Modification of spectrum of heavylight mesons in nuclear medium

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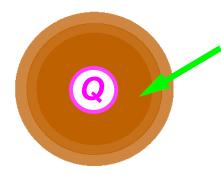
at "Hadrons and Hadron Interactions in QCD" (YITP, Kyoto, March 2, 2015)

Based on

 D.Suenaga, B.-R.He, Y.L.Ma and M.H., Phys. Rev. C 89, 068201 (2014); Phys. Rev. D 91, 036001 (2015)

INTRODUCTION

Heavy-Light Mesons (Qq type) Baryons (Qqq)



"Light-quark cloud" (Brown Muck) ... made of light quarks and gluons typical energy scale $\sim \Lambda_{QCD} << M_{Q}$

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© Heavy mesons ···· 3 or 3^{bar}, ... of SU(3)<sub>I</sub>

◎ Heavy baryons ···· 6, ... of SU(3)<sub>I</sub>
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Flavor representations, which do not exist in the light quark sector, give a new clue to understand the hadron structure.

Heavy-light mesons as a probe to the structure (symmetries) of nuclear matter

- In this talk, I shall show main results of D.Suenaga, B.-R.He, Y.L.Ma and M.H., Phys. Rev. C 89, 068201 (2014); Phys. Rev. D 91, 036001 (2015)
- In PRC89, 068201 (2014), we propose to study the mass spectrum of the heavy-light mesons to probe the structure of the spin-isospin correlation in nuclear matter.
 - The spin-isospin correlation generates a mixing among the heavy-light mesons carrying different spins and isospin such as D⁺, D⁰, D^{*+} and D^{*0} mesons.
 - The structure of the mixing reflects the pattern of the correlation, i.e. the remaining symmetry.
 - The magnitude of the mass modification provides information of the strength of the correlation.

Mass degeneracy of heavy-light mesons as a probe to the half-Skyrmion phase

- In D.Suenaga, B.-R.He, Y.L.Ma and M.H., PRC89, 068201 (2014), we explored the mass splitting of heavy-light mesons with chiral partner structure in nuclear matter.
 - We regard $[D(J^P=0^-), D^*(1^-)]$ and $[D^*_0(0^+), D_1(1^+)]$ as chiral partners.
 - We construct the nuclear matter by putting Skyrmions onto a crystal.
 - Our results show that the masses of chiral partners are degenerated in the half-Skyrmion phase in which the chiral symmetry is restored globally.

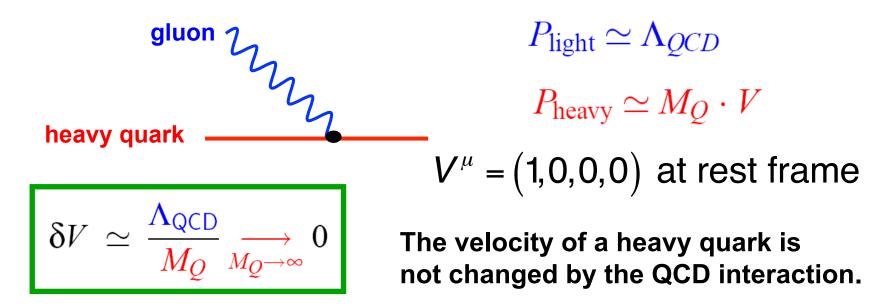
Outline

- 1. Introduction
- 2. Heavy meson effective fields
- 3. D-D* mixing in Spin-Isospin correlated Nuclear matter
- 4. Mass degeneracy in the half-Skyrmion phase
- 5. Summary

Heavy meson effective fields

★ Heavy quark symmetry •••• a symmetry of QCD at $M_Q \rightarrow \infty$ limit

O velocity super-selection rule



O Heavy quark number conservation

No pair production of heavy quarks by QCD interaction.

◎ SU(2) spin symmetry

QCD interaction cannot flip the spin of heavy quarks.

☆ Heavy-Light Mesons (Qq type)

"Light-quark cloud" (Brown Muck) ··· made of light quarks and gluons typical energy scale ~ Λ_{QCD} << M_{O} $\vec{J} = \vec{S}_Q + \vec{J}_l$ angular momentum carried by "Brown muck" **O** spin of meson t spin of heavy quark • $M_{Q} \rightarrow \infty$ limit same J_{I} $\begin{bmatrix} \vec{S}_Q, H \end{bmatrix} = 0$ $\begin{bmatrix} \vec{J}, H \end{bmatrix} = 0$ $\begin{bmatrix} \vec{J}_l, H \end{bmatrix} = 0$ **Q(**↑) $J = J_l + \frac{1}{2}$ $J = J_l - \frac{1}{2}$ conservation of J_{μ} **Heavy Meson Multiplet** \Rightarrow classification of hadrons by J_{μ}

••• degenerate masses

O Parity of Mesons

1

• Quark model ••• Brown muck $\Leftrightarrow \overline{q}$

$$\vec{J_l} = \vec{S_l} + \vec{L}$$
 angular momentum
 $\vec{J_l} = \vec{S_l} + \vec{L}$ $P_{\text{light}} = -(-1)^L$
spin of light quark

Parity of meson
$$\cdots P = (-1)^{L+1}$$

© classification of mesons

A heavy meson multiplet includes $J_{\pm} = J_l \pm \frac{1}{2}$ states

•
$$J_l = \frac{1}{2}$$
 \Rightarrow $J^P = \begin{cases} (0^-, 1^-) & \cdots & \text{ground states} \\ (0^+, 1^+) & \cdots & \text{excited states} \end{cases}$
• $J_l = \frac{3}{2}$ \Rightarrow $J^P = \begin{cases} (1^+, 2^+) & \cdots & \text{excited states} \\ \cdots & \cdots & \text{excited states} \end{cases}$

Heavy meson effective fields

☆ Ground states · · · $J_1 = 1/2$; $J^P = (0^-, 1^-)$

Pseudoscalar meson **D** ; Vector meson **D***

$$D = (D^0, D^+)$$
 $D^* = (D^{*0}, D^{*+})$

- Bi-spinor field $H \sim Q \overline{\Psi}$; $\Psi \cdots$ light constiuent quark field

$$H = \frac{1+\not p}{2} \left[D^{*\mu} \gamma_{\mu} + i D \gamma_5 \right]$$

annihilates heavy mesons (not generate)

☆ First excited states · · · $J_1 = 1/2$; $J^P = (0^+, 1^+)$

Scalar meson D_0^* ; Axial-vector meson D_1

$$D_0^* = (D_0^{*0}, D_0^{*+}) \quad D_1 = (D_1^0, D_1^+)$$

$$G = \frac{1+\not p}{2} \left[-iD_1^{\mu} \gamma_{\mu} \gamma_5 + D_0^* \right]$$

D-D* MIXING IN SPIN-ISOSPIN CORRELATED NUCLEAR MATTER

based on D.Suenaga, B.-R.He, Y.L.Ma and M.H., Phys. Rev. C 89, 068201 (2014)

Model (heavy-light mesons)

 Heavy-light mesons made of a heavy c quark and (u, d) quarks

$$H_a = \frac{1 + \gamma_v V^v}{2} \left[D_a^{*;\mu} \gamma_\mu + i D_a \gamma_5 \right]; \quad (a = 1, 2: \text{ isospin index})$$

- Symmetries (at rest frame: $V^{v} = (1,0,0,0)$)
 - heavy-quark spin : SU(2)
 - representation : $S_h = \frac{1}{2}$
 - spin of Brown-muck : SU(2)
 - representation : $J_1 = \frac{1}{2}$
 - isospin : SU(2)
 - representation : $I = \frac{1}{2}$
- Degeneracy
 - $2 \times (2J_{1}+1) \times 2 = 8$ states make $(2J_{1}+1) \times 2 = 4$ HQ pairs
 - An HQ pair belongs to the representation $S_h = 1/2$.

Chiral Lagrangian for pion

O Chiral symmetry breaking

 $SU(N_f) \times SU(N_f)_R \rightarrow SU(N_f)_V$

O Basic Quantity

$$\boldsymbol{U} = e^{2i\pi^{a}T_{a}/F_{\pi}} \rightarrow \boldsymbol{g}_{L} \boldsymbol{U} \boldsymbol{g}_{R}^{\dagger} ; \quad \boldsymbol{g}_{L,R} \in \mathrm{SU}(N_{f})_{L,R}$$

O Lagrangian

$$\mathcal{L} = \frac{F_{\pi}^{2}}{4} \operatorname{tr} \left[\partial_{\mu} U \partial^{\mu} U^{\dagger} \right]$$
$$U = \xi_{L}^{\dagger} \xi_{R}; \quad \xi_{R} = \xi_{L}^{\dagger} = e^{i\pi^{a}T_{a}/F_{\pi}};$$
$$\xi_{R}^{\dagger} \rightarrow g_{R} \xi_{R}^{\dagger} h^{\dagger} (g_{R}, g_{L}, \pi);$$
$$\xi_{L} \rightarrow h(g_{R}, g_{L}, \pi) \xi_{L} g_{L}^{\dagger} \quad ; \quad h(g_{R}, g_{L}, \pi) \in SU(N_{f})_{V}$$
$$L = F_{\pi}^{2} \operatorname{tr} \left[\alpha_{\perp}^{\mu} \alpha_{\perp \mu} \right]; \quad \alpha_{\perp \mu} = \left(\partial_{\mu} \xi_{R} \cdot \xi_{R}^{\dagger} - \partial_{\mu} \xi_{L} \cdot \xi_{L}^{\dagger} \right) / (2i)$$

Interaction to the light sector

• Lagrangian

$$L = \operatorname{tr}\left[H(iV^{\mu}D_{\mu})\overline{H}\right] + g_{A}\operatorname{tr}\left[H\gamma^{\mu}\gamma_{5}\alpha_{\perp\mu}\overline{H}\right]$$

– $g_A = 0.56$ determine from D* \rightarrow D π decay

$$\alpha^{\mu}_{\perp} = \frac{1}{2i} \left(\partial^{\mu} \xi_R \cdot \xi^{\dagger}_R - \partial^{\mu} \xi_L \cdot \xi^{\dagger}_L \right)$$

covariant derivative

$$\alpha_{\perp\mu} = \sum_{A=1}^{3} \alpha_{\perp\mu}^{A} T_{A}; \quad T_{A} = \frac{1}{2} \tau_{A}$$

$$D_{\mu}\overline{H} = \partial_{\mu}\overline{H} - i\alpha_{\mu}\overline{H}$$
$$\alpha^{\mu}_{\parallel} = \frac{1}{2i} \left(\partial^{\mu}\xi_{R} \cdot \xi^{\dagger}_{R} + \partial^{\mu}\xi_{L} \cdot \xi^{\dagger}_{L} \right)$$

position dependent pion condensation

- Spin-isospin correlation in the nuclear medium will cause the position dependent pion condensation.
- In the equilibrium case, the pion condensation does not depend on the time.

$$\left\langle \alpha_{\perp 0}^{A}\left(\vec{x}\right)\right\rangle = \left\langle \alpha_{\rm H0}^{A}\left(\vec{x}\right)\right\rangle = 0$$

- $\langle \alpha_{III}^{A}(\vec{x}) \rangle$ does not contribute at the rest frame.
- We are interested in the mass spectrum of the heavy-light mesons, so that we take the spatial components of the residual momentum of the heavy-light mesons to be zero.
- Only the space average of α_{\perp} contributes.

$$\left\langle \alpha_{\perp i}^{A} \right\rangle = \int_{V} d^{3}x \left\langle \alpha_{\perp i}^{A} \left(\vec{x} \right) \right\rangle : \left\langle \alpha_{\perp i}^{A} \right\rangle = \left\langle \frac{1}{f_{\pi}} \partial_{i} \pi^{A} + \cdots \right\rangle$$

P-wave pion condensation

Inverse propagator matrix

Lagrangian
$$\rightarrow (D_a, D_a^{*i}) \Delta_{ab}^{[i,j]} \begin{pmatrix} D_b \\ D_b^{*j} \end{pmatrix}$$

 $\Delta^{|}$

$$\begin{bmatrix} i,j \\ ab \end{bmatrix} = \begin{pmatrix} \Delta_{ab} & -\Delta_{ab}^{j} \\ \Delta_{ab}^{i} & \Delta_{ab}^{ij} \end{pmatrix}$$

$$\Delta_{ab} = p_{0} \delta_{ab}$$

$$\Delta_{ab}^{i} = -i g_{A} \langle \alpha_{\perp}^{iA} \rangle (\tau_{A})_{ab}$$

$$\Delta_{ab}^{ij} = p_{0} \delta_{ab} - i g_{A} \varepsilon^{ijk} \langle \alpha_{\perp}^{kA} \rangle (\tau_{A})_{ab}$$
causes a mixing among different modes of D* mesons

Mass spectrum (pattern 1)

• $\langle \alpha_{\perp}^{iA} \rangle$ = $\alpha \delta_{iA}$ motivated by the hedgehog ansatz of skyrmion

Nucleon as Skyrme soliton

Skyrme model

$$\mathcal{L}_{\text{Skyr}} = \frac{F_{\pi}^2}{4} \text{Tr} \left[\partial_{\mu} U \partial^{\mu} U^{\dagger} \right] + \frac{\epsilon^2}{4} \text{Tr} \left\{ \left[U^{\dagger} \partial_{\mu} U, U^{\dagger} \partial_{\nu} U \right] \left[U^{\dagger} \partial^{\mu} U, U^{\dagger} \partial^{\nu} U \right] \right\}$$

hedgehog ansatz

$$U(\mathbf{x}) = \exp\left(i\boldsymbol{\tau} \cdot \hat{\mathbf{x}}F(r)\right) = \cos F(r) + i\boldsymbol{\tau} \cdot \hat{\mathbf{x}}\sin F(r)$$

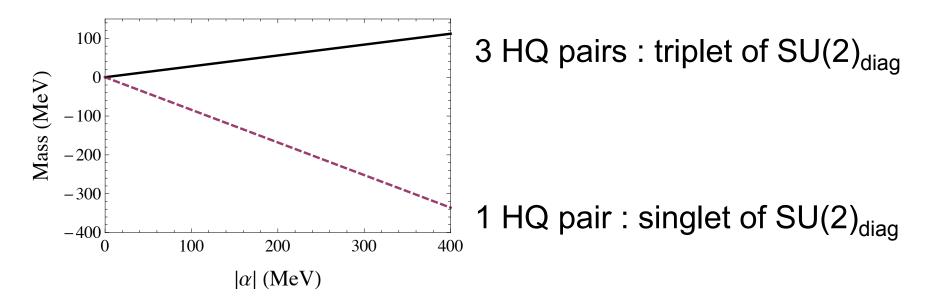
$$\left\langle \alpha_{i}^{a} \right\rangle = \delta_{i}^{a} \frac{1}{3} \int d^{3}x \left[\frac{2\sin F}{r} + F' \cos F + \frac{F' (\sin F)^{2}}{1 + \cos F} \right]$$

Mass spectrum (pattern 1)

• $\langle \alpha_{\perp}^{iA} \rangle$ = $\alpha \delta_{iA}$ motivated by the hedgehog ansatz of skyrmion

$$\det\Delta = (p_0 - g_A \alpha)^6 (p_0 + 3g_A \alpha)^2$$

 $SU(2)_{diag}$ [diagonal subgroup of $SU(2)_{spin} \times SU(2)_{isospin}$] exists.



Mass spectrum (pattern 2)

• $\langle \alpha_{\perp}^{iA} \rangle$ = $\alpha \delta_{i3} \delta_{A3}$ motivated by the chiral density wave

See, e.g.,

A. Heinz, F. Giacosa, and D. H. Rischke, NPA933, 34 (2014);

D. Mller, M. Buballa, and J.Wambach, PLB727, 240 (2013);

T. Kojo, Y. Hidaka, L. McLerran, and R. D. Pisarski, NPA843, 37 (2010);

E. Nakano and T. Tatsumi, Phys. Rev. D 71, 114006 (2005)

Chiral density wave

- Several literatures pointed that the chiral field *U* has the position dependent expectation value in the nuclear medium.
- Here I take the following ansatz as an example:

$$\left\langle U\left(\vec{x}\right)\right\rangle = \cos(2\alpha z) - i\tau^{3}\sin(2\alpha z)$$

$$\Leftrightarrow \left\langle U\left(\vec{x}\right)\right\rangle = 1 \text{ at vacuum}$$

$$\xi_{R} = \xi_{L}^{\dagger} = \cos(\alpha z) + i\tau^{3}\sin(\alpha z)$$

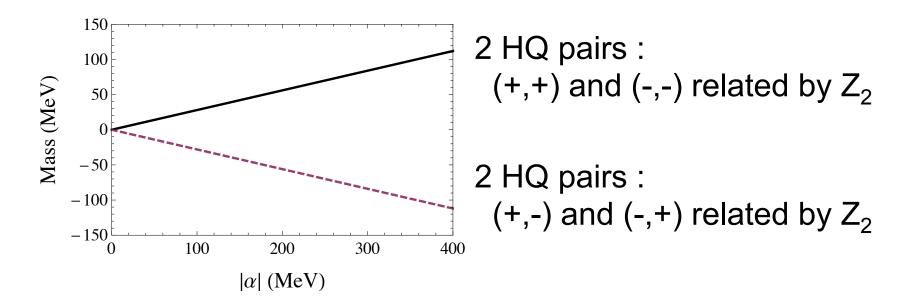
• This leads to $\langle \alpha_i^a \rangle = \alpha \, \delta_{i3} \delta_{a3}$

Mass spectrum (pattern 2)

• $\langle \alpha_{\perp}^{iA} \rangle$ = $\alpha \delta_{i3} \delta_{A3}$ motivated by the chiral density wave

$$\det \Delta = \left(p_0 - g_A \alpha \right)^4 \left(p_0 + g_A \alpha \right)^4$$

 $U(1)_{s} \times U(1)_{I} \times Z_{2}$ [subgroup of SU(2)_{spin} $\times SU(2)_{isospin}$] exists.



Inclusion of mass difference at the vacuum

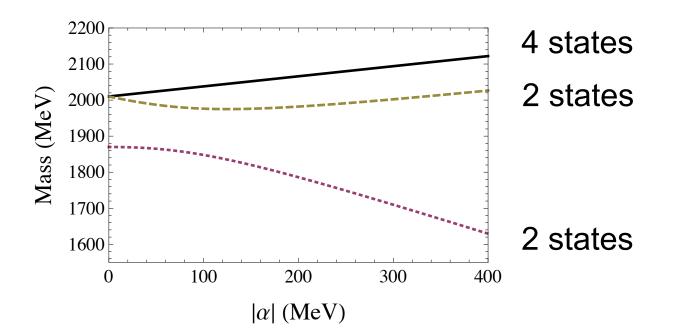
- We include the violation of the heavy quark symmetry by the mass difference between D and D* mesons at vacuum.
- Meson spin denoted by SU(2)_J together with the isospin SU(2)_I is conserved at vacuum.
- In the spin-isospin correlated matter, the SU(2)_J X SU(2)_I is broken to a subgroup depending on the symmetry structure of the correlation.

Inclusion of mass difference at the vacuum (pattern 1)

Pattern 1 : SU(2)_J X SU(2)_I → SU(2)_{diag}
 > 6 D* mesons are split into 2 and 4 of SU(2)diag:

 $3\otimes 2=2\oplus 4$

2 D mesons make 2 of SU(2)_{diag} which mixes with another 2 from D*.



Inclusion of mass difference at the vacuum (pattern 2)

Pattern 2 : $SU(2)_{I} \times SU(2)_{I} \rightarrow U(1)_{I} \times U(1)_{I} \times Z_{2}$ • \geq 8 states are split into 4 pairs of Z₂ symmetry. \geq 2 D mesons provide 1 pair {(0,+), (0,-) } \succ 6 D* provide 3 pairs : \succ {(0,+), (0,-)}, {(+,+), (-,-)}, {(+,-), (-,+)}. \geq 2 of {(0,+), (0,-)} are mixed with each other 2200 1 pair 2100 Mass (MeV) 1 pair 2000 1 pair 1900 1800 1 pair

300

400

1700

0

100

200

 $|\alpha|$ (MeV)

MASS DEGENERACY IN THE HALF-SKYRMION PHASE

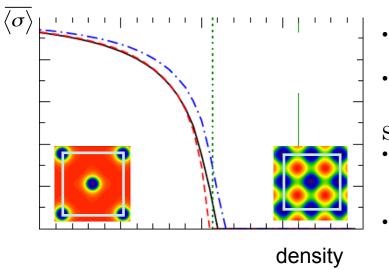
based on D.Suenaga, B.-R.He, Y.L.Ma and M.H., Phys. Rev. D 91, 036001 (2015)

Nuclear matter from Skyrmions on a crystal

We put Skyrmions onto the face-centered cubic (FCC) crystal and regard the Skyrmion matter as nuclear matter.

e.g. M. Kugler and S. Shtrikman, PLB 208, 491 (1988); PRD 40, 3421 (1989). H. -J. Lee, B. -Y. Park, D. -P. Min, M. Rho and V. Vento, NPA 723, 427 (2003).

• There appears the "half-Skyrmion phase", which is characterized by



$$\overline{\sigma} = \int \frac{d^3 x}{V} \langle U \rangle = 0 \Leftrightarrow \int d^3 x \left\langle \overline{q} q \right\rangle = 0$$

- B.-Y. Park, H.-J. Lee, V. Vento, J.-I. Kim, D.-P. Min and M. Rho, NPB. Suppl. 141, 267 (2005).
- H. -J. Lee, B. -Y. Park, D. -P. Min, M. Rho and V. Vento, NPA 723, 427 (2003).

See also

- Y.-L. Ma, M. Harada, H. K. Lee, Y. Oh, B.-Y. Park, M. Rho, PRD88, 014016 (2013); PRD90, 034015 (2014).
- M. Harada and H.K.Lee, Y.-L. Ma and M. Harada, arXiv:1502.02508.

<u>Chiral partner structure of Heavy-</u> light mesons

- To Probe the half-Skyrmion phase, we use the chiral parner structure of heavy-light mesons.
- Ground states · · · $J_1 = 1/2$; $J^P = (0^-, 1^-)$
 - Pseudoscalar meson D ; Vector meson D*
 - > $D = (D^0, D^+, D_s)$ $D^* = (D^{*0}, D^{*+}, D_s^*)$

chiral partner

M.A.Nowak, M.Rho and I.Zahed, PRD**48**, 4370 (1993); W. A. Bardeen and C. T. Hill, PRD**49**, 409 (1994)

- First excited states $\cdots J_1 = 1/2$; $J^P = (0^+, 1^+)$
 - > Scalar meson D_0^* ; Axial-vector meson D_1
 - > $D_0^* = (D_0^{*0}, D_0^{*+}, D_{s0}^*)$ $D_1 = (D_1^0, D_1^+, D_{s1})$

Heavy meson effective fields

☆ Ground states · · · $J_1 = 1/2$; $J^P = (0^-, 1^-)$

Pseudoscalar meson **D** ; Vector meson **D***

$$D = (D^0, D^+)$$
 $D^* = (D^{*0}, D^{*+})$

- Bi-spinor field $H \sim Q \overline{\Psi}$; $\Psi \cdots$ light constiuent quark field

$$H = \frac{1+\not p}{2} \left[D^{*\mu} \gamma_{\mu} + i D \gamma_5 \right]$$

annihilates heavy mesons (not generate)

☆ First excited states · · · $J_1 = 1/2$; $J^P = (0^+, 1^+)$

Scalar meson D_0^* ; Axial-vector meson D_1

$$D_0^* = (D_0^{*0}, D_0^{*+}) \quad D_1 = (D_1^0, D_1^+)$$

$$G = \frac{1+\not p}{2} \left[-iD_1^{\mu} \gamma_{\mu} \gamma_5 + D_0^* \right]$$

Heavy meson Lagrangian

© chiral doubler fields for heavy mesons

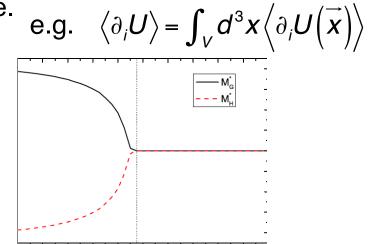
$$\begin{aligned} \mathcal{H}_{L} &= \frac{1}{\sqrt{2}}[G + iH\gamma_{5}], \quad \mathcal{H}_{R} = \frac{1}{\sqrt{2}}[G - iH\gamma_{5}] \\ \mathcal{H}_{L} &\to \mathcal{H}_{L}g_{L}^{\dagger}, \quad \mathcal{H}_{R} \to \mathcal{H}_{R}g_{R}^{\dagger} \qquad \boldsymbol{g}_{_{L,R}} \in SU(2)_{_{L,R}} \\ & \textcircled{O} \text{ chiral field for pion } \quad \boldsymbol{U} = \boldsymbol{e}^{2i\pi/f_{\pi}} = \boldsymbol{\xi}^{2} \to \boldsymbol{g}_{L} \ \boldsymbol{U}g_{R}^{\dagger} \\ & \bigstar \text{ model Lagrangian} \\ \mathcal{L} = \operatorname{tr}[\mathcal{H}_{L}(iv \cdot \partial)\bar{\mathcal{H}}_{L}] + \operatorname{tr}[\mathcal{H}_{R}(iv \cdot \partial)\bar{\mathcal{H}}_{R}] \\ &\quad + \frac{1}{2}\Delta_{M}\operatorname{tr}[\mathcal{H}_{L}U\bar{\mathcal{H}}_{R} + \mathcal{H}_{R}U^{\dagger}\bar{\mathcal{H}}_{L}] \\ &\quad - i\frac{g_{A_{1}}}{2}\operatorname{tr}[\mathcal{H}_{R}\gamma_{5}\gamma^{\mu}\partial_{\mu}U^{\dagger}\bar{\mathcal{H}}_{L} - \mathcal{H}_{L}\gamma_{5}\gamma^{\mu}\partial_{\mu}U\bar{\mathcal{H}}_{R}] \\ &\quad - i\frac{g_{A_{2}}}{2}\operatorname{tr}[\mathcal{H}_{L}\gamma_{5}\partial UU^{\dagger}\bar{\mathcal{H}}_{L} - \mathcal{H}_{R}\gamma_{5}\partial U^{\dagger}U\bar{\mathcal{H}}_{R}], \end{aligned}$$

Mideum effects to the heavy meson chiral doublet fields

 \bigstar Lagrangian written in the parity eigenstates

$$\begin{split} \mathcal{L}_{\text{eff}} &= \text{tr}[G(iv \cdot \partial)\bar{G} - H(iv \cdot \partial)\bar{H}] \\ &+ \frac{1}{4}\Delta_M \text{tr}[G\langle U + U^{\dagger}\rangle\bar{G} + H\langle U + U^{\dagger}\rangle\bar{H}] \\ &- \frac{ig_{A_1}}{4}\text{tr}[G\gamma_5\langle \partial U^{\dagger} - \partial U\rangle\bar{G} - H\gamma_5\langle \partial U^{\dagger} - \partial U\rangle\bar{H}] \\ &- \frac{ig_{A_2}}{4}\text{tr}[G\gamma_5\langle \partial UU^{\dagger} - \partial U^{\dagger}U\rangle\bar{G} + H\gamma_5\langle \partial UU^{\dagger} - \partial U^{\dagger}U\rangle\bar{H}] \end{split}$$

- We are interested in the mass spectrum of the heavy-light mesons at the rest frame, so that we take the space average.
- We can show $\langle \partial_i U \partial_i U^{\dagger} \rangle = \langle \partial_i U U^{\dagger} \partial_i U^{\dagger} U \rangle = 0$ so that only Δ_M term contributes.
- $\langle U + U^{\dagger} \rangle \rightarrow 0$ in the half-Skyrmion phase, so that *G* doublet and *H* doublet are degenerate with each other.



<u>Summary</u>

- We proposed to study the mass spectrum of heavy-light mesons to probe the structure of the spin-isospin correlation in the nuclear medium, which is expected to occur in the skyrmion crystal or the chiral density wave phase.
- Our result shows that the structure of the mixing among D and D* mesons reflects the pattern of the correlation, i.e. the remaining symmetry.
- Furthermore, the magnitude of the mass modification provides information of the strength of the correlation.

Summary 2

- We explore the mass splitting of heavy-light mesons with chiral partner structure in nuclear matter.
- We regard [D(JP=0-),D*(1-)] and [D*0(0+),D1(1+)] as chiral partners.
- The nuclear matter is constructed by putting Skyrmions onto a crystal.
- The masses of chiral partners are degenerated in the half-Skyrmion phase in which the chiral symmetry is restored globally.
- Our results show that the medium modified mass splitting of *H* and *G* doublets can be used as a probe of the existence of the half-Skyrmion phase.

<u>Remarks</u>

• [B. Friman, J. Phys. G 30, S895 (2004)] and [C. Sasaki, Phys. Rev. D 90, 114007 (2014)] pointed that the degeneracy occurs when the chiral symmetry is restored. In the half-Skyrmion phase, although the chiral symmetry is not restored, the masses are degenerated since the space average of the quark condensate vanishes.

$$\overline{\langle \sigma \rangle} = \int \frac{d^3 x}{V} \langle U \rangle = 0 \Leftrightarrow \int d^3 x \left\langle \overline{q} q \right\rangle = 0$$

- In the present analysis, we included only the pion effect. The effects of the heavier resonances, such as ρ , ω , σ and so on which are essential for nuclear matter properties, on the heavy-light meson spectrum will be reported elsewhere.
- In the present analysis, we studied heavy meson mass defined at the rest frame to probe the global structure of the nuclear matter. It would be interesting to see the dispersion relation of the heavy meson to probe the local structure of the matter, i.e., the potential provided by the nuclear matter. (D. Suenaga, Y.-L. Ma and M. Harada, work in progress.)

