Chiral EFT for few-nucleon systems at the precision frontier

Introduction
Chiral EFT for nuclear forces
A new generation of NN potentials up to N^4LO
Quantification of theoretical uncertainties
Applications to the 3N system
Summary
Weinberg introduces nuclear chiral EFT

first real calculations by van Kolck et al (N^2LO)

first Nd scattering calculations with 3NF (N^3LO), EE et al.

Kaiser derives NN potential at N^3LO

N^3LO NN potentials by EE-Glöckle-Meißner, Entem-Machleidt

derivation of 3NF, 4NF at N^3LO and beyond

lattice EFT, Lee et al.
The 25 years of nuclear chiral EFT

Some milestones

1990: Weinberg introduces nuclear chiral EFT

1990: First real calculations by van Kolck et al. (N^2LO)

2000: KSW

2000: First Nd scattering calculations with 3NF (N^3LO), EE et al.

2010: Kaiser derives NN potential at N^3LO

2010: N^3LO NN potentials by EE-Glöckle-Meißner, Entem-Machleidt

Today: Derivation of 3NF, 4NF at N^3LO and beyond

Many new insights including

- Explanation of the observed hierarchy of many-body forces
- Promising results using 2NF at N^3LO combined with 3NF at N^2LO
- Electroweak structure of light nuclei, pion-deuteron scattering, Compton, …
The 25 years of nuclear chiral EFT

Some milestones

- 1990: Weinberg introduces nuclear chiral EFT
- 1990: First real calculations by van Kolck et al. (N^2LO)
- 2000: KSW first Nd scattering calculations with 3NF (N^2LO), EE et al.
- 2000: Kaiser derives NN potential at N^3LO
- 2010: N^3LO NN potentials by EE-Glöckle-Meißner, Entem-Machleidt
- Today: Derivation of 3NF, 4NF at N^3LO and beyond
- Today: Lattice EFT, Lee et al.

Many new insights including
- Explanation of the observed hierarchy of many-body forces
- Promising results using 2NF at N^3LO combined with 3NF at N^2LO
- Electroweak structure of light nuclei, pion-deuteron scattering, Compton, …

Today: precision frontier…
- Addressing unsolved problems (especially 3NF)
In spite of decades of effort, the spin structure of the 3NF is NOT properly described by 3NF models…

Kalantar-Nayestanaki et al., Rept. Prog. Phys. 75 (2012) 016301

Phenomenological 3NF models
Fujita-Miyazawa, Brasil, Tucson-Melbourne, Urbana, Illinois,…

In Nd scattering, large discrepancies between theory and data are observed at higher energies especially for spin observables

Today: precision frontier…
● addressing unsolved problems (especially 3NF)
In spite of decades of effort, the spin structure of the 3NF is NOT properly described by 3NF models...

Kalantar-Nayestanaki et al., Rept. Prog. Phys. 75 (2012) 016301

**Phenomenological 3NF models**

Fujita-Miyazawa, Brasil, Tucson-Melbourne, Urbana, Illinois,...

In Nd scattering, large discrepancies between theory and data are observed at higher energies especially for spin observables

**Today: precision frontier...**

- addressing **unsolved problems** (especially 3NF)
- **predictive theory** with quantified uncertainties
Today: precision frontier…

- addressing unsolved problems (especially 3NF)
- predictive theory with quantified uncertainties
Today: precision frontier…

- addressing **unsolved problems** (especially 3NF)
- **predictive theory** with quantified uncertainties
NCSM calculation based on chiral 2NF@N3LO + 3NF@N2LO
Navratil et al. '07

Today: precision frontier…
- addressing unsolved problems (especially 3NF)
- predictive theory with quantified uncertainties
The 25 years of nuclear chiral EFT

NCSM calculation based on chiral $2\text{NF}_{\text{N3LO}} + 3\text{NF}_{\text{N2LO}}$

Navratil et al. '07

Today: precision frontier…

- addressing unsolved problems (especially 3NF)
- predictive theory with quantified uncertainties
- chiral EFT meets ab-initio many-body methods: testing chiral forces in nuclei…

www.lenpic.org
Chiral perturbation theory

- **Ideal world** \( m_u = m_d = 0 \), **zero-energy limit**: non-interacting massless GBs (+ strongly interacting massive hadrons)

- **Real world** \( m_u, m_d \ll \Lambda_{QCD} \), **low energy**: weakly interacting light GBs (+ strongly interacting massive hadrons)

expand about the ideal world (ChPT)
Chiral Perturbation Theory: expansion of the scattering amplitude in powers of momenta of pions and nucleons or $M_\pi \sim 140$ MeV

\[ Q = \text{hard scales [at best } \Lambda_\chi = 4\pi F_\pi \sim 1 \text{ GeV}] \]

Manohar, Georgi '84

Tool: Feynman calculus using the effective chiral Lagrangian

\[
\mathcal{L}_{\pi} = \mathcal{L}_{\pi}^{(2)} + \mathcal{L}_{\pi}^{(4)} + \ldots
\]

\[
\mathcal{L}_{\pi N} = \bar{N} \left( i \gamma^\mu D_\mu[\pi] - m + \frac{g_A}{2} \gamma^\mu \gamma^5 u_\mu[\pi] \right) N + \sum_i c_i \bar{N} \hat{O}_i^{(2)}[\pi] N + \sum_i d_i \bar{N} \hat{O}_i^{(3)}[\pi] N + \ldots
\]
Chiral Perturbation Theory: expansion of the scattering amplitude in powers of 
Weinberg, Gasser, Leutwyler, Meißner, ... 

\[ Q = \text{momenta of pions and nucleons or } M_\pi \sim 140 \text{ MeV} \]

\[ \text{hard scales [at best } \Lambda_\chi = 4\pi F_\pi \sim 1 \text{ GeV}] \]
Manohar, Georgi '84

Tool: Feynman calculus using the effective chiral Lagrangian

\[ \mathcal{L}_\pi = \mathcal{L}^{(2)}_\pi + \mathcal{L}^{(4)}_\pi + \ldots \]

\[ \mathcal{L}_{\pi N} = \bar{N} \left( i\gamma^\mu D_\mu [\pi] - m + \frac{g_A}{2} \gamma^\mu \gamma_5 u_\mu [\pi] \right) N + \sum_i c_i \bar{N} \tilde{O}_i^{(2)} [\pi] N + \sum_i d_i \bar{N} \tilde{O}_i^{(3)} [\pi] N + \ldots \]

Pion-nucleon scattering up to \( Q^4 \) in heavy-baryon ChPT
Fettes, Meißner '00; Krebs, Gasparayan, EE '12

Order \( Q \):

Order \( Q^2 \):

Order \( Q^3 \):

Order \( Q^4 \):
Chiral EFT for nuclei

$m_N$  
$m_{\rho,\omega}$  
$m_{\Delta} - m_N$  
$M_\pi$

hard scales  
chiral EFT
A new, soft scale associated with nuclear binding $Q \sim 1/a_S \simeq 8.5$ MeV (36 MeV) in $^1S_0$ ($^3S_1$) to be generated dynamically (need resummations...).
A new, soft scale associated with nuclear binding

\[ Q \sim 1/a_S \simeq 8.5 \text{ MeV} (36 \text{ MeV}) \text{ in } {}^1S_0 ({}^3S_1) \]

to be generated dynamically (need resummations...)

**Pionless EFT** (valid for \( \sqrt{m_N E_B} \ll Q \ll M_\pi \))

- zero-range forces between nucleons
- for 2N equivalent to Effective Range Theory
- universality, Efimov physics, cold gases, halos,...
Chiral EFT for nuclei

A new, soft scale associated with nuclear binding
\[ Q \sim 1/a_S \simeq 8.5 \text{ MeV} \left(36 \text{ MeV}\right) \] in \(^1\!S_0 \left(^3\!S_1\right)\)
to be generated dynamically (need resumptions...)

**Pionless EFT** (valid for \( \sqrt{m_N E_B} \ll Q \ll M_\pi \))
- zero-range forces between nucleons
- for 2N equivalent to Effective Range Theory
- universality, Efimov physics, cold gases, halos,...

**Chiral EFT** (valid for \( Q \sim M_\pi \))
Weinberg, van Kolck, EE, Glöckle, Meißner, Machleidt, Entem...
- Schrödinger equation for nucleons
interacting via contact forces and
long-range potentials (pion exchanges)

\[
\left[ \left( \sum_{i=1}^A \frac{\vec{V}_i}{2m_N} + \mathcal{O}(m_N^{-3}) \right) + V_{2N} + V_{3N} + V_{4N} + \ldots \right] |\Psi\rangle = E |\Psi\rangle
\]
- access to heavier nuclei (ab initio few-/many-body methods)
Chiral dynamics and nuclear forces

conventional picture

\[
\text{Internucleon potential (MeV)} \quad \text{Separation (fm)}
\]

\[
\text{Repulsive core} \quad \rho, \omega, \sigma \quad 2\pi \quad \pi
\]

\[
0 \quad 0.5 \quad 1.0 \quad 1.5 \quad 2.0 \quad 2.5
\]

\[
-100 \quad 0 \quad 100 \quad 200 \quad 300
\]

chiral EFT

\[
\text{Internucleon potential (MeV)} \quad \text{Separation (fm)}
\]

\[
\text{contact interactions}
\]

\[
\text{multiple GB exchange (ChPT)}
\]
## Chiral expansion of nuclear forces

<table>
<thead>
<tr>
<th>Order (Q^n)</th>
<th>Two-nucleon force</th>
<th>Three-nucleon force</th>
<th>Four-nucleon force</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO (Q^0)</td>
<td><img src="image1" alt="Graph" /></td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>NLO (Q^2)</td>
<td><img src="image2" alt="Graph" /></td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>N^2LO (Q^3)</td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
<td>—</td>
</tr>
<tr>
<td>N^3LO (Q^4)</td>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
<td><img src="image7" alt="Graph" /></td>
</tr>
<tr>
<td>N^4LO (Q^5)</td>
<td><img src="image8" alt="Graph" /></td>
<td><img src="image9" alt="Graph" /></td>
<td><img src="image10" alt="Graph" /></td>
</tr>
</tbody>
</table>
### Chiral expansion of nuclear forces

<table>
<thead>
<tr>
<th></th>
<th>Two-nucleon force</th>
<th>Three-nucleon force</th>
<th>Four-nucleon force</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO $(Q^0)$</td>
<td><img src="#" alt="Diagram" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NLO $(Q^2)$</td>
<td><img src="#" alt="Diagram" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N^2$LO $(Q^3)$</td>
<td><img src="#" alt="Diagram" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N^3$LO $(Q^4)$</td>
<td><img src="#" alt="Diagram" /></td>
<td><img src="#" alt="Diagram" /></td>
<td><img src="#" alt="Diagram" /></td>
</tr>
<tr>
<td>$N^4$LO $(Q^5)$</td>
<td><img src="#" alt="Diagram" /></td>
<td><img src="#" alt="Diagram" /></td>
<td><img src="#" alt="Diagram" /></td>
</tr>
</tbody>
</table>

---

- have been worked out and employed
### Chiral expansion of nuclear forces

<table>
<thead>
<tr>
<th>LO (Q^0)</th>
<th>NLO (Q^2)</th>
<th>N^2LO (Q^3)</th>
<th>N^3LO (Q^4)</th>
<th>N^4LO (Q^5)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Two-nucleon force" /></td>
<td><img src="image" alt="Three-nucleon force" /></td>
<td><img src="image" alt="Four-nucleon force" /></td>
<td><img src="image" alt="Three-nucleon force" /></td>
<td><img src="image" alt="Four-nucleon force" /></td>
</tr>
</tbody>
</table>

- have been worked out and employed
- have been worked out but not employed yet
### Chiral expansion of nuclear forces

<table>
<thead>
<tr>
<th>LO (Q⁰)</th>
<th>NLO (Q²)</th>
<th>N²LO (Q³)</th>
<th>N³LO (Q⁴)</th>
<th>N⁴LO (Q⁵)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="two-nucleon-force.png" alt="Diagram" /></td>
<td><img src="three-nucleon-force.png" alt="Diagram" /></td>
<td><img src="four-nucleon-force.png" alt="Diagram" /></td>
<td><img src="three-nucleon-force.png" alt="Diagram" /></td>
<td><img src="four-nucleon-force.png" alt="Diagram" /></td>
</tr>
</tbody>
</table>

- **LO (Q⁰)**: Have been worked out and employed
- **NLO (Q²)**: Have been worked out but not employed yet
- **N²LO (Q³)**: Have not been completely worked out yet
- **N³LO (Q⁴)**: Have not been completely worked out yet
- **N⁴LO (Q⁵)**: Have not been completely worked out yet
The long-range part

Ordonez et al.; Kaiser; EE, Krebs, Meiβner, ...

\[ g_A \]

\[ \pi N \text{ amplitude up to order } Q^4 \]

\[ \pi N \text{ amplitude up to order } Q^3 \]

Order-\( Q^2 \) \( \pi N \) amplitude

(Leading) \( 3\pi \)-exchange potential is weak and probably can be well described by contacts (can be neglected)
The long-range part

Ordonez et al.; Kaiser; EE, Krebs, Meißner, …

\[ g_A \]

\[ \pi N \text{ amplitude up to order } Q^4 \ (c_i, d_i, e_i) \]

\[ \pi N \text{ amplitude up to order } Q^3 \ (c_i, d_i) \]

order-\( Q^2 \) \( \pi N \) amplitude (\( c_i \))

(Leading) \( 3\pi \)-exchange potential is weak and probably can be well described by contacts (can be neglected)

The determined values of LECs

Krebs, Gasparian, EE ’12

<table>
<thead>
<tr>
<th></th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_3 )</th>
<th>( c_4 )</th>
<th>( \tilde{d}_1 + \tilde{d}_2 )</th>
<th>( \tilde{d}_3 )</th>
<th>( \tilde{d}_5 )</th>
<th>( \tilde{d}<em>{14} - \tilde{d}</em>{15} )</th>
<th>( \tilde{e}_{14} )</th>
<th>( \tilde{e}_{15} )</th>
<th>( \tilde{e}_{16} )</th>
<th>( \tilde{e}_{17} )</th>
<th>( \tilde{e}_{18} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q^4 ) fit to GW</td>
<td>-1.13</td>
<td>3.69</td>
<td>-5.51</td>
<td>3.71</td>
<td>5.57</td>
<td>-5.35</td>
<td>0.02</td>
<td>10.26</td>
<td>1.75</td>
<td>-5.80</td>
<td>1.76</td>
<td>-0.58</td>
<td>0.96</td>
</tr>
<tr>
<td>( Q^4 ) fit to KH</td>
<td>-0.75</td>
<td>3.49</td>
<td>-4.77</td>
<td>3.34</td>
<td>6.21</td>
<td>-6.83</td>
<td>0.78</td>
<td>12.02</td>
<td>1.52</td>
<td>-10.41</td>
<td>6.08</td>
<td>-0.37</td>
<td>3.26</td>
</tr>
</tbody>
</table>
The long-range part

Ordonez et al.; Kaiser; EE, Krebs, Meißner, ...

\[ g_A \]

$\pi N$ amplitude up to order $Q^4$ ($c_i, d_i, e_i$)

$\pi N$ amplitude up to order-$Q^3$ ($c_i, d_i$)

Order-$Q^2$ $\pi N$ amplitude ($c_i$)

(Leading) $3\pi$-exchange potential is weak and probably can be well described by contacts (can be neglected)

The determined values of LECs

<table>
<thead>
<tr>
<th>$Q^4$ fit to GW</th>
<th>$-1.13$</th>
<th>$3.69$</th>
<th>$-5.51$</th>
<th>$3.71$</th>
<th>$5.57$</th>
<th>$-5.35$</th>
<th>$0.02$</th>
<th>$-10.26$</th>
<th>$1.75$</th>
<th>$-5.80$</th>
<th>$1.76$</th>
<th>$-0.58$</th>
<th>$0.96$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q^4$ fit to KH</td>
<td>$-0.75$</td>
<td>$3.49$</td>
<td>$-4.77$</td>
<td>$3.34$</td>
<td>$6.21$</td>
<td>$-6.83$</td>
<td>$0.78$</td>
<td>$-12.02$</td>
<td>$1.52$</td>
<td>$-10.41$</td>
<td>$6.08$</td>
<td>$-0.37$</td>
<td>$3.26$</td>
</tr>
</tbody>
</table>

The short-range part

(contact terms)

LO [$Q^0$]: 2 operators (S-waves)

NLO [$Q^2$]: + 7 operators (S-, P-waves and $\varepsilon_1$)

$N^2$LO [$Q^3$]: no new isospin-conserving operators + 2 IB terms ($^1S_0$)

$N^3$LO [$Q^4$]: + 15 operators (S-, P-, D-waves and $\varepsilon_1, \varepsilon_2$)

$N^4$LO [$Q^5$]: no new isospin-conserving operators + 1 IB term ($^1S_0$)
Regularization, renormalization and all that...

\[ T(\vec{p}', \vec{p}) = V_{2N}(\vec{p}', \vec{p}) + m \int \frac{d^3k}{(2\pi)^3} \frac{V_{2N}(\vec{p}', \vec{k}) T(\vec{k}, \vec{p})}{p^2 - k^2 + i\epsilon} \]

with \[ V_{2N} = \frac{\vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q}}{q^2 + M^2} \vec{\tau}_1 \cdot \vec{\tau}_2 + \ldots \]

→ Lippmann-Schwinger eq. is linearly divergent, need infinitely many CTs to absorb UV divergences from iterations!
Regularization, renormalization and all that...

$$T(\vec{p}', \vec{p}) = V_{2N}(\vec{p}', \vec{p}) + m \int \frac{d^3k}{(2\pi)^3} \frac{V_{2N}(\vec{p}', \vec{k})}{p^2 - k^2 + i\epsilon} T(\vec{k}, \vec{p})$$

with

$$V_{2N} = \alpha \frac{\vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q}}{q^2 + M^2} \tau_1 \cdot \tau_2 + \ldots$$

→ Lippmann-Schwinger eq. is linearly divergent, need infinitely many CTs to absorb UV divergences from iterations!

Possible approaches:

- Use a manifestly Lorentz-invariant approach (3D-eqs. fulfilling relativistic unitarity)
  EE, Gegelia’12,’13; EE, Gasparyan, Gegelia, Krebs, Schindler ’14,’15
  — integral eq. is renormalizable at LO (only log-divergences), \( \Lambda \) can be removed!
  — Caveat: calculations are complicated, hard to go beyond the NN system…
Regularization, renormalization and all that...

\[ T(\vec{p}', \vec{p}) = V_{2N}(\vec{p}', \vec{p}) + m \int \frac{d^3k}{(2\pi)^3} \frac{V_{2N}(\vec{p}', \vec{k})}{p^2 - k^2 + i\epsilon} \quad \text{with} \quad V_{2N} = \alpha \frac{\vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^2 + M^2} \tau_1 \cdot \tau_2 + \ldots \]

→ Lippmann-Schwinger eq. is linearly divergent, need infinitely many CTs to absorb UV divergences from iterations!

Possible approaches:

- **Use a manifestly Lorentz-invariant approach** (3D-eqs. fulfilling relativistic unitarity)  
  EE, Gegelia’12,’13; EE, Gasparian, Gegelia, Krebs, Schindler ’14,’15
  - integral eq. is renormalizable at LO (only log-divergences), \( \Lambda \) can be removed!
  - Caveat: calculations are complicated, hard to go beyond the NN system...

- **Use a finite UV cutoff** (standard)  
  Lepage ’97
  - simple, well suited for few- and many-body calculations
  - Caveat: finite-cutoff artifacts…
  - we use a local regulator for long-range terms  
    (maintains analytic structure of the amplitude)
    and choose \( R = 0.8 \ldots 1.2 \text{ fm} \)
    \[ V(r) \rightarrow V(r) \left[ 1 - \exp \left( -\frac{r^2}{R^2} \right) \right]^6 \]
Regularization, renormalization and all that...

\[
T(\vec{p}', \vec{p}) = V_{2N}(\vec{p}', \vec{p}) + m \int \frac{d^3k}{(2\pi)^3} \frac{V_{2N}(\vec{p}', \vec{k}) T(\vec{k}, \vec{p})}{p^2 - k^2 + i\epsilon}
\]

with \( V_{2N} = \alpha \frac{\vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q}}{q^2 + M_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 + \ldots \)

→ Lippmann-Schwinger eq. is linearly divergent, need infinitely many CTs to absorb UV divergences from iterations!

Possible approaches:

- Use a manifestly Lorentz-invariant approach (3D-eqs. fulfilling relativistic unitarity)
  EE, Gegelia'12,'13; EE, Gasparyan, Gegelia, Krebs, Schindler '14,'15
  — integral eq. is renormalizable at LO (only log-divergences), \( \Lambda \) can be removed!
  — Caveat: calculations are complicated, hard to go beyond the NN system...

- Use a finite UV cutoff (standard) Lepage '97
  — simple, well suited for few- and many-body calculations
  — Caveat: finite-cutoff artifacts...
  — we use a local regulator for long-range terms
    (maintains analytic structure of the amplitude)
    and choose \( R = 0.8 \ldots 1.2 \text{ fm} \)

(Implicit) renormalization: express bare LECs in terms of observables (phase shifts)
TABLE III:

<table>
<thead>
<tr>
<th>Order</th>
<th>Neutron-proton phase shifts</th>
<th>Proton-proton phase shifts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0–100</td>
<td>360</td>
</tr>
<tr>
<td></td>
<td>0–200</td>
<td>480</td>
</tr>
</tbody>
</table>

Only those channels are included which have been used in the N$_3$LO/N$_4$LO fits, namely the S-, P- and D-waves and the mixing angles $\epsilon_1$ and $\epsilon_2$.

FIG. 1: Chiral expansion of the NN phase shifts in comparison with the NPWA [25] (solid dots) and the GWU single-energy np partial wave analysis [58] (open triangles). Dotted, dashed, dashed-dotted, dashed-double-dotted and solid lines show the results at LO, NLO, N$_2$LO, N$_3$LO and N$_4$LO, respectively, calculated using $R = 0.9$ fm.

Convergence of the chiral expansion for neutron-proton phase shifts [using $R = 0.9$ fm]
Convergence of the chiral expansion for neutron-proton phase shifts [using R = 0.9 fm]

\( \chi^2_{\text{datum}} \) for the reproduction of the Nijmegen phase shifts [using R = 0.9 fm]

<table>
<thead>
<tr>
<th>( E_{\text{lab}} ) bin</th>
<th>LO [( Q^0 )]</th>
<th>NLO [( Q^2 )]</th>
<th>( N^2\text{LO} ) [( Q^3 )]</th>
<th>( N^3\text{LO} ) [( Q^4 )]</th>
<th>( N^4\text{LO} ) [( Q^5 )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>neutron-proton phase shifts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0–100</td>
<td>360</td>
<td>31</td>
<td>4.5</td>
<td>0.7</td>
<td>0.3</td>
</tr>
<tr>
<td>0–200</td>
<td>480</td>
<td>63</td>
<td>21</td>
<td>0.7</td>
<td>0.3</td>
</tr>
<tr>
<td>proton-proton phase shifts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0–100</td>
<td>5750</td>
<td>102</td>
<td>15</td>
<td>0.8</td>
<td>0.3</td>
</tr>
<tr>
<td>0–200</td>
<td>9150</td>
<td>560</td>
<td>130</td>
<td>0.7</td>
<td>0.6</td>
</tr>
</tbody>
</table>
Convergence of the chiral expansion for neutron-proton phase shifts [using R = 0.9 fm]

$\chi^2_{\text{datum}}$ for the reproduction of the Nijmegen phase shifts [using R = 0.9 fm]

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>neutron-proton phase shifts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0–100</td>
<td>360</td>
<td>31</td>
<td>4.5</td>
<td>0.7</td>
<td>0.3</td>
</tr>
<tr>
<td>0–200</td>
<td>480</td>
<td>63</td>
<td>21</td>
<td>0.7</td>
<td>0.3</td>
</tr>
<tr>
<td>proton-proton phase shifts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0–100</td>
<td>5750</td>
<td>102</td>
<td>15</td>
<td>0.8</td>
<td>0.3</td>
</tr>
<tr>
<td>0–200</td>
<td>9150</td>
<td>560</td>
<td>130</td>
<td>0.7</td>
<td>0.6</td>
</tr>
</tbody>
</table>
**Convergence of the chiral expansion for neutron-proton phase shifts** [using \( R = 0.9 \text{ fm} \)]

<table>
<thead>
<tr>
<th>( E_{\text{lab}} ) bin</th>
<th>LO [( Q^0 )]</th>
<th>NLO [( Q^2 )]</th>
<th>( N^2 \text{LO} [Q^3] )</th>
<th>( N^3 \text{LO} [Q^4] )</th>
<th>( N^4 \text{LO} [Q^5] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>neutron-proton phase shifts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0–100</td>
<td>360</td>
<td>31</td>
<td>4.5</td>
<td>0.7</td>
<td>0.3</td>
</tr>
<tr>
<td>0–200</td>
<td>480</td>
<td>63</td>
<td>21</td>
<td>0.7</td>
<td>0.3</td>
</tr>
<tr>
<td>proton-proton phase shifts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0–100</td>
<td>5750</td>
<td>102</td>
<td>15</td>
<td>0.8</td>
<td>0.3</td>
</tr>
<tr>
<td>0–200</td>
<td>9150</td>
<td>560</td>
<td>130</td>
<td>0.7</td>
<td>0.6</td>
</tr>
</tbody>
</table>

\( \chi^2_{\text{datum}} \) for the reproduction of the Nijmegen phase shifts [using \( R = 0.9 \text{ fm} \)]

- No new LECs
- 1 LEC \( |S_0| \)
- No new LECs

---

**TABLE III:**

**Proton-proton phase shifts**

| Energy bin | \( S\) | \( P\) | Mixing angle | \( |S_0|\) |
|------------|--------|--------|--------------|---------|
| 0–100 MeV  | 9150   | 560    | 130          | 0.7     |
| 0–200 MeV  | 9150   | 560    | 130          | 0.7     |

**Neutron-proton phase shifts**

| Energy bin | \( S\) | \( P\) | Mixing angle | \( |S_0|\) |
|------------|--------|--------|--------------|---------|
| 0–100 MeV  | 5750   | 102    | 15           | 0.8     |
| 0–200 MeV  | 9150   | 560    | 130          | 0.7     |
Expansion parameter: \[ Q = \max \left( \frac{p}{\Lambda_b}, \frac{M_\pi}{\Lambda_b} \right) \]

the breakdown scale is estimated to be \( \Lambda_b \sim 600 \text{ MeV} \) (for \( R = 0.8 \ldots 1.0 \text{ fm} \))
Convergence of the chiral expansion

Expansion parameter:

\[ Q = \max \left( \frac{p}{\Lambda_b}, \frac{M_\pi}{\Lambda_b} \right) \]

\[ \Lambda_b \sim 600 \text{ MeV} \quad (\text{for } R = 0.8\ldots1.0 \text{ fm}) \]

Example: neutron-proton total cross section \( R=0.9 \text{ fm} \)

\[ E_{\text{lab}} = 96 \text{ MeV} \quad [p = 212 \text{ MeV}]: \]

\[ \sigma_{\text{tot}} = \frac{Q^0}{Q^2} + \frac{Q^3}{Q^4} + \frac{Q^4}{Q^5} \]

\[ \sigma_{\text{tot}} = \frac{84.8}{-9.7} + \frac{3.2}{0.8} + \frac{0.5}{0.5} = 78.0 \text{ mb} \]
Convergence of the chiral expansion

Expansion parameter: \[ Q = \max \left( \frac{p}{\Lambda_b}, \frac{M_\pi}{\Lambda_b} \right) \]

\[ Q = 212 / 600 \sim 0.35 \]

\[ \sigma_{\text{tot}} = 84.8 - 9.7 + 3.2 - 0.8 + 0.5 = 78.0 \text{ mb} \]

Example: neutron-proton total cross section \( R=0.9 \text{ fm} \)

\( E_{\text{lab}} = 96 \text{ MeV} \) \( [p = 212 \text{ MeV}] \):

\[ \sigma_{\text{tot}} = \sum_{Q}^{5} Q^{n} \]

\[ Q = 212 / 600 \sim 0.35 \]

\[ \sigma_{\text{tot}} = 84.8 - 9.7 + 3.2 - 0.8 + 0.5 = 78.0 \text{ mb} \]

The breakdown scale is estimated to be \( \Lambda_b \sim 600 \text{ MeV} \) (for \( R = 0.8 \ldots 1.0 \text{ fm} \))
Convergence of the chiral expansion

Expansion parameter: \[ Q = \max \left( \frac{p}{\Lambda_b}, \frac{M_\pi}{\Lambda_b} \right) \]

The breakdown scale is estimated to be \( \Lambda_b \sim 600 \text{ MeV} \) (for \( R = 0.8...1.0 \text{ fm} \))

Example: neutron-proton total cross section \( R=0.9 \text{ fm} \)

\[ E_{\text{lab}} = 96 \text{ MeV} \ [p = 212 \text{ MeV}]: \quad \sigma_{\text{tot}} = 84.8 - 9.7 + 3.2 - 0.8 + 0.5 = 78.0 \text{ mb} \]
\[ Q = 212 / 600 \sim 0.35 \quad \text{expect:} \quad \sim 11 \quad \sim 4 \quad \sim 1.3 \quad \sim 0.5 \]

\[ E_{\text{lab}} = 200 \text{ MeV} \ [p = 307 \text{ MeV}]: \quad \sigma_{\text{tot}} = 34.9 + 1.0 + 6.7 + 0.6 - 0.5 = 42.7 \text{ mb} \]
\[ Q = 307 / 600 \sim 0.5 \quad \text{expect:} \quad \sim 9 \quad \sim 5 \quad \sim 2.4 \quad \sim 1.2 \]

\[ \rightarrow \text{ good convergence of the chiral expansion} \]
Let $X(p)$ be some observable with $p$ denoting the corresponding momentum scale and $X^{(n)}(p), \ n = 0, 2, 3, 4, \ldots$ a prediction at order $Q^n$ in the chiral expansion:

$$X^{(n)} = X^{(0)} + \Delta X^{(2)} + \ldots + \Delta X^{(n)}$$

calculated in the chiral expansion

For the order-$n$ contribution one expects $\Delta X^{(n)} \sim O\left(Q^n X^{(0)}\right)$ with $Q = \max\left(\frac{M_\pi}{\Lambda_b}, \frac{p}{\Lambda_b}\right)$
Let $X(p)$ be some observable with $p$ denoting the corresponding momentum scale and $X^{(n)}(p)$, $n = 0, 2, 3, 4, \ldots$ a prediction at order $Q^n$ in the chiral expansion:

$$X^{(n)} = X^{(0)} + \Delta X^{(2)} + \ldots + \Delta X^{(n)}$$

calculated in the chiral expansion

For the order-$n$ contribution one expects $\Delta X^{(n)} \sim O(Q^n X^{(0)})$ with $Q = \max\left(\frac{M_\pi}{\Lambda_b}, \frac{p}{\Lambda_b}\right)$

Theoretical uncertainty $\delta X^{(n)}$ estimated via the size of neglected higher-order contributions*:

$$\delta X^{(0)} = Q^2 |X^{(0)}|,$$
$$\delta X^{(2)} = \max(\frac{Q^3 |X^{(0)}|}{Q |\Delta X^{(2)}|}, \frac{Q |\Delta X^{(2)}|}{|X^{(0)}|}),$$
$$\delta X^{(3)} = \max\left(\frac{Q^4 |X^{(0)}|}{Q |\Delta X^{(2)}|}, \frac{Q^2 |\Delta X^{(2)}|}{Q |\Delta X^{(3)}|}\right),$$
$$\ldots$$

(*Also demand that $\delta X^{(n)}$ is not smaller than the actual higher-order contributions whenever known)
Let $X(p)$ be some observable with $p$ denoting the corresponding momentum scale and $X^{(n)}(p), \ n = 0, 2, 3, 4, \ldots$ a prediction at order $Q^n$ in the chiral expansion:

$$X^{(n)} = X^{(0)} + \Delta X^{(2)} + \ldots + \Delta X^{(n)}$$

For the order-$n$ contribution one expects $\Delta X^{(n)} \sim O\left(Q^n X^{(0)}\right)$ with $Q = \max\left(\frac{M_\pi}{\Lambda_b}, \frac{p}{\Lambda_b}\right)$

Theoretical uncertainty $\delta X^{(n)}$ estimated via the size of neglected higher-order contributions*:

$$\delta X^{(0)} = Q^2 |X^{(0)}|,$$

$$\delta X^{(2)} = \max\left(Q^3 |X^{(0)}|, \ Q |\Delta X^{(2)}|\right),$$

$$\delta X^{(3)} = \max\left(Q^4 |X^{(0)}|, \ Q^2 |\Delta X^{(2)}|, \ Q |\Delta X^{(3)}|\right),$$

$$\ldots$$

(*Also demand that $\delta X^{(n)}$ is not smaller than the actual higher-order contributions whenever known)

→ a simple approach applicable for any observable and any choice of the regulator
np total cross section for $R_1,\ldots,5 = \{0.8, 0.9, 1.0, 1.1, 1.2\}$ fm

![Graphs showing total cross section for np at different incident energies.](image)
Selected neutron-proton scattering observables at 50 MeV $R=0.9$ fm

\[ \text{d} \sigma / \text{d} \Omega \text{ [mb/sr]} \]

\[ A_y \]

\[ D \]

\[ A \]

\[ A_{xx} \]

\[ A_{yy} \]

\[ \theta_{CM} \text{ [deg]} \]
Selected neutron-proton scattering observables at 50 MeV \( R=1.2 \text{fm} \)

- The theoretical predictions for different cutoff choices are consistent with each other.
- Softer cutoffs lead to larger theoretical uncertainties.
Selected neutron-proton scattering observables at 200 MeV \( R = 0.9 \text{ fm} \)

- Accurate results even at the energy of \( E_{\text{lab}} = 200 \text{ MeV} \) (for \( R = 0.9 \text{ fm} \))
Deuteron properties \( R=0.9 \text{ fm} \)

<table>
<thead>
<tr>
<th></th>
<th>LO</th>
<th>NLO</th>
<th>N</th>
<th>N</th>
<th>N</th>
<th>empirical</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B )</td>
<td>2.0235</td>
<td>2.1987</td>
<td>2.2311</td>
<td>2.2246*</td>
<td>2.2246*</td>
<td>2.224575(9)</td>
</tr>
<tr>
<td>( A )</td>
<td>0.8333</td>
<td>0.8772</td>
<td>0.8865</td>
<td>0.8845</td>
<td>0.8844</td>
<td>0.8846(9)</td>
</tr>
<tr>
<td>( \eta )</td>
<td>0.0212</td>
<td>0.0256</td>
<td>0.0256</td>
<td>0.0255</td>
<td>0.0255</td>
<td>0.0256(4)</td>
</tr>
<tr>
<td>( r_d )</td>
<td>1.990</td>
<td>1.968</td>
<td>1.966</td>
<td>1.972</td>
<td>1.972</td>
<td>1.97535(85)</td>
</tr>
<tr>
<td>( Q \ [\text{fm}] )</td>
<td>0.230</td>
<td>0.273</td>
<td>0.270</td>
<td>0.271</td>
<td>0.271</td>
<td>0.2859(3)</td>
</tr>
<tr>
<td>( P_D )</td>
<td>2.54</td>
<td>4.73</td>
<td>4.50</td>
<td>4.19</td>
<td>4.29</td>
<td></td>
</tr>
</tbody>
</table>

— fast convergence of the chiral expansion (\( P_D \) is not observable)

— error estimation (assuming \( Q=M_\pi/\Lambda_b \))

\textbf{\( A_S \):} \( \text{LO: } 0.83(5) \rightarrow \text{NLO: } 0.878(13) \rightarrow \text{N}^2\text{LO: } 0.887(3) \rightarrow \text{N}^3\text{LO: } 0.8845(8) \rightarrow \text{N}^4\text{LO: } 0.8844(2) \)

\textbf{\( \eta \):} \( \text{LO: } 0.021(5) \rightarrow \text{NLO: } 0.026(1) \rightarrow \text{N}^2\text{LO: } 0.0256(3) \rightarrow \text{N}^3\text{LO: } 0.0255(1) \rightarrow \text{N}^4\text{LO: } 0.0255 \)

→ theoretical results for \( A_S, \eta \) at \( N^4\text{LO} \) are more accurate than empirical numbers

— results for \( r_d \) and \( Q \) do not take into account MECs and relativistic corrections:

\textbf{\( r_d \):} \( |\Delta r_d| \approx 0.004 \text{ fm} \) \cite{Kohno '83} \rightarrow predictions in agreement with the data

\textbf{\( Q \):} rel. corrections + \( 1\pi \)-exchange MEC: \( \Delta Q \approx +0.008 \text{ fm}^2 \) \cite{Phillips '07} \rightarrow \( Q \approx 0.279 \text{ fm}^2 \)

the remaining deviation of 0.007 fm\(^2\) agrees with the expected size of \( \chi \) \cite{Phillips '07}
Intermediate summary

A new generation of chiral NN potentials up to $N^4$LO is developed

- chiral expansion for NN scattering shows good convergence
- excellent description of NN scattering observables & deuteron properties at $N^3$LO, $N^4$LO

A simple approach for uncertainty quantification is introduced

- applicable to any observable and for any choice of the regulator
- results in the NN system at all orders and for all cutoffs are consistent with each other and with experimental data (within uncertainties) → seems to work reliably

With these developments, we are ready to take up the 3NF challenge
(work in progress by the LENPIC collaboration)
Evidence for missing 3N forces
LENPIC, in preparation

While no complete calculations based on the new 2N+3N forces are available yet, we performed incomplete calculations based on 2N forces only in order to:

— identify observables/kinematics best suitable for searches of 3NF effects
— estimate the achievable accuracy of of chiral EFT
Chiral expansion of the 3N force

<table>
<thead>
<tr>
<th></th>
<th>Long - range</th>
<th>Short - range</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLO</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N^2LO</td>
<td></td>
<td></td>
</tr>
<tr>
<td>van Kolck '94, Epelbaum et al. '02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N^3LO</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ishikawa, Robilotta, PRC76 (07); Bernard, Epelbaum, HK, Meißner, PRC77 (08); PRC84 (11)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N^4LO</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HK, Gasparyan, Epelbaum PRC85 (12); PRC87 (13)</td>
<td>Work in progress</td>
<td>Girlanda, Kievsy, Viviani, PRC84 (11)</td>
</tr>
</tbody>
</table>

2π-1π  ring  2π
Notice: 3N force at large distance is completely determined by the chiral symmetry of QCD + experimental information on the πN system (parameter-free!)

\[ V(r_{12}, r_{23}, r_{31}) = \sum_{i=1}^{20} \tilde{G}_i F_i(r_{12}, r_{23}, r_{31}) + \text{permutations} \]

Examples of the operators:

\[ \tilde{G}_4 = \tau_1 \cdot \tau_3 \bar{\sigma}_1 \cdot \bar{\sigma}_3 \]
\[ \tilde{G}_{16} = \tau_2 \cdot \tau_3 \hat{r}_{12} \cdot \hat{\sigma}_2 \hat{r}_{12} \cdot \hat{\sigma}_3 \]
A new generation of chiral NN potentials up to N⁴LO is being developed

- excellent description of NN data
- good convergence of the chiral expansion

A simple approach to estimate theoretical uncertainty at a given order

- applicable to any observable and for a particular choice of the regulator
- results in the NN system at all orders and for all cutoffs are consistent with each other and with experimental data (within uncertainties)

Application to the 3N system:

- clear evidence for missing 3NF effects
- expect accurate results for Nd scattering up to E_{lab} \sim 200 MeV (at N⁴LO)
- Nd scattering at intermediate (E_{lab} \sim 50…200 MeV): a golden window to test/probe the 3NF in chiral EFT

Next step: explicit inclusion of the 3NF

Goal: reliable ab initio few- and many-body calculations based on chiral EFT with quantified theoretical uncertainties!