\right\rangle$$

Ch. 2: $K^{bar}NN$, NN: ^1O

- Basis function = Correlated Gaussian
…including 3-types Jacobi-coordinates

$$G_a^{(KNN,i)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)=N_a^{(KNN,i)}\exp\left[-\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)A_a^{(KNN,i)}\begin{pmatrix}\mathbf{x}_1^{(3)}\\ \mathbf{x}_2^{(3)}\end{pmatrix}\right]$$

Chiral SU(3) potential with a Gaussian form

- Weinberg-Tomozawa term of effective chiral Lagrangian
- Gaussian form in r-space
- Semi-rela. / Non-rela.
- Based on Chiral SU(3) theory
  → **Energy dependence**

A non-relativistic potential (NRv2c)

$$V_{ij}^{(I=0,1)}(r) = -\frac{C_{ij}^{(I=0,1)}}{8 f_\pi^2} \left(\omega_i + \omega_j\right) \sqrt{\frac{1}{m_i m_j}}\, g_{ij}(r)$$

$$g_{ij}(r) = \frac{1}{\pi^{3/2} d_{ij}^3} \exp\left[-\left(r/d_{ij}\right)^2\right]$$: Gaussian form

ω_i: meson energy

Constrained by $K^{bar}N$ scattering length

$a_{KN(I=0)} = -1.70+i0.67fm$, $a_{KN(I=1)} = 0.37+i0.60fm$ A. D. Martin, NPB179, 33(1979)

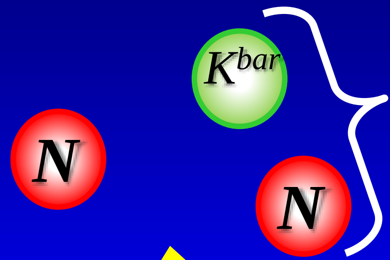
A resonance pole corresponding to Λ(1405) at
(M, -Γ/2) ~ (1420, -20) MeV **B_{KN} ~ 15MeV**

A. D., T. Inoue, T. Myo, Nucl. Phys. A 912, 66 (2013)

Double-pole structure is confirmed.
Lower pole ~ (1395, -138) MeV at f_π=110 MeV

A. D., T. Myo, Nucl. Phys. A 930, 86 (2014)

Self-consistency for complex $K^{bar}N$ energy



E_{KN}

$$U^{Eff}_{K^{bar}N_i(I)}\left(E_{K^{bar}N}\right)$$

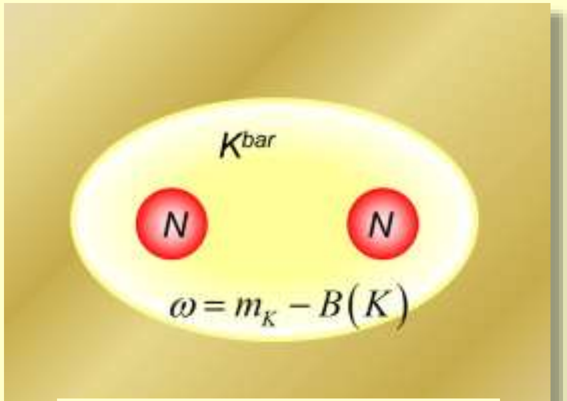
How to determine the two-body energy in the three-body system?

A. D., T. Hyodo, W. Weise, PRC79, 014003 (2009)

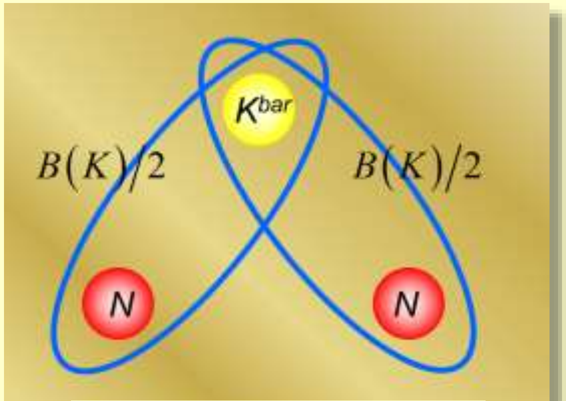
1. Kaon's binding energy: $B(K) \equiv -\left\{\langle H\rangle - \langle H_{NN}\rangle\right\}$ H_{NN} : Hamiltonian of two nucleons

2. Define a $K^{bar}N$-bond energy in two ways

$$E_{KN} = M_N + \omega = \begin{cases} M_N + m_K - B(K) & \text{: Field picture} \\ M_N + m_K - B(K)/2 & \text{: Particle picture} \end{cases}$$



Field picture

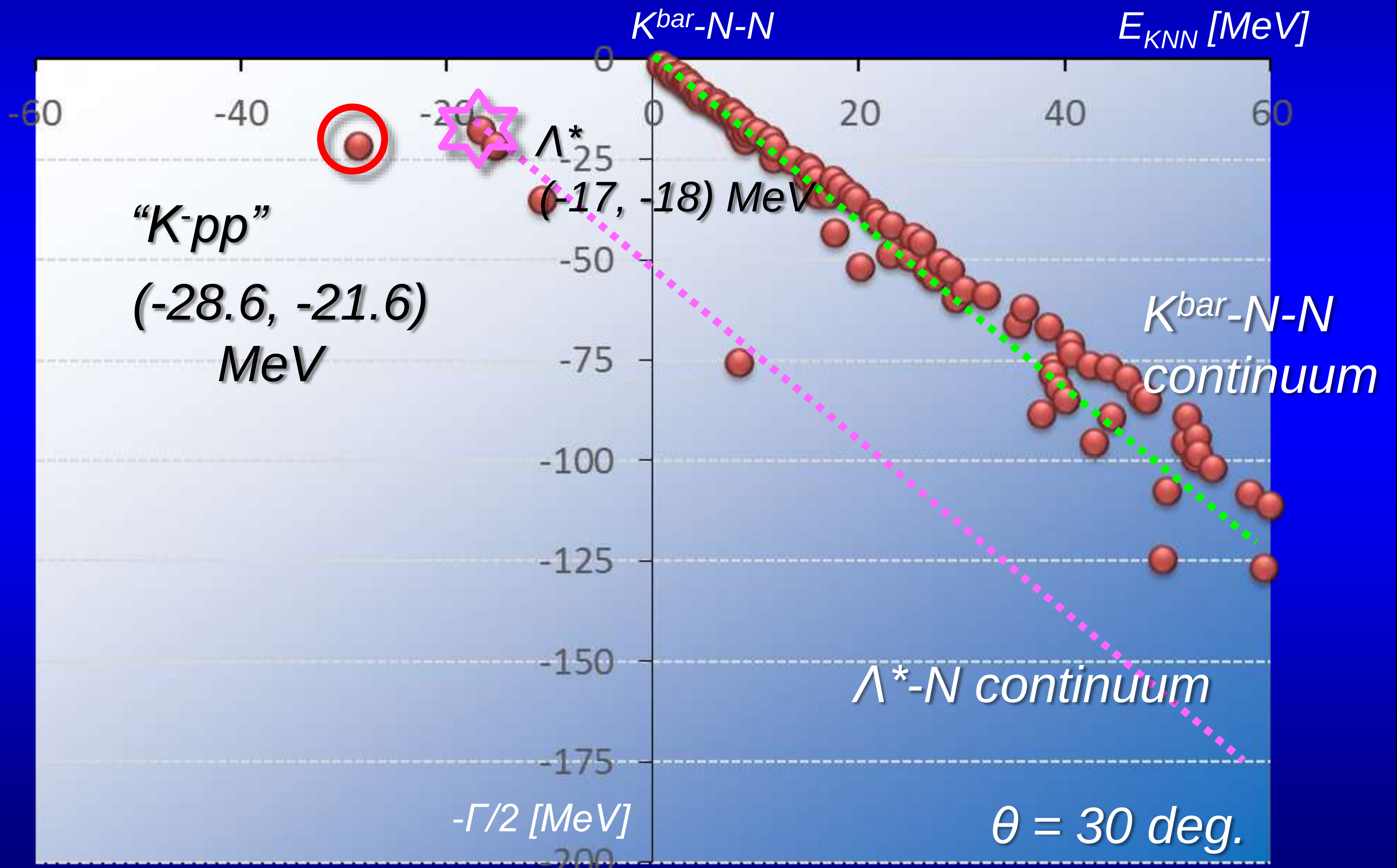


Particle picture

Result

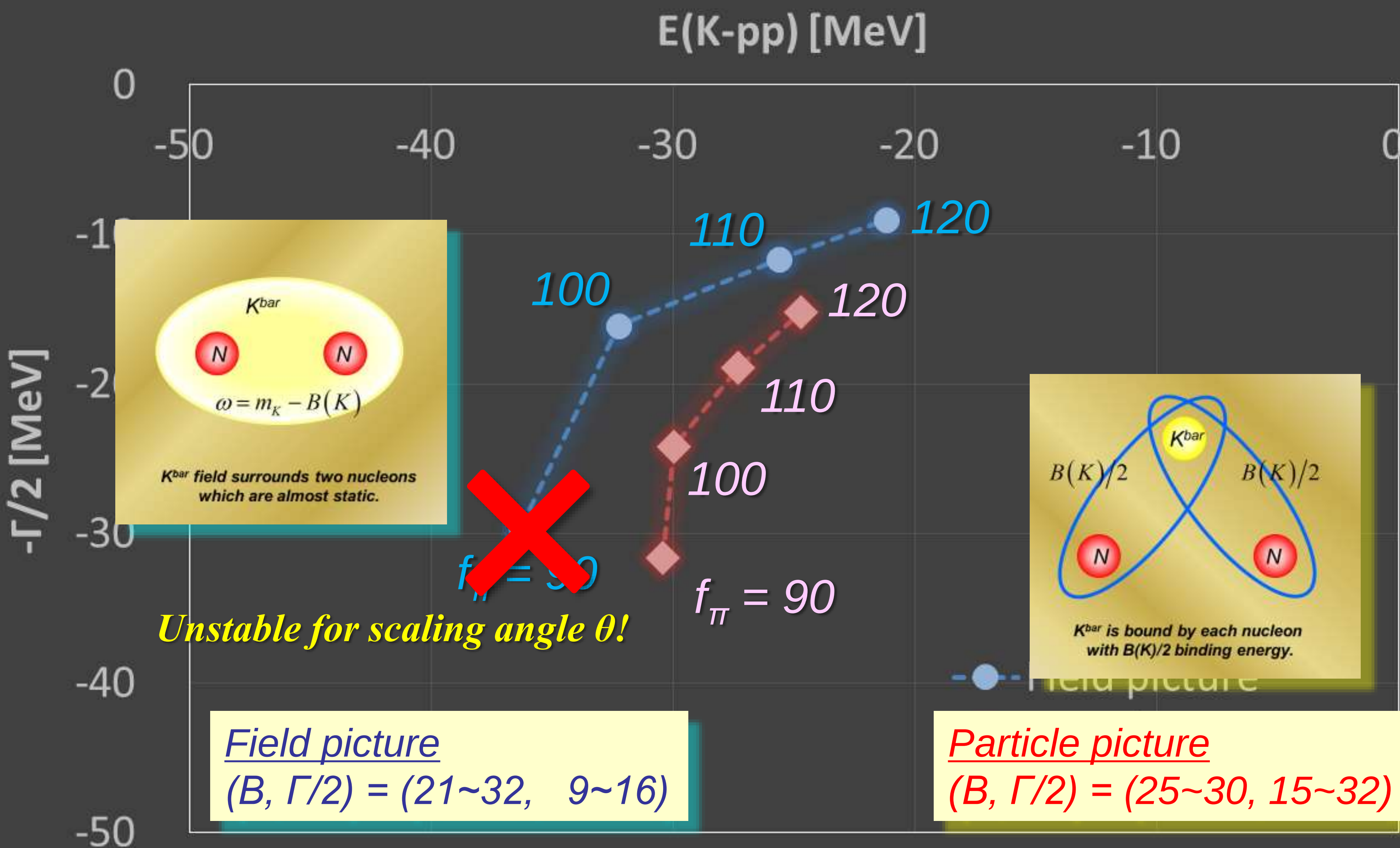
NN pot. : Av18 (Central)
$K^{bar}N$ pot. : NRv2c potential (f_π=110MeV)

Fix the $K^{bar}N$ energy at Λ*
… self-consistent for Λ* in free space



Self-consistent results f_π=90~120MeV

NN pot. : Av18 (Central)
K^{bar}N pot. : NRv2c potential
(f_π=90 - 120MeV)



Field picture
(B, Γ/2) = (21~32, 9~16)

Particle picture
(B, Γ/2) = (25~30, 15~32)

NN correlation density

NN pot. : Av18 (Central)
$K^{bar}N$ pot. : NRv2c potential
f_π=110, Particle pict.

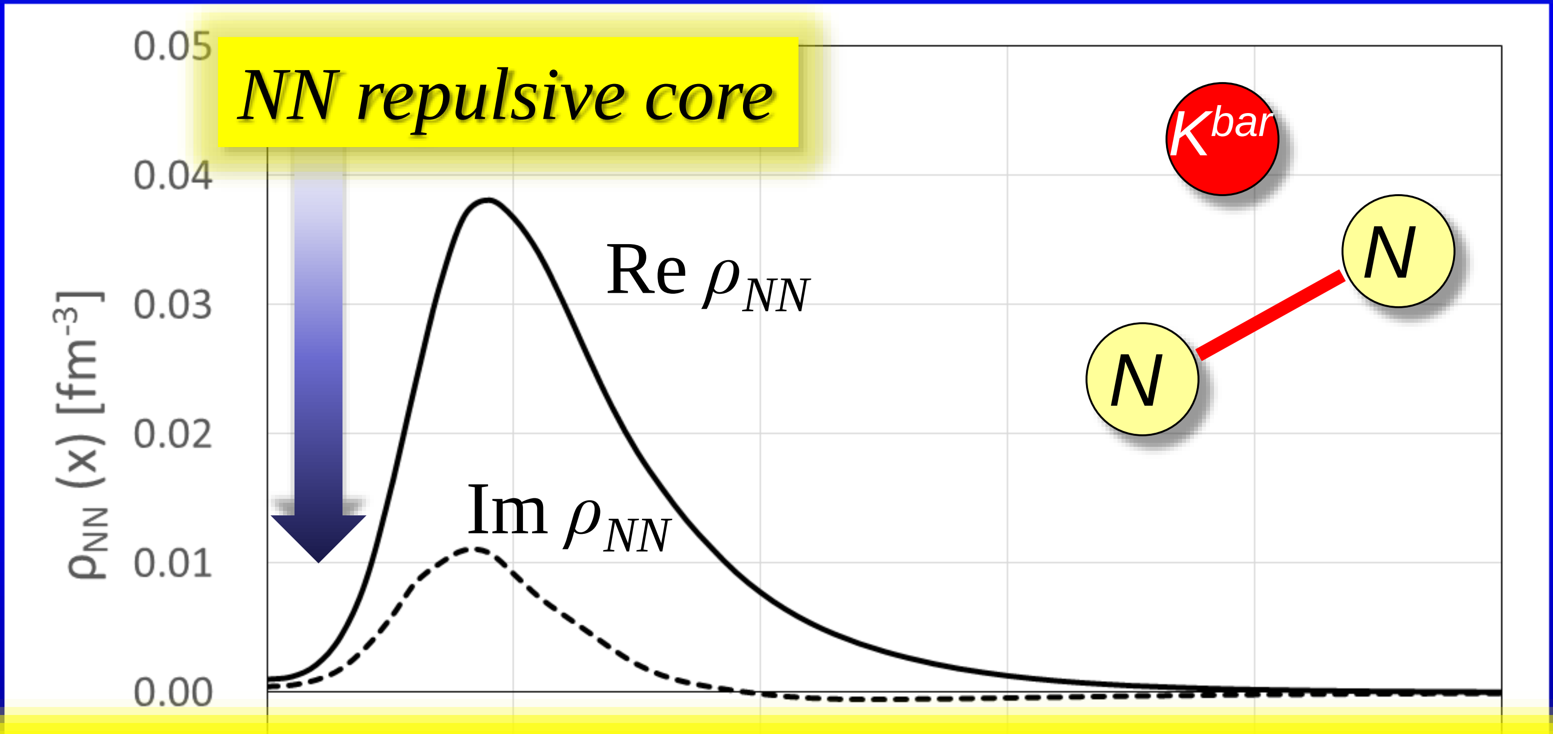
Correlation density in Complex Scaling Method

$$\rho_{NN,\theta}(\mathbf{x}) = \delta^3\left(\hat{\mathbf{r}}_{NN,\theta} - \mathbf{x}\right)$$

$$\hat{\mathbf{r}}_{XN,\theta} = \hat{\mathbf{r}}_{XN}\, e^{i\theta}$$

$$\rho_{NN}(\mathbf{x}) \equiv \left\langle \Phi_\theta \middle| \rho_{XN,\theta}(\mathbf{x}) \middle| \Phi_\theta \right\rangle$$

$$= e^{-3i\theta} \int d^3\mathbf{R}\, \Phi_\theta^2\left(\mathbf{x}e^{-i\theta}, \mathbf{R}\right)$$

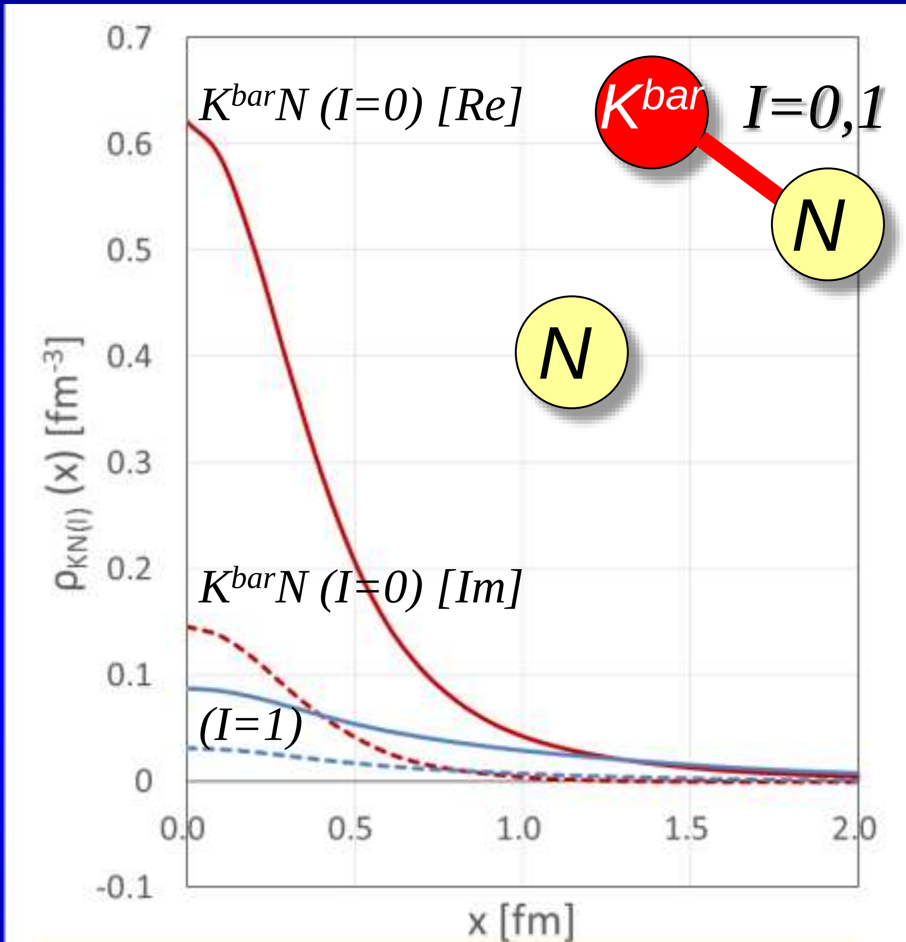


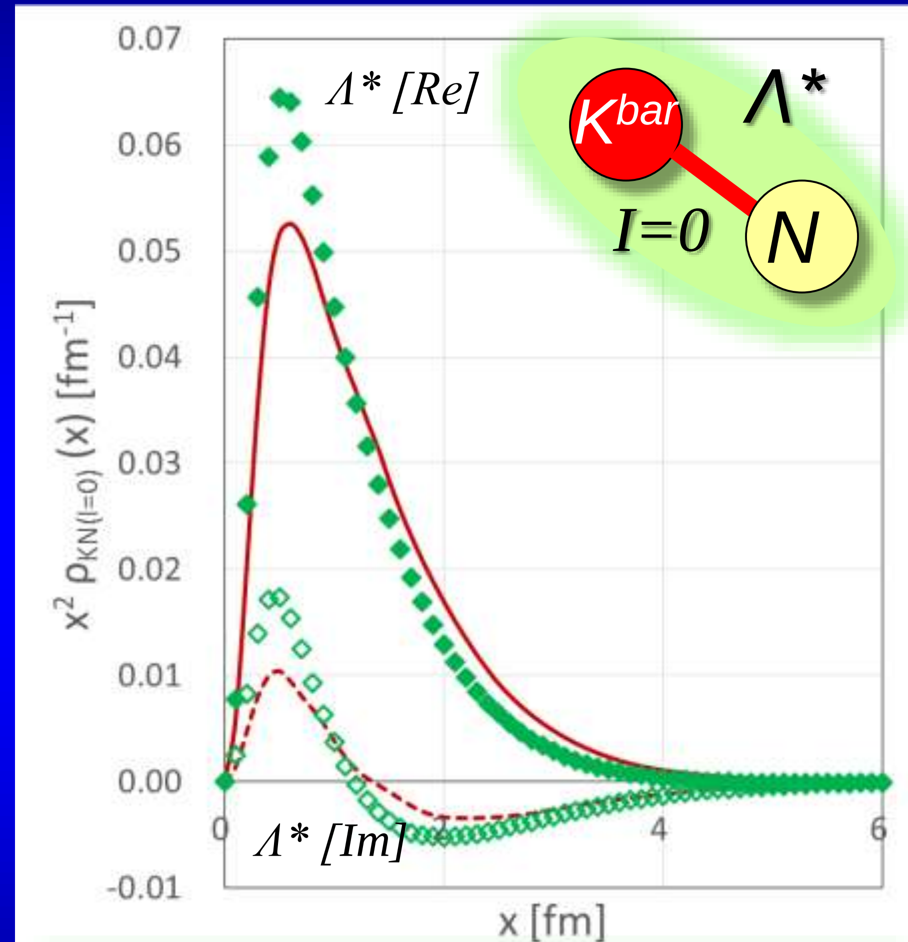
NN distance = 2.1 - i 0.3 fm

~ Mean distance of 2N in nuclear matter at normal density!

$K^{bar}N$ correlation density

NN pot. : Av18 (Central)
$K^{bar}N$ pot. : NRv2c potential
f_π=110, Particle pict.





$I=0$ $K^{bar}N$ compacter than $I=1$ one

Strong $K^{bar}N$ attraction in $I=0$

$I=0$ $K^{bar}N$ seems similar to Λ*

Λ* survives in K^-pp

4. Further investigation

- *$K^{bar}NN$ with $J^{\pi}=1^{-}$ state*

  … $S_{NN}=1$

- *$K^{-}pp$ with SIDDHARTA data*

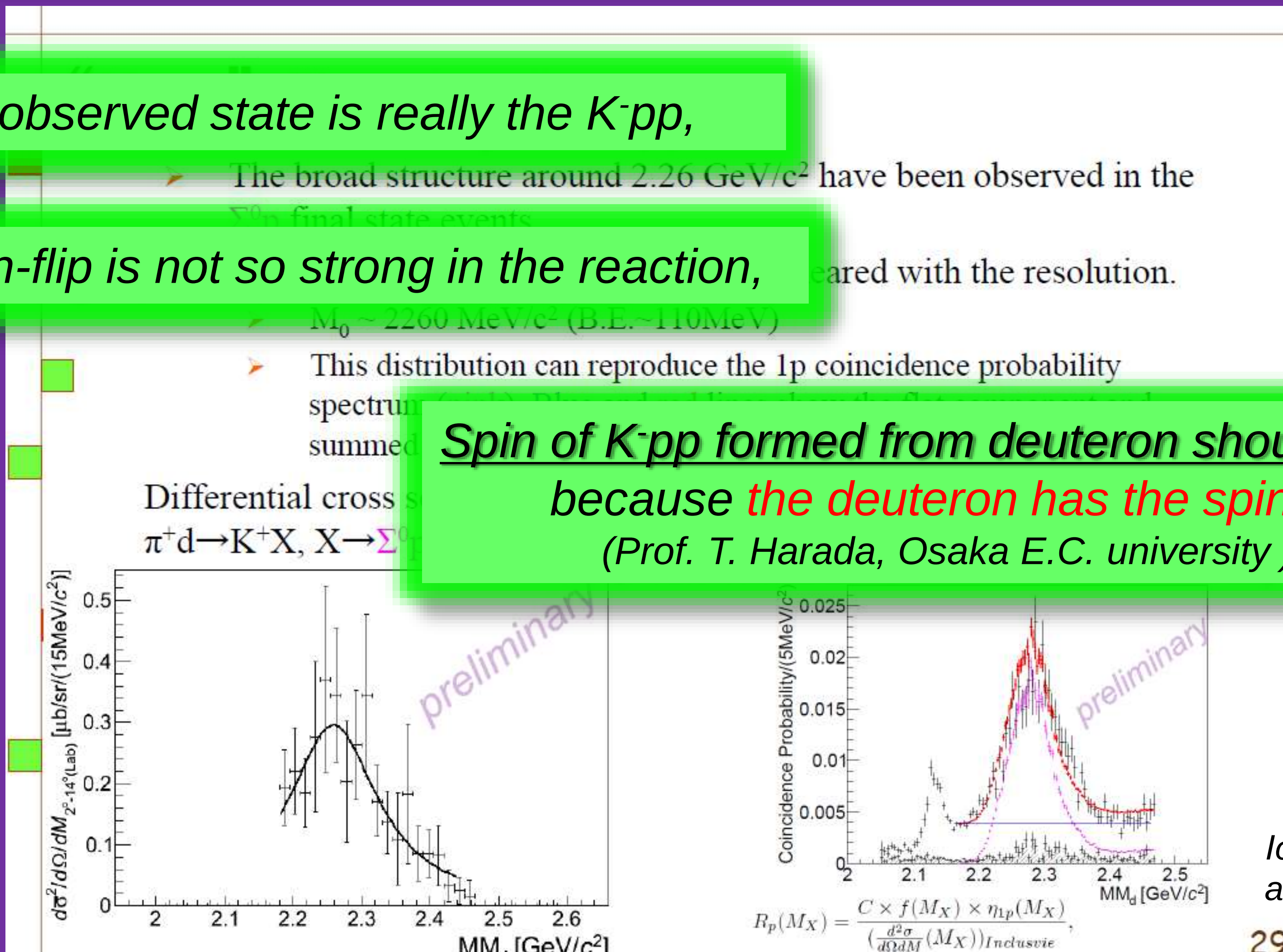
J-PARC E27 experiment

K^-pp search by d (π^+, K^+) reaction at 1.69 GeV/c

Inclusive spectrum: Ichikawa et al., PTEP 101D03 (2014)

✓*If the observed state is really the K^-pp,*

✓*If spin-flip is not so strong in the reaction,*



Spin of K^-pp formed from deuteron should be 1,

because the deuteron has the spin 1.

(Prof. T. Harada, Osaka E.C. university)

Ichikawa's talk at EXA2014

How is $J^\pi = 1^-$ state … S_{NN}=1?

$$\left|"K^- pp"\right\rangle \approx \left|L_{KNN}=0, NN: s-wave\right\rangle \left|S_{NN}=0\right\rangle \left|\left[K\left[NN\right]_1\right]_{T=1/2}\right\rangle$$

$J^\pi=0^-$, $T=1/2$

$$\left|"K^- d"\right\rangle \approx \left|L_{KNN}=0, NN: s-wave\right\rangle \left|\mathbf{S_{NN}=1}\right\rangle \left|\left[K\left[NN\right]_{\mathbf{0}}\right]_{T=1/2}\right\rangle$$

$J^\pi=1^-$, $T=1/2$

"K^- + Deuteron"-like channel

"K^- d" studied ***simply*** with …

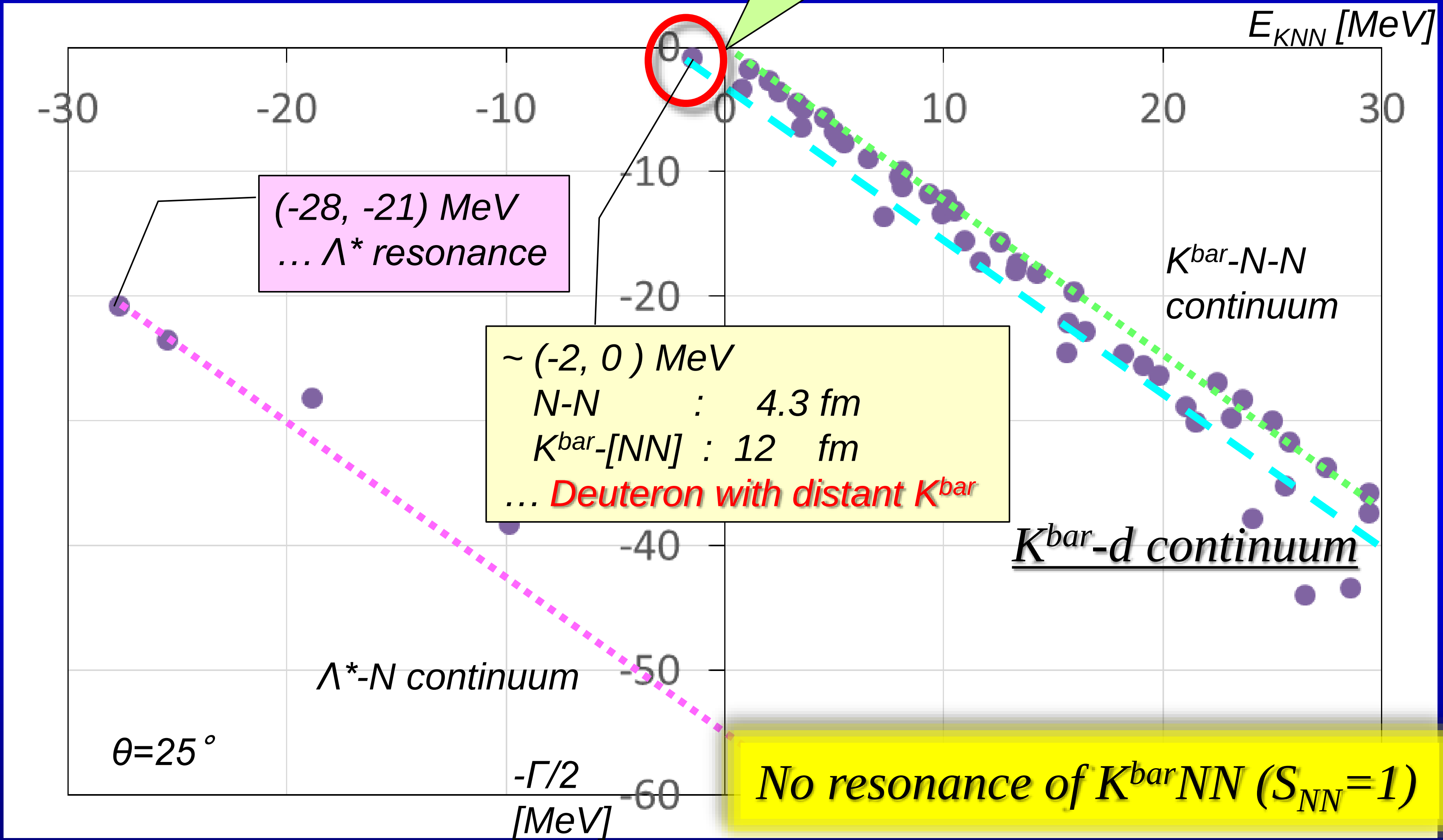
- NN potential: Av4' (^{3}E, ^{3}O) (fitted with 5-range Gaussian functions)
  … *Tensor force is incorporated into central potential.*
- K^{bar}N potential: A phenomenological potential
  … *Energy independent*

Y. Akaishi and T. Yamazaki, PRC 52 (2002) 044005

- Λ*-fixed ansatz
  … *E(K^{bar}N) in effective K^{bar}N potential is fixed to the Λ* energy.*

How is $J^{\pi} = 1^{-}$ state … $S_{NN}=1$?

NN pot.   : Av4' (5 Gauss)
$K^{bar}N$ pot. : AY potential
Λ^* fixed



No resonance of $K^{bar}NN$ ($S_{NN}=1$)

4. Further investigation

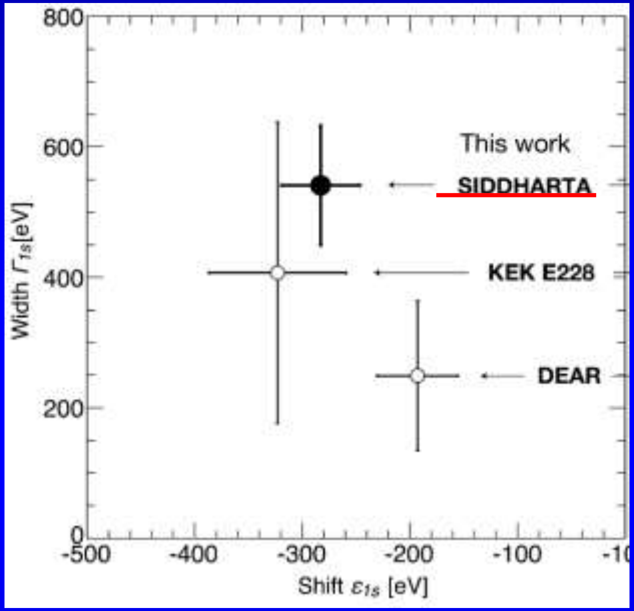
- $K^{bar}NN$ with $J^{\pi}=1^{-}$ state

  … $S_{NN}=1$

- $K^{-}pp$ with SIDDHARTA data

K^-pp with SIDDHARTA data

Precise measurement of 1s level shift of kaonic hydrogen



Strong constraint for the $K^{bar}N$ interaction!

$$\epsilon_{1s} = -283 \pm 36(\text{stat}) \pm 6(\text{syst})\ \text{eV}$$
$$\Gamma_{1s} = 541 \pm 89(\text{stat}) \pm 22(\text{syst})\ \text{eV},$$

M. Bazzi et al. (SIDDHARTA collaboration), NPA 881, 88 (2012)

K^-p scattering length (with improved Deser-Truman formula)

U. -G. Meissner, U. Raha and A. Rusetsky, Eur. Phys. J. C 35, 349 (2004)

$$\text{Re}\, a(K^- p) = -0.65 \pm 0.10\ \text{fm}, \qquad \text{Im}\, a(K^- p) = 0.81 \pm 0.15\ \text{fm}$$

Y. Ikeda, T. Hyodo and W. Weise, NPA 881, 98 (2012)

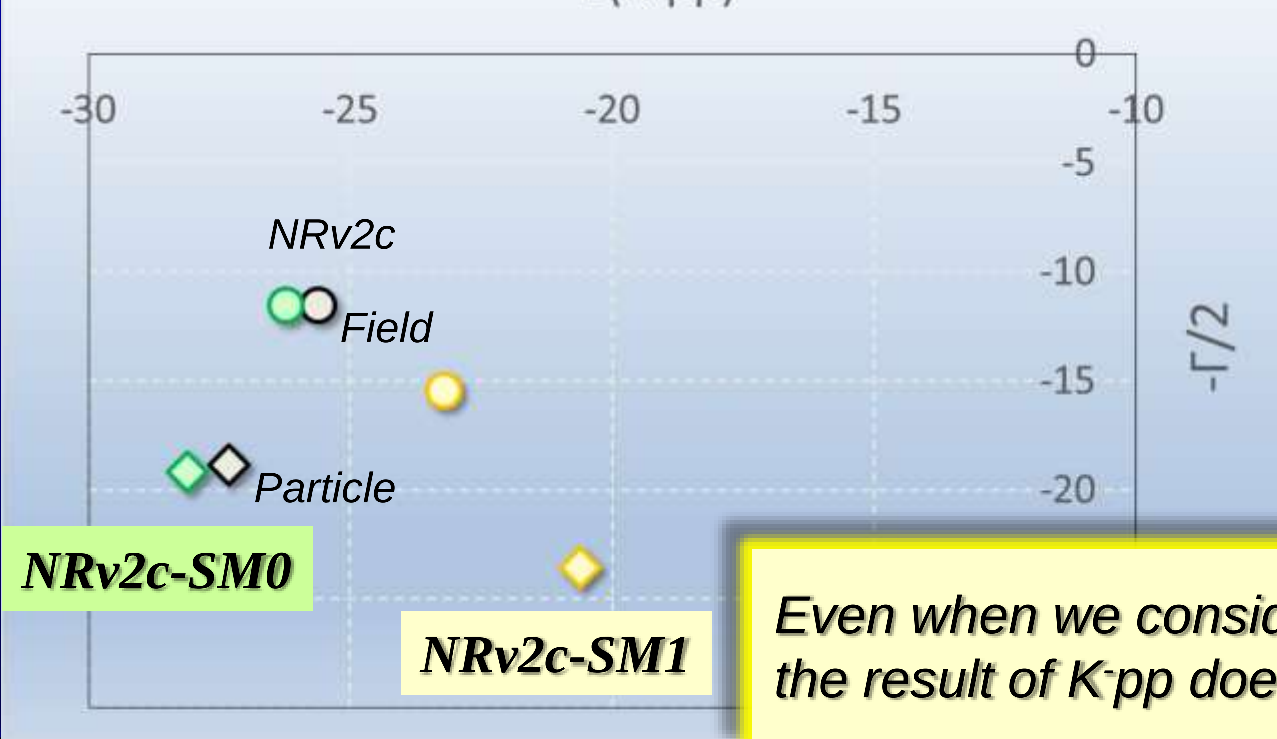
K^-pp with SIDDHARTA data

K^-p scattering length is an average of I=0 and I=1 scattering lengths.

To obtain a(I=0) and a(I=1) separately, we combine SIDDHARTA data with the old Martin value.

	a(K−p) [fm]	a(I=0) [fm]	a(I=1) [fm]
NRv2c	---	Martin −1.70 + 0.68i	Martin 0.37 + 0.60i (0.66 + 0.60i)
NRv2c−SM1	*SIDDHARTA* −0.65 + 0.81i	determined −1.67 + 1.02i	*Martin* 0.37 + 0.60i (0.64 + 0.60i)
NRv2c−SM0	*SIDDHARTA* −0.65 + 0.81i	*Martin* −1.70 + 0.68i	determined 0.40 + 0.94i (0.69 + 0.94i)

Case: f_π=110 MeV



Even when we consider the SIDDHARTA data, the result of K^-pp does not change so much!

5. Summary
and future plans

5. Summary and future plans

A prototype of K^{bar} nuclei "K^-pp" = Resonance state of $K^{bar}NN$-πYN coupled system

"coupled-channel Complex Scaling Method + Feshbach projection"

… Represent the Q-space Green function with the Extended Complete Set well approximated by Gaussian base

⇒ *Eliminate πY channels to reduce the problem to a $K^{bar}NN$ single channel problem.*

K^-pp studied with ccCSM+Feshbch method

- *Used a Chiral SU(3)-based potential (Gaussian form in r-space)*
- *Self-consistency for kaon's complex energy*
- *Correlation density in CSM shows effect of NN repulsive core and Λ^* survival in K^-pp resonance.*
- *$J^\pi=1^-$ state ("Deuteron+K^-"-like channel) seems not to exist as a resonance state.*
- *When the SIDDARTA data for K^-p scattering length is taken into account, the result of K^-pp does not change so much.*

K^-pp ($J^\pi=0^-$, $T=1/2$) --- NRv2c potential case

(B, $\Gamma/2$) = (21~31, 9~16) MeV : "Field picture"
(25~30, 15~32) MeV : "Particle pict."

Mean NN distance ~ 2.2 fm → Normal density

Future plans

- Full-coupled channel calculation of K^-pp
- Application to resonances of other hadronic systems

Kaonic nuclei

Thank you very much!

Prototype system = K^-pp

Reference:

A. D., T. Inoue, T. Myo, arXiv: 1411.0348, to be published in Prog. Theor. Exp. Phys.

at J-PARC