SU(3) Thermodynamics from Yang-Mills Gradient Flow

FlowQCD Collaboration

M. Asakawa (Osaka), T. Hatsuda (RIKEN), T. Iritani (YITP),
E. Itou (KEK), M. Kitazawa (Osaka), H. Suzuki (Kyushu)

Phys. Rev. D90 (2014) 011501 & arXiv:1412.4508[hep-lat]

1. Energy-momentum tensor : why should we care ?
2. Yang-Mills gradient flow : From 4D boundary to(4+1)D bulk
3. Yang-Mills thermodynamics : From (4+1)D bulk to 4D boundary
4. Summary and Future

Related talks: March 16 (Mon)
L. Del Debbio, H.Suzuki

Energy-Momentum Tensor : why should we care?

$T_{\mu\nu}$ Generator of Poincare group (Translation + Lorentz)

Conservation law

$$\partial^\mu T_{\mu\nu} = 0$$

YM Trace anomaly

$$T_\mu{}^\mu = \frac{\beta}{2g} G_{\mu\nu} G^{\mu\nu}$$

Yang-Mills theory at finite T

$$\langle T_{\mu\nu} \rangle \Longleftrightarrow \varepsilon(T), P(T), s(T)$$

Bulk thermodynamics

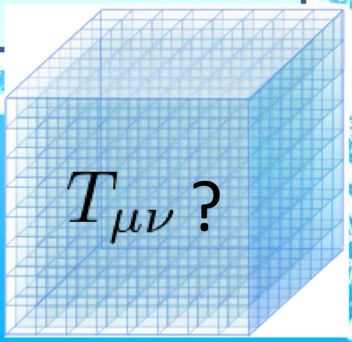
$$\langle (T_{\mu\nu})^n \rangle \Longleftrightarrow C_V(T) \text{ etc}$$

Fluctuations

$$\langle T_{\mu\nu}(x) T_{\lambda\rho}(0) \rangle \Longleftrightarrow \eta(T), \zeta(T)$$

Transports

However,
apparent conflict with Lattice discretization

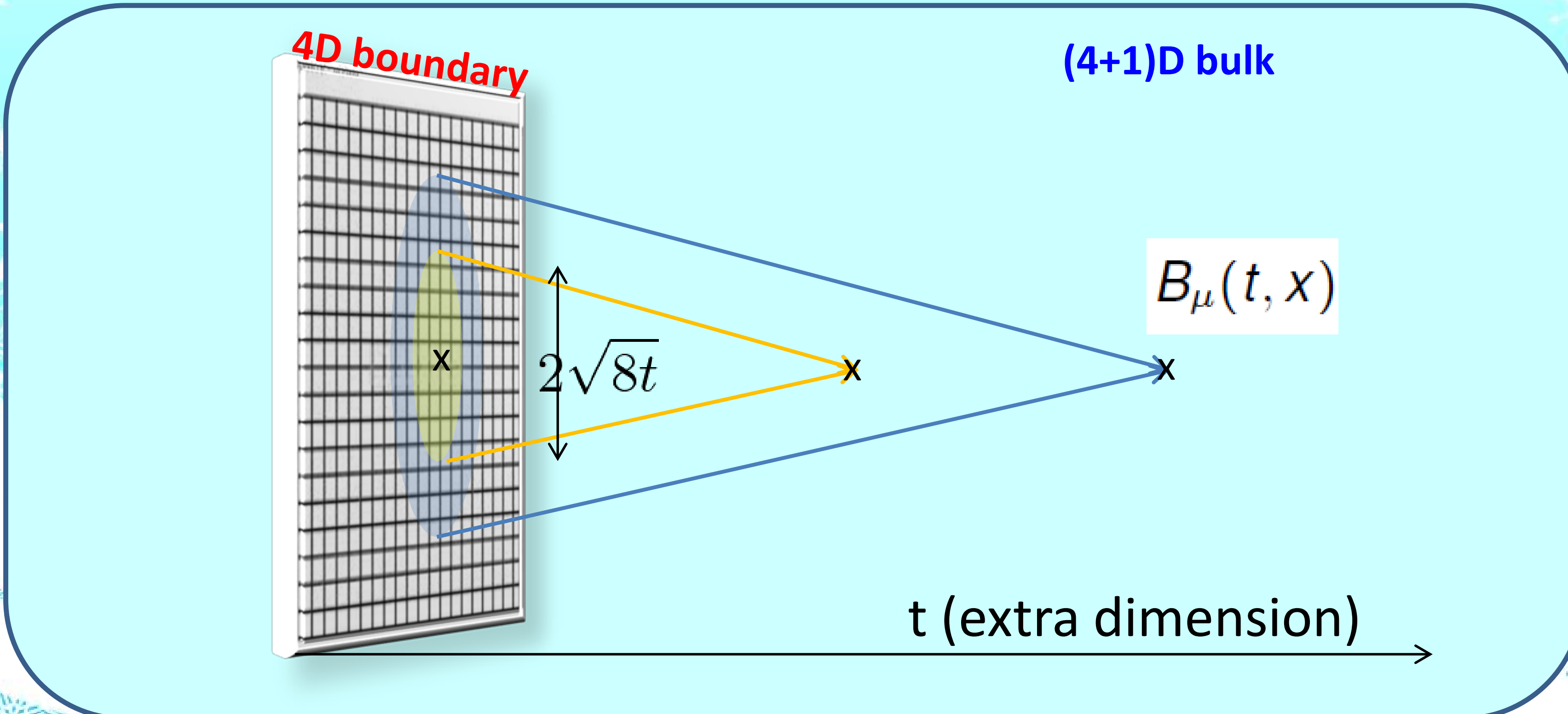


Yang-Mills Gradient Flow (1): 4D world from (4+1)D bulk

a=0

Diffusion equation towards extra dimension

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x), \quad B_\mu(0,x) = A_\mu^{\rm bare}(x)$$



$$\left\langle \tilde{B}^a_\mu(t,p)\tilde{B}^b_\nu(s,q) \right\rangle \sim \frac{1}{(p^2)^2}\left[(\delta_{\mu\nu}p^2 - p_\mu p_\nu) e^{-(t+s)p^2} \right]$$

Luscher & Weisz, JHEP 1102 (2011) 051

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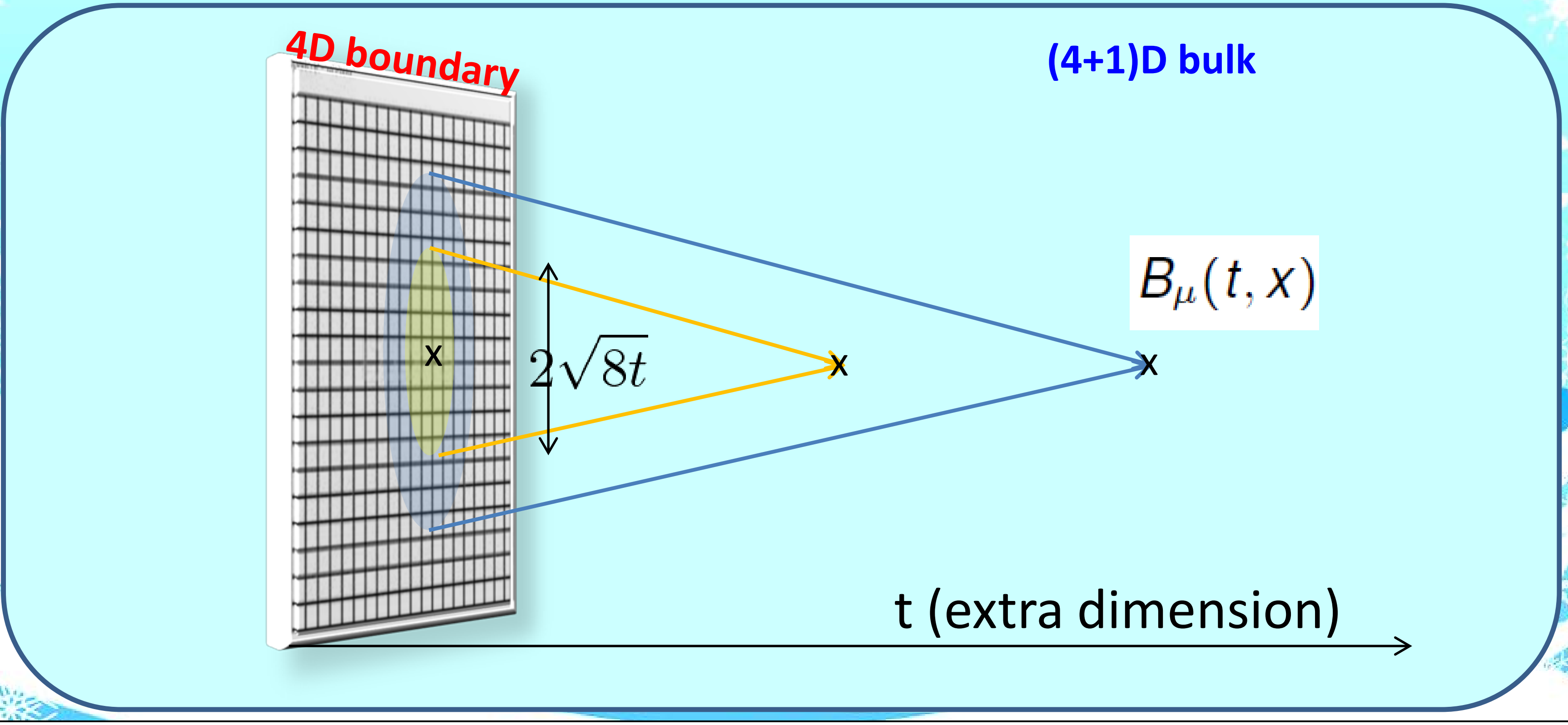

$$\langle B_{\mu_1}(t_1,x_1) B_{\mu_2}(t_2,x_2) \cdots B_{\mu_n}(t_n,x_n) \rangle\,, \qquad t_1 > 0, \ldots, t_n > 0,$$

remains finite even for the equal-point product

Luscher & Weisz, JHEP 1102 (2011) 051

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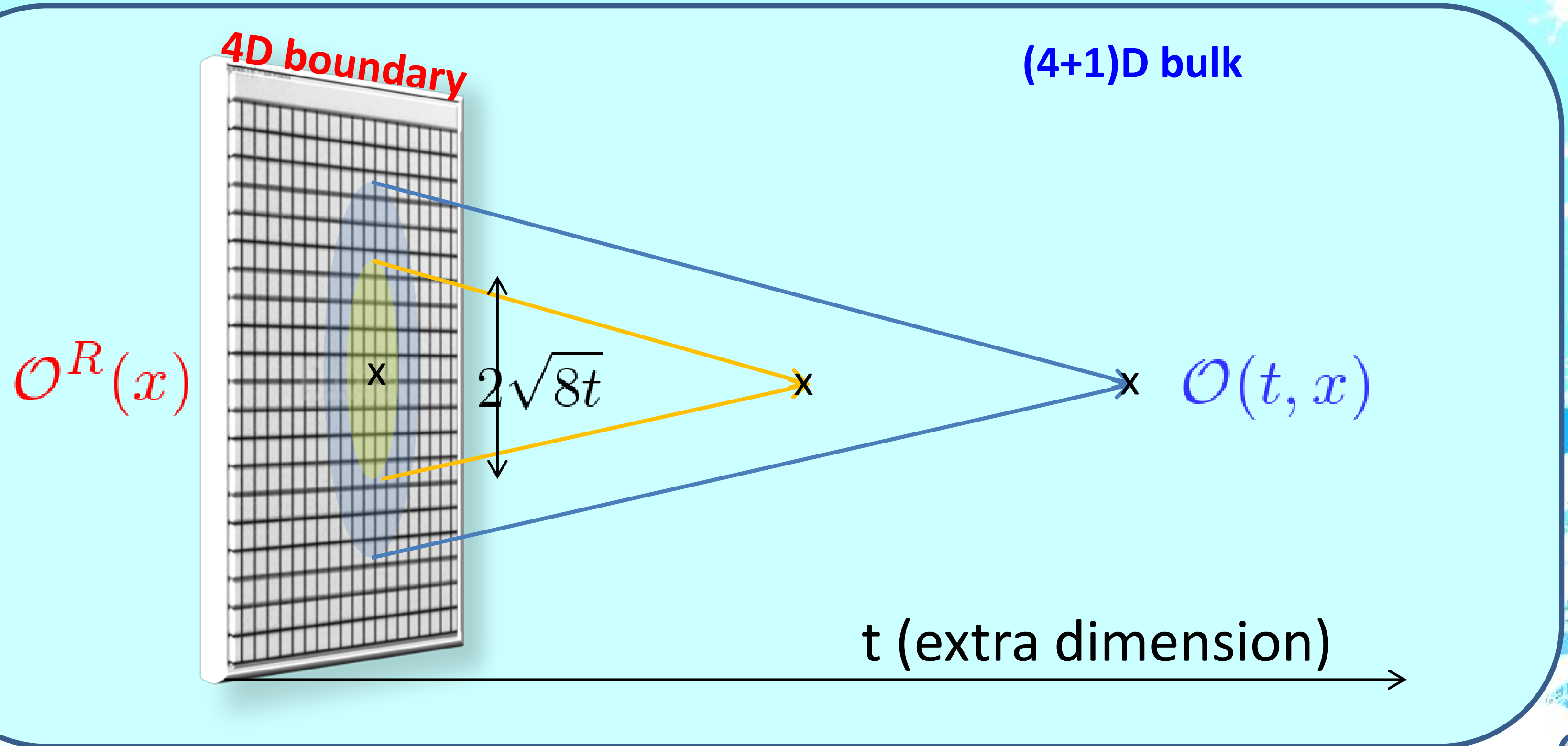


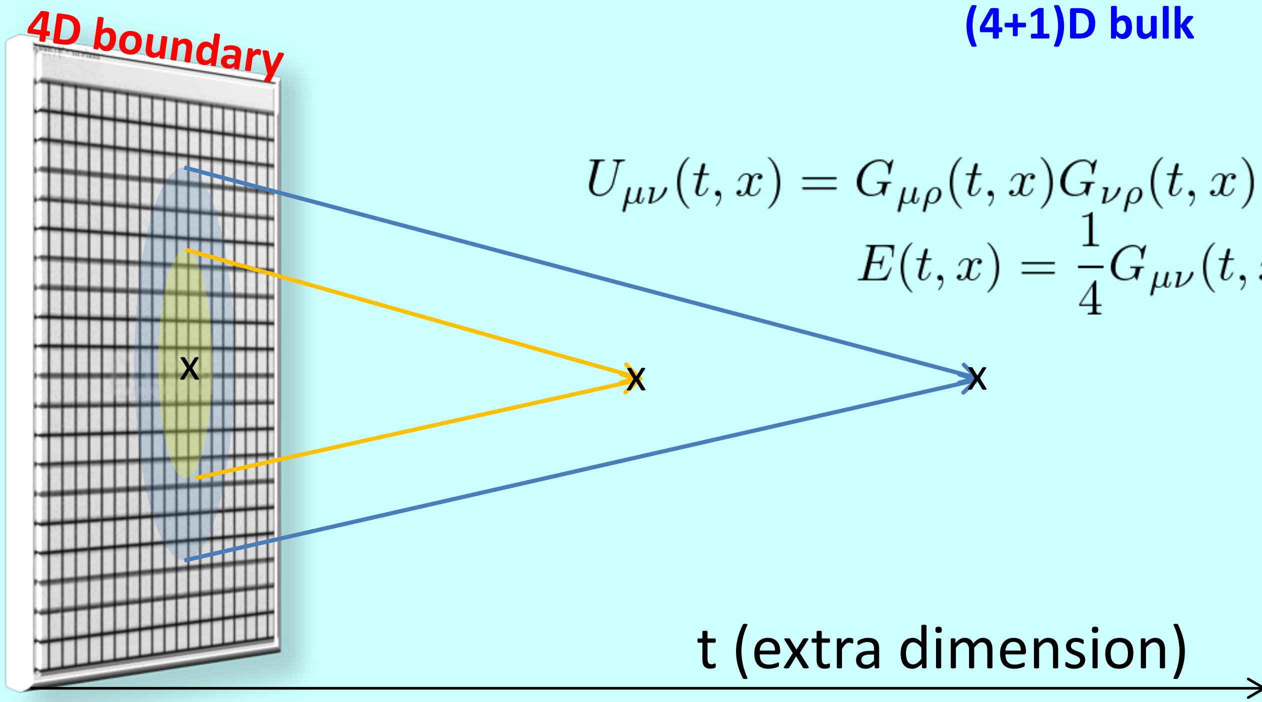
$$\mathcal{O}(t,x) \xrightarrow[t\to 0]{} \sum_k c_k(t)\mathcal{O}_k^R(x) + (\text{powers in } t)$$

Luscher & Weisz, JHEP 1102 (2011) 051

Yang-Mills Gradient Flow (2): 4D world from (4+1)D bulk

a=0

$T^R_{\mu\nu}(x)$



$$U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - E(t,x)\delta_{\mu\nu}$$

$$E(t,x) = \frac{1}{4}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x)$$

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[\{T_{\mu\nu}\}_R(x) - \frac{1}{4}\delta_{\mu\nu}\{T_{\rho\rho}\}_R(x)\right] + O(t),$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)\{T_{\rho\rho}\}_R(x) + O(t),$$

$$\alpha_U(t) = \bar{g}(1/\sqrt{8t})^2\left[1 + 2b_0\bar{s}_1\bar{g}(1/\sqrt{8t})^2 + O(\bar{g}^4)\right],$$

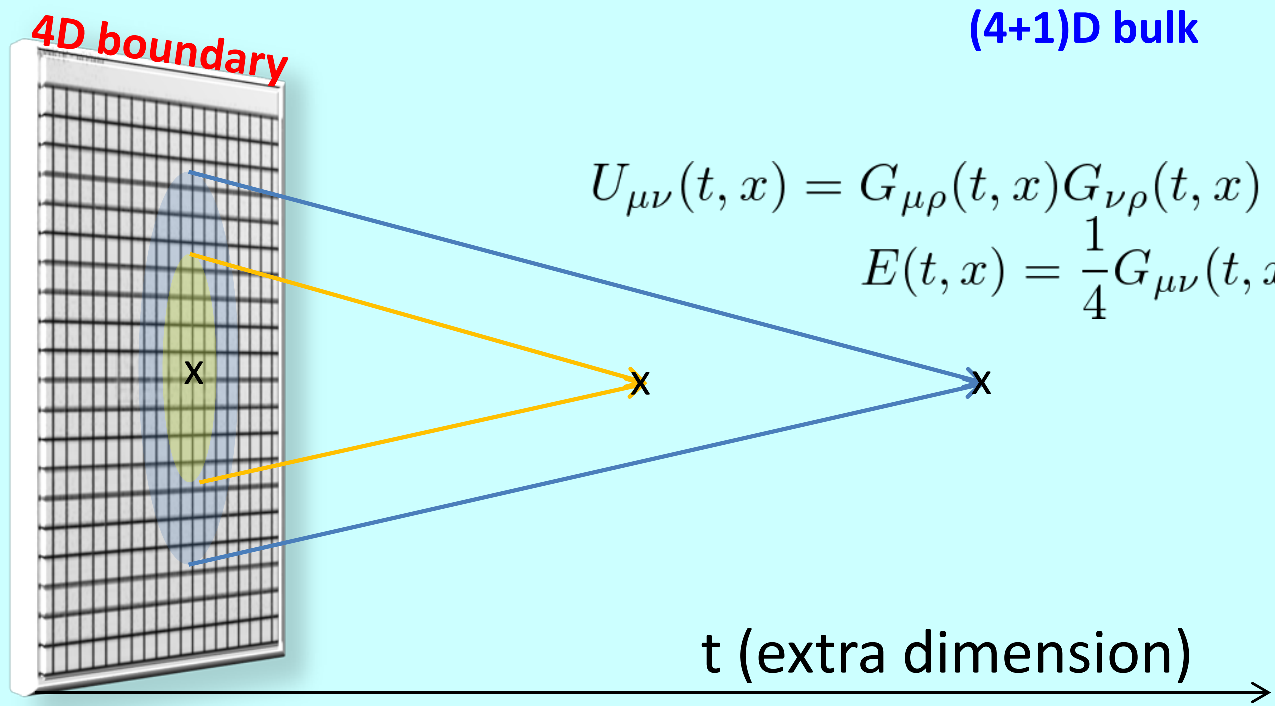
$$\alpha_E(t) = \frac{1}{2b_0}\left[1 + 2b_0\bar{s}_2\bar{g}(1/\sqrt{8t})^2 + O(\bar{g}^4)\right]$$

H. Suzuki, PTEP 2013 (2013) 083B03

Yang-Mills Gradient Flow (2): 4D world from (4+1)D bulk

a=0

$T^R_{\mu\nu}(x)$



$$U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - E(t,x)\delta_{\mu\nu}$$

$$E(t,x) = \frac{1}{4}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x)$$

$$\{T_{\mu\nu}\}_R(x) = \frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{1}{4\alpha_E(t)}\delta_{\mu\nu}\left[E(t,x) - \langle E(t,x)\rangle\right] + O(t)$$

Do NOT confuse this with the standard smearing

H. Suzuki, PTEP 2013 (2013) 083B03

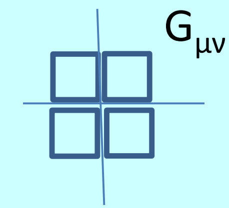
Yang-Mills thermodynamics (1): Four steps to go

a≠0

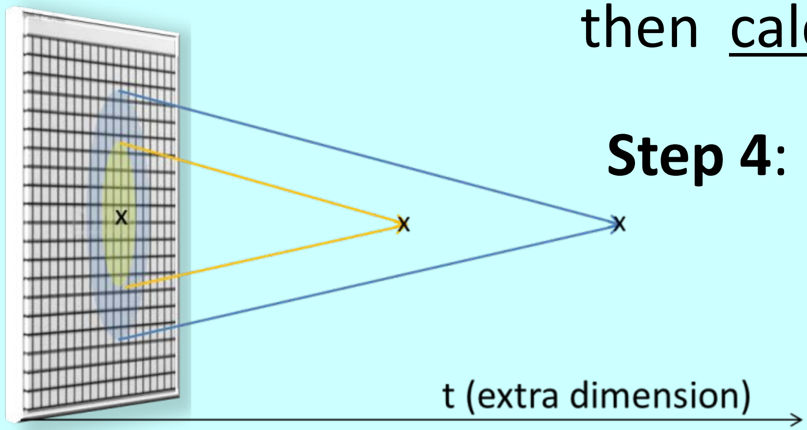
Step 1: Generate <u>gauge configuration</u> at t=0 as usual

Step 2: Solve the <u>gradient flow</u> for each configuration

Step 3: Construct $U_{\mu\nu}(t,x)$ & $E(t,x)$, then <u>calculate</u> $\langle U_{\mu\nu}(t,x)\rangle$ & $\langle E(t,x)\rangle$



Step 4: Take <u>the limit</u> (t→0 after a→0) to obtain $\langle T_{\mu\nu}(x)\rangle$

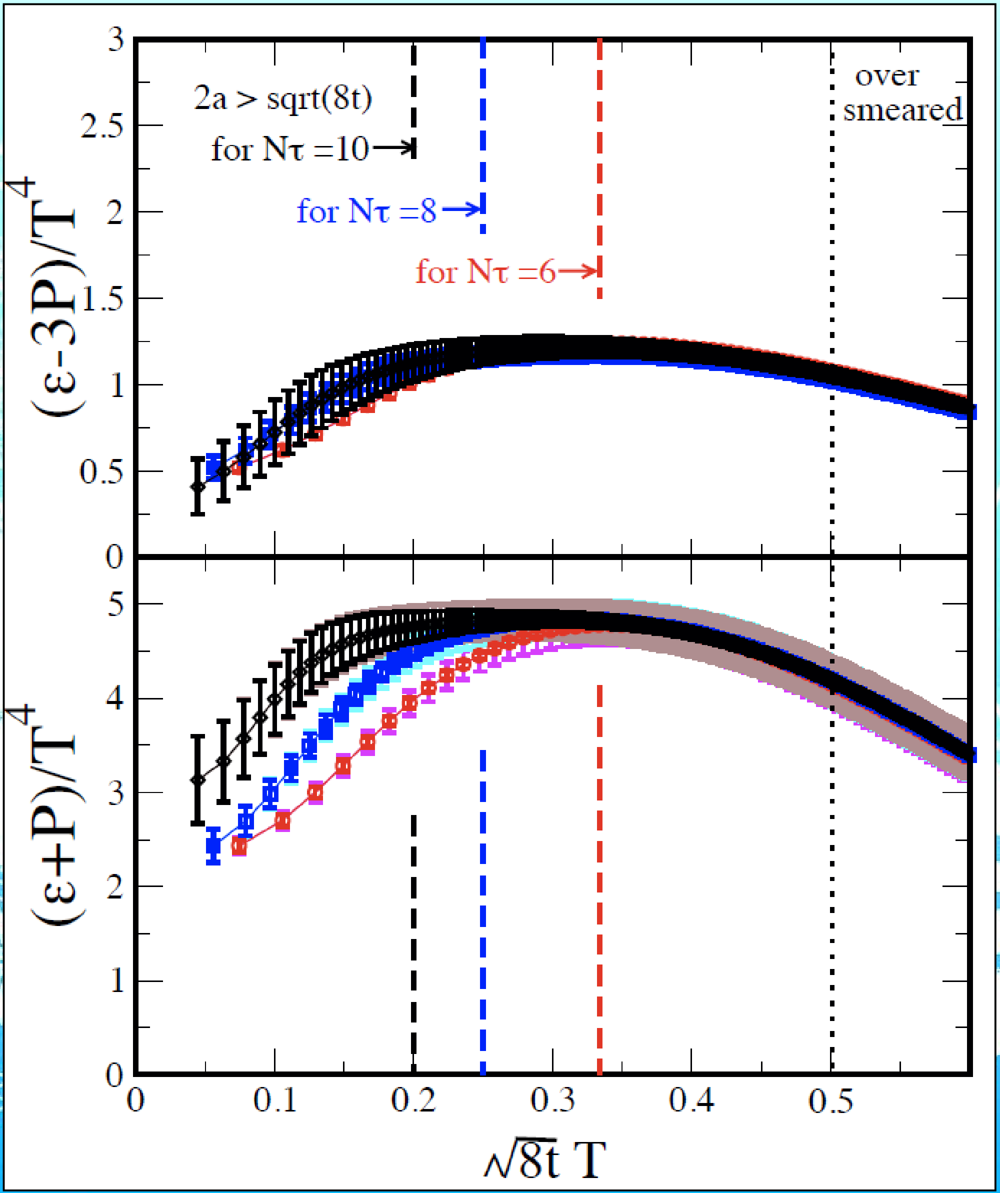


$$2a \ll \sqrt{8t} \ll R$$

$$\Delta = \varepsilon - 3P = -\langle T^R_{\mu\mu}(x)\rangle$$

$$sT = \varepsilon + P = -\langle T^R_{00}(x)\rangle + \frac{1}{3}\sum_{i=1,2,3}\langle T^R_{ii}(x)\rangle$$

Yang-Mills thermodynamics (2) : extrapolation back to t=0



Flow QCD Coll., PRD90 (2014) 011501

- SU(3) YM theory
- Wilson action
- $32^3 \times (6,8,10)$
- β=5.89-6.56
- 100-300 config.

$$2a \ll \sqrt{8t} \ll R$$

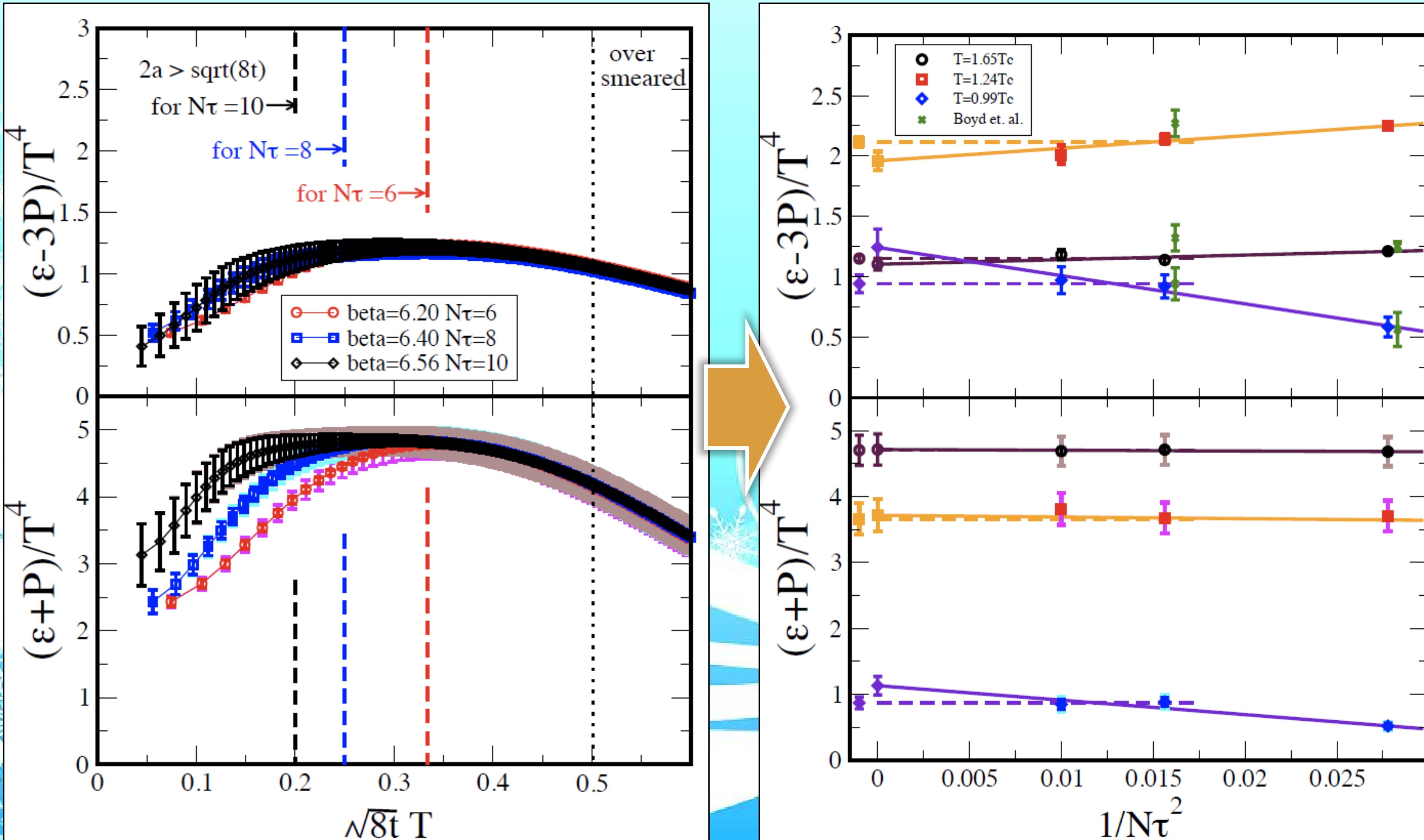
$T/T_c=1.65$

a = 0.074 fm

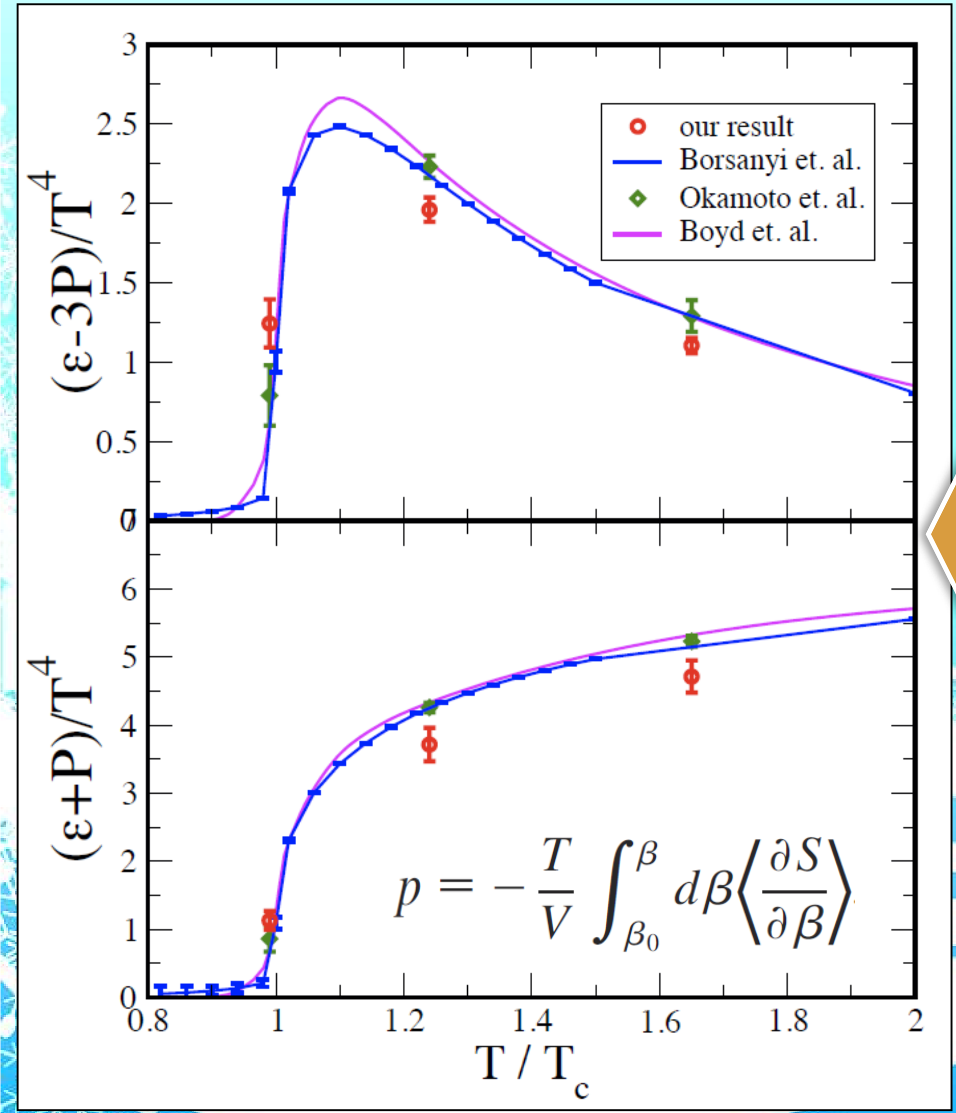
0.055 fm

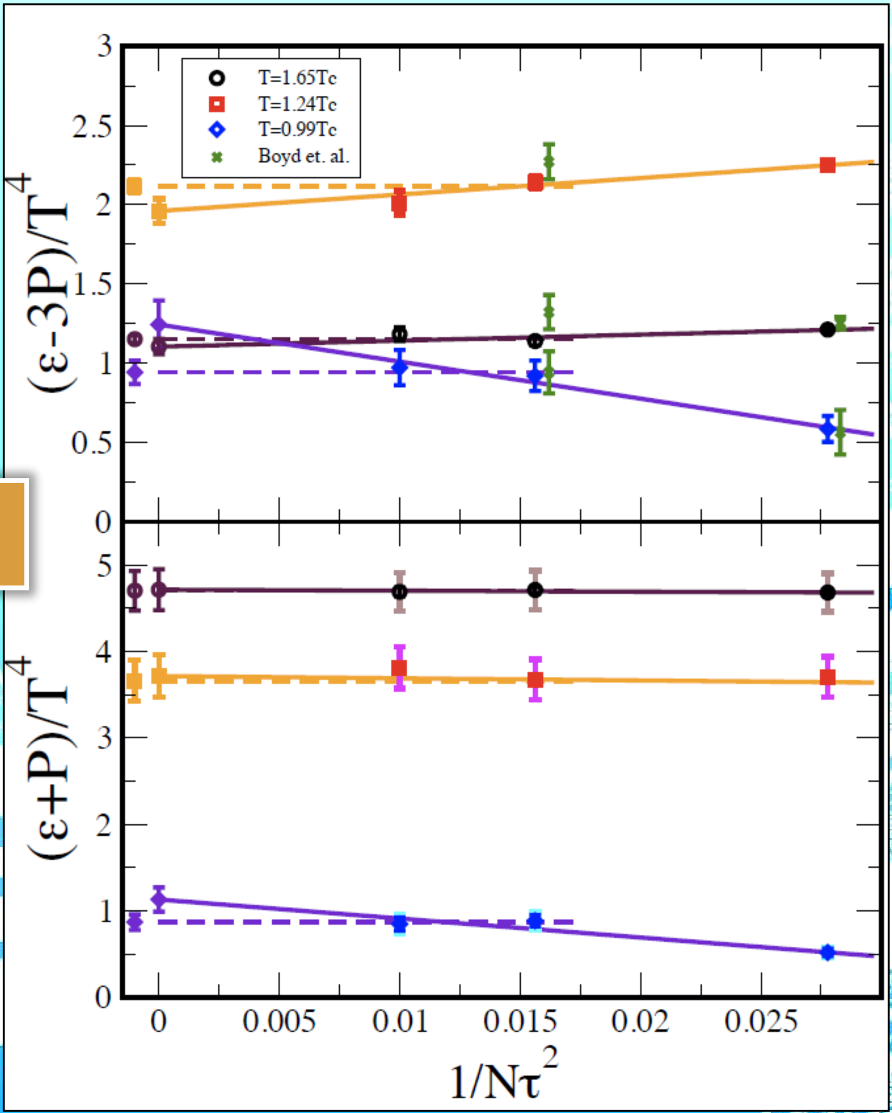
0.043 fm

Yang-Mills thermodynamics (3) : continuum extrapolation



Yang-Mills thermodynamics (3) : continuum extrapolation





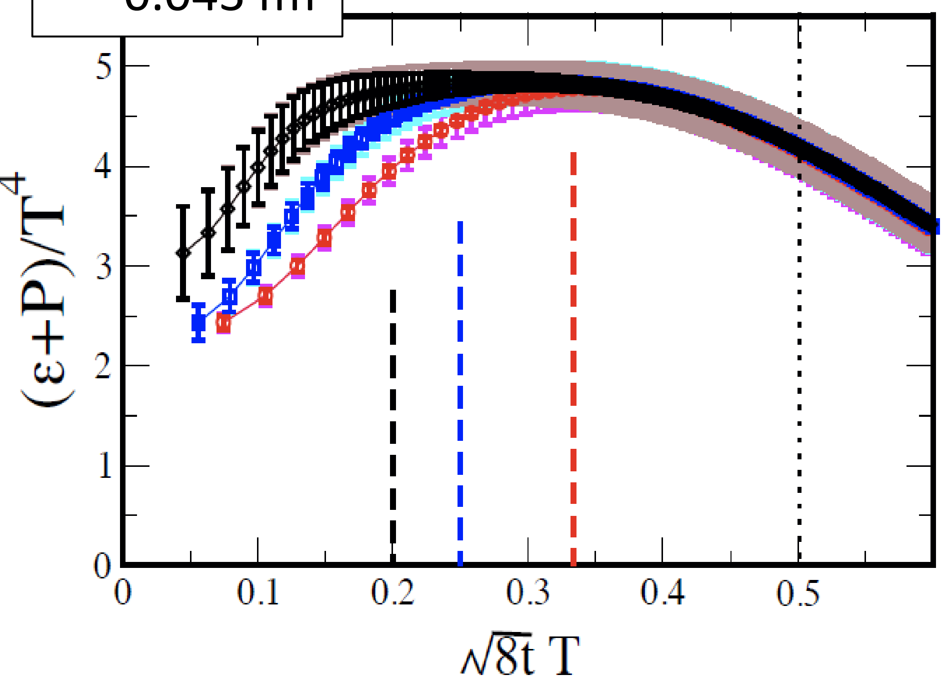
Yang-Mills thermodynamics (4) : larger & finer lattices

- SU(3) YM theory
- Wilson action
- 32^3 x (6,8,10)
- β=5.89-6.56
- 100-300 config.

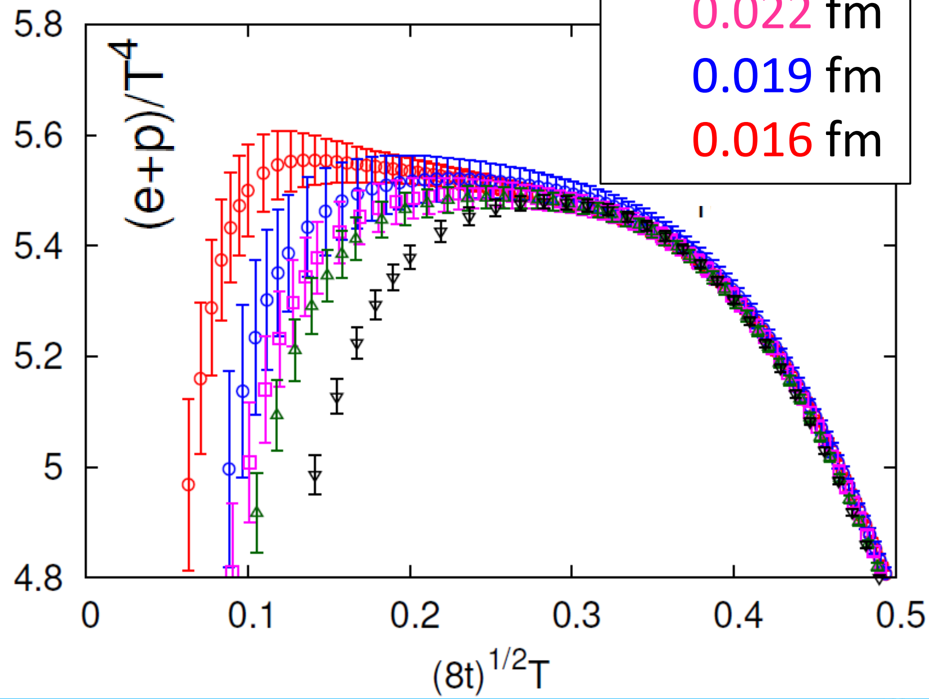
→

- SU(3) YM theory
- Wilson action
- 64^3 x (10,12,14,16), 128^3x20
- β=6.83-7.38
- ～2000 config.

T/T_c=1.65
a = 0.074 fm
0.055 fm
0.043 fm



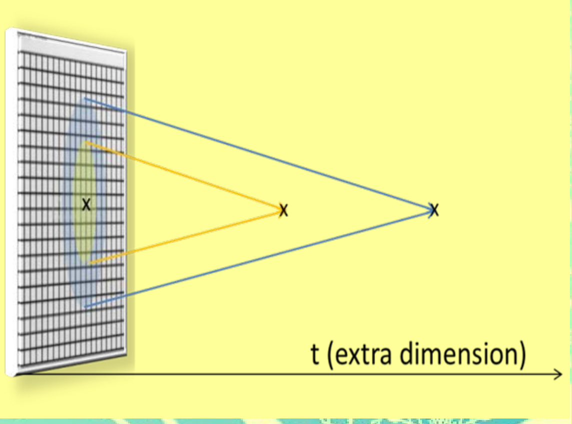
T/T_c=2.31
a = 0.031 fm
0.026 fm
0.022 fm
0.019 fm
0.016 fm



Summary

<u>Energy-momentum tensor from Yang-Mills gradient flow</u>
provides a new way to study the hot plasma

1. UV free definition
2. small statistical error
3. nice t-window as a→0
4. no need of "integration"



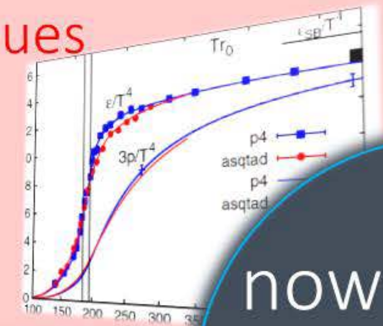
- YM thermodynamics in high precision
  V=64^3, 96^3, 128^3, T/T_c= 0.9, 1.1, 1.4, 1.7, 2.1, 2.3, 2.7
  [FlowQCD Collaboration]
- (2+1)-flavor QCD Thermodynamics
  [FlowQCD + WHOT QCD Collaborations]

Future

Thermodynamics

direct measurement of expectation values

$\langle T_{00} \rangle, \langle T_{ii} \rangle$



Fluctuations and Correlations

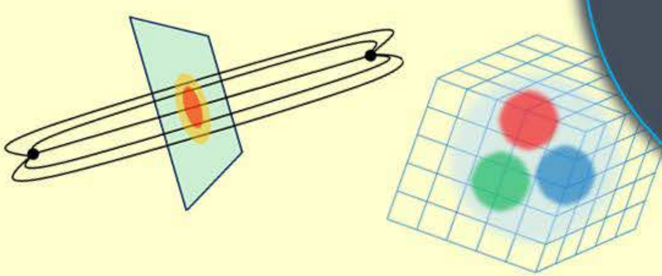
viscosity, specific heat, ...

$c_V \sim \langle \delta T_{00}^2 \rangle$

$\eta = \langle T_{12} ; T_{12} \rangle$

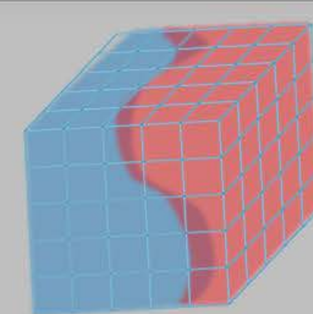
now we have

$T_{\mu\nu}$



- confinement string
- EM distribution in hadrons

Hadron Structure



- vacuum configuration
- mixed state on 1st transition

Vacuum Structure