Evgeny Epelbaum, Ruhr-Univ. Bochum

HHIQCD Workshop, Feb 15 - Mar 21, 2015, YITP, Kyoto Univ, Japan

RUB

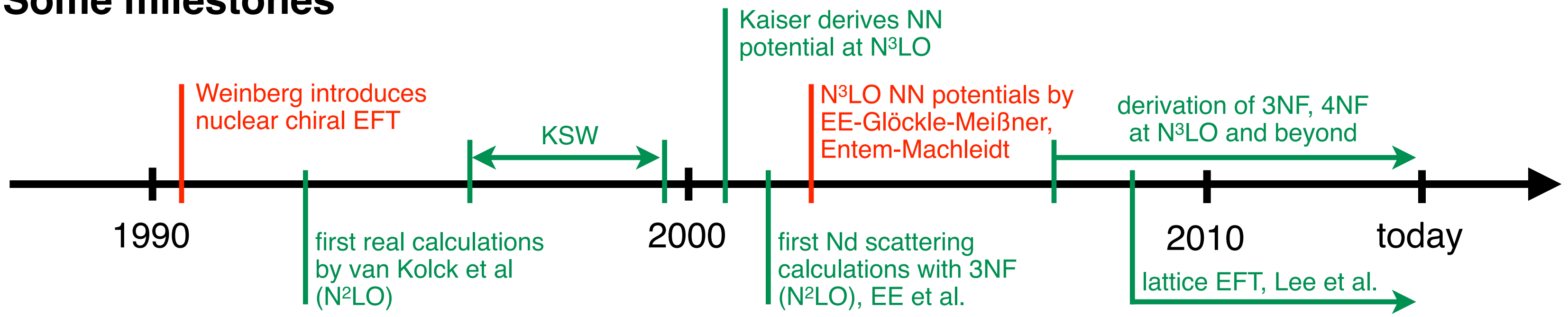
Chiral EFT for few-nucleon systems at the precision frontier

based on: EE, Krebs, Meißner, arXiv:1412.0142[nucl-th]
arXiv:1412.4623[nucl-th]

- Introduction
- Chiral EFT for nuclear forces
- A new generation of NN potentials up to N^{4}LO
- Quantification of theoretical uncertainties
- Applications to the 3N system
- Summary

The 25 years of nuclear chiral EFT

Some milestones



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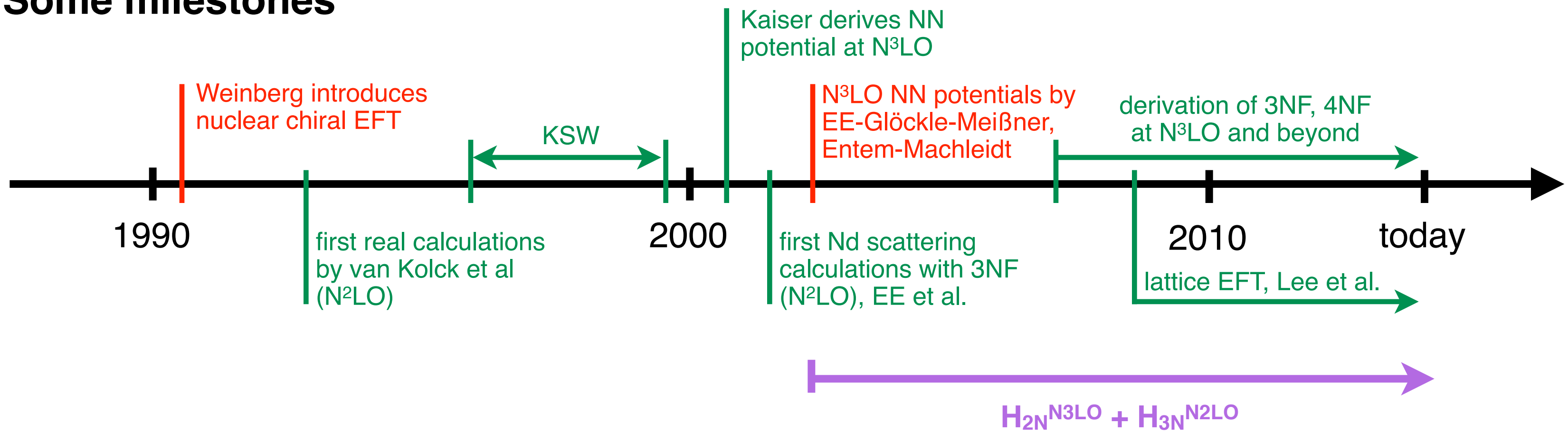

Many new insights including

- explanation of the observed hierarchy of many-body forces
- promising results using 2NF at N³LO combined with 3NF at N²LO
- electroweak structure of light nuclei, pion-deuteron scattering, Compton, …

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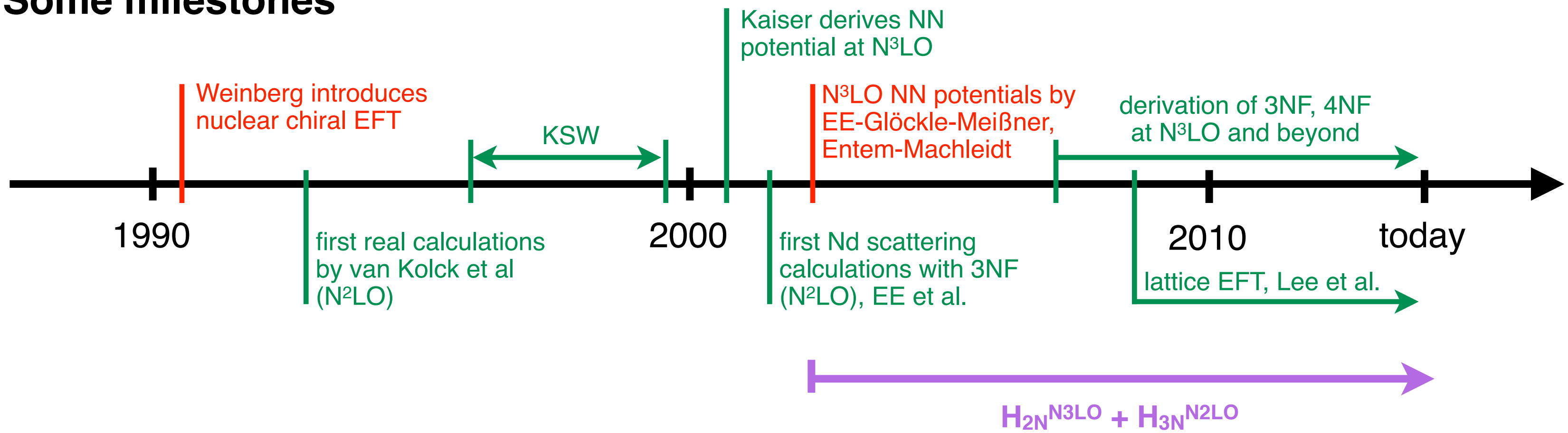

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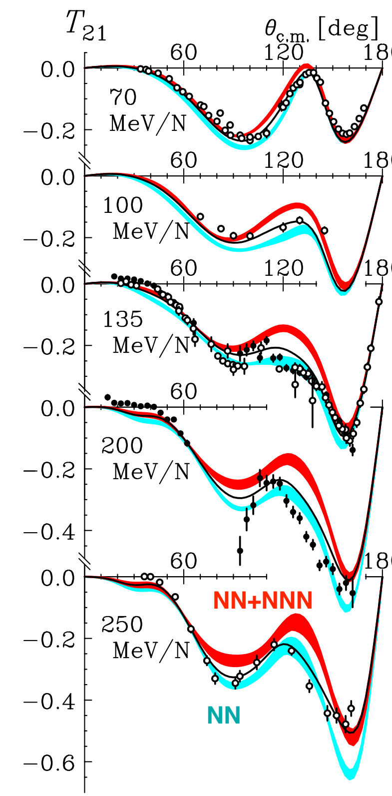
Today: precision frontier…

- addressing **unsolved problems** (especially **3NF**)

The 25 years of nuclear chiral EFT

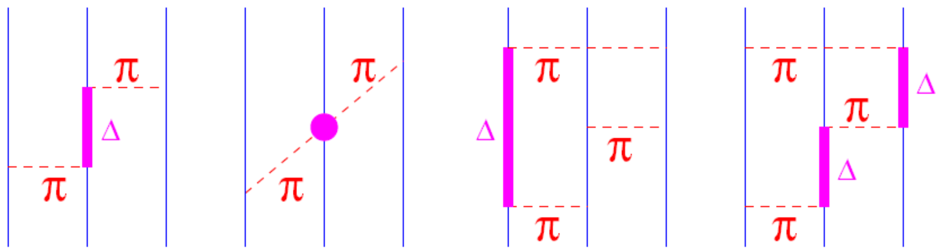
In spite of decades of effort, the spin structure of the 3NF is NOT properly described by 3NF models…

Kalantar-Nayestanaki et al., Rept. Prog. Phys. 75 (2012) 016301



Phenomenological 3NF models

Fujita-Miyazawa, Brasil, Tucson-Melbourne, Urbana, Illinois,…



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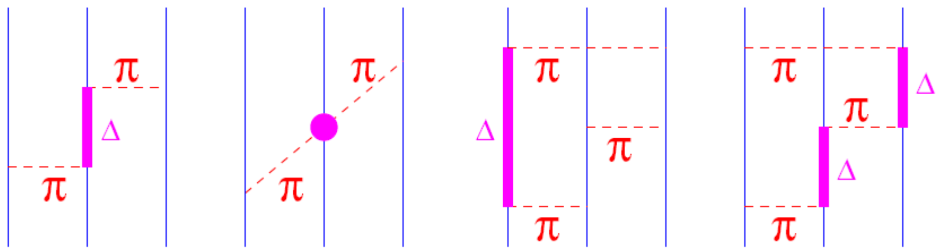

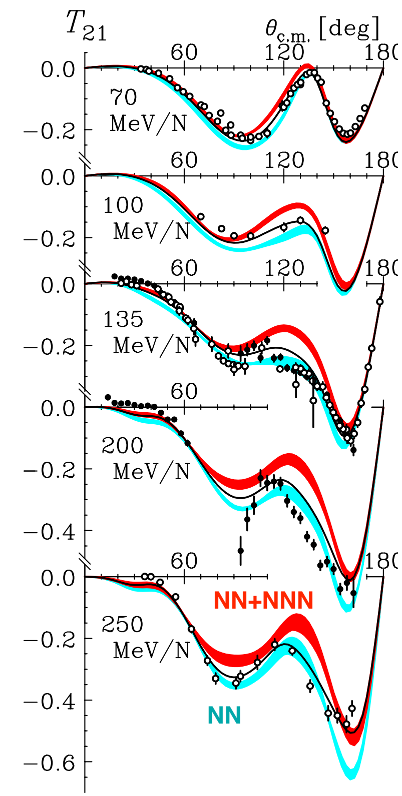

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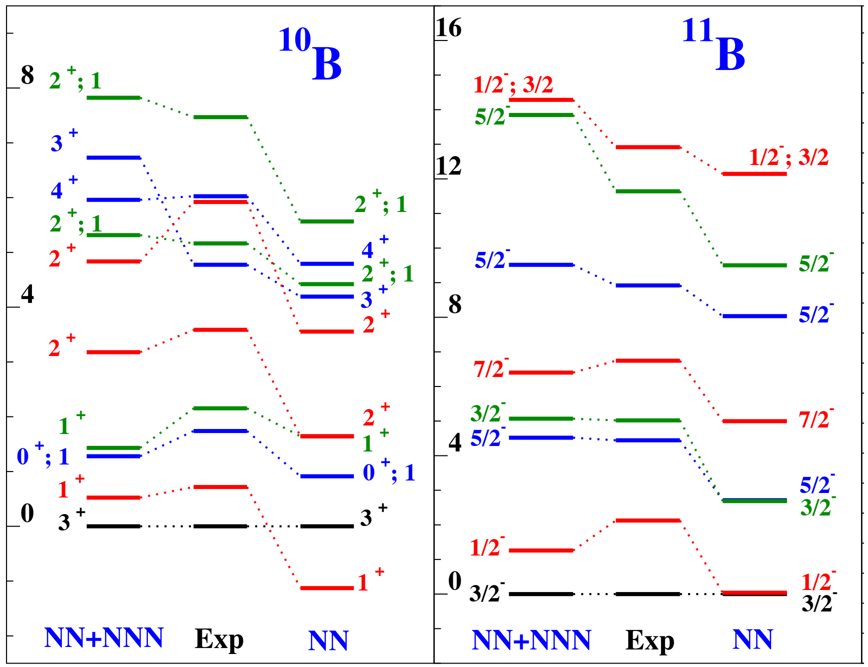
NCSM calculation based on chiral 2NF$_{@N3LO}$ + 3NF$_{@N2LO}$

Navratil et al. ’07



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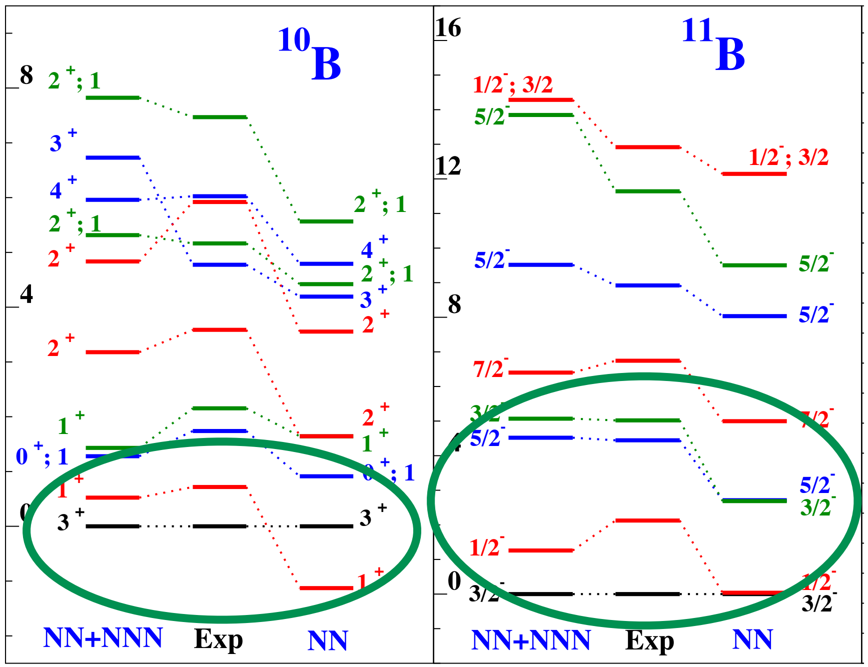

Success

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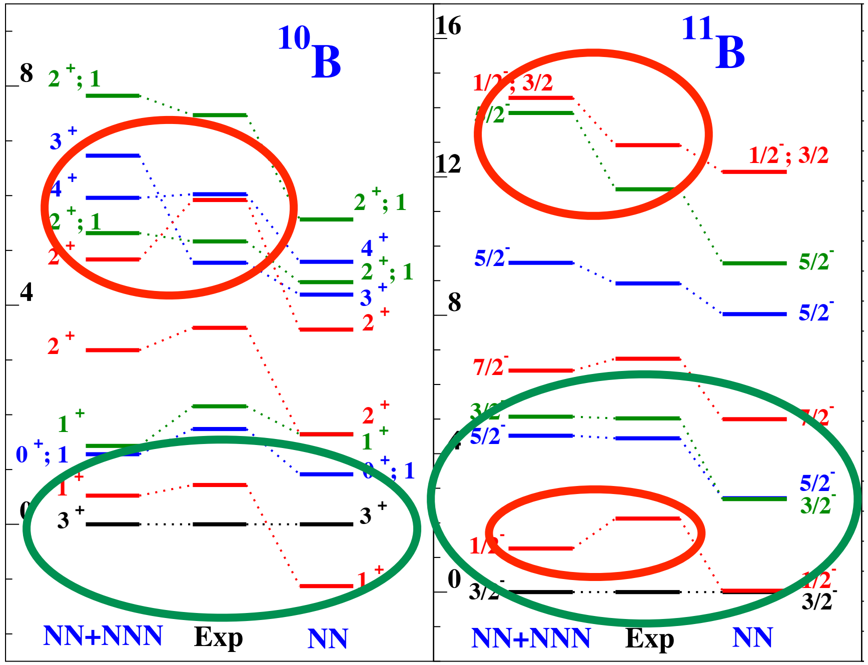

Success **or failure?**

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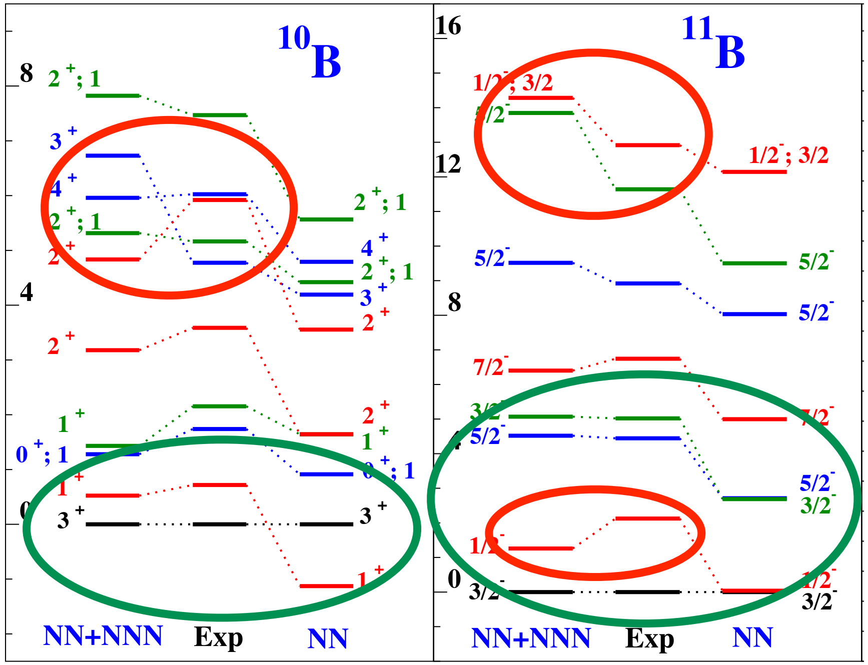
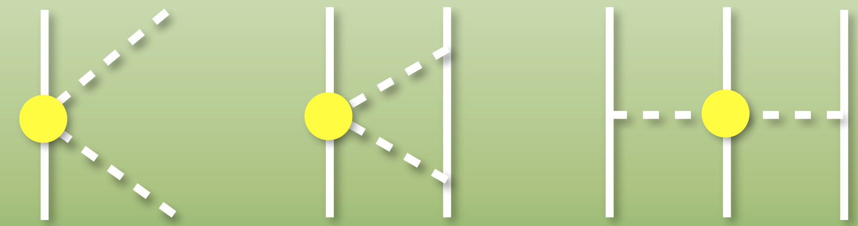


Success **or failure?**

Today: precision frontier…

- addressing **unsolved problems** (especially **3NF**)
- **predictive theory** with quantified uncertainties
- chiral EFT meets ab-initio many-body methods: **testing chiral forces in nuclei**…



Chiral perturbation theory

- **Ideal world [$m_u = m_d = 0$], zero-energy limit:** non-interacting massless GBs
  (+ strongly interacting massive hadrons)

- **Real world [$m_u,\ m_d \ll \Lambda_{QCD}$], low energy:** weakly interacting light GBs
  (+ strongly interacting massive hadrons)

→ expand about the ideal world (ChPT)

Chiral Perturbation Theory

Chiral Perturbation Theory: expansion of the scattering amplitude in powers of

Weinberg, Gasser, Leutwyler, Meißner, ...

$$Q = \frac{\text{momenta of pions and nucleons or } M_\pi \sim 140 \text{ MeV}}{\text{hard scales [at best } \Lambda_\chi = 4\pi F_\pi \sim 1 \text{ GeV]}}$$

Manohar, Georgi '84

Tool: Feynman calculus using the effective chiral Lagrangian

low-energy constants

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots$$

$$\mathcal{L}_{\pi N} = \underbrace{\bar N \left(i\gamma^\mu D_\mu[\pi] - m + \frac{g_A}{2}\gamma^\mu \gamma_5 u_\mu[\pi]\right) N}_{\mathcal{L}_{\pi N}^{(1)}} + \underbrace{\sum_i \boldsymbol{c_i}\, \bar N \hat O_i^{(2)}[\pi] N}_{\mathcal{L}_{\pi N}^{(2)}} + \underbrace{\sum_i \boldsymbol{d_i}\, \bar N \hat O_i^{(3)}[\pi] N}_{\mathcal{L}_{\pi N}^{(3)}} + \ldots$$

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Pion-nucleon scattering up to Q⁴ in heavy-baryon ChPT

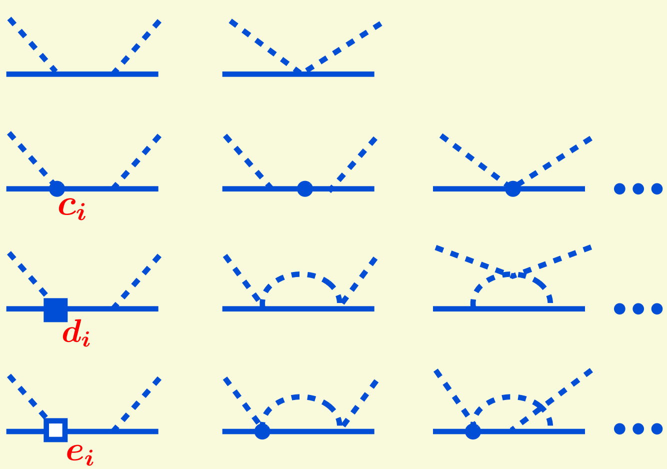
Fettes, Meißner '00; Krebs, Gasparyan, EE '12

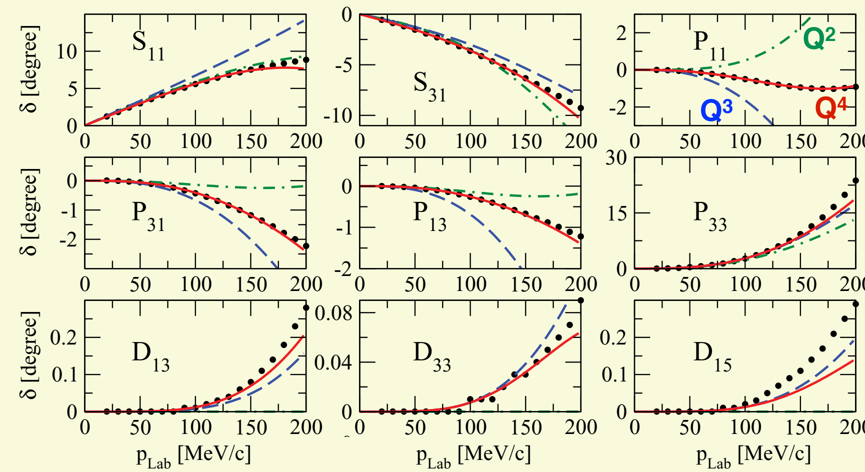
Order Q:

Order Q²:

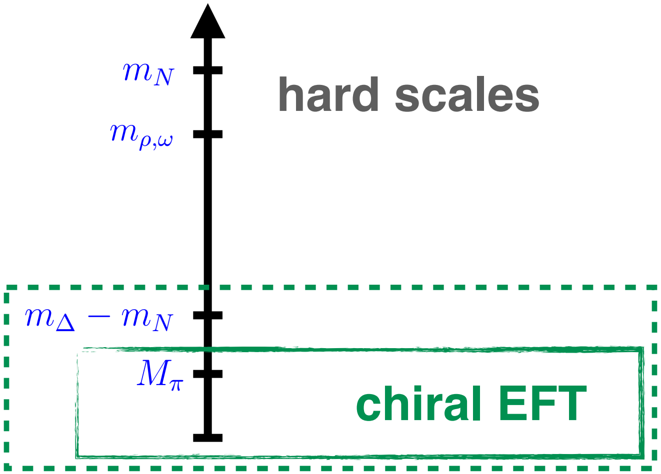
Order Q³:

Order Q⁴:





Chiral EFT for nuclei



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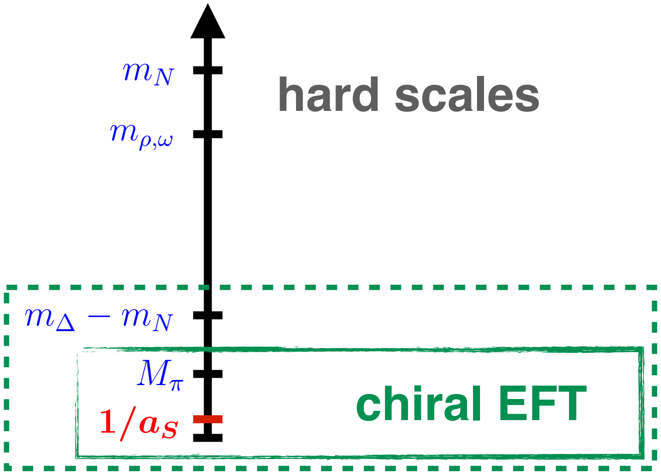

A new, soft scale associated with nuclear binding

$$Q \sim 1/a_S \simeq 8.5 \text{ MeV}(36 \text{ MeV}) \text{ in } {}^1S_0 \; ({}^3S_1)$$

to be generated dynamically (need resummations...)

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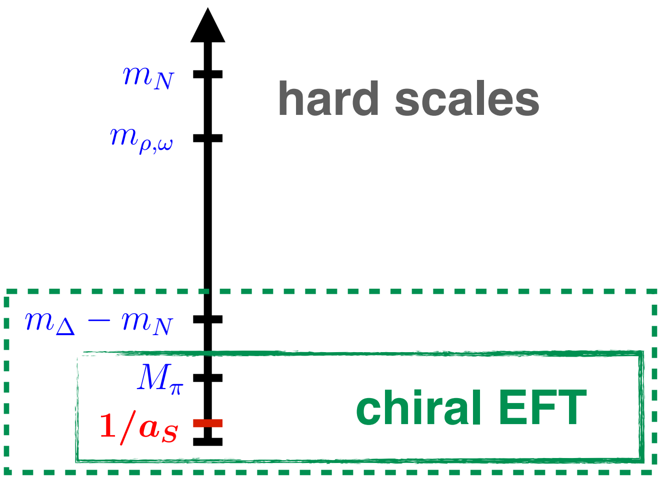

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Pionless EFT (valid for $\sqrt{m_N E_B} \ll Q \ll M_\pi$)

- zero-range forces between nucleons
- for 2N equivalent to Effective Range Theory
- universality, Efimov physics, cold gases, halos,...

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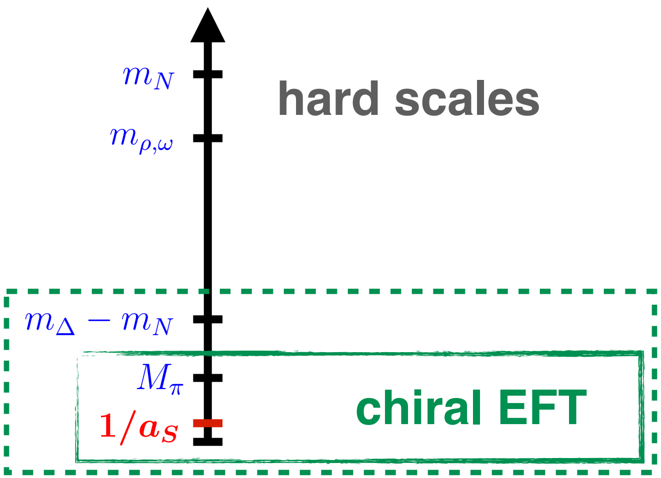

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- zero-range forces between nucleons
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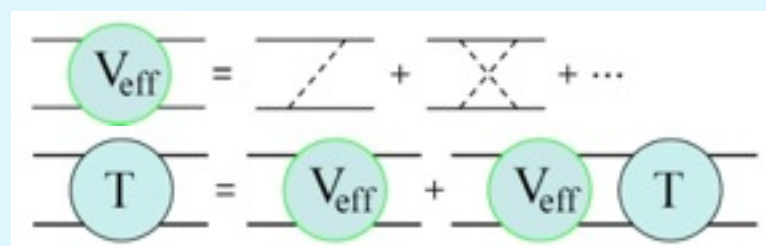
Chiral EFT (valid for $Q \sim M_\pi$)

Weinberg, van Kolck, EE, Glöckle, Meißner, Machleidt, Entem...

- Schrödinger equation for nucleons interacting via contact forces and long-range potentials (pion exchanges)

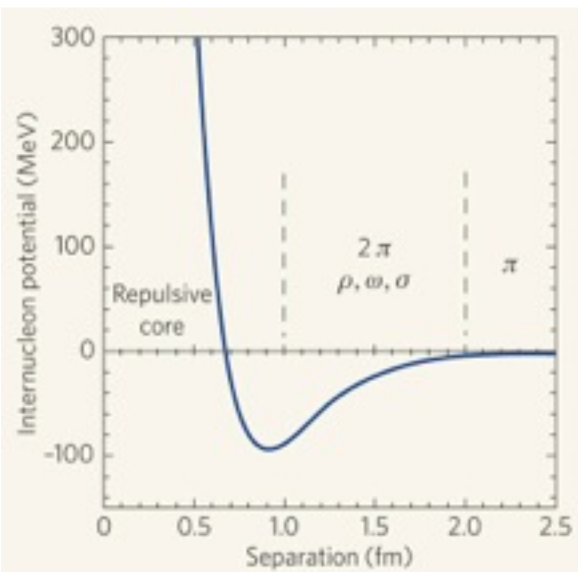


$$\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\text{derived in ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

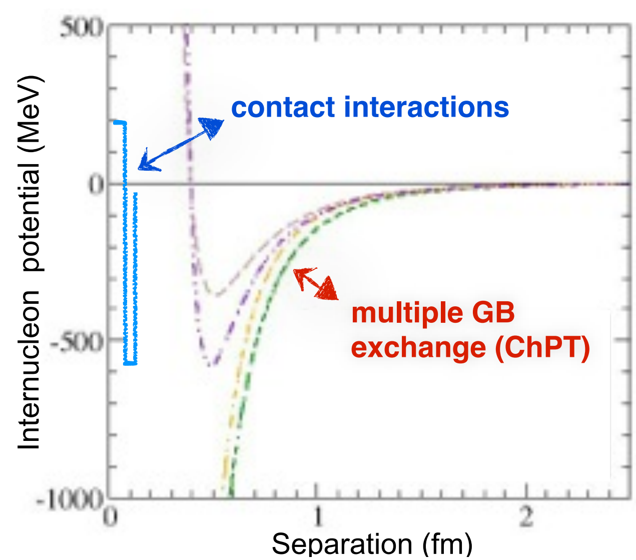
- access to heavier nuclei (ab initio few-/many-body methods)

Chiral dynamics and nuclear forces

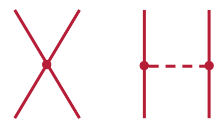
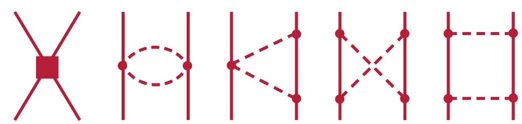
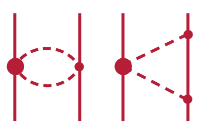
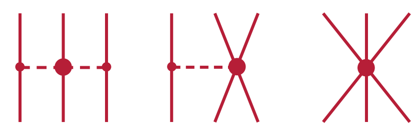
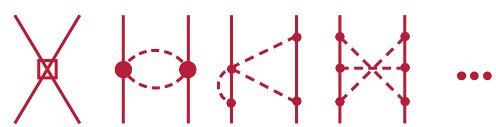
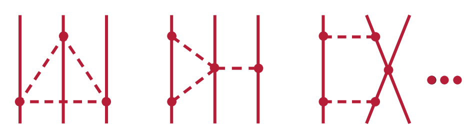
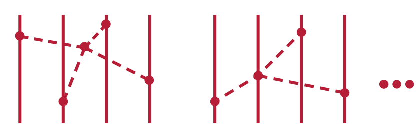
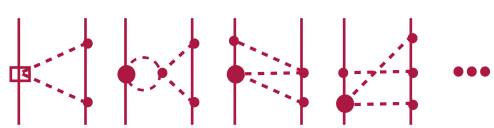
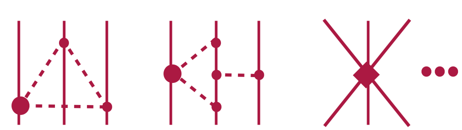
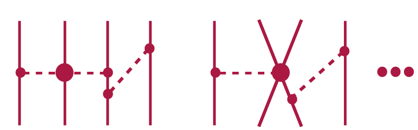
conventional picture



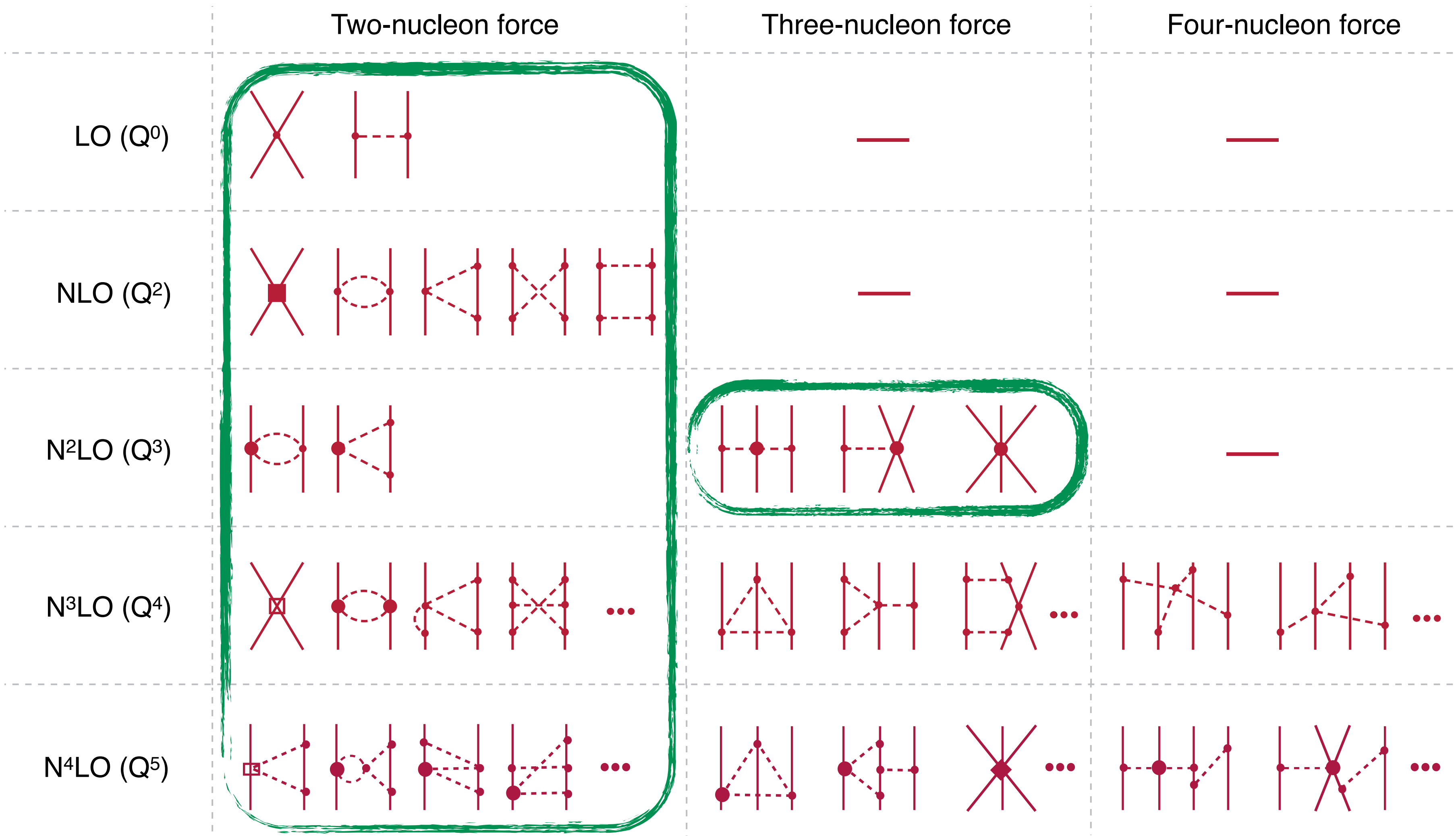
chiral EFT



Chiral expansion of nuclear forces

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)		—	—
NLO (Q^2)		—	—
N^2LO (Q^3)			—
N^3LO (Q^4)			
N^4LO (Q^5)			

Chiral expansion of nuclear forces



— **have been worked out and employed**

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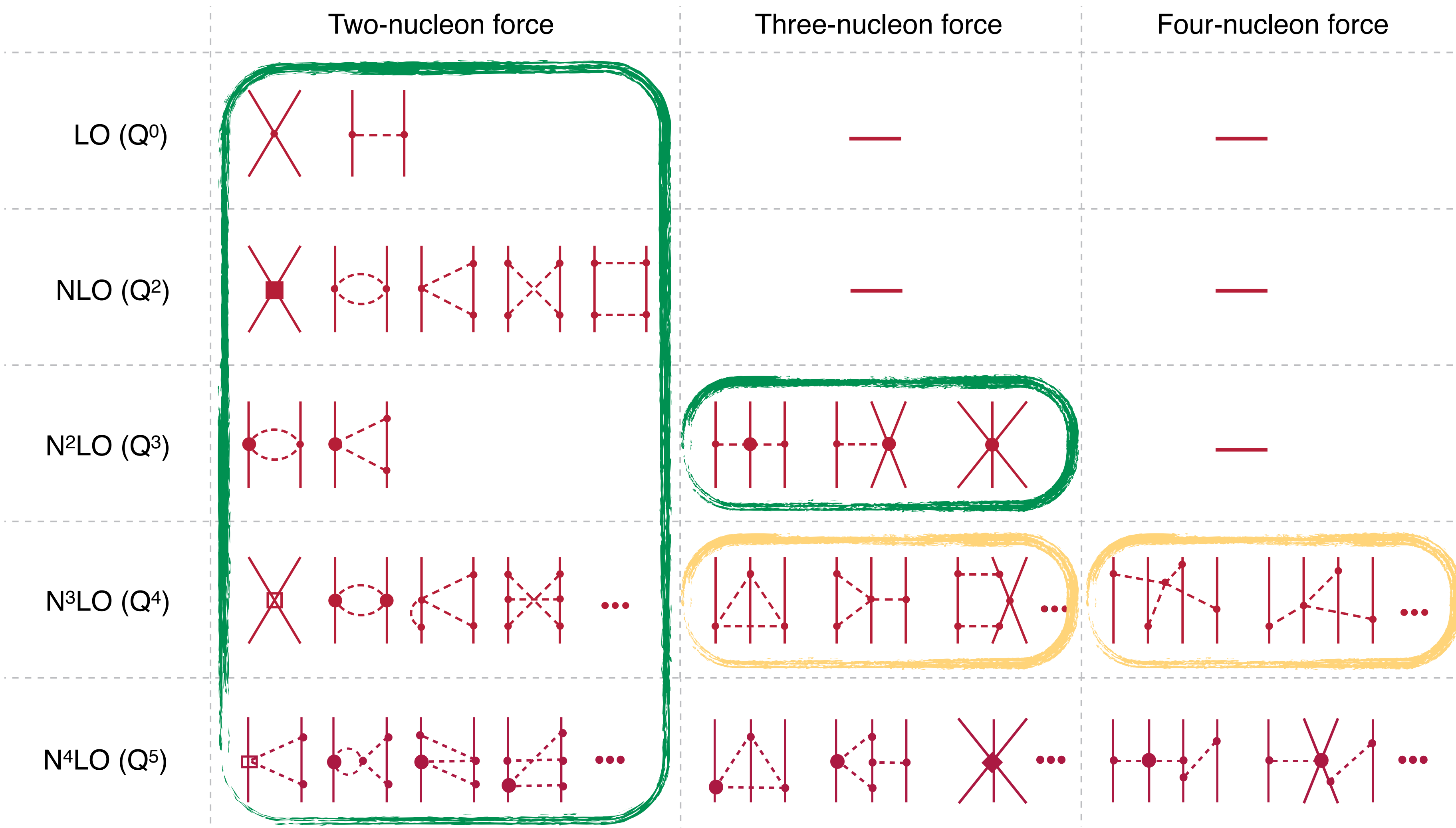

— have been worked out and employed

— have been worked out but not employed yet

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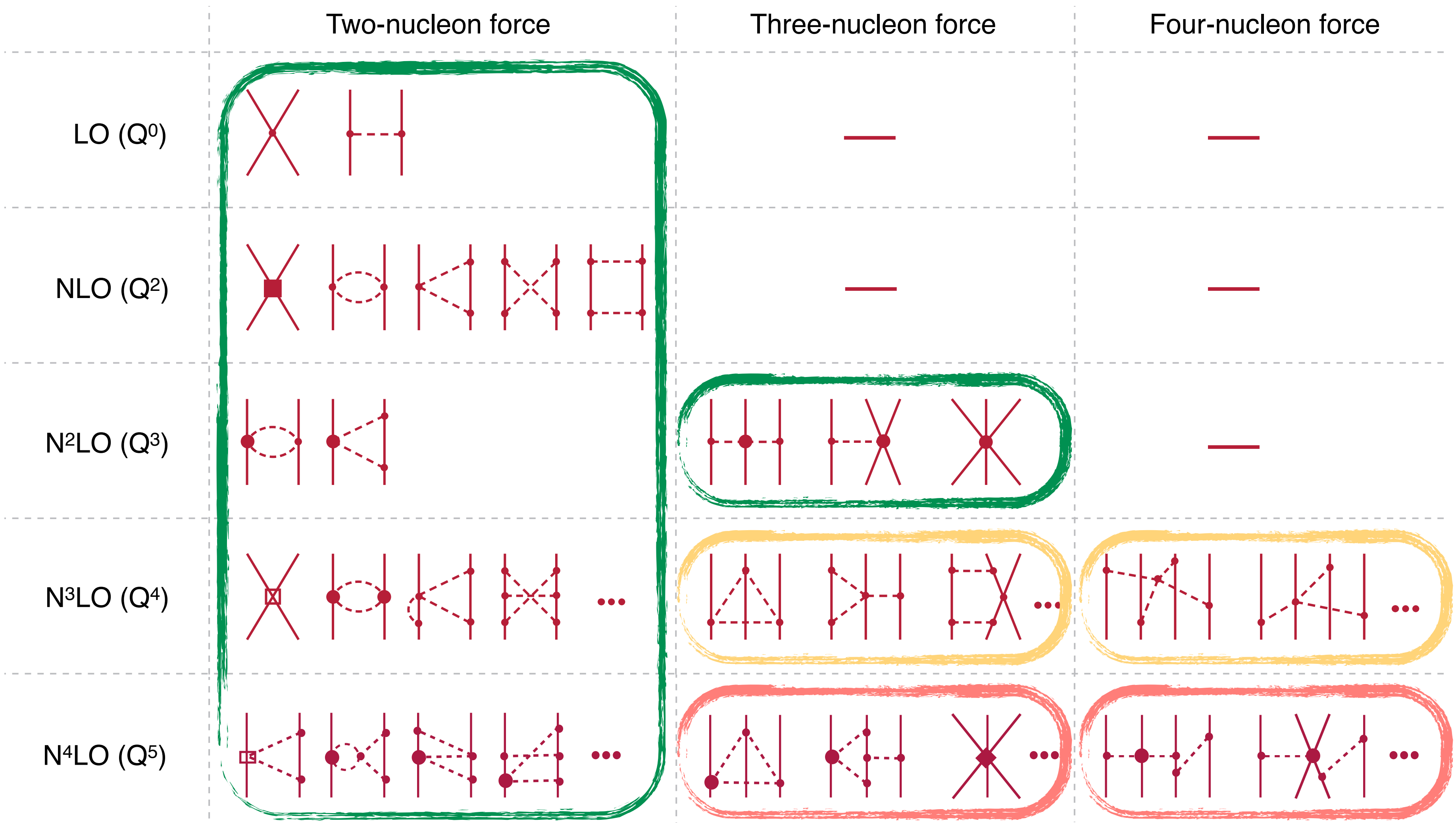


— have been worked out and employed

— have been worked out but not employed yet

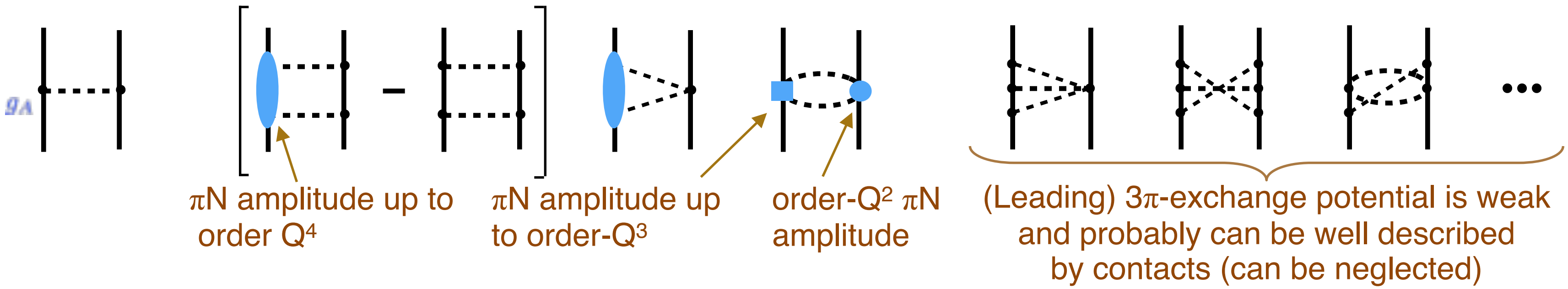
— have not been completely worked out yet

Nucleon-nucleon force up to N^4LO

EE, Krebs, Meißner, arXiv:1412.0142 [nucl-th], arXiv:1412.4623 [nucl-th]

The long-range part

Ordonez et al.; Kaiser; EE, Krebs, Meißner, …



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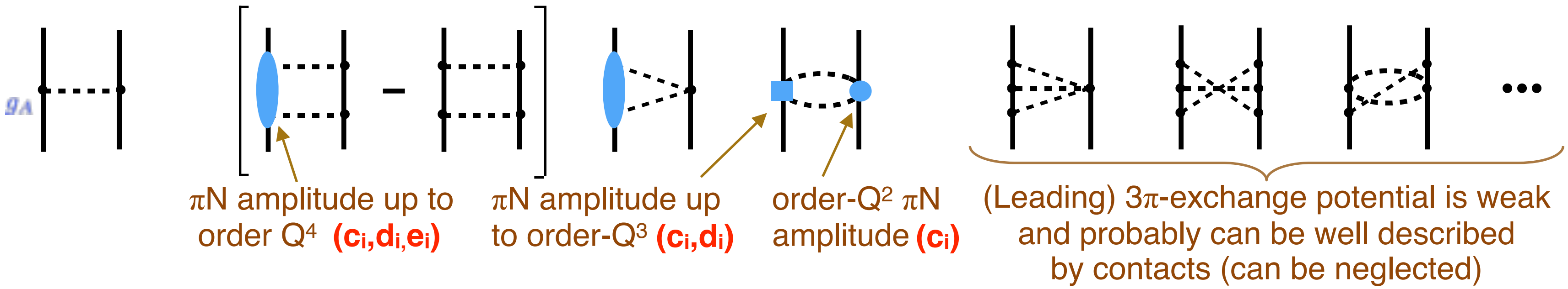

The determined values of LECs Krebs, Gasparyan, EE ’12

	c_1	c_2	c_3	c_4	$\bar{d}_1 + \bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14} - \bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
Q^4 fit to GW	-1.13	3.69	-5.51	3.71	5.57	-5.35	0.02	-10.26	1.75	-5.80	1.76	-0.58	0.96
Q^4 fit to KH	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-10.41	6.08	-0.37	3.26

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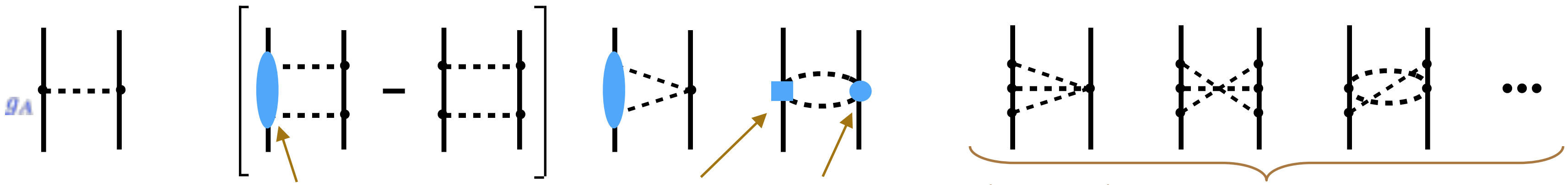

πN amplitude up to order Q^4 **(c_i,d_i,e_i)**

πN amplitude up to order-Q^3 **(c_i,d_i)**

order-Q^2 πN amplitude **(c_i)**

(Leading) 3π-exchange potential is weak and probably can be well described by contacts (can be neglected)

The determined values of LECs

Krebs, Gasparyan, EE '12

	c_1	c_2	c_3	c_4	$\bar d_1 + \bar d_2$	$\bar d_3$	$\bar d_5$	$\bar d_{14} - \bar d_{15}$	$\bar e_{14}$	$\bar e_{15}$	$\bar e_{16}$	$\bar e_{17}$	$\bar e_{18}$
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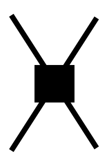
The short-range part (contact terms)



LO [Q^0]: 2 operators (S-waves)

NLO [Q^2]: + 7 operators (S-, P-waves and ε_1)

N^{2}LO [Q^3]: no new isospin-conserving operators + 2 IB terms ($^1S_0$)

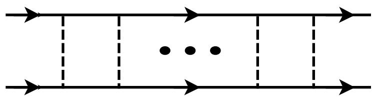
N^{3}LO [Q^4]: + 15 operators (S-, P-, D-waves and ε_1, ε_2)

N^{4}LO [Q^5]: no new isospin-conserving operators + 1 IB term ($^1S_0$)

Regularization, renormalization and all that...

$$T(\vec{p}\,',\,\vec{p}) = V_{\rm 2N}(\vec{p}\,',\,\vec{p}) + m \int \frac{d^3k}{(2\pi)^3} \frac{V_{\rm 2N}(\vec{p}\,',\,\vec{k})\,T(\vec{k},\,\vec{p})}{p^2 - k^2 + i\epsilon} \quad \text{with} \quad V_{\rm 2N} = \alpha \frac{\vec{\sigma}_1 \cdot \vec{q}\,\vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + M_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \ldots$$

→ Lippmann-Schwinger eq. is linearly divergent, need infinitely many CTs to absorb UV divergences from iterations!



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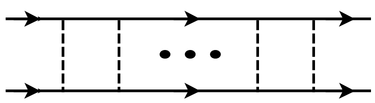


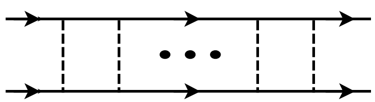
Possible approaches:

- Use a manifestly Lorentz-invariant approach (3D-eqs. fulfilling relativistic unitarity)
  EE, Gegelia'12,'13; EE, Gasparyan, Gegelia, Krebs, Schindler '14,'15
  - — integral eq. is renormalizable at LO (only log-divergences), Λ can be removed!
  - — Caveat: calculations are complicated, hard to go beyond the NN system…

Regularization, renormalization and all that…

$$T(\vec{p}\,', \vec{p}) = V_{2N}(\vec{p}\,', \vec{p}) + m \int \frac{d^3k}{(2\pi)^3} \frac{V_{2N}(\vec{p}\,', \vec{k})\, T(\vec{k}, \vec{p})}{p^2 - k^2 + i\epsilon} \quad \text{with} \quad V_{2N} = \alpha \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}\,^2 + M_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \ldots$$

→ Lippmann-Schwinger eq. is linearly divergent, need infinitely many CTs to absorb UV divergences from iterations!



Possible approaches:

- Use a manifestly Lorentz-invariant approach (3D-eqs. fulfilling relativistic unitarity)
  EE, Gegelia'12,'13; EE, Gasparyan, Gegelia, Krebs, Schindler '14,'15
  - — integral eq. is renormalizable at LO (only log-divergences), Λ can be removed!
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- Use a finite UV cutoff (standard) Lepage '97
  - — simple, well suited for few- and many-body calculations
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  - — we use a local regulator for long-range terms (maintains analytic structure of the amplitude) and choose R = 0.8…1.2 fm

$$V(r) \longrightarrow V(r) \left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^6$$

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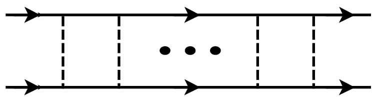

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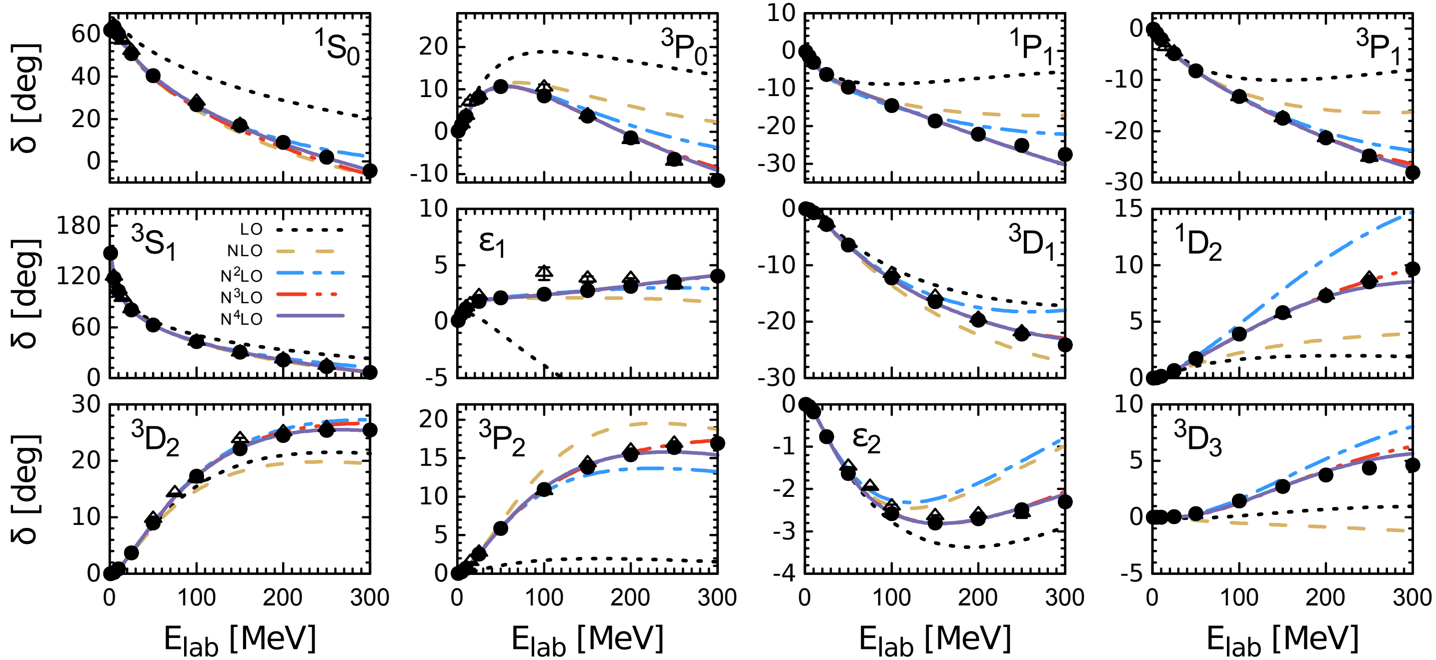
(Implicit) renormalization: express bare LECs in terms of observables (phase shifts)

NN phase shifts order by order

Convergence of the chiral expansion for neutron-proton phase shifts [using R = 0.9 fm]



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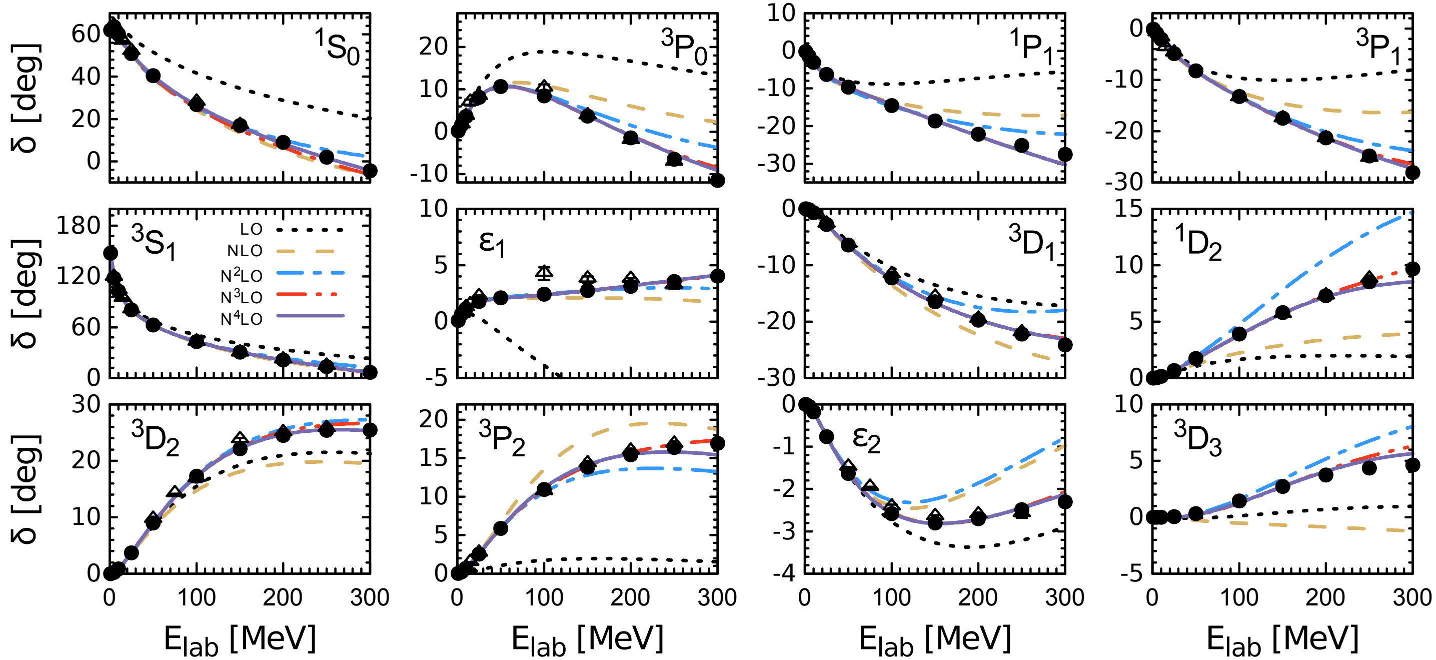

χ^2_{datum} for the reproduction of the Nijmegen phase shifts [using R = 0.9 fm]

$E_{\rm lab}$ bin	LO [Q^0]	NLO [Q^2]	N^{2}LO [Q^3]	N^{3}LO [Q^4]	N^{4}LO [Q^5]
neutron-proton phase shifts					
0–100	360	31	4.5	0.7	0.3
0–200	480	63	21	0.7	0.3
proton-proton phase shifts					
0–100	5750	102	15	0.8	0.3
0–200	9150	560	130	0.7	0.6

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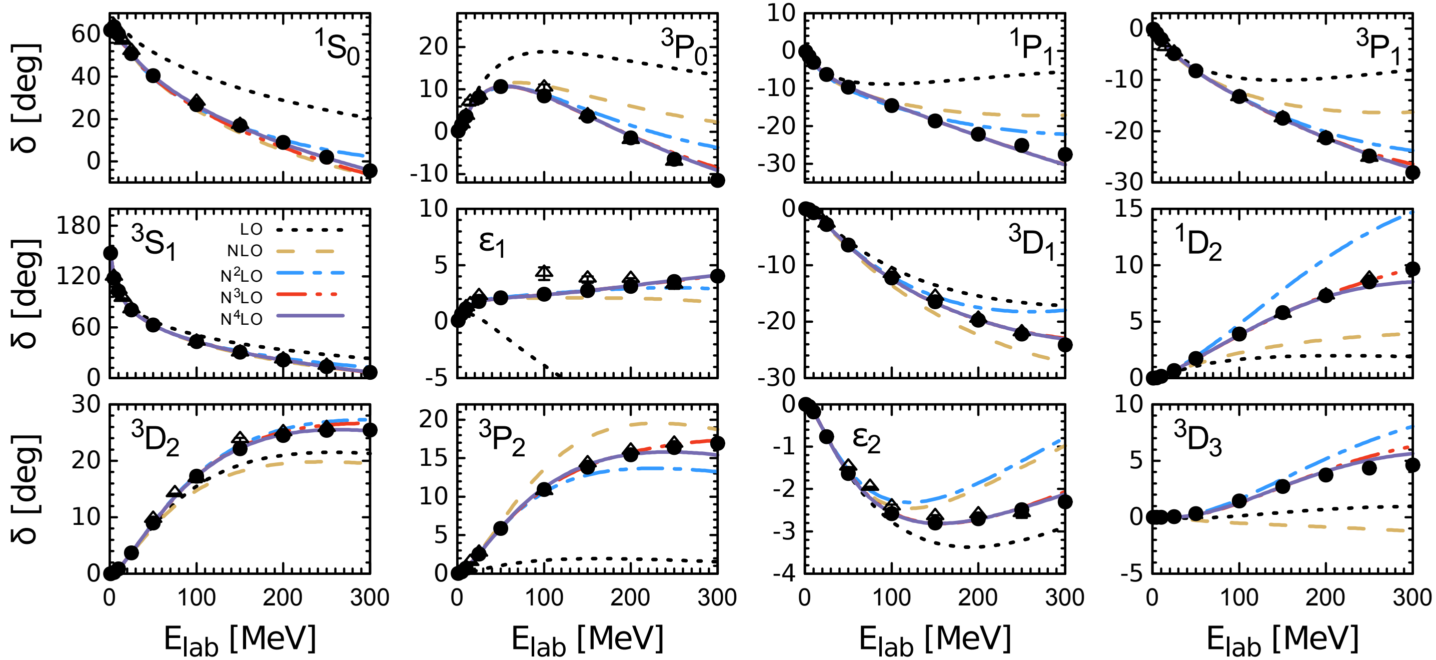

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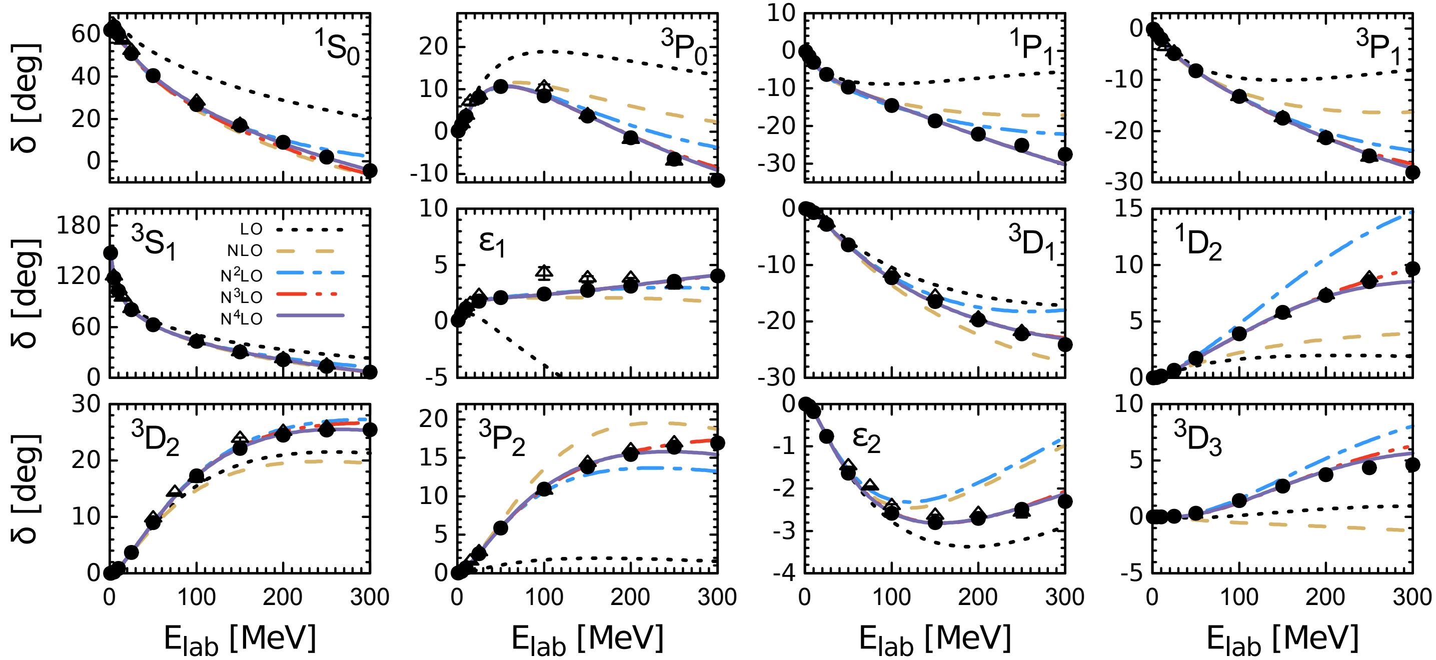

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no new LECs (NLO → N^{2}LO)

1 LEC ($^1S_0$) (N^{3}LO → N^{4}LO, neutron-proton)

no new LECs (N^{3}LO → N^{4}LO, proton-proton)

Convergence of the chiral expansion

EE, Krebs, Meißner, arXiv:1412.0142 [nucl-th], arXiv:1412.4623 [nucl-th]

Expansion parameter: $Q = \max \left(\frac{p}{\Lambda_b}, \frac{M_\pi}{\Lambda_b} \right)$

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Q = 212 / 600 ~ 0.35 → expect: ~ 11, ~ 4, ~ 1.3, ~ 0.5

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Q = 212 / 600 ~ 0.35 ⟶ expect: ~ 11, ~ 4, ~ 1.3, ~ 0.5

$E_{lab} = 200$ MeV [p = 307 MeV]: $\sigma_{tot} = 34.9 + 1.0 + 6.7 + 0.6 - 0.5 = 42.7$ mb

Q = 307 / 600 ~ 0.5 ⟶ expect: ~ 9, ~ 5, ~ 2.4, ~ 1.2

⟶ good convergence of the chiral expansion

Uncertainty quantification

EE, Krebs, Meißner, arXiv:1412.0142 [nucl-th], arXiv:1412.4623 [nucl-th]

Let $X(p)$ be some observable with p denoting the corresponding momentum scale and $X^{(n)}(p), \quad n = 0, 2, 3, 4, \ldots$ a prediction at order Q^n in the chiral expansion:

$$X^{(n)} = X^{(0)} + \Delta X^{(2)} + \ldots + \Delta X^{(n)}$$

calculated in the chiral expansion

For the order-n contribution one expects $\Delta X^{(n)} \sim \mathcal{O}\big(Q^n \, X^{(0)}\big)$ with $Q = \max\left(\frac{M_\pi}{\Lambda_b}, \; \frac{p}{\Lambda_b}\right)$

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Theoretical uncertainty $\delta X^{(n)}$ estimated via the size of neglected higher-order contributions*:

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(*Also demand that $\delta X^{(n)}$ is not smaller than the actual higher-order contributions whenever known)

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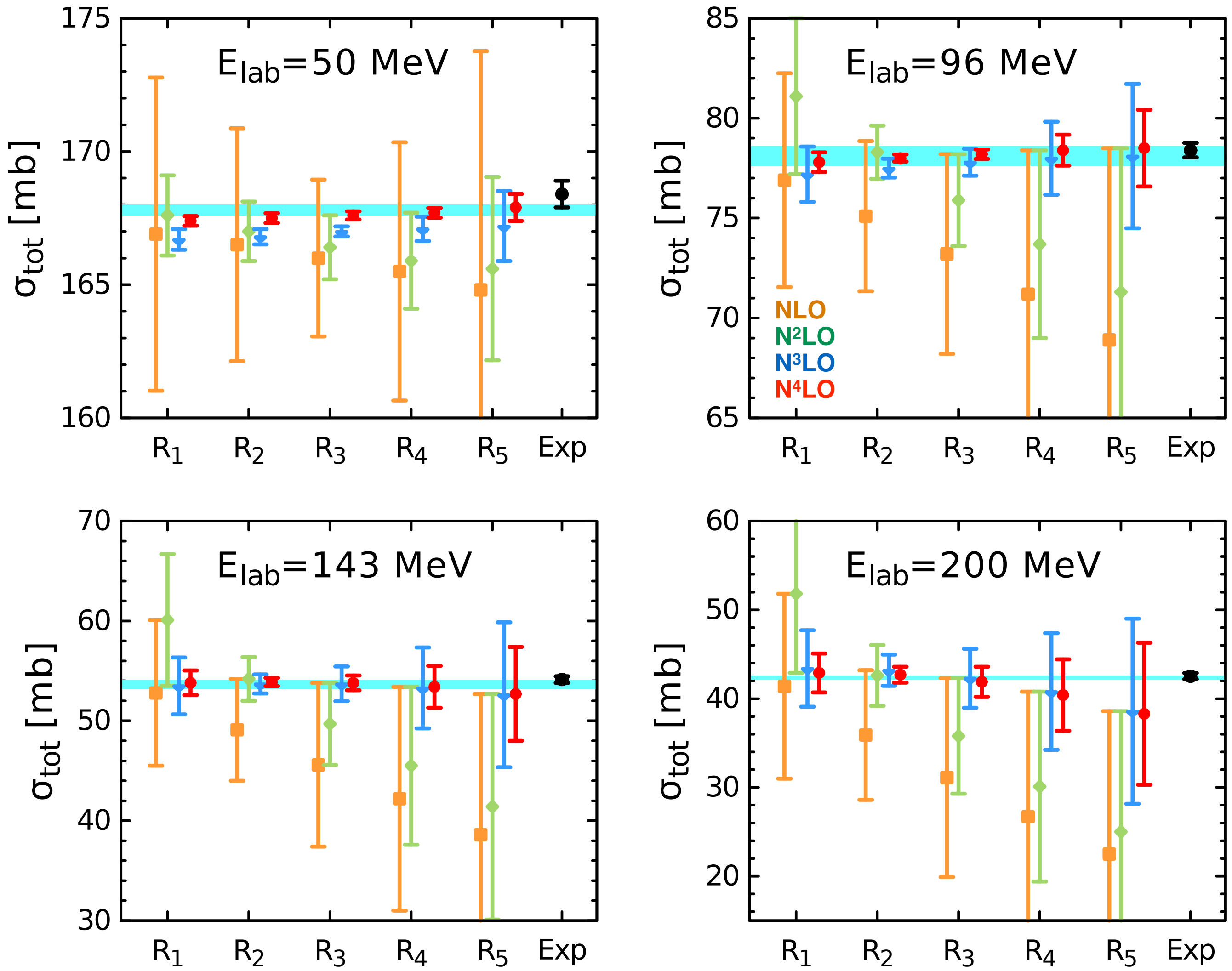
→ a simple approach applicable for any observable and any choice of the regulator

Uncertainty quantification

EE, Krebs, Meißner, arXiv:1412.0142 [nucl-th], arXiv:1412.4623 [nucl-th]

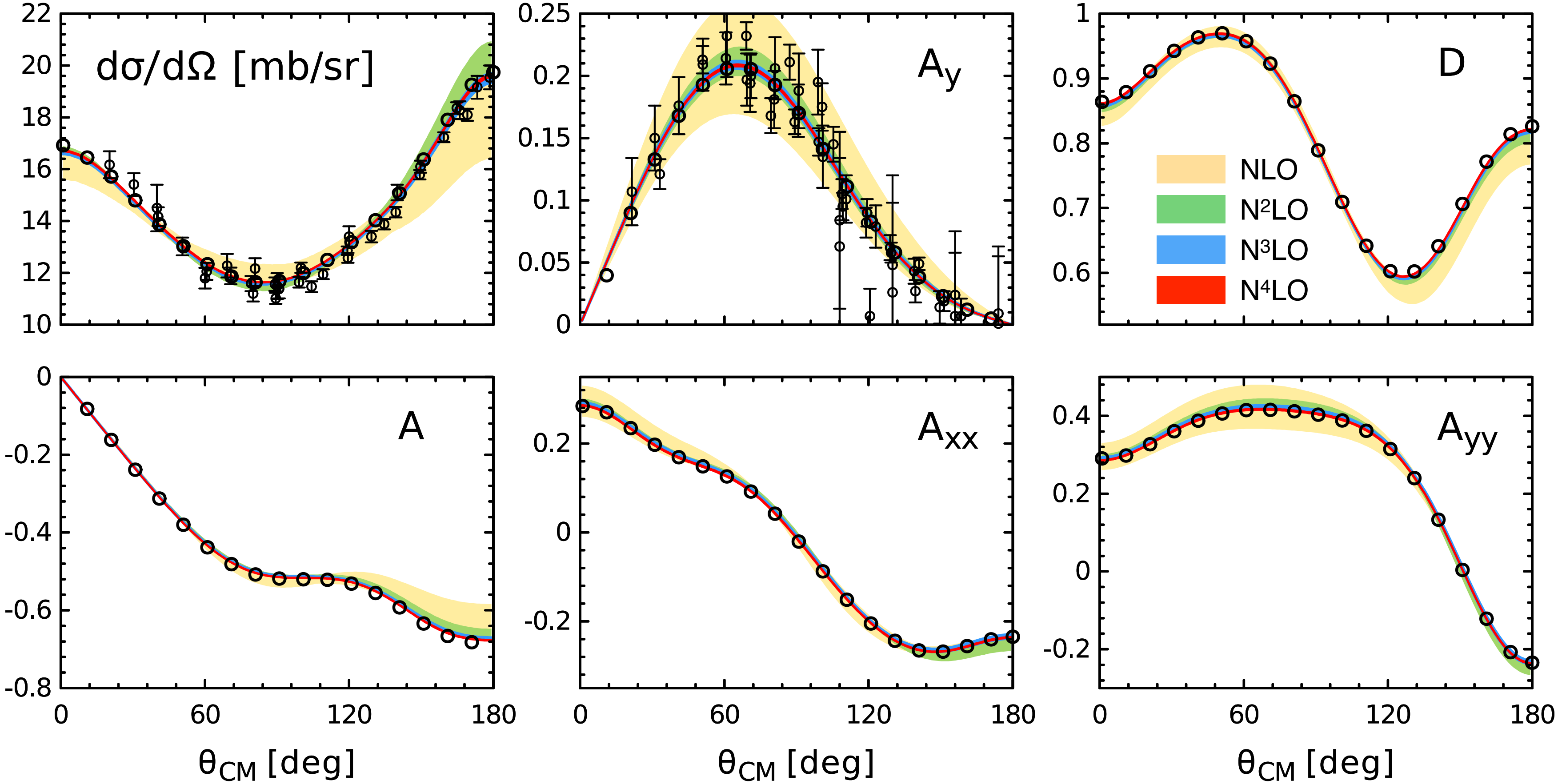
np total cross section for $R_{1,\ldots,5} = \{0.8, 0.9, 1.0, 1.1, 1.2\}$ fm



Neutron-proton scattering

EE, Krebs, Meißner, arXiv:1412.0142 [nucl-th], arXiv:1412.4623 [nucl-th]

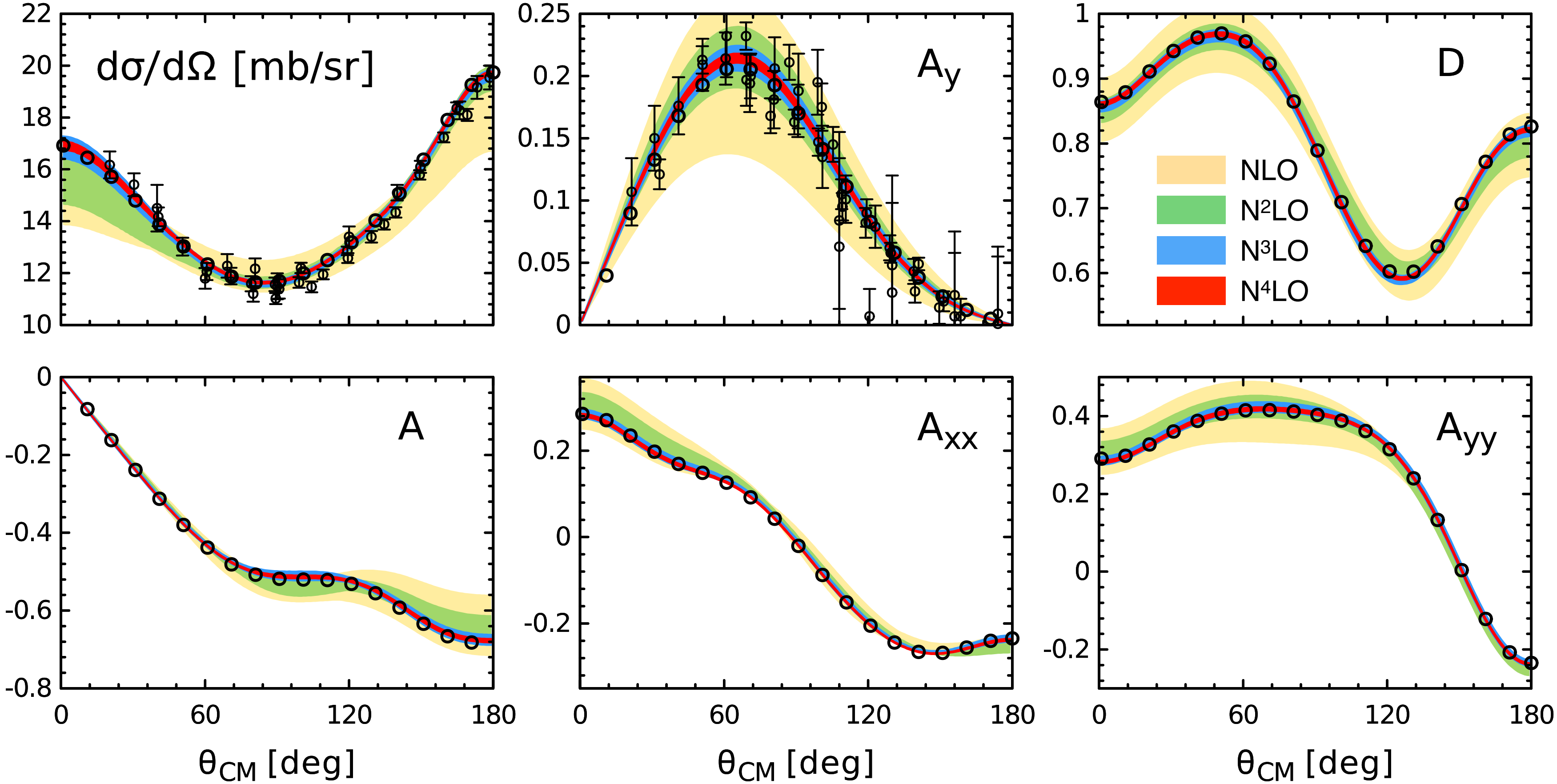
Selected neutron-proton scattering observables at 50 MeV R=0.9fm



Neutron-proton scattering

EE, Krebs, Meißner, arXiv:1412.0142 [nucl-th], arXiv:1412.4623 [nucl-th]

Selected neutron-proton scattering observables at 50 MeV R=1.2fm

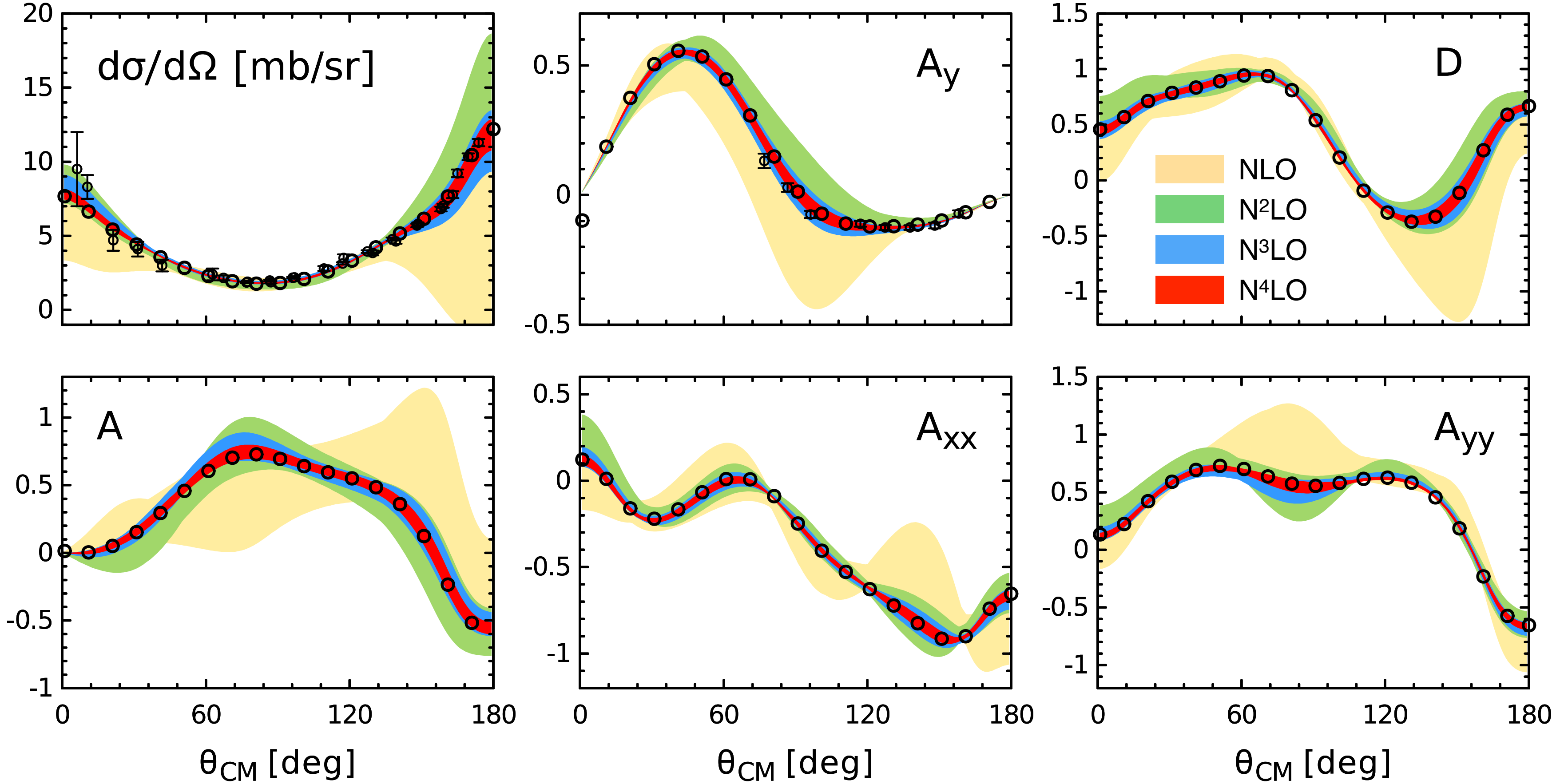


- The theoretical predictions for different cutoff choices are consistent with each other
- Softer cutoffs lead to larger theoretical uncertainties

Neutron-proton scattering

EE, Krebs, Meißner, arXiv:1412.0142 [nucl-th], arXiv:1412.4623 [nucl-th]

Selected neutron-proton scattering observables at 200 MeV R=0.9fm



- Accurate results even at the energy of E_{lab} = 200 MeV (for R = 0.9 fm)

Deuteron properties R=0.9 fm

EE, Krebs, Meißner, arXiv:1412.0142 [nucl-th], arXiv:1412.4623 [nucl-th]

	LO	NLO	N	N	N	empirical
B	2.0235	2.1987	2.2311	2.2246*	2.2246*	2.224575(9)
A	0.8333	0.8772	0.8865	0.8845	0.8844	0.8846(9)
η	0.0212	0.0256	0.0256	0.0255	0.0255	0.0256(4)
r_d	1.990	1.968	1.966	1.972	1.972	1.97535(85)
Q [fm	0.230	0.273	0.270	0.271	0.271	0.2859(3)
P_D	2.54	4.73	4.50	4.19	4.29	

— fast convergence of the chiral expansion (P_D is not observable)

— error estimation (assuming $Q=M_\pi/\Lambda_b$)

A_S: LO: 0.83(5) → NLO: 0.878(13) → N²LO: 0.887(3) → N³LO: 0.8845(8) → N⁴LO: 0.8844(2)

η: LO: 0.021(5) → NLO: 0.026(1) → N²LO: 0.0256(3) → N³LO: 0.0255(1) → N⁴LO: 0.0255

→ theoretical results for A_S,η at N⁴LO are more accurate than empirical numbers

— results for r_d and Q do not take into account MECs and relativistic corrections:

r_d: $|\Delta r_d| \simeq 0.004$ fm [Kohno '83] → predictions in agreement with the data

Q: rel. corrections + 1π-exchange MEC: $\Delta Q \simeq +0.008\ \mathrm{fm}^2$ [Phillips '07] → $Q \simeq 0.279\ \mathrm{fm}^2$

the remaining deviation of 0.007 fm² agrees with the expected size of  [Phillips '07]

Intermediate summary

A new generation of chiral NN potentials up to N^4LO is developed

— chiral expansion for NN scattering shows good convergence

— excellent description of NN scattering observables & deuteron properties at N^3LO, N^4LO

A simple approach for uncertainty quantification is introduced

— applicable to any observable and for any choice of the regulator

— results in the NN system at all orders and for all cutoffs are consistent with each other and with experimental data (within uncertainties) → seems to work reliably

With these developments, we are ready to take up the 3NF challenge

(work in progress by the LENPIC collaboration)

Evidence for missing 3N forces

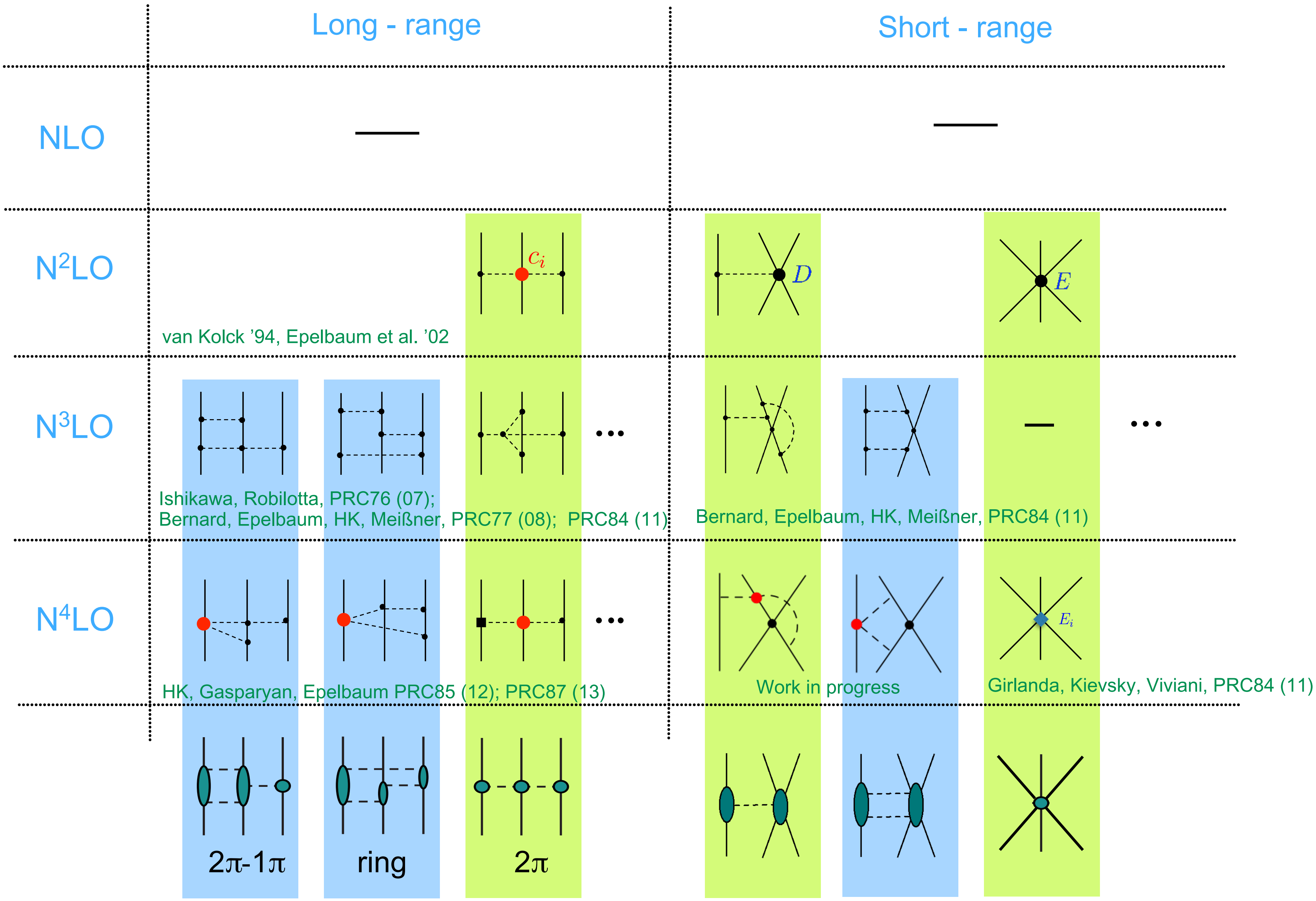
LENPIC, in preparation

While no complete calculations based on the new 2N+3N forces are available yet, we preformed **incomplete calculations based on 2N forces only** in order to:

— identify observables/kinematics best suitable for searches of 3NF effects

— estimate the achievable accuracy of of chiral EFT

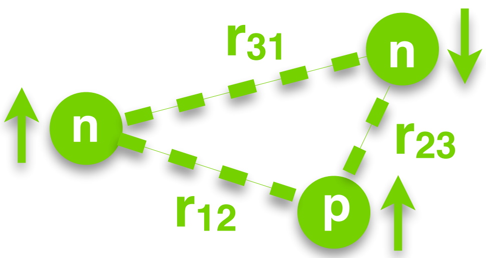
Chiral expansion of the 3N force



Chiral expansion of the 3N force

EE, Gasparyan, Krebs, Schat, Eur. Phys. J. A51 (2015) 3, 26

Notice: 3N force at large distance is completely determined by the chiral symmetry of QCD + experimental information on the πN system (parameter-free!)

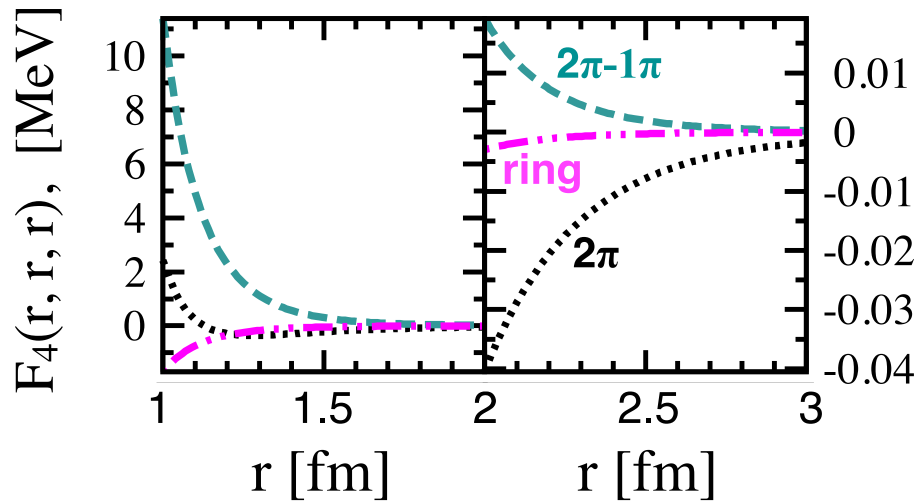


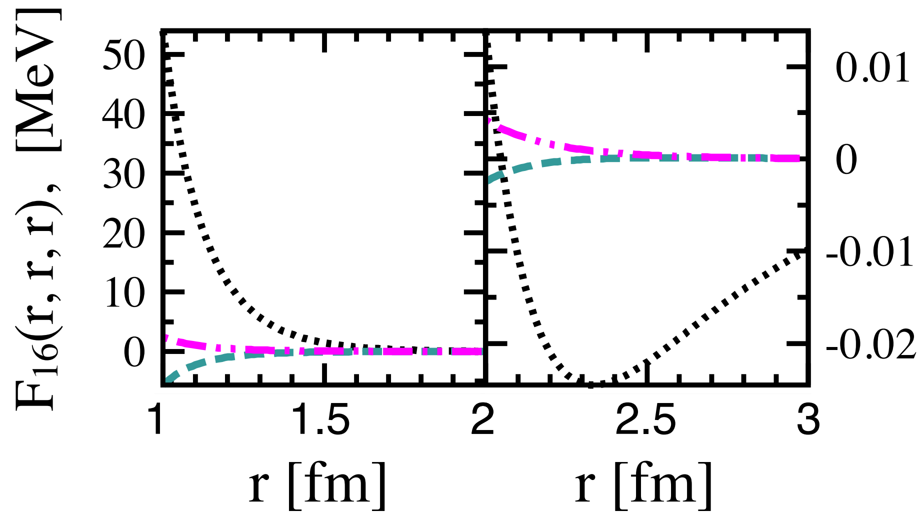
$$V(r_{12}, r_{23}, r_{31}) = \sum_{i=1}^{20} \tilde{\mathcal{G}}_i \, F_i(r_{12}, r_{23}, r_{31}) + \text{permutations}$$

Examples of the operators:

$$\tilde{\mathcal{G}}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \vec{\sigma}_1 \cdot \vec{\sigma}_3$$

$$\tilde{\mathcal{G}}_{16} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{12} \cdot \vec{\sigma}_2 \, \hat{r}_{12} \cdot \vec{\sigma}_3$$





Summary

A new generation of chiral NN potentials up to N^{4}LO is being developed

— excellent description of NN data
— good convergence of the chiral expansion

A simple approach to estimate theoretical uncertainty at a given order

— applicable to any observable and for a particular choice of the regulator
— results in the NN system at all orders and for all cutoffs are consistent with each other and with experimental data (within uncertainties)

Application to the 3N system:

— clear evidence for missing 3NF effects
— expect accurate results for Nd scattering up to $E_{lab} \sim 200$ MeV (at N^{4}LO)
— Nd scattering at intermediate ($E_{lab} \sim 50\ldots200$ MeV): a golden window to test/probe the 3NF in chiral EFT

Next step: explicit inclusion of the 3NF

Goal: reliable ab initio few- and many-body calculations based on chiral EFT with quantified theoretical uncertainties!