Charmed Tetraquarks from Lattice QCD

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HAL QCD (Hadrons to Atomic nuclei from Lattice QCD)

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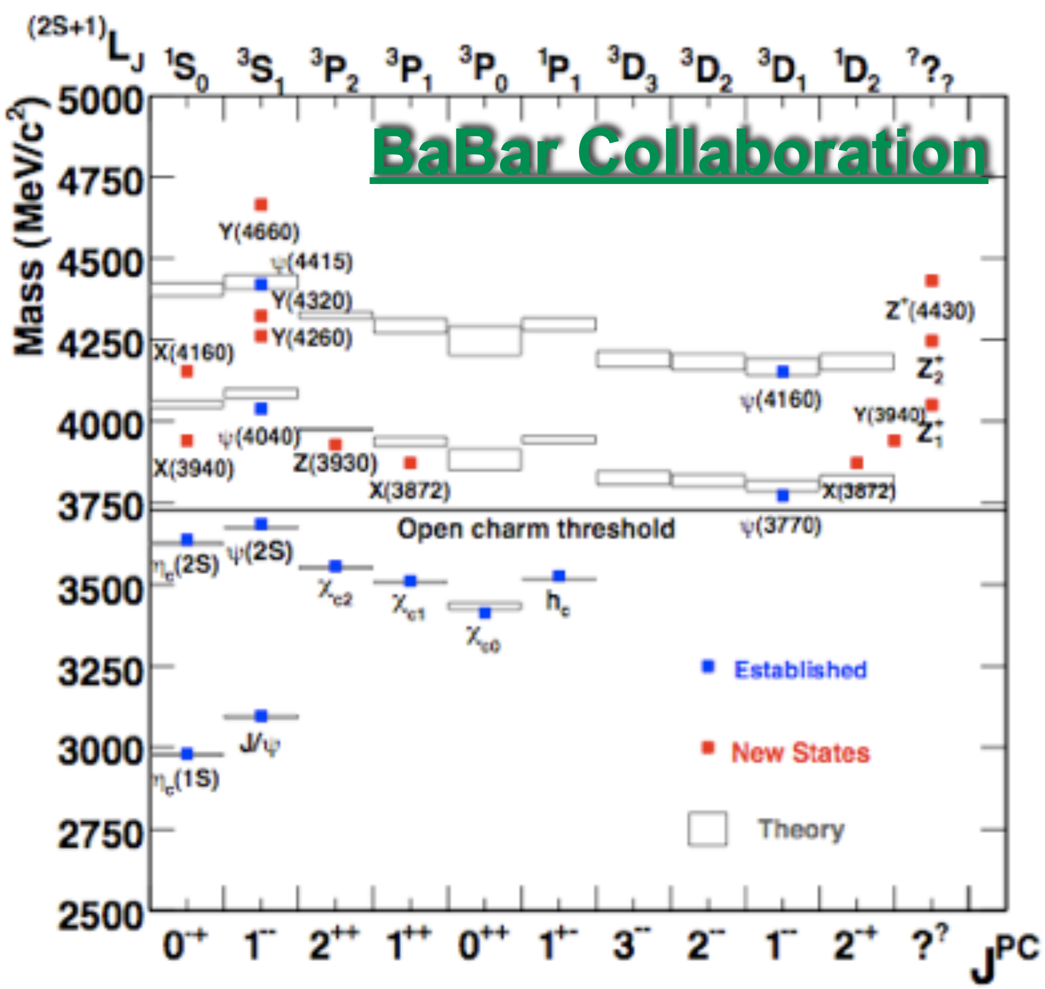
Takumi Doi, Tetsuo Hatsuda, Yoichi Ikeda, Vojtech Krejcirik (RIKEN)
Takashi Inoue (Nihon Univ.)
Noriyoshi Ishii, Keiko Murano (RCNP, Osaka Univ.)
Hidekatsu Nemura, Kenji Sasaki,
Masanori Yamada, Takaya Miyamoto (Univ. Tsukuba)
Faisal Etminan (Univ. Birjand)

Long-term workshop on "Hadrons and Hadron Interactions in QCD 2015 (HHIQCD2015)"
@YITP, 3 Mar. 2015.

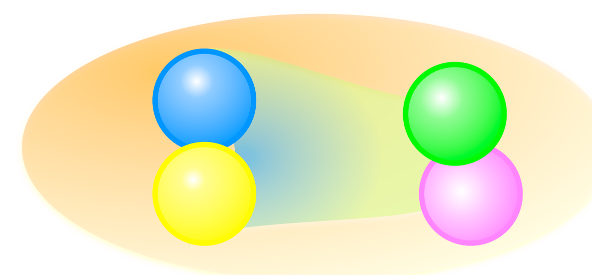
Spectrum of charmonium(-like) system



- Quark potential models well describe mass spectra below open charm threshold

Godfrey, Isgur, PRD 32 (1985).
Barnes, Godfrey, Swanson, PRD 72 (2005).

- "**NEW**" charmonium-like (X, Y, Z) states:
  ➡ not within quark model spectrum
  ➡ candidates of exotic hadrons

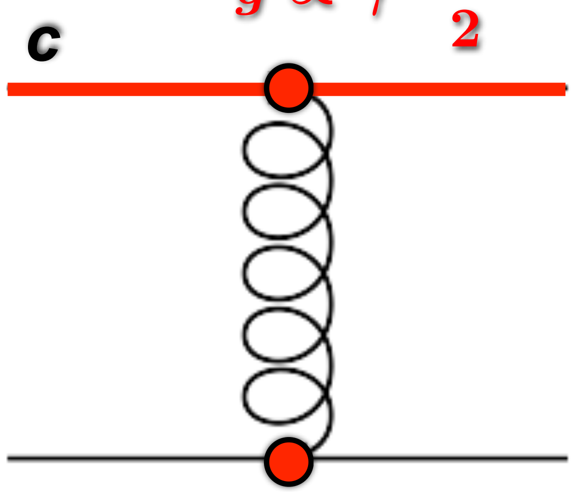


- "**Other**" exotic candidates (expected from quark models):
  ➡ doubly charmed tetra-quark, but experimentally not observed so far

Our target: tetra-quark channels

- "**Tetraquark**" Tcc ($ccu^{bar}d^{bar}$) is manifest 4-quark channel
- "**Charged**" charmonium-like states ($cc^{bar} + \pi^{+/-}$) require at least 4 quarks

Tcc bound state



$g \propto \gamma^\mu \frac{\lambda^a}{2}$

$\sim \bar{c}\gamma^\mu \frac{\lambda^a}{2} c A^a$ ➡ color magnetic + color electric forces

magnetic interactions << electric interactions

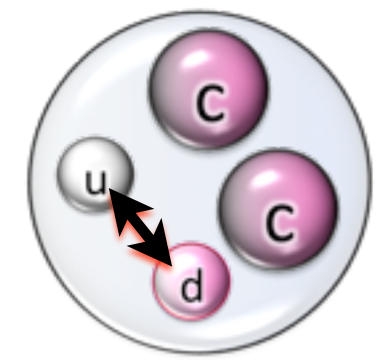
➡ magnetic gluon coupling is suppressed by $O(1/m_c)$

✓ Heavy quark spin symmetry

- Color magnetic interaction is enhanced in light-quark sector

$$V_{ij}^{\mathrm{CMI}} \propto -\frac{(\vec{\lambda}(i)\cdot\vec{\lambda}(j))(\vec{\sigma}(i)\cdot\vec{\sigma}(j))}{M_i M_j}$$

H. J. Lipkin, PLB172, 242 (1986).



- Color-spin matrix elements : $\langle v_{ij}\rangle = -\langle(\vec{\lambda}(i)\cdot\vec{\lambda}(j))(\vec{\sigma}(i)\cdot\vec{\sigma}(j))\rangle$

$\langle v_{ij}\rangle$	C=1	C=8	C=3	C=6^{bar}
S=0	-16	2	-8	4
S=1	16/3	-2/3	8/3	-4/3

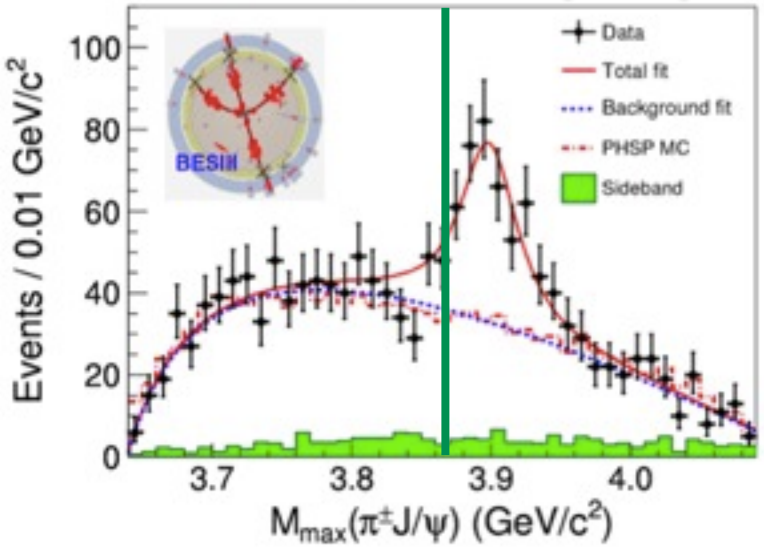
- C=3, S=0 (I=0) : -8
- C=6^{bar}, S=1 (I=0) : -4/3
- C=3, S=1 (I=1) : 8/3
- C=6^{bar}, S=0 (I=1) : 4

attractive ↕ repulsive

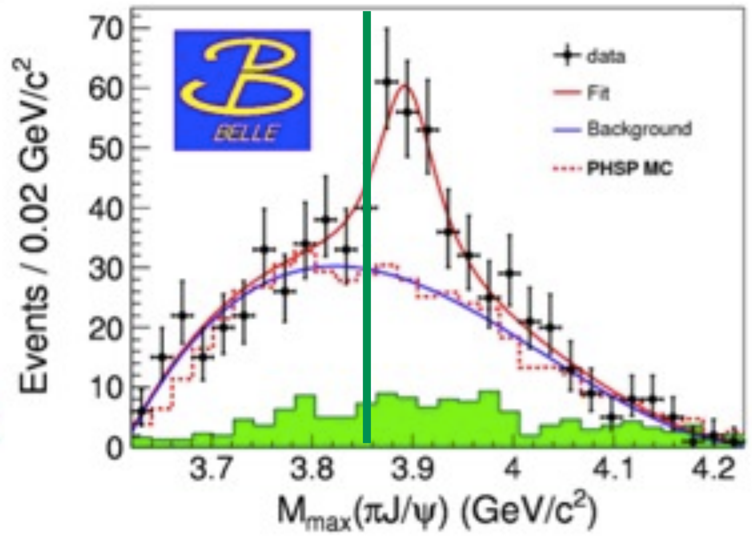
➡ I=0 [ud]-diquark correlation (good diquark) --> Tcc bound state?

Charmonium-like Zc(3900)

BESIII Coll., PRL110 (2013).



Belle Coll., PRL110 (2013).



$D^{bar}D^*$ = 3872 **Zc(3900)**

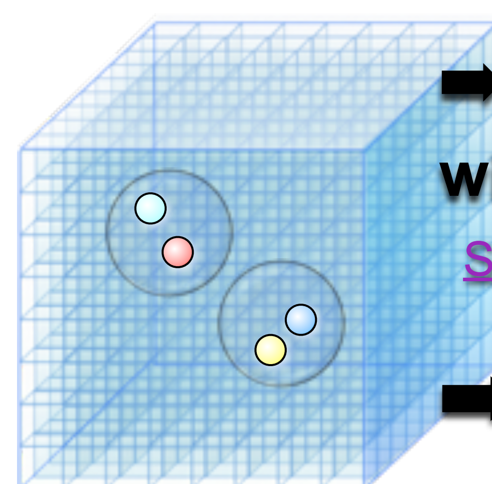
Δ = 640

$\pi J/\Psi$ = 3232

- **observed in $\pi^{\pm}J/\Psi$ invariant mass (confirmed by CLOE-c)** (Xiao et al., PLB727 (2013).)
- **$J^P = 1^+$ seems most probable** (BESIII Coll., PRL112 (2014).)

★ Structure of Zc(3900)

- **Tetra-quark?** Maiani et al. (2013).
- **$D^{bar}D^*$ molecule?** Nieves et al. (2011) + many others
- **cc^{bar} + meson cloud?** Voloshin (2008).
- **pole + meson cloud?** Wang et al. (2013).
- **cusp?** Chen et al. (2013), Swanson (2014).

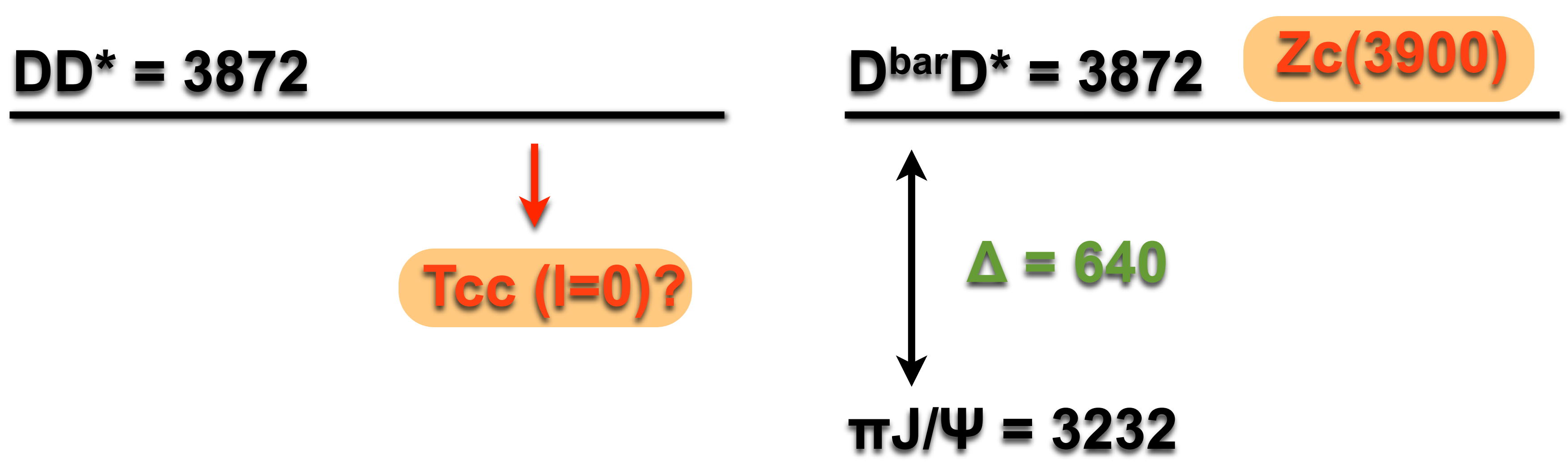


➡ **energy spectrum consistent with scattering**
S. Prelovsek et al., PRD91, 014504 (2015).

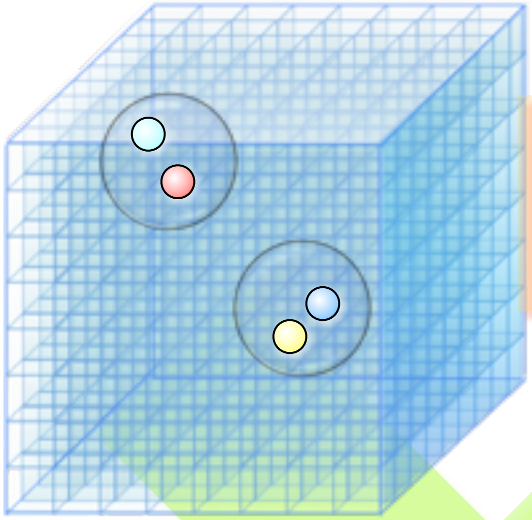
➡ **coupled-channel scattering from LQCD**

Contents

- Introduction
- HAL QCD method to define (coupled-channel) potentials
- Tcc in $I(J^P)=0,1(1^+)$ channels [DD^* single-channel]
- Zc(3900) in $I(J^P)=1(1^+)$ [$\pi J/\Psi$-$\rho\eta_c$-$D^{bar}D^*$ coupled-channel]
- Summary



Two identical methods for scattering



- ***Lüscher's finite size formula***

  interaction energy --> phase shift

  Lüscher, NPB354, 531 (1991).

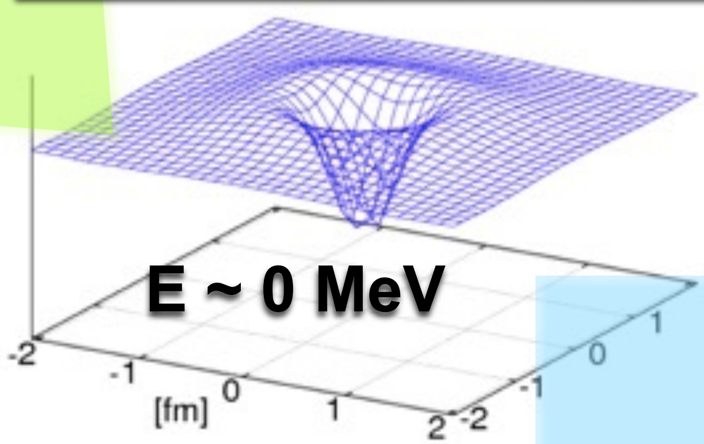
- ***Scattering parameters***

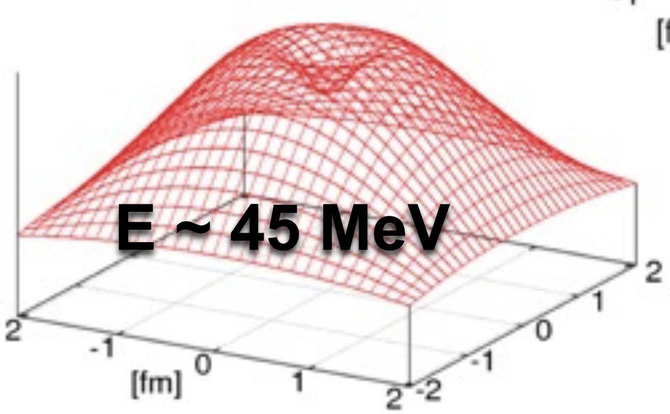
$$k\cot\delta(k) = \frac{1}{a} - \frac{1}{2} r_e k^2 + \cdots$$

- **Guaranteed to be the same**

  Kurth et al., JHEP 1312 (2013) 015.

- ***NBS wave function***

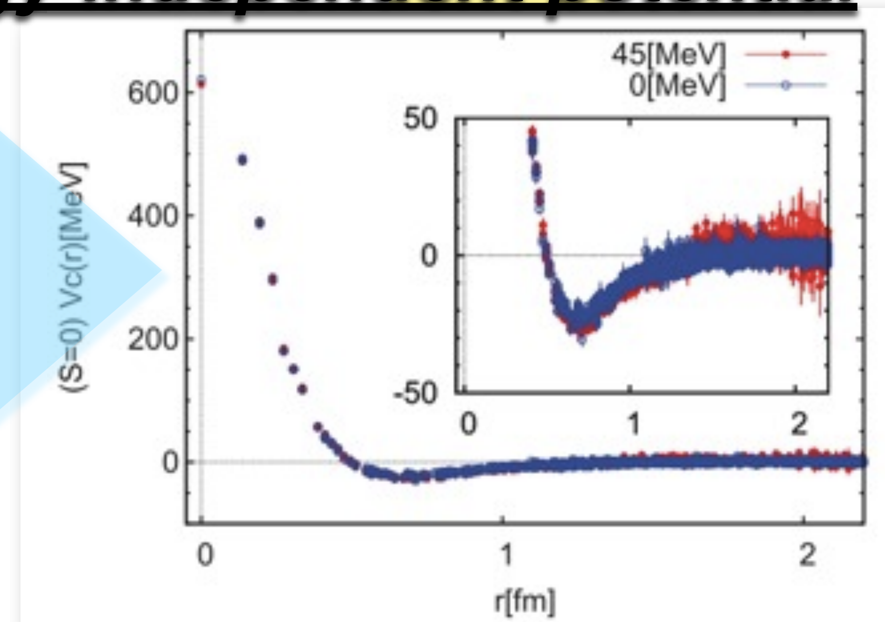




Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).

Aoki, Hatsuda, Ishii, PPTP123, 89 (2010).

- ***Energy-independent potential***



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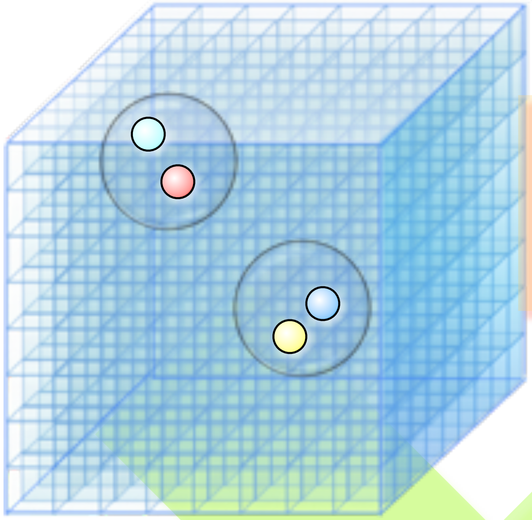

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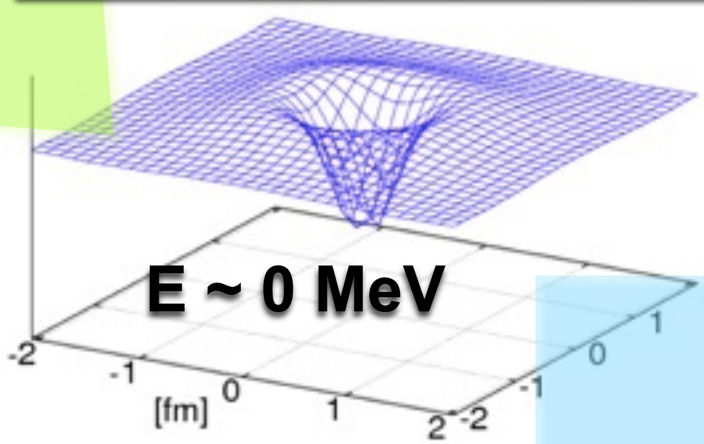

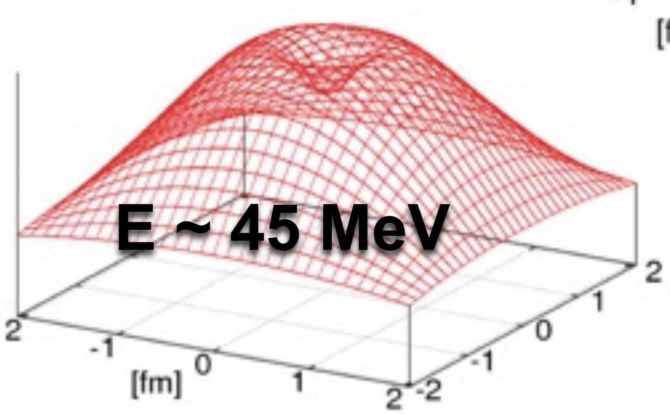

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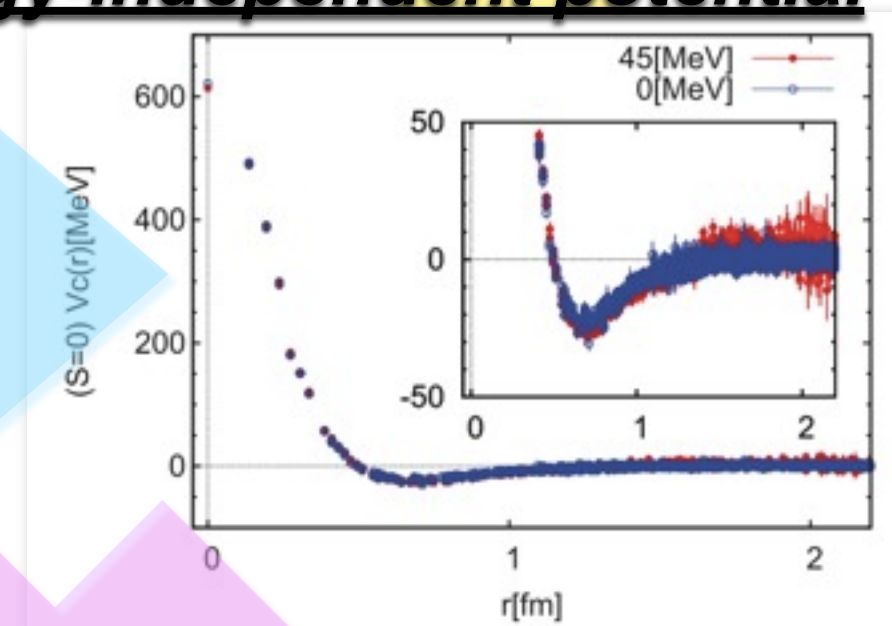


✓ **LQCD potentials can be applied to...**
properties of hadrons & nuclei, construction of EOS, etc.

Resonance from LQCD

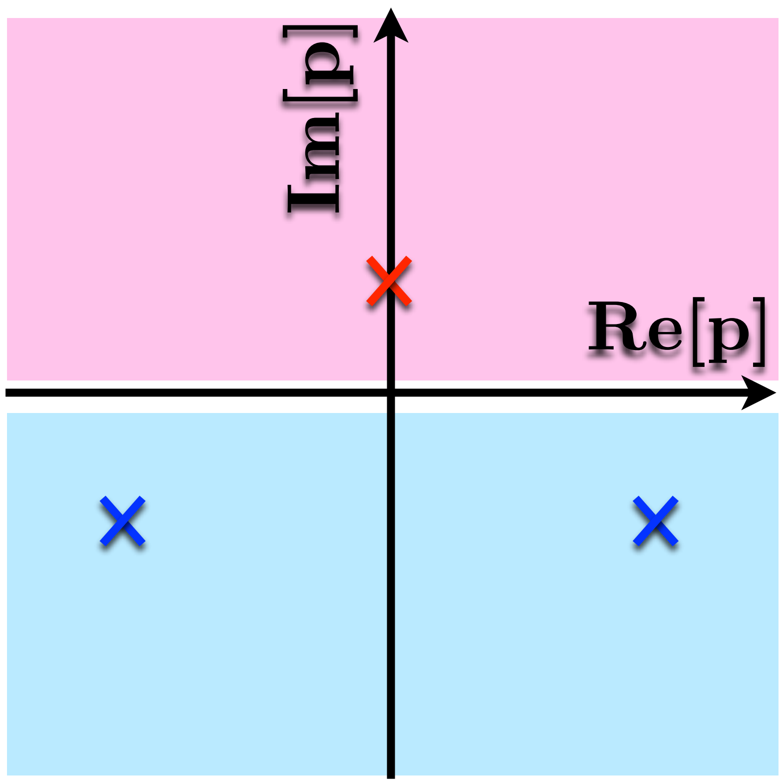
T-matrix in formal scattering theory (N/D method)

$$T^{-1}(\sqrt{s}) = V^{-1} + \frac{1}{2\pi}\int_{s_+}^{\infty} ds' \frac{\rho(s')}{s'-s}$$

Interaction part is not determined within scattering theory

➡ interactions faithful to phase shift in QCD

Analyticity of T-matrix is uniquely determined



Bound states (physical sheet, 1st)

- binding energy --> T-matrix pole position
- coupling --> residue of pole

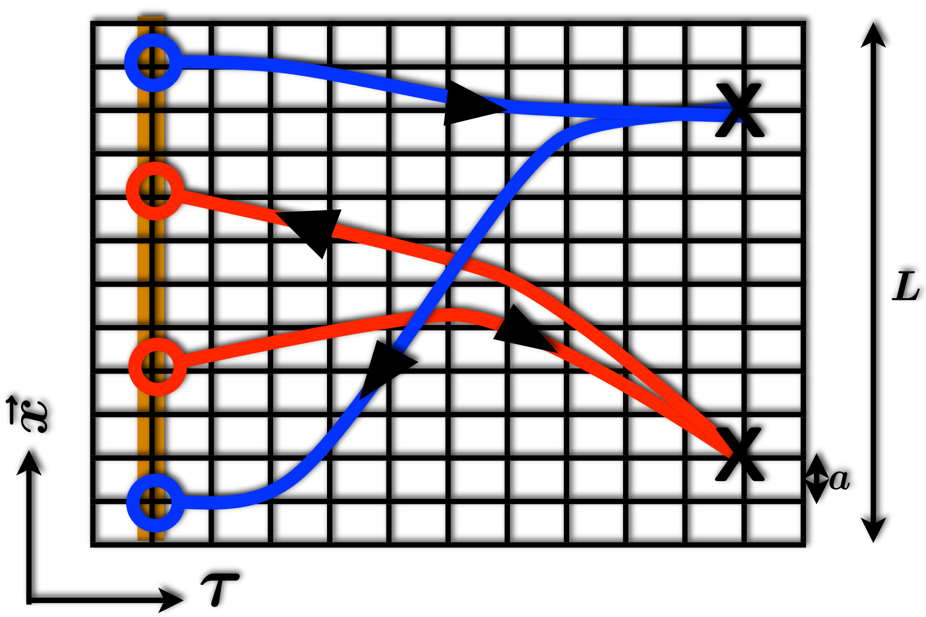
Resonance/Virtual states (unphysical sheet, 2nd)

- Analytic continuation of T-matrix
- resonance energy --> T-matrix pole position
- coupling --> (complex) residue of pole?

"Potentials" in QCD

Hadron 4pt functions & Nambu-Bethe-Salpeter (NBS) wave function

$$\psi^{ab}(\vec{r},\tau) = \sum_{\vec{x}} \langle 0|\phi_1^a(\vec{x}+\vec{r},\tau)\phi_2^a(\vec{x},\tau)\mathcal{J}^{b\dagger}(\tau=0)|0\rangle$$
$$= \sum_n A_n^b \exp\left[-W_n\tau\right]\sqrt{Z_1^a}\sqrt{Z_2^a}\,\psi_n^a(\vec{r})$$



- Helmholtz eq. of NBS wave func.

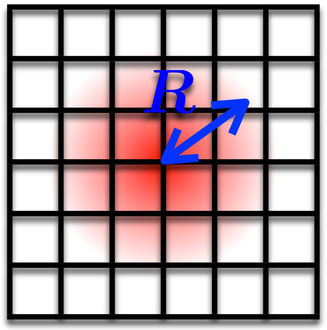
$$(\nabla^2 + (\vec{k}^a)^2)\psi^a_{W(\vec{k})}(\vec{r}) = 0 \quad (|\vec{r}| > R)$$

$$\psi^{(l)}_{W(\vec{k})}(r) \sim \frac{e^{i\delta_l(k)}}{kr}\sin(kr + \delta_l(k) - l\pi/2)$$

- NBS wave func. in QFT ~ wave func. in Q.M.

- Coupled-channel potential matrix (faithful to phase shifts)

$$\left(\nabla^2 + (\vec{k}^a)^2\right)\psi_n^a(\vec{r}) = 2\mu^a \sum_b \int d\vec{r}' U^{ab}(\vec{r},\vec{r}')\psi_n^b(\vec{r}')$$



Aoki et al. [HAL QCD Coll.], Proc. Jpn. Acad., Ser. B, 87 (2011); PTEP 2012, 01A105 (2012).

- Coupled-channel potentials are energy-independent (non-local in general)

HAL QCD method

✓ Definition of energy-independent coupled-channel potentials :

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

$$\psi_n(\vec{r}) = \langle 0|\phi_1^a(\vec{r}+\vec{x})\phi_2^a(\vec{x})|W_n; J^P\rangle$$

$$\left(\nabla^2 + (\vec{k}^a)^2\right)\psi_n^a(\vec{r}) = 2\mu \sum_b \int d\vec{r}' U^{ab}(\vec{r},\vec{r}')\psi_n^b(\vec{r}')$$

✤ Since energy-independent potential can produce all scattering states, single-state saturations in simulations is not required

✓ Extract energy-independent potential from time-dependent Schrödinger-type eq.

Ishii et al.(HAL QCD), PLB712, 437(2012).

$$R^{ab}(\vec{r},\tau) \equiv \psi^{ab}(\vec{r},\tau)\frac{e^{(m_1^a+m_2^a)\tau}}{\sqrt{Z_1^a}\sqrt{Z_2^a}} \qquad \delta = \frac{m_1^a - m_2^a}{m_1^a + m_2^a} \qquad \Delta^{ac} = \frac{e^{(m_1^a+m_2^a)\tau}}{e^{(m_1^c+m_2^c)\tau}}$$

$$\left[-\partial_\tau + \nabla^2/2\mu^a + \partial_\tau^2/8\mu^a + \mathcal{O}(\delta^2)\right]R^{ab}(\vec{r},\tau) = \sum_c \int d\vec{r}' \Delta^{ac} U^{ac}(\vec{r},\vec{r}')R^{cb}(\vec{r}',\tau)$$

✓ Velocity expansion:

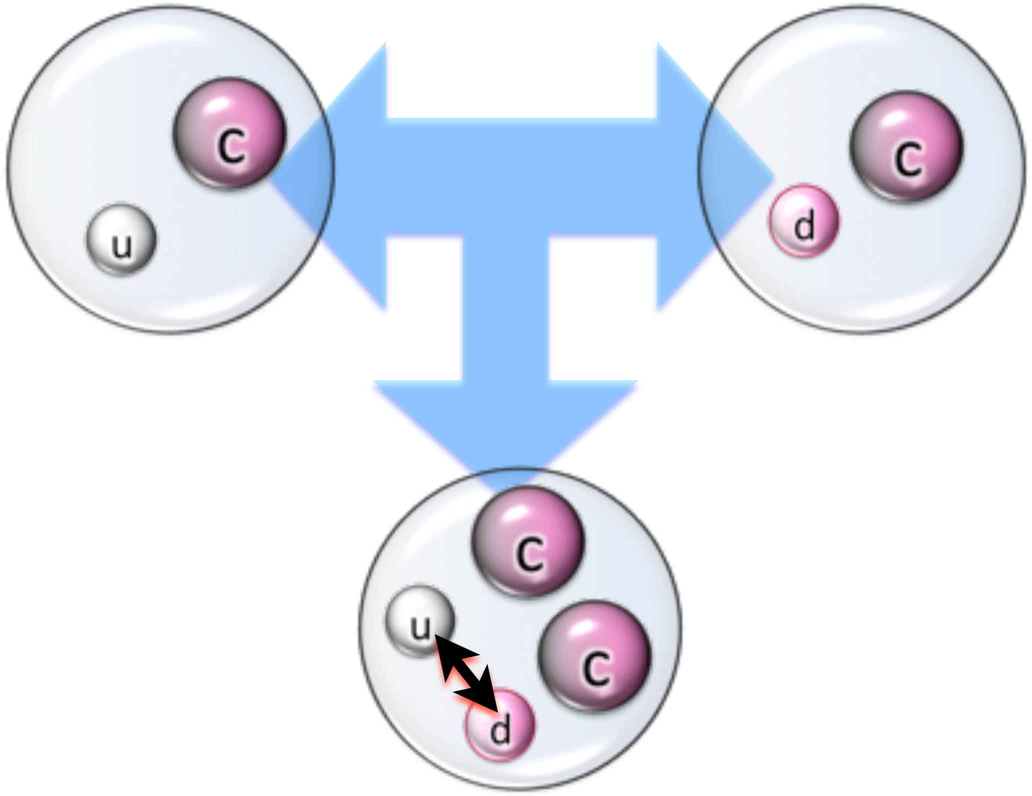
$$U(\vec{r},\vec{r}') = V(\vec{r},\nabla)\delta(\vec{r}-\vec{r}')$$

(LO) (NLO)

$$\Rightarrow V(\vec{r},\nabla) = V_C(\vec{r}) + \vec{L}\cdot\vec{S}V_{LS}(\vec{r}) + \mathcal{O}(\nabla^2)$$

✓ Calculate observable: phase shift, binding energy, pole position, ...

Tcc in $I(J^P)=0,1(1^+)$



Asymptotic states : DD* (s-wave)

Lattice QCD Setup

N_f=2+1 full QCD configurations generated by PACS-CS Coll.

PACS-CS Coll., S. Aoki et al., PRD79, 034503, (2009).

- Iwasaki gauge & O(a)-improved Wilson quark actions
- a=0.0907(13) fm --> L~2.9 fm (32^3 x 64)

Light meson mass [conf.1, conf.2, conf.3] (MeV)
M_π=699(1), 572(2), 411(2) [PDG:135 (π^0)]
M_K=787(1), 714(1), 635(2) [PDG:498 (K^0)]

Tsukuba-type Relativistic Heavy Quark (RHQ) action for charm quark

S. Aoki et al., PTP109, 383 (2003)

➡ remove leading cutoff errors O(m_c a), O(Λ_{QCD} a), ...

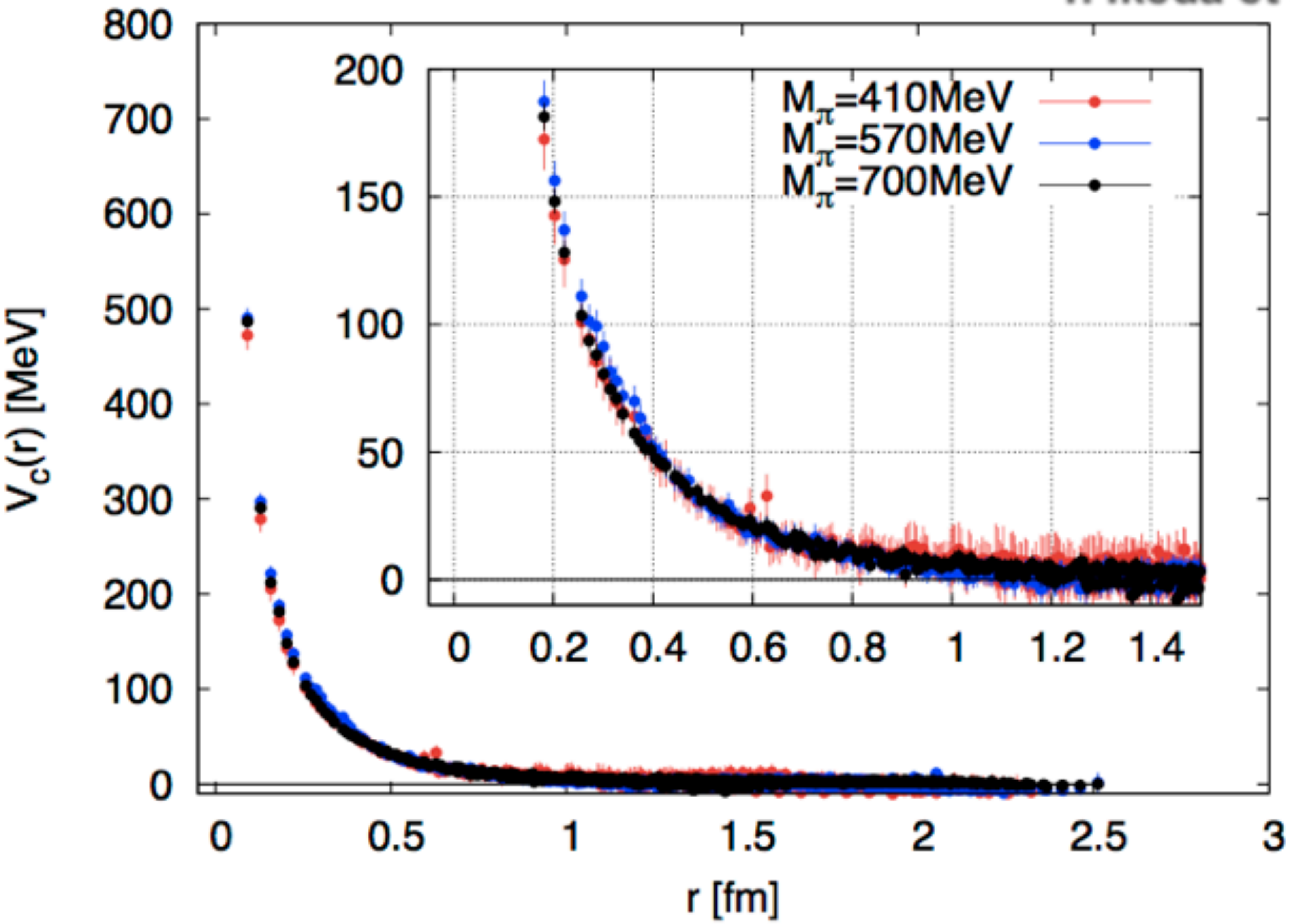
- We are left with O($(a\Lambda_{QCD})^2$) error (~ a few %)
- We employ RHQ parameters tuned by Namekawa et al.

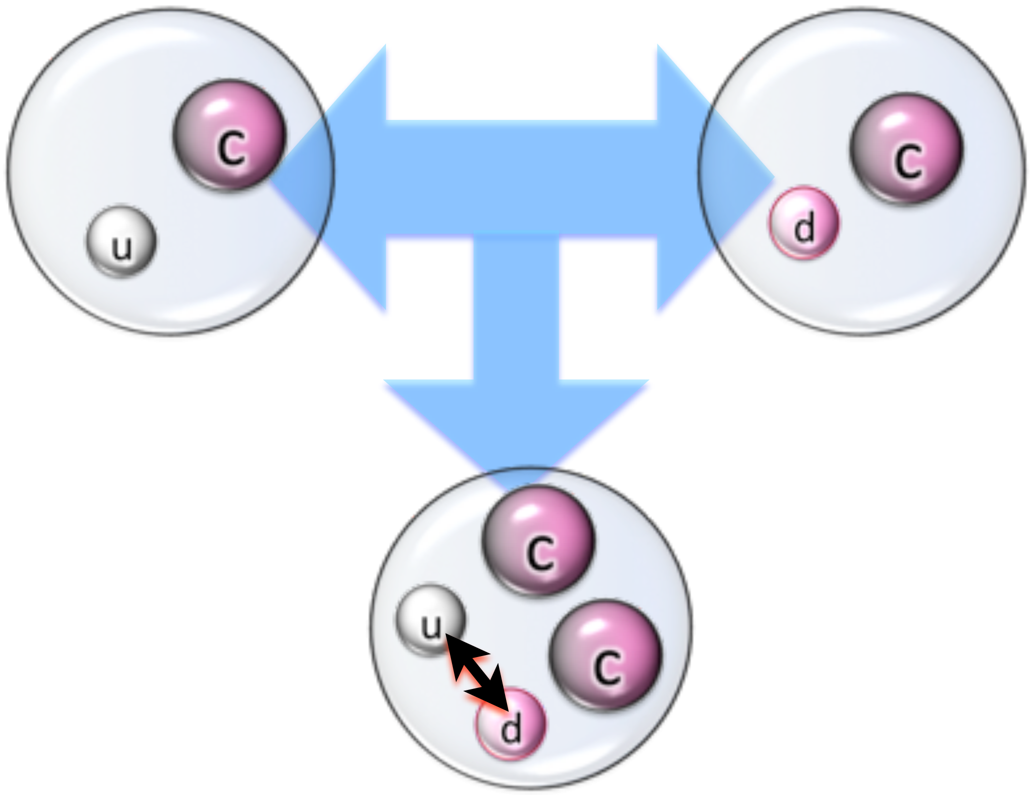
Y. Namekawa et al., PRD84, 074505 (2011)

Charmed meson mass [conf.1, conf.2, conf.3] (MeV)
$M_{\eta c}$=3024(1), 3005(1), 2988(2) [PDG:2981]
$M_{J/\psi}$=3142(1), 3118(1), 3097(2) [PDG:3097]
M_D=1999(1), 1946(1), 1902(3) [PDG:1865 (D^0)]
M_{D^*}=2159(4), 2099(6), 2048(12) [PDG:2007 (D^{*0})]

S-wave DD* in I=1 : "bad" diquark

Y. Ikeda et al. (HAL QCD), PLB729, 85 (2014).

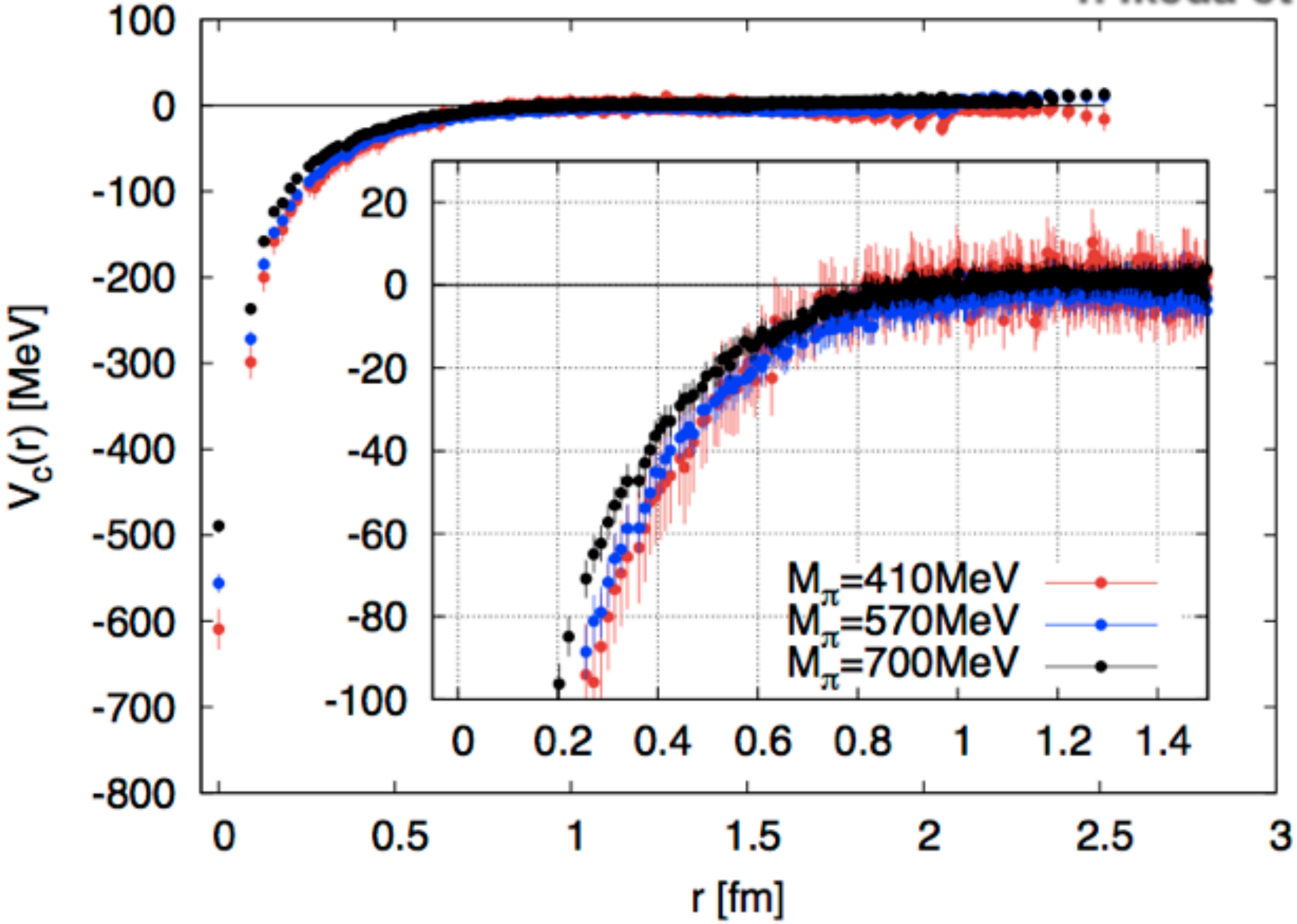


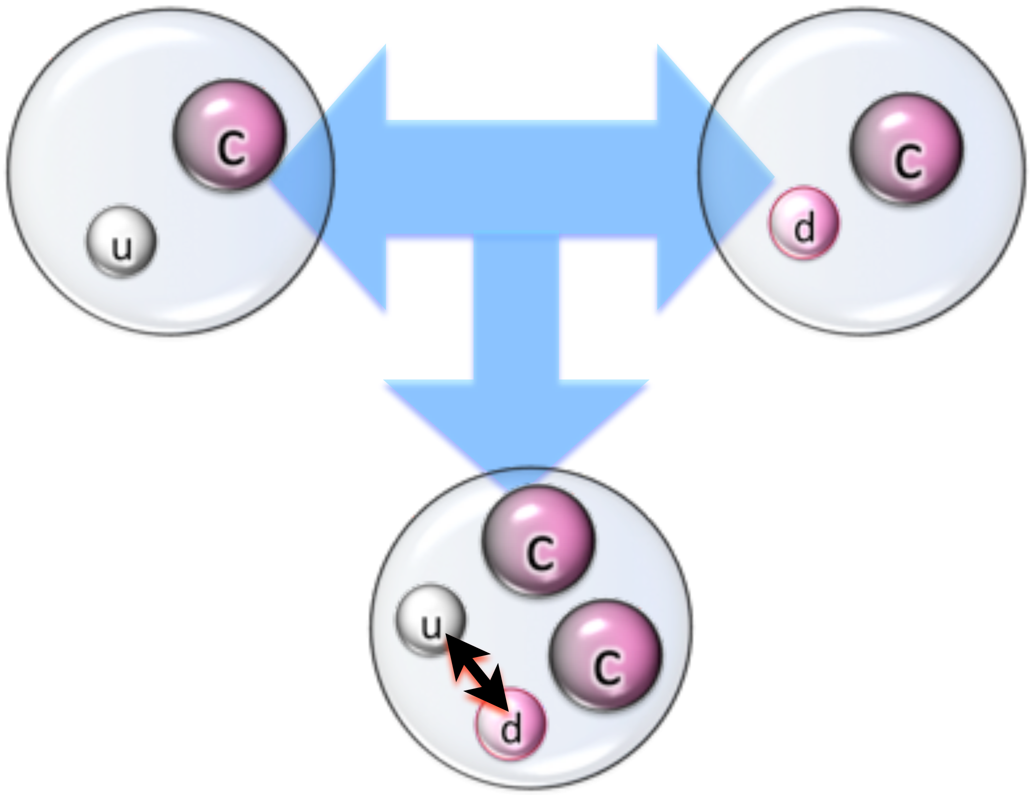


- **Repulsive** s-wave potentials of DD*
- Weak quark mass dependence
- It is unlikely to form bound state even at physical point

S-wave DD* in I=0 : “good” diquark

Y. Ikeda et al. (HAL QCD), PLB729, 85 (2014).



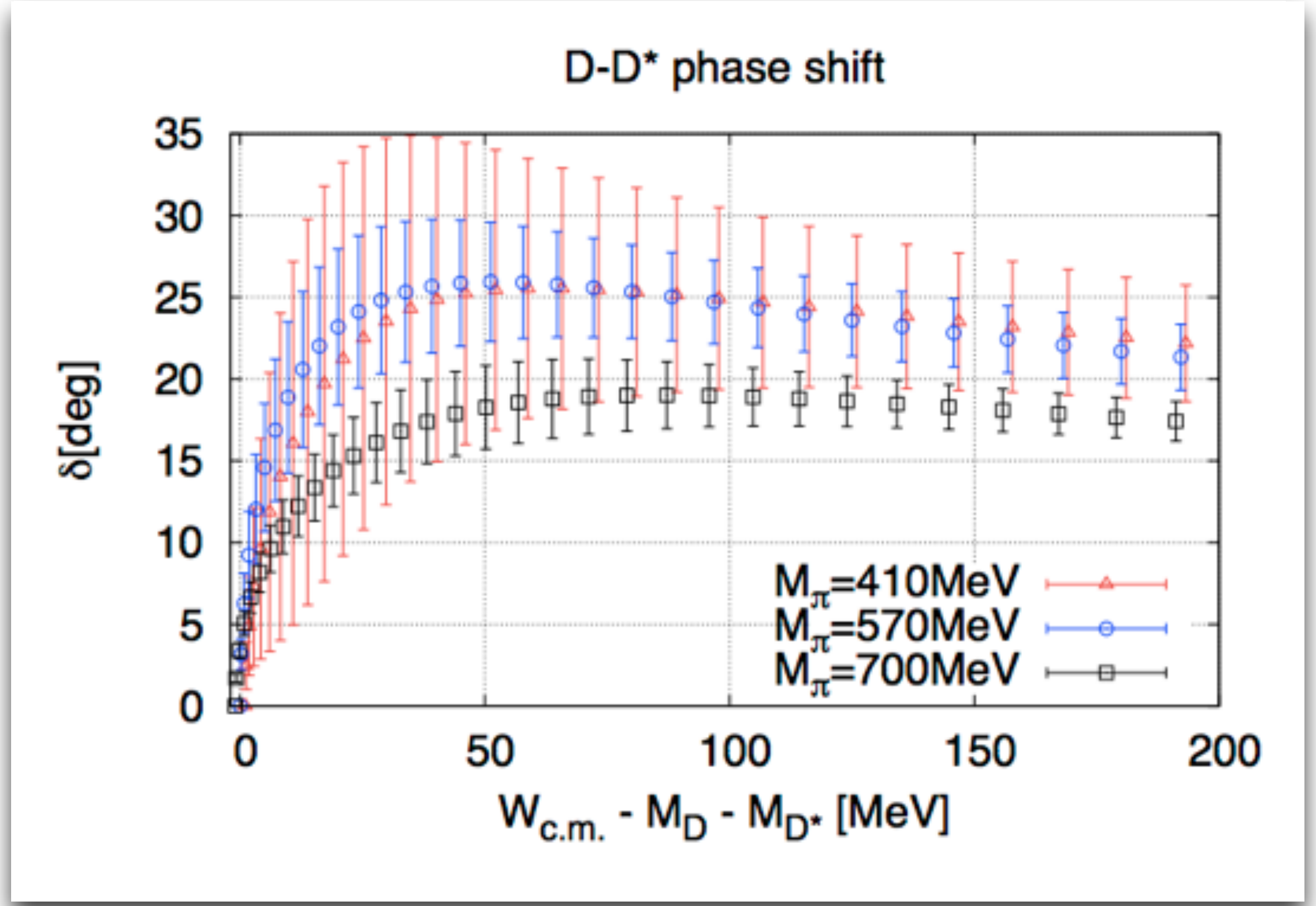


- **Attractive** S-wave potentials
- Attraction increases, as m_q decreases
- Check whether bound T_{cc} exist or not --> phase shift analysis

S-wave phase shifts : Tcc in I=0

Y. Ikeda et al. (HAL QCD), PLB729, 85 (2014).

- solve Schrödinger equation --> phase shifts

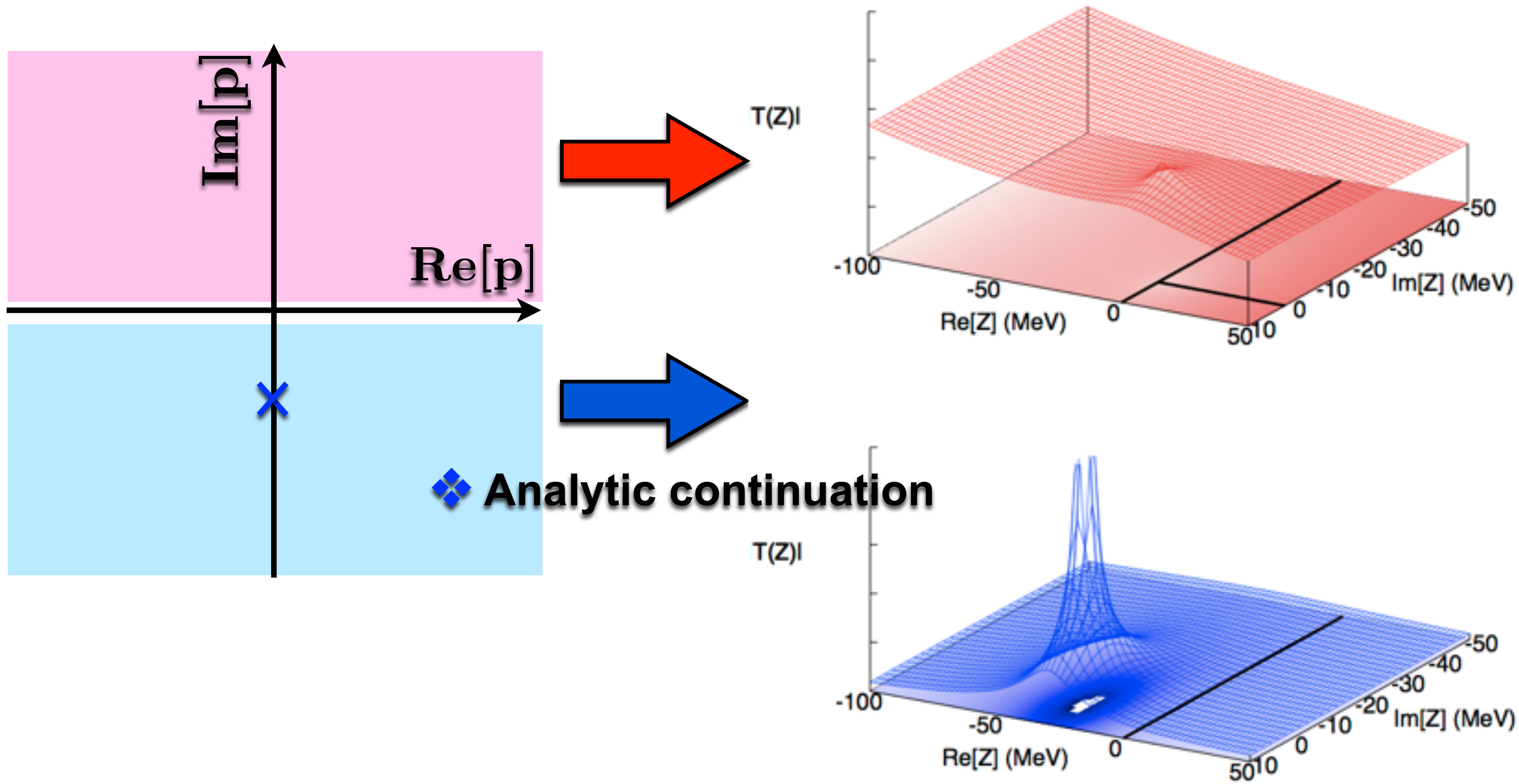


- Attraction is not sufficiently strong to generate bound state
- Rapid increase at threshold of DD* phase shift --> effect of virtual state?

➡ examine pole position

I=0 DD* T-matrix on complex energy plane

- Pole search w/ LQCD potential@m_π=410MeV



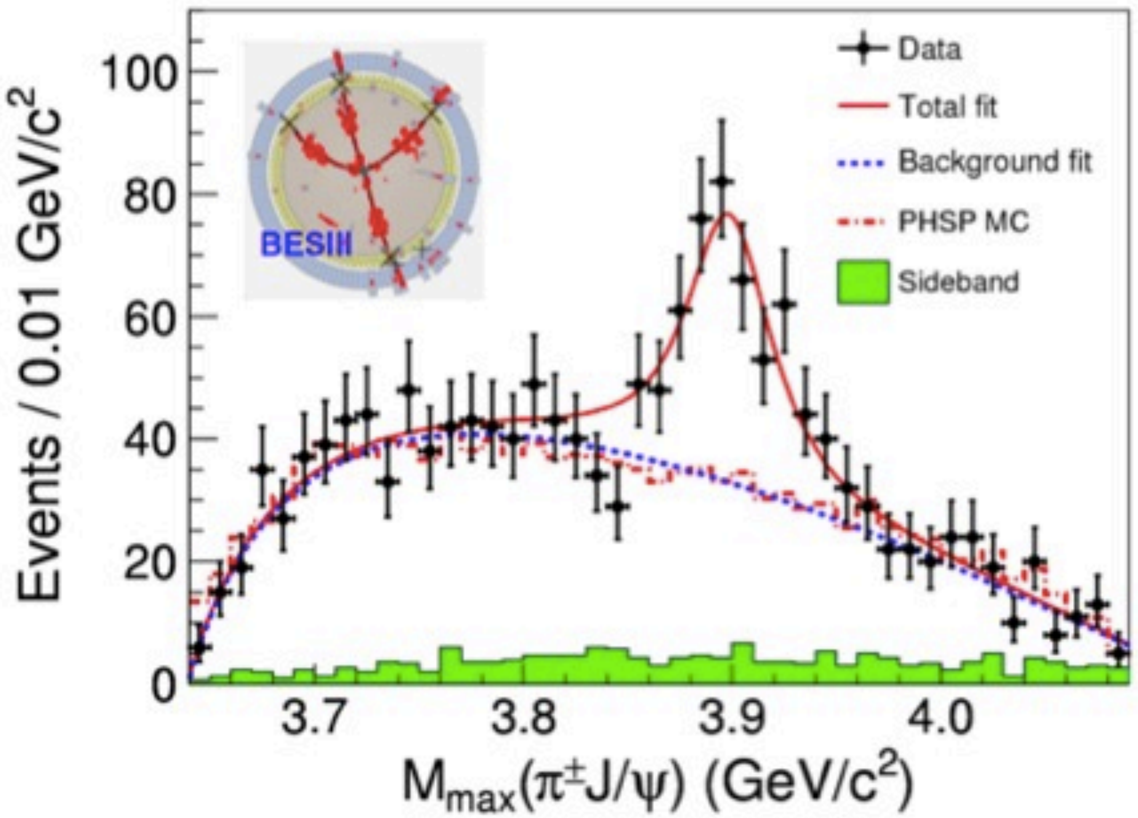
- **Virtual** pole on the DD* unphysical energy plane

➡ threshold cusp of the amplitude

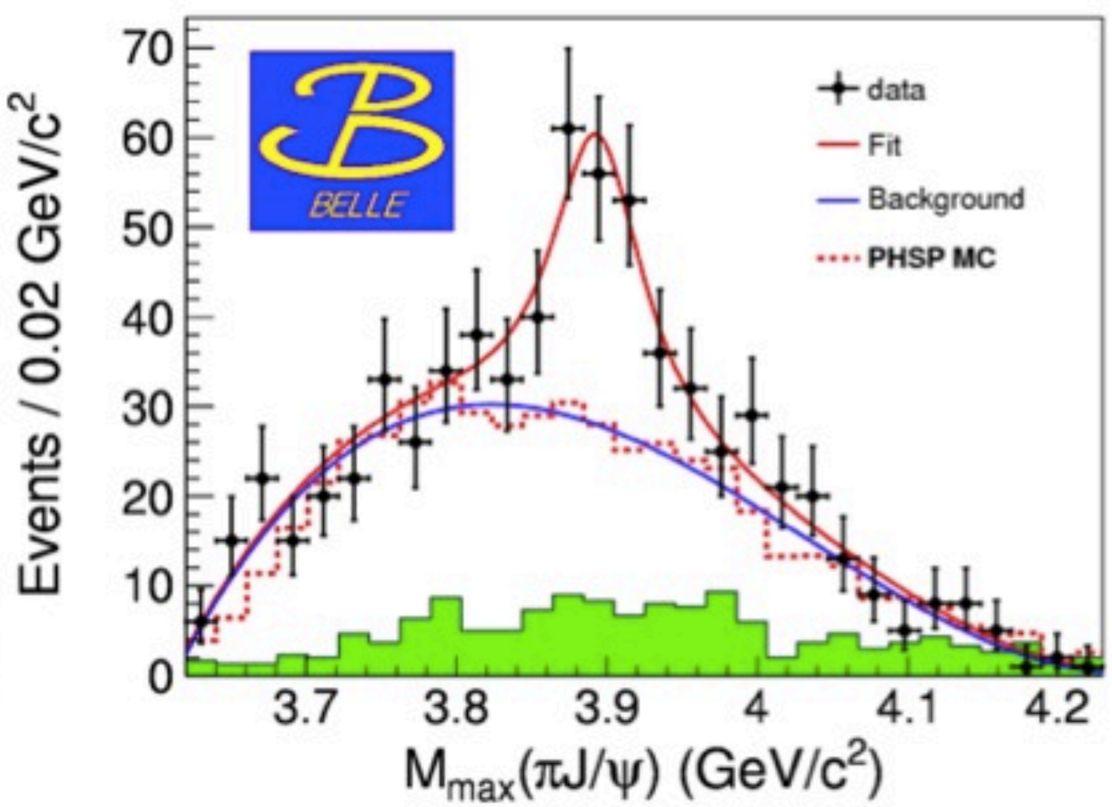
➡ rapid increase of scattering phase shift

Zc(3900) in $I^G(J^P)=1^+(1^+)$

BESIII Coll., PRL110, 252001, (2013).



Belle Coll., PRL110, 252002, (2013).



Lattice QCD setup

- N_f=2+1 full QCD configurations (PACS-CS) w/ L=2.9fm

  S. Aoki et al. (PACS-CS Coll.), PRD79, 034503, (2009).

- Tsukuba-type RHQ action for charm quark

  S. Aoki et al., PTP109, 383 (2003)

  Y. Namekawa et al., PRD84, 074505 (2011)

Light meson mass (MeV)

M_π= 411(2) [PDG:135 (π^0)]

M_ρ= 895(14) [PDG:775]

Charmed meson mass (MeV)

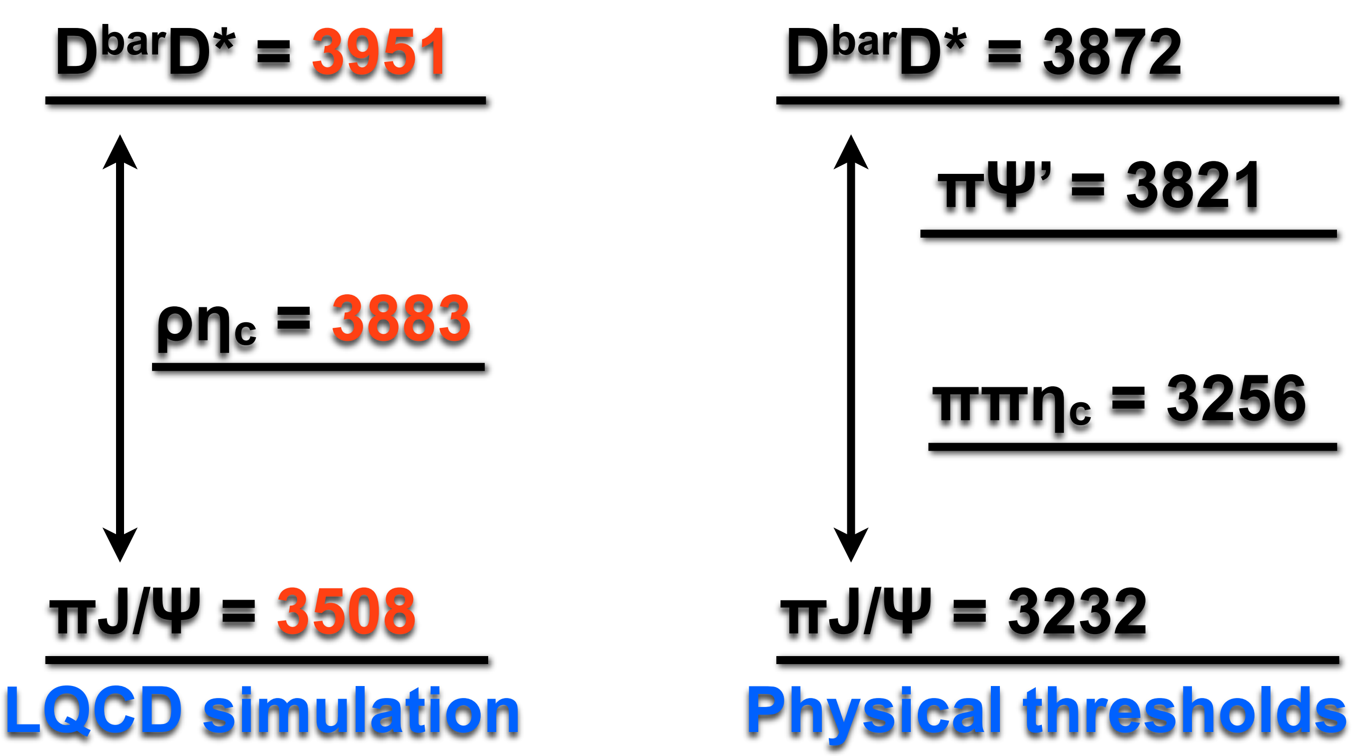
M_{η_c}= 2988(2) [PDG:2981]

$M_{J/\Psi}$= 3097(2) [PDG:3097]

M_D= 1902(3) [PDG:1865 (D^0)]

M_{D^*}= 2048(12) [PDG:2007 (D^{*0})]

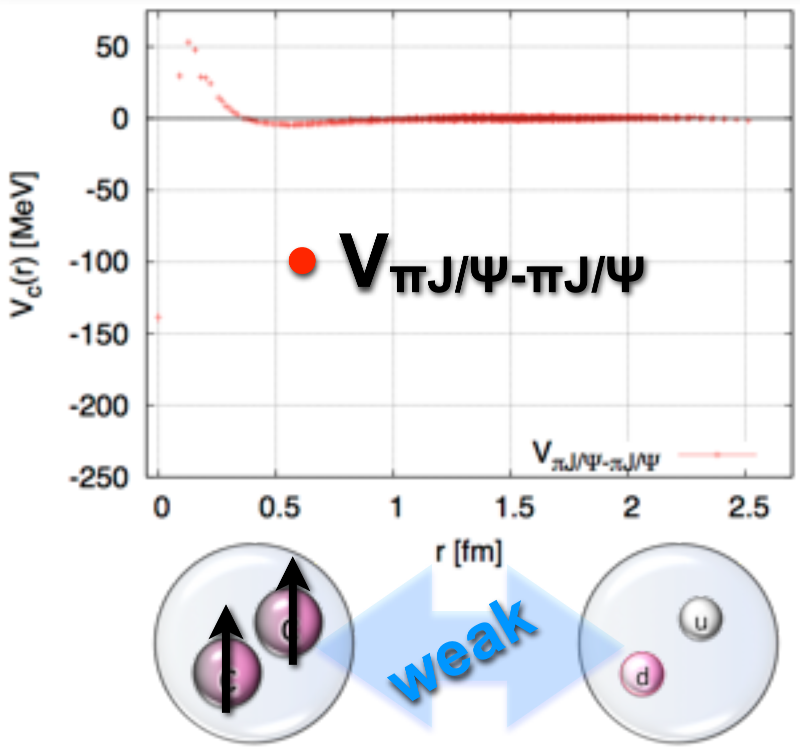
✦ **Thresholds in $I^GJ^P=1^+1^+$ channel**

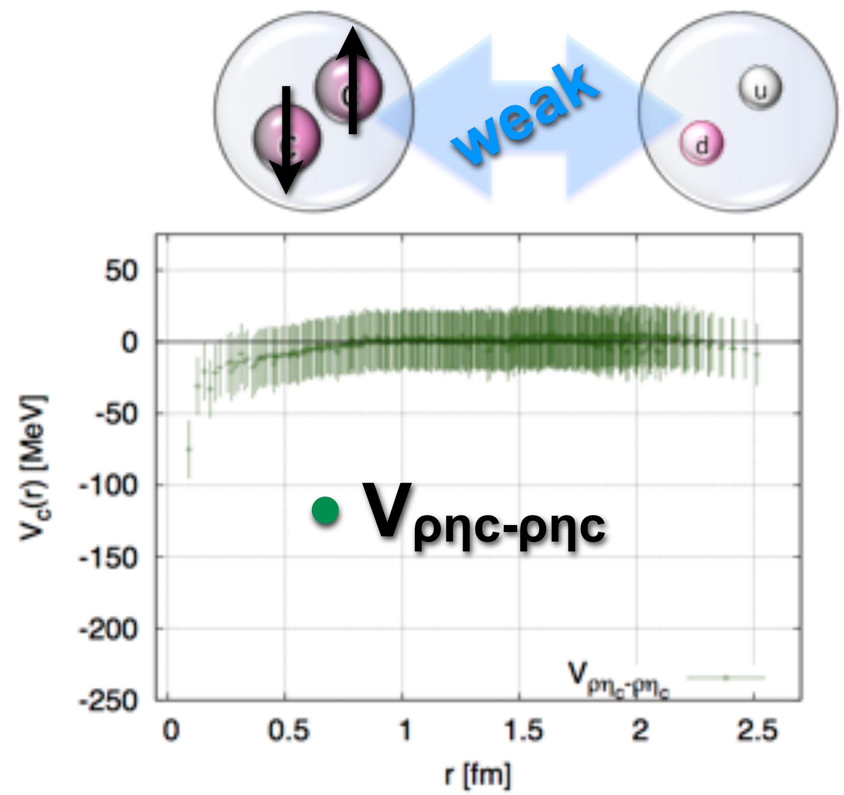


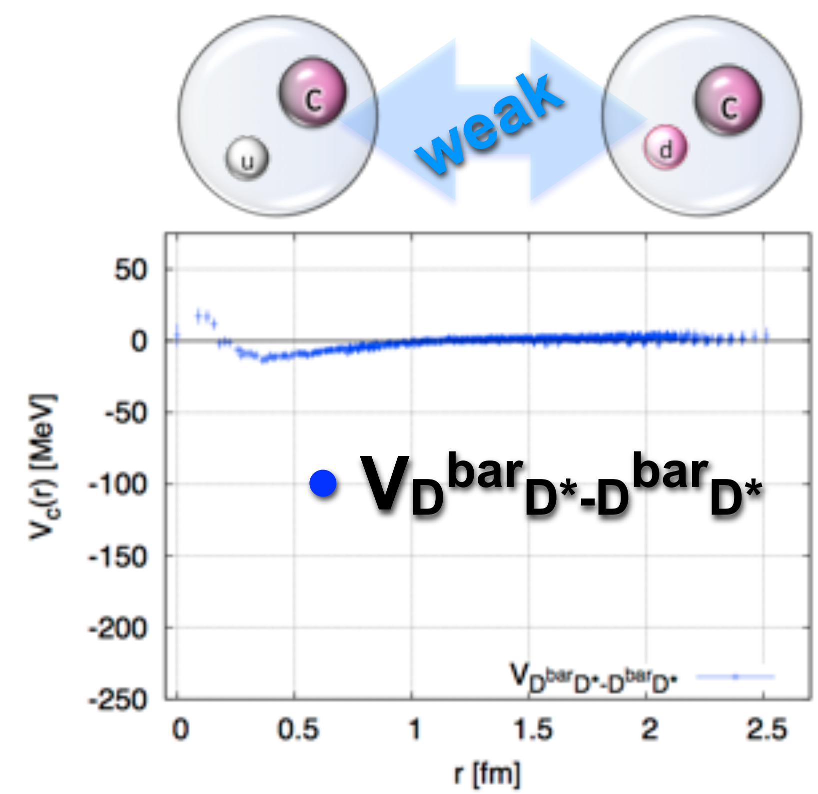
- $M_{\pi\Psi'} > M_{D^{bar}D^*}$ due to heavy pion mass
- ρ-->ππ decay not allowed in our setup

- S-wave πJ/Ψ - $\rho\eta_c$ - $D^{bar}D^*$ coupled-channel analysis is performed

Potential matrix ($\pi J/\Psi$ - $\rho\eta_c$ - $D^{bar}D^*$)



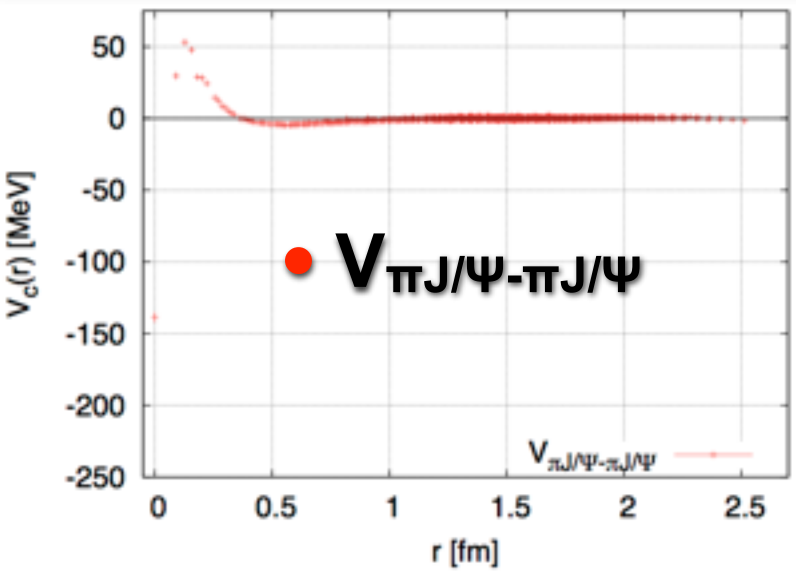


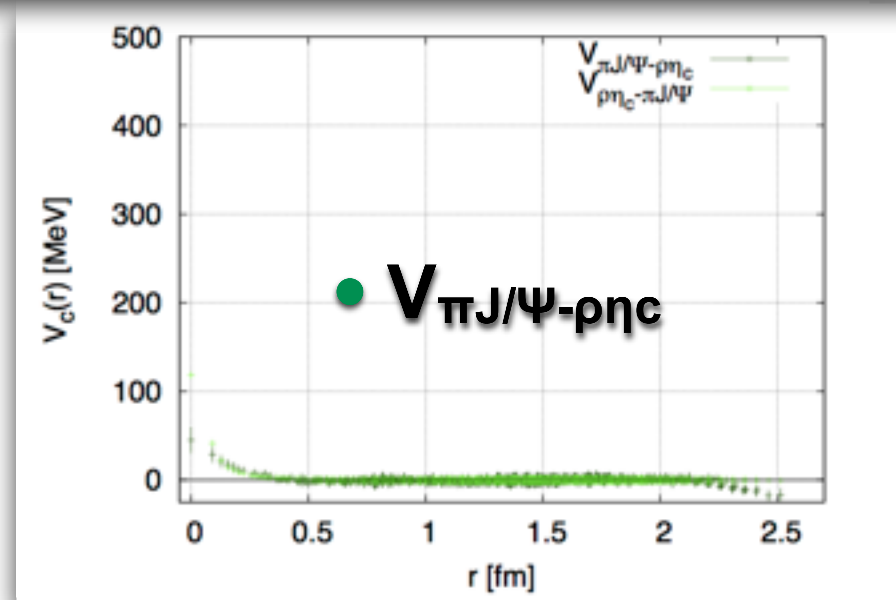


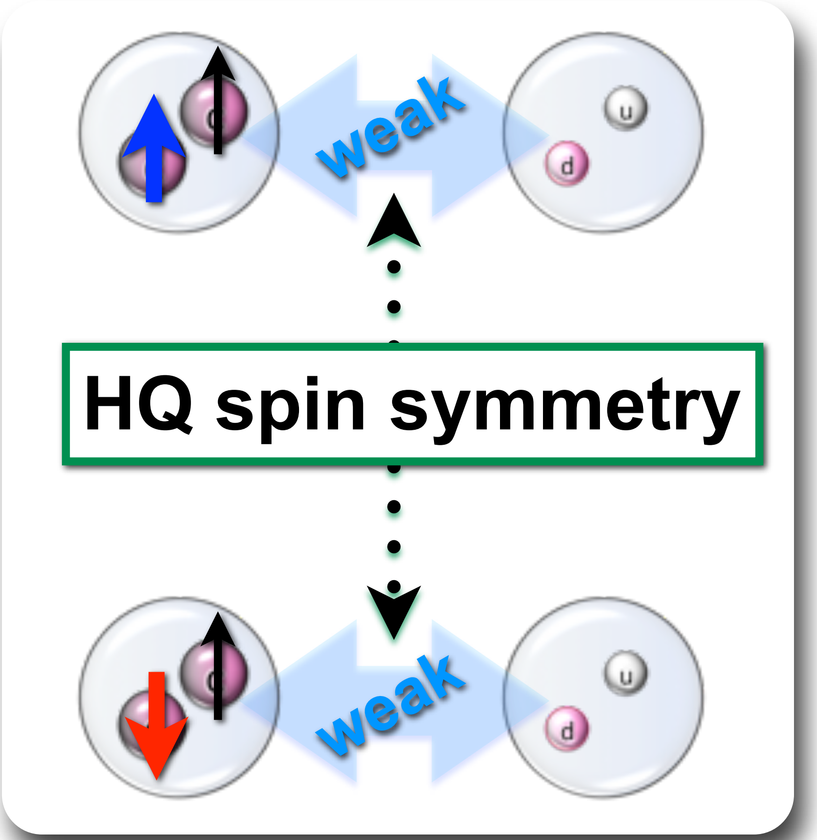
- **All diagonal potentials are weak**

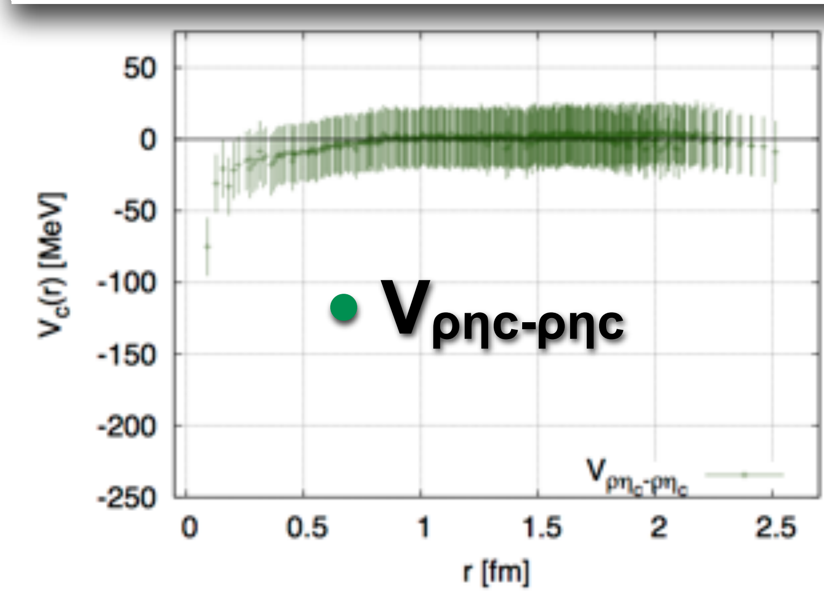
➡ **no bound $D^{bar}D^*$**

Potential matrix (πJ/Ψ - $\rho\eta_c$ - $D^{bar}D^*$)





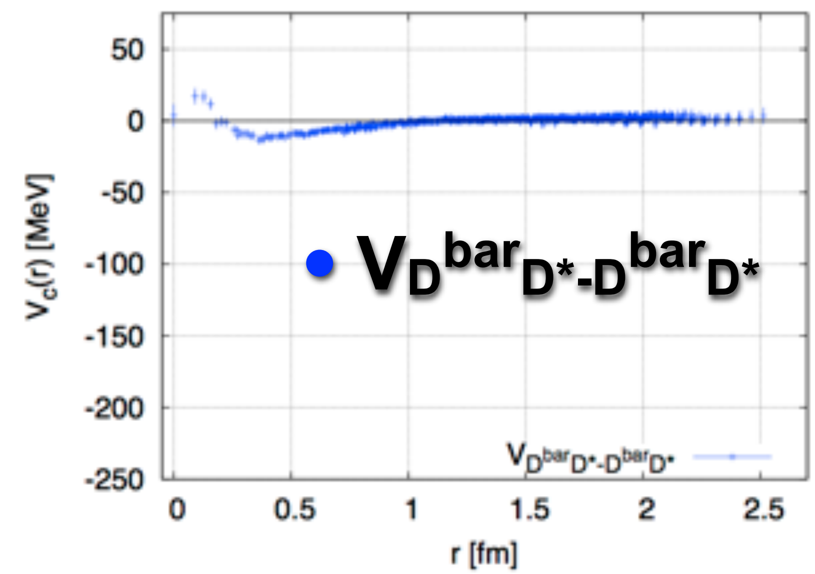




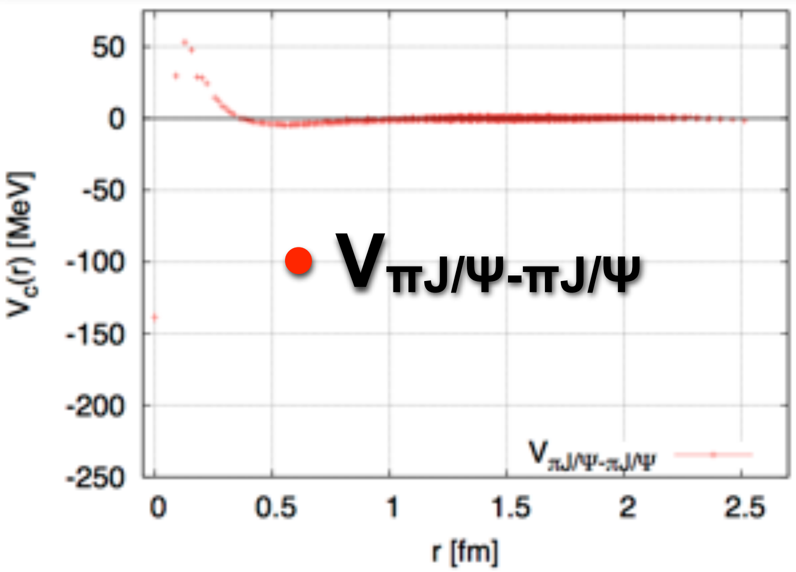
- Weak charm spin-flip potential

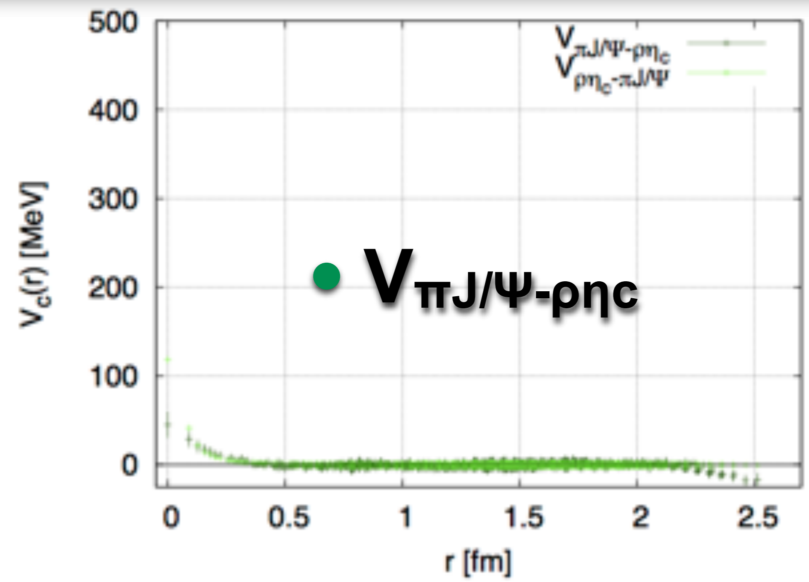
➡ heavy quark spin symmetry

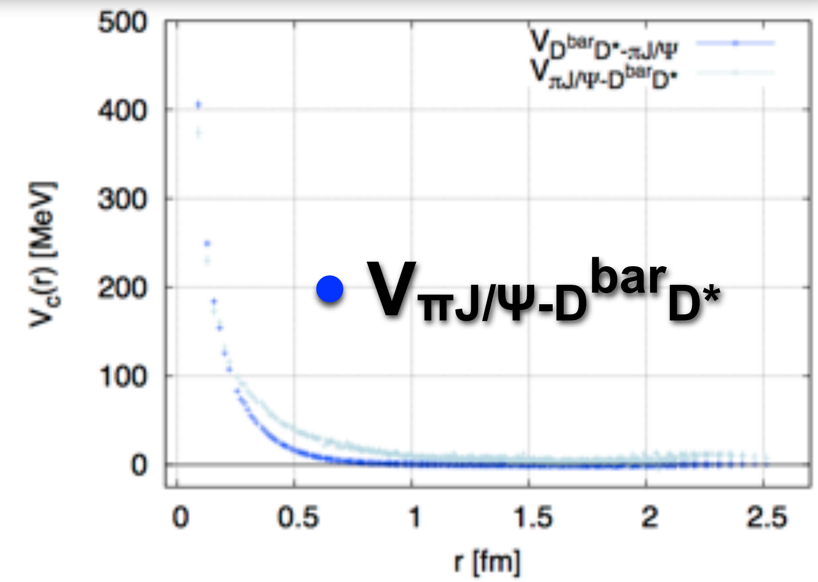
(charm quark spin-flip amplitude is suppressed)

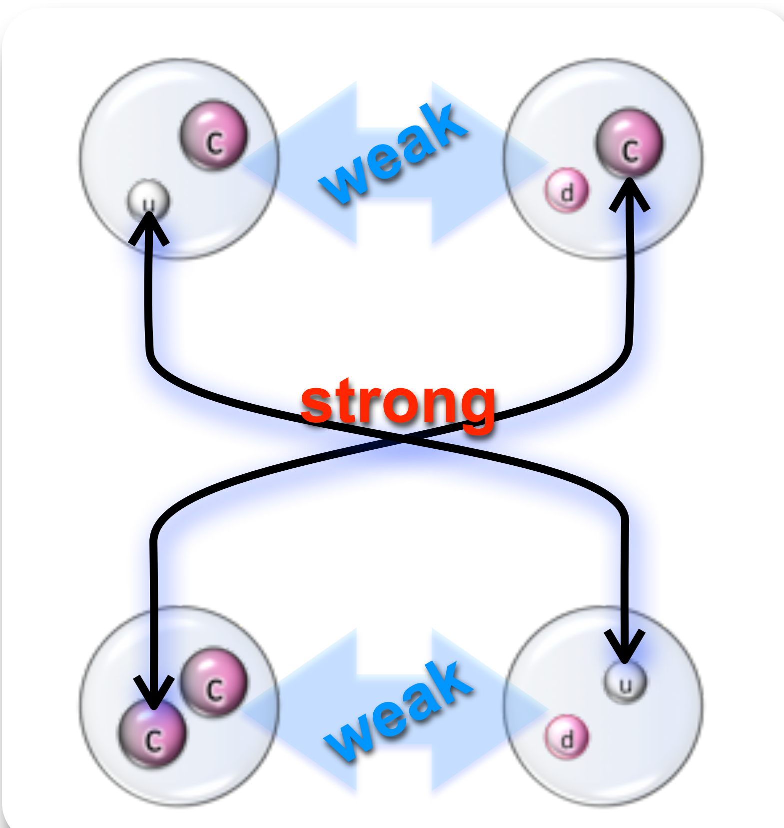


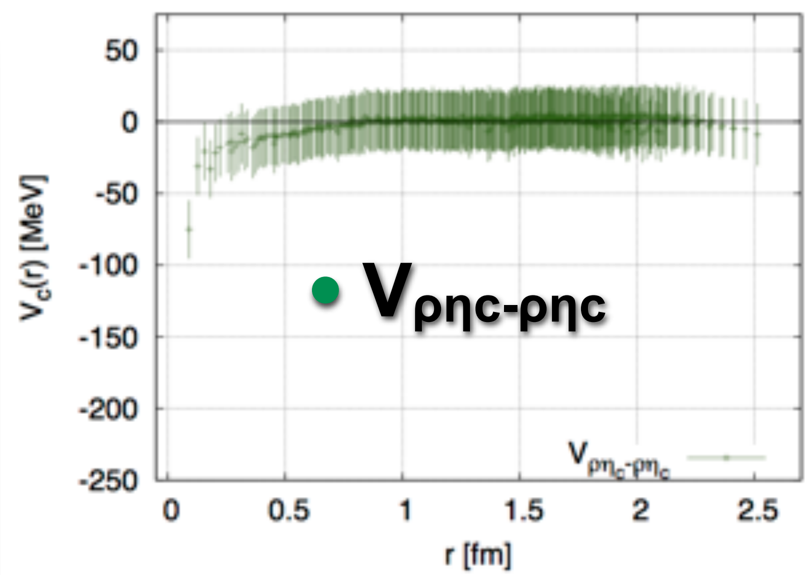
Potential matrix ($\pi J/\Psi$ - $\rho\eta_c$ - $D^{bar}D^*$)

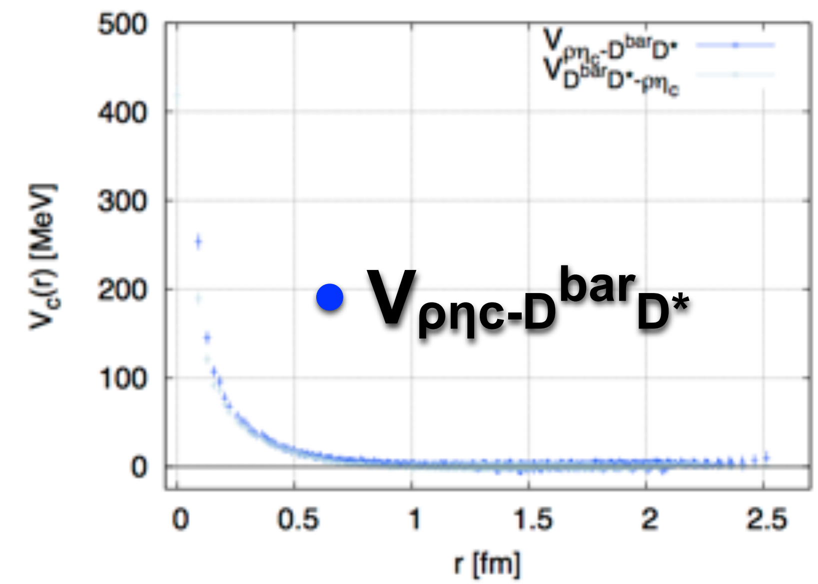


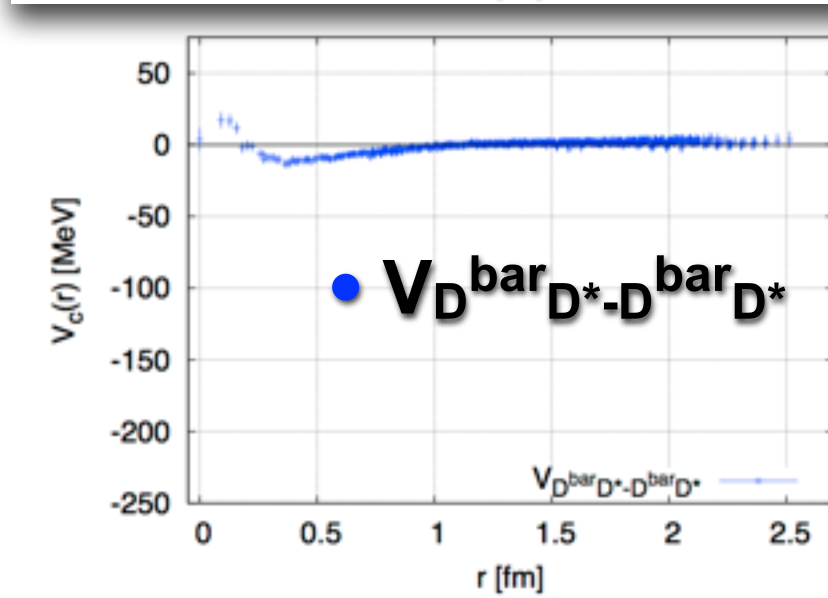






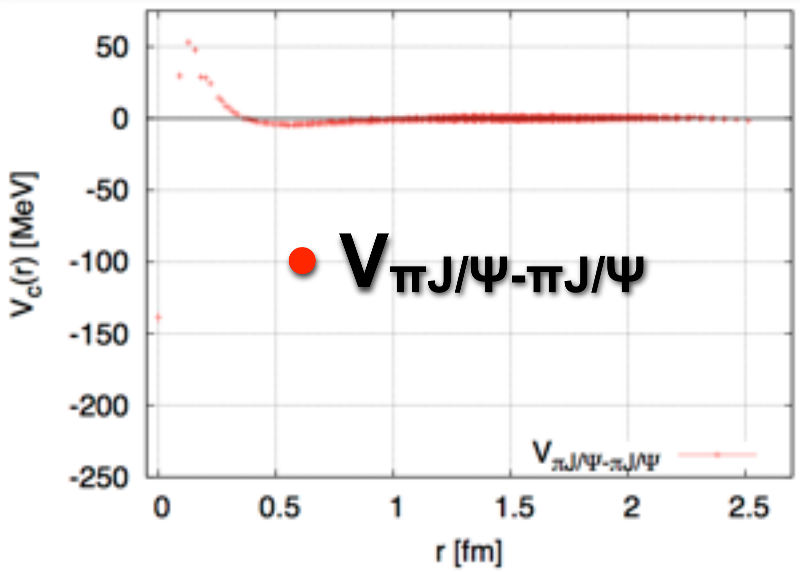


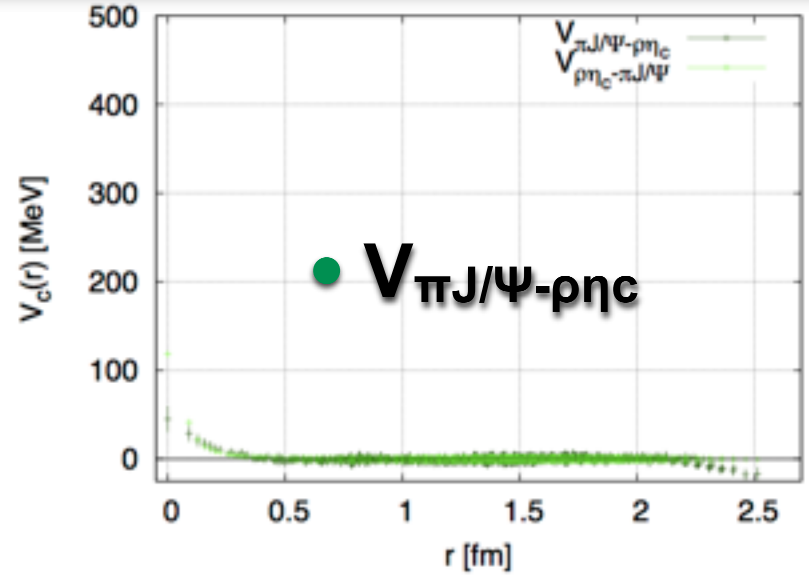


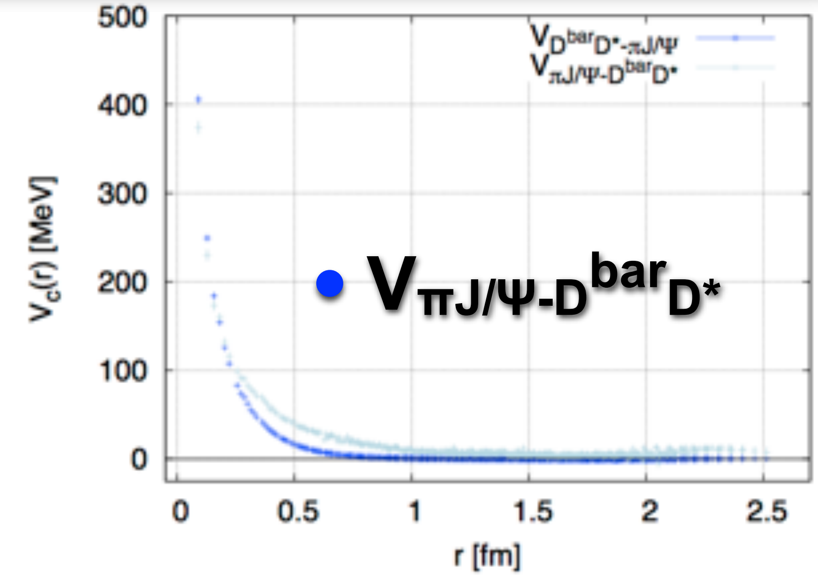


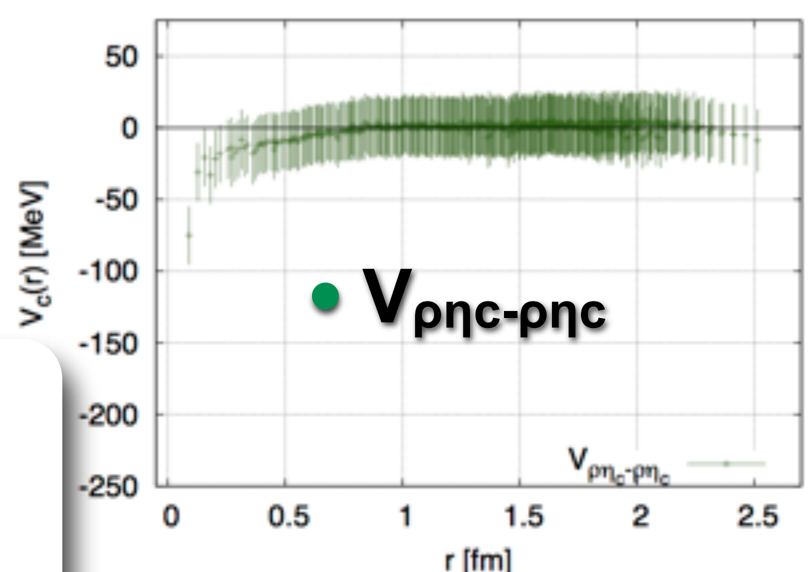
- **Strong off-diagonal $D^{bar}D^*$ potentials**
  - ✓ **strong charm-quark-exchange interactions**

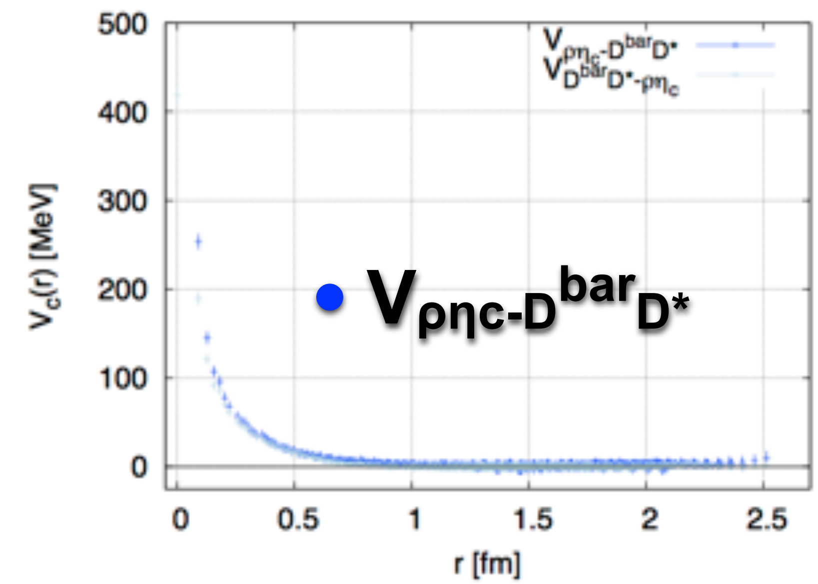
Potential matrix ($\pi J/\Psi$ - $\rho\eta_c$ - $D^{bar}D^*$)

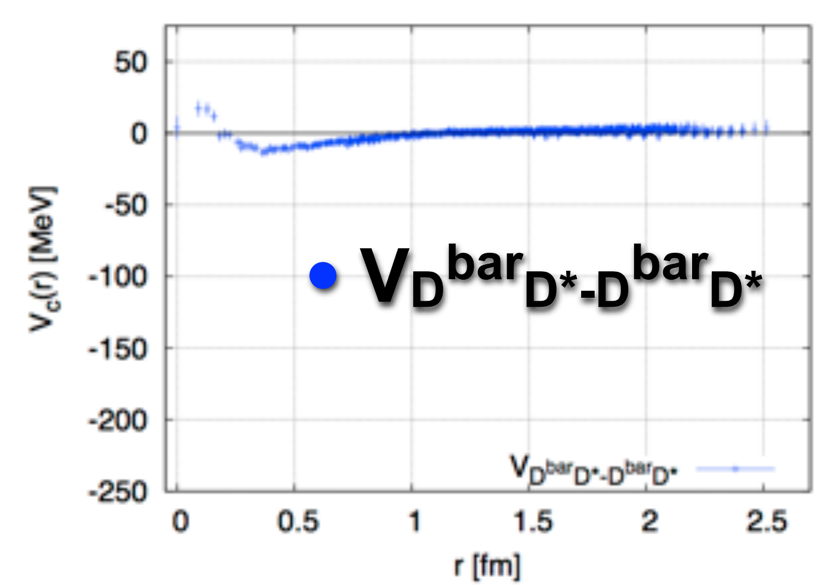


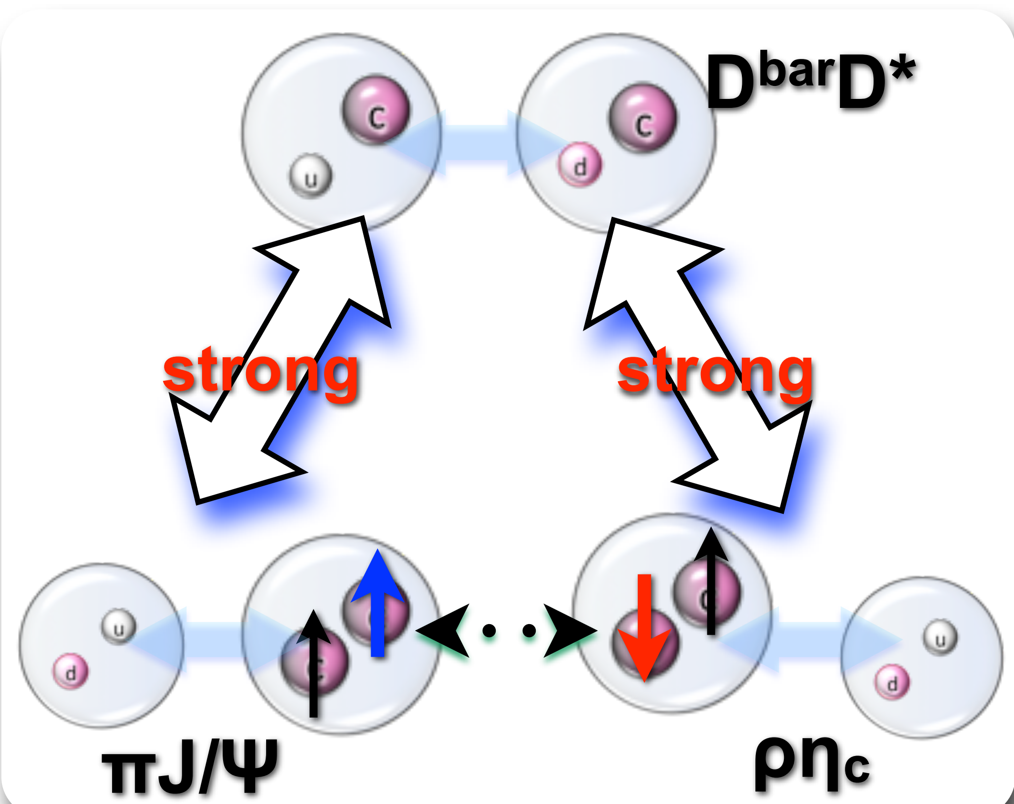






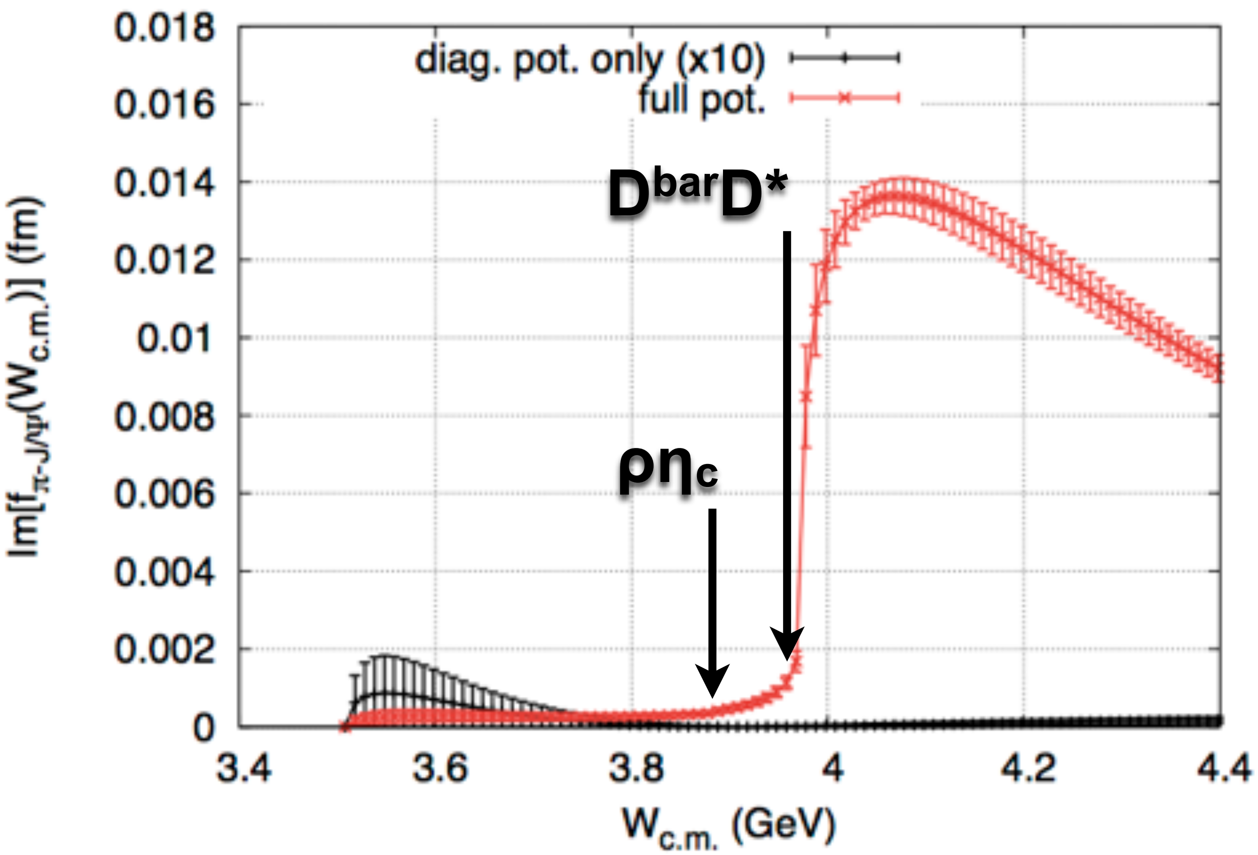




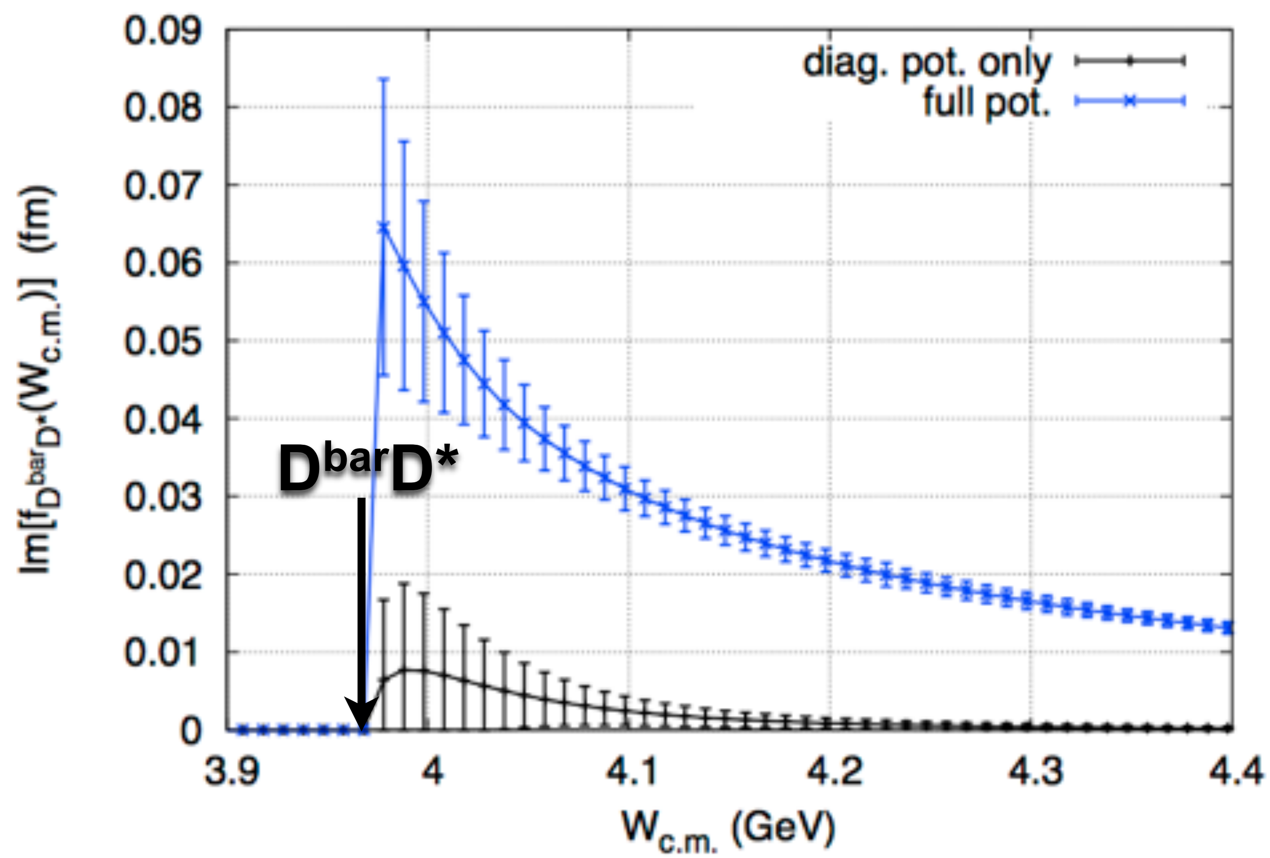


Invariant mass spectra of πJ/Ψ & $D^{bar}D^*$

- πJ/Ψ invariant mass (m_π=410MeV)



- $D^{bar}D^*$ invariant mass (m_π=410MeV)

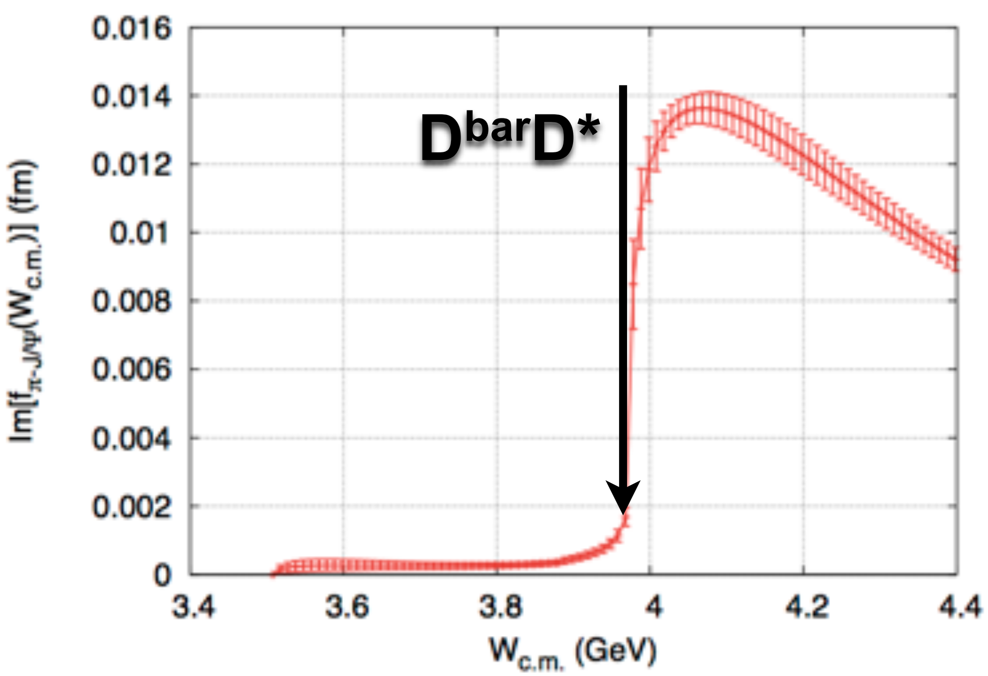


✓ enhancement near $D^{bar}D^*$ threshold due to large πJ/Ψ-$D^{bar}D^*$ coupling

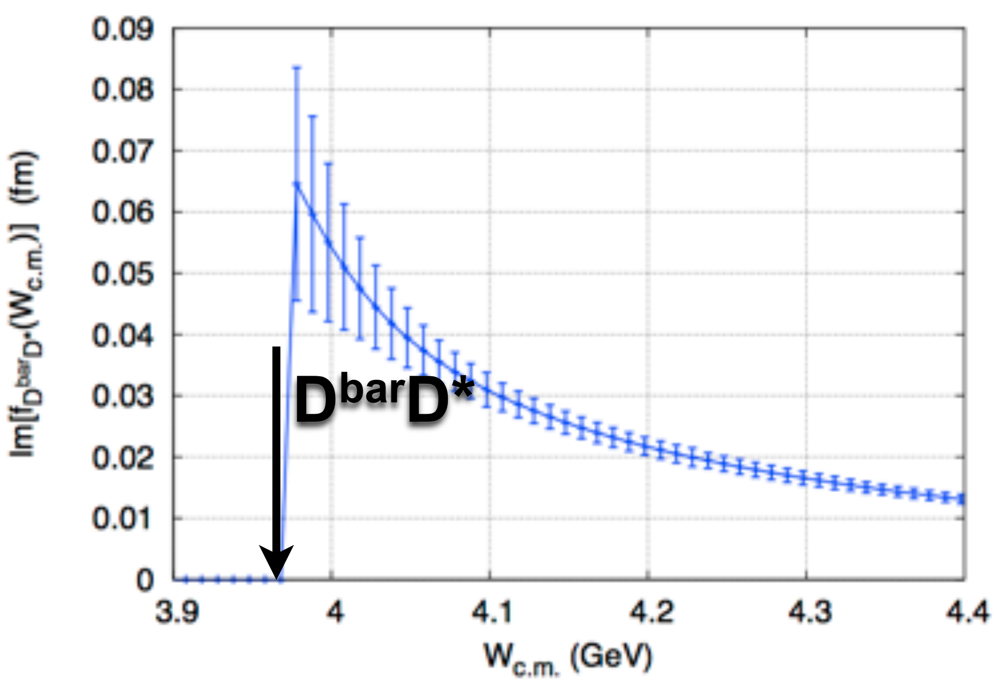
- peak in πJ/Ψ invariant mass
- enhancement (cusp?) in $D^{bar}D^*$ invariant mass

LQCD results & EXP. results

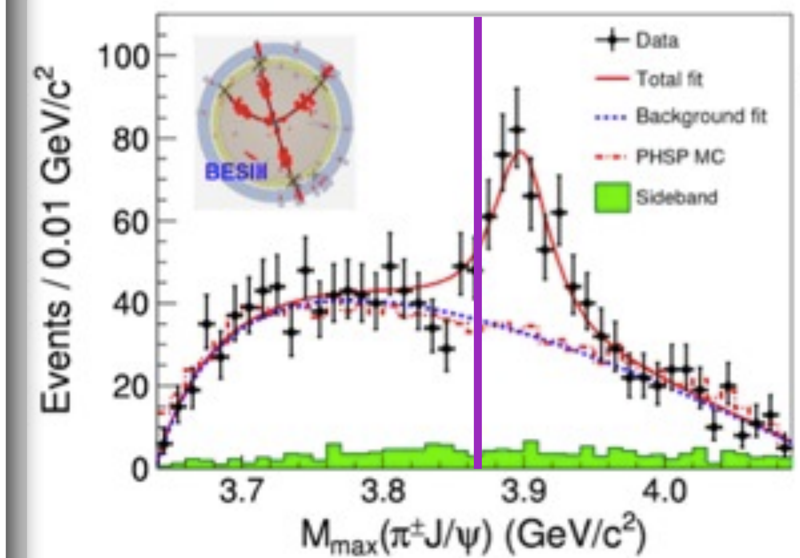
- $\pi J/\Psi$ invariant mass (m_π=410MeV)



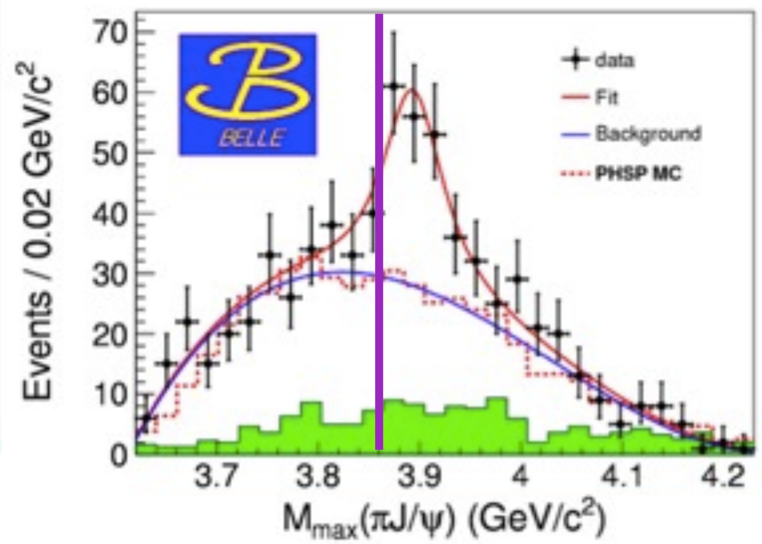
- $D^{bar}D^*$ invariant mass (m_π=410MeV)



- e^+e^- --> $\pi(\pi J/\Psi)$ @ 4.26GeV

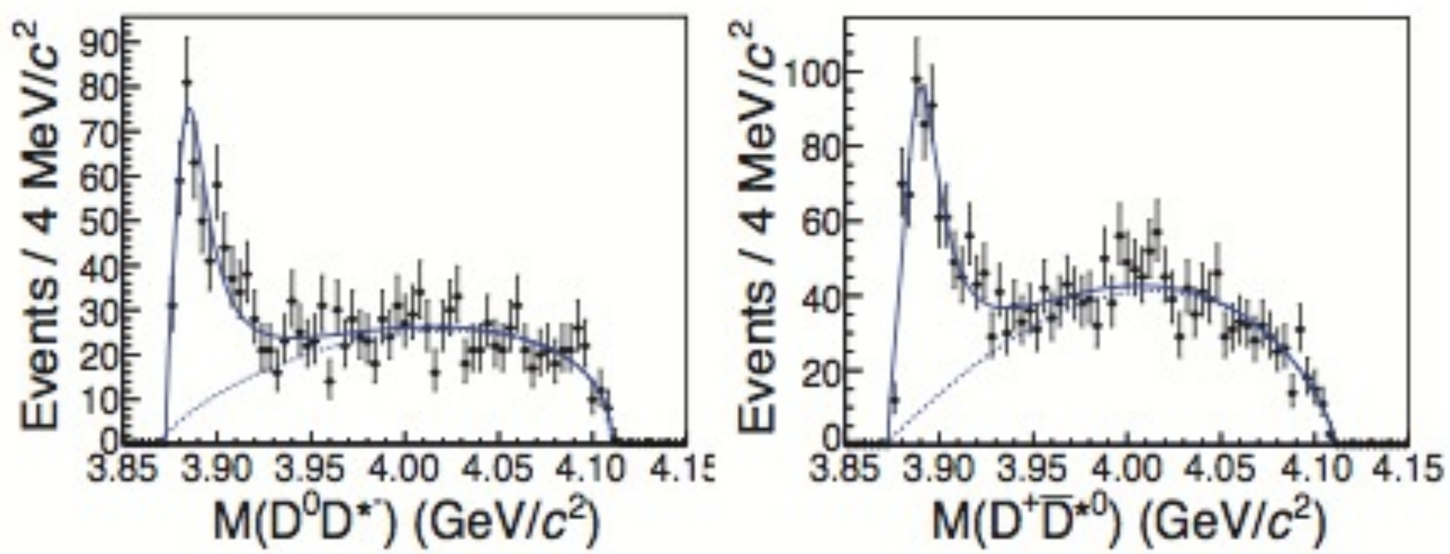


BESIII Coll., PRL110 (2013).



Belle Coll., PRL110 (2013).

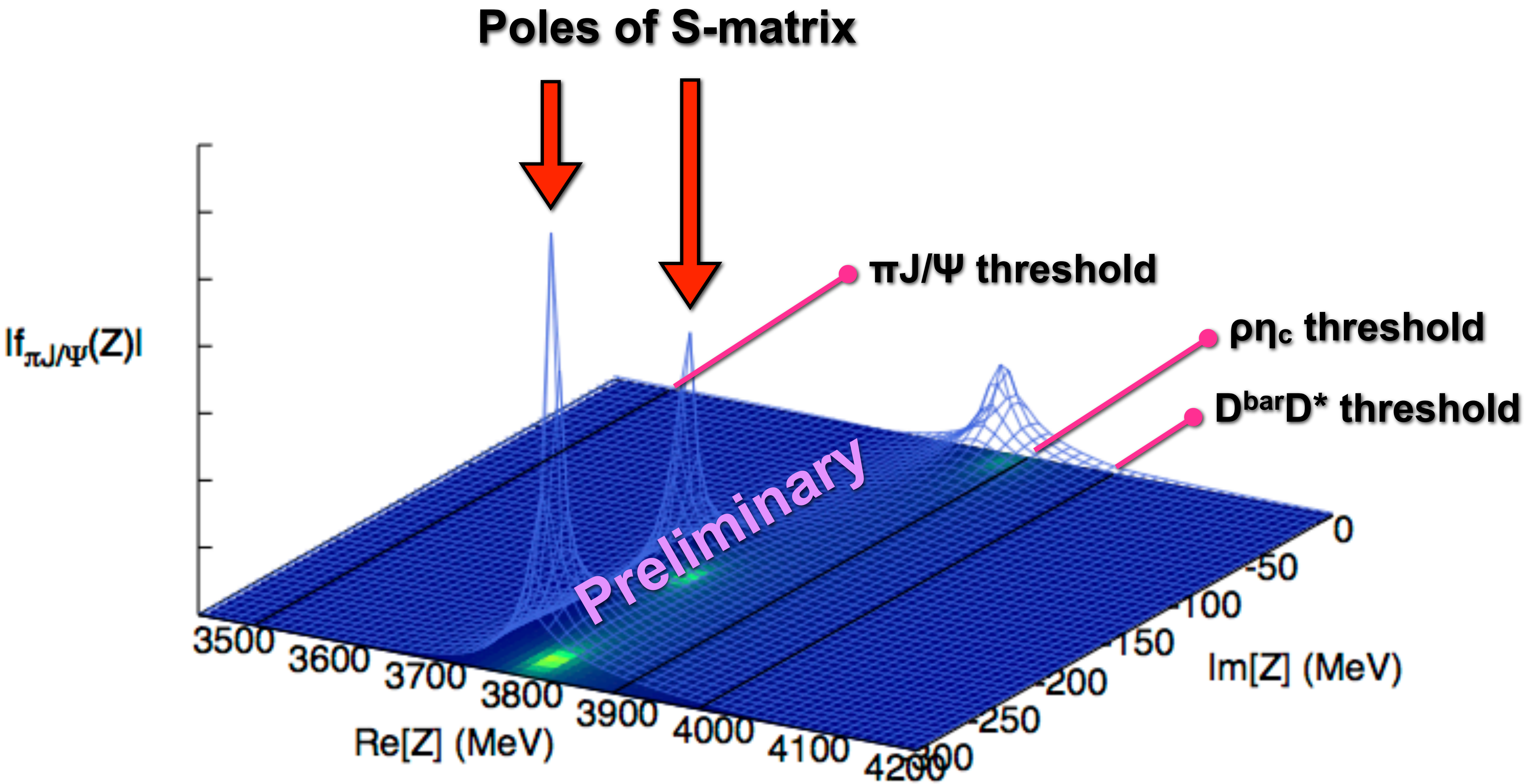
- e^+e^- --> $\pi^{+/-}$ $(D^{bar}D^*)^{-/+}$



BESIII Coll., PRL112 (2014).

✓ We observe similar line shapes of $\pi J/\Psi$ & $D^{bar}D^*$ inv. mass

Pole search ($\pi J/\Psi$:2nd, $\rho\eta_c$:2nd, $D^{bar}D^*$:2nd)



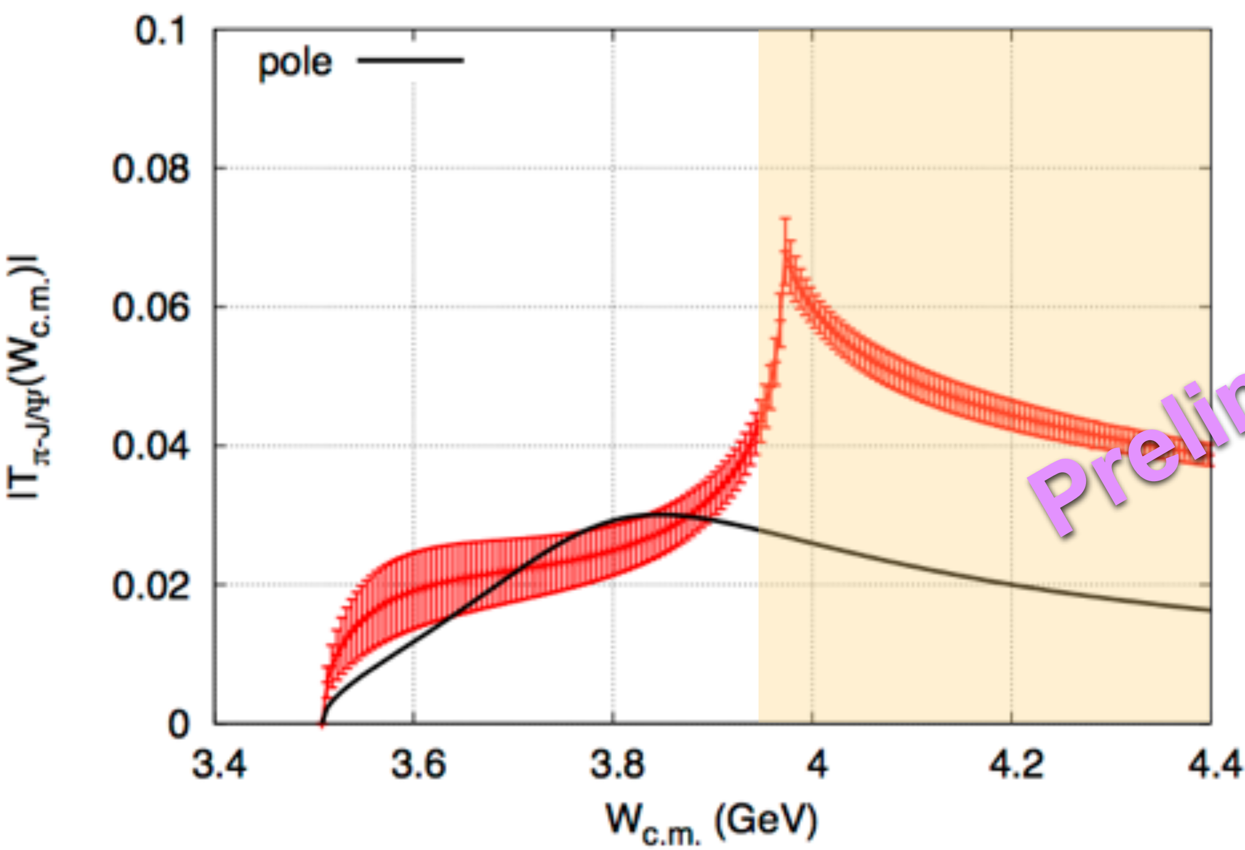
- ✓ Poles on the most adjacent complex energy plane for $Z_c(3900)$ are found
- ✓ How do these poles contribute to enhancement in T-matrix?

T-matrix of πJ/Ψ & $D^{bar}D^*$

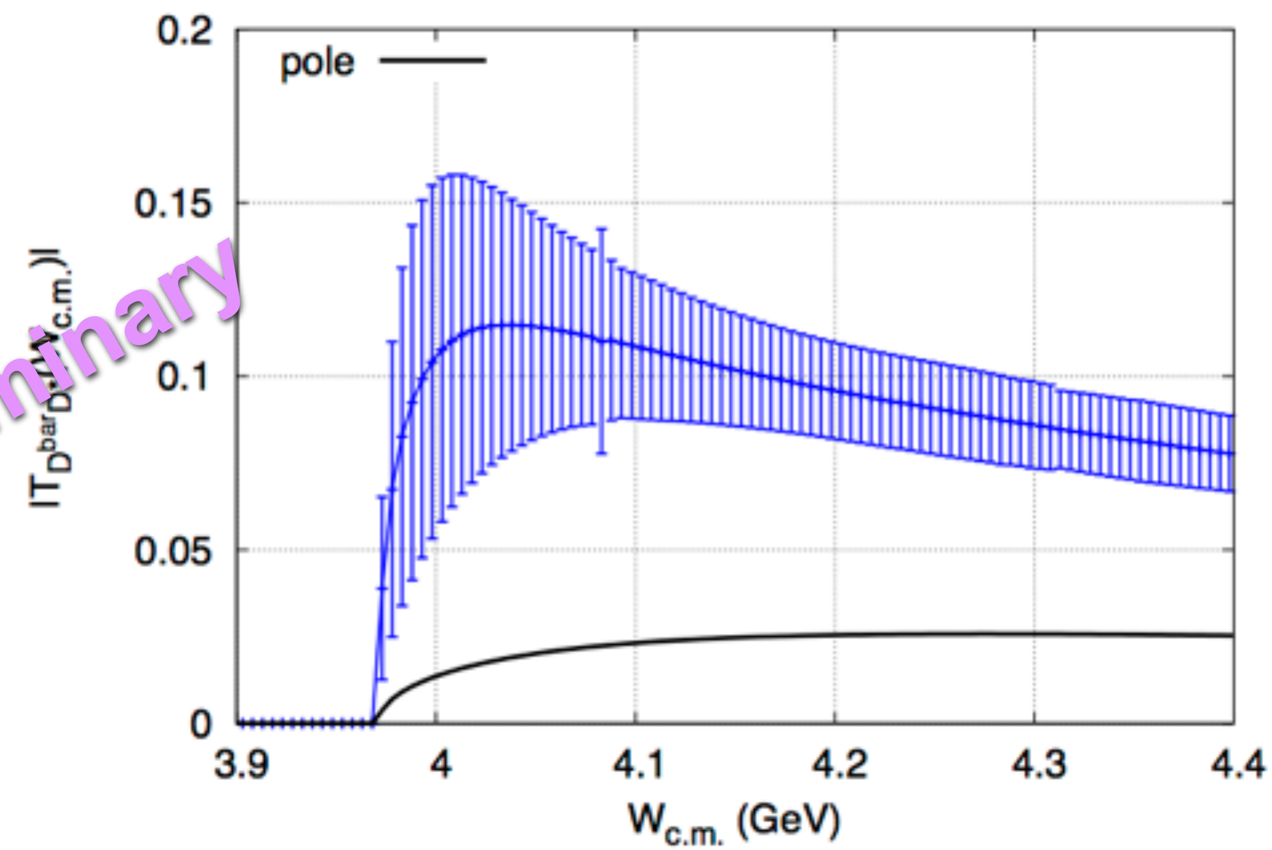
- calculate residues of T-matrix in each channel

$$S(k) = 1 + 2iT(k)$$

- πJ/Ψ-πJ/Ψ T-matrix (m_π=410MeV)



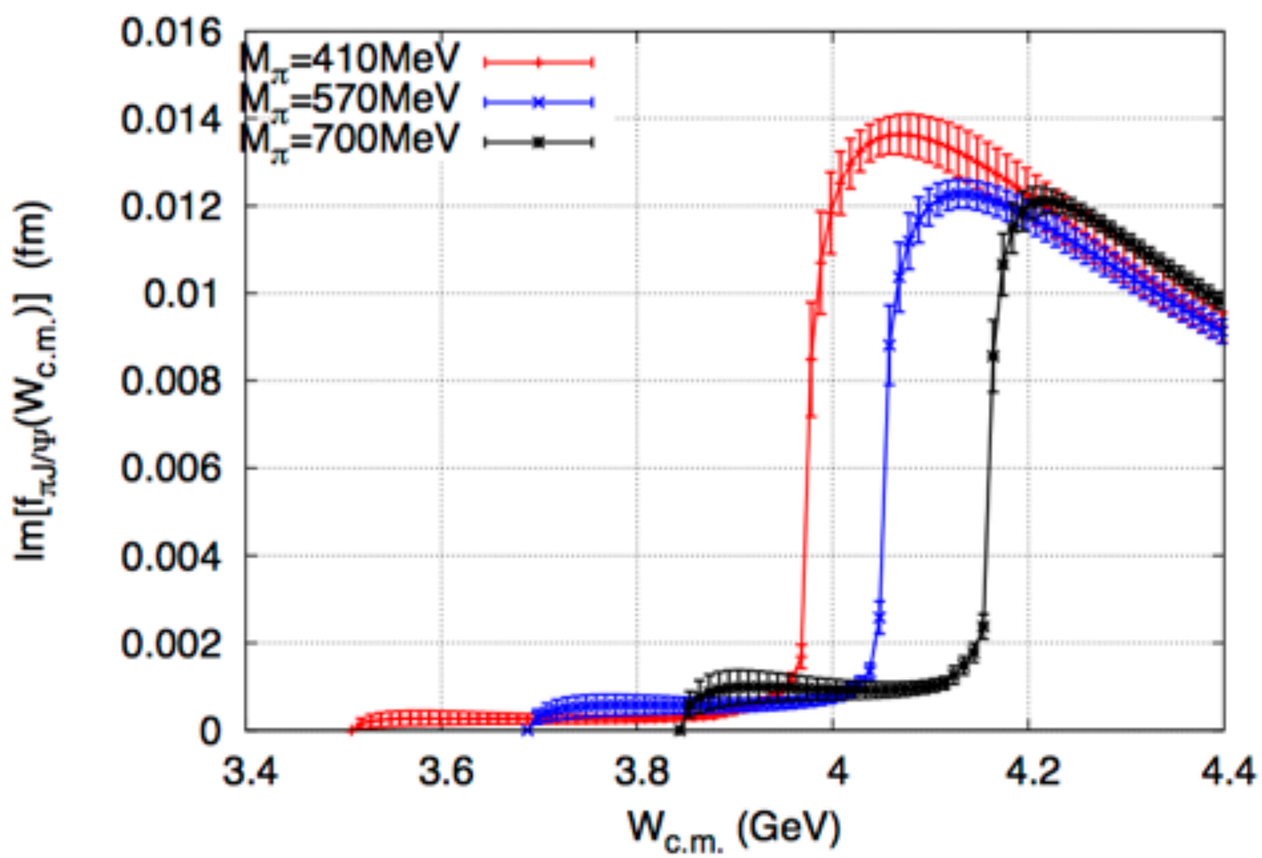
- $D^{bar}D^*$-$D^{bar}D^*$ T-matrix (m_π=410MeV)



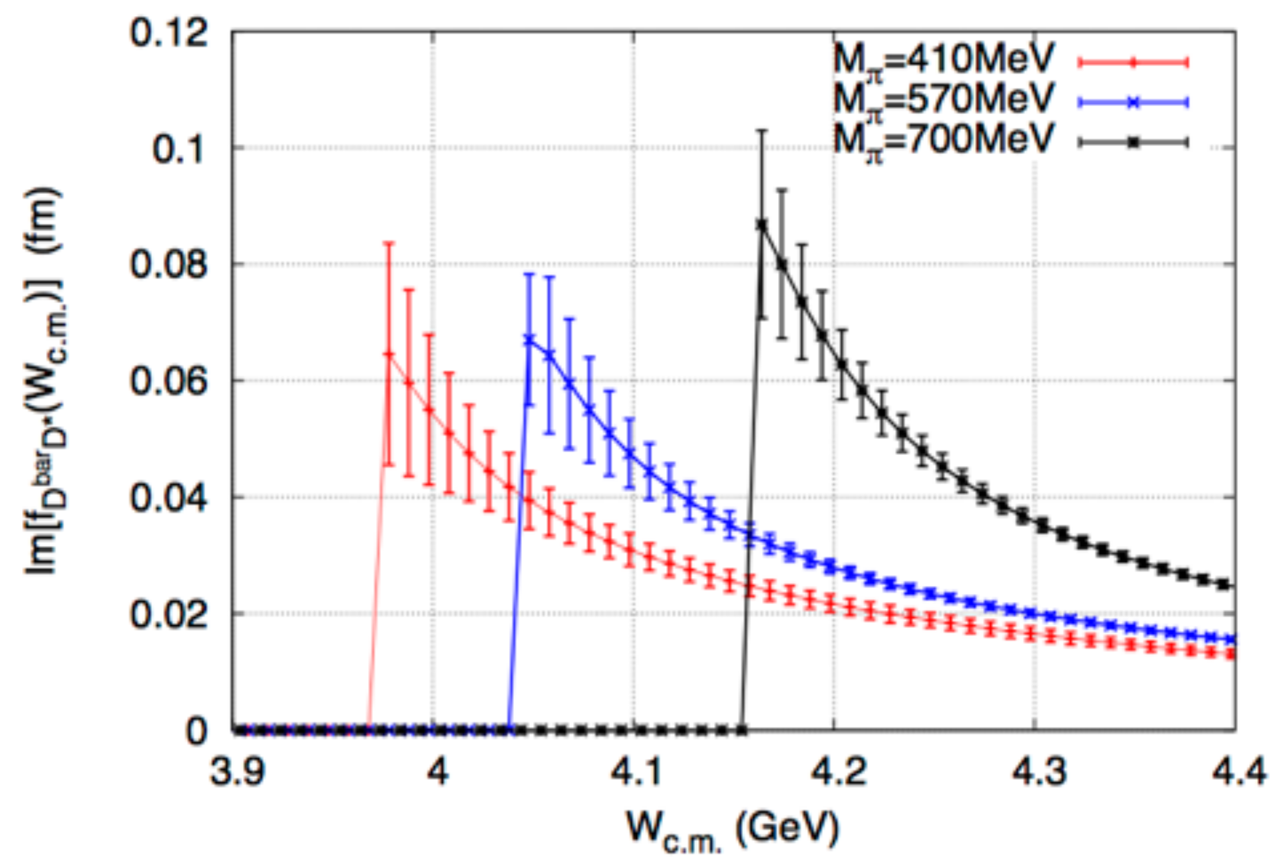
✓ sizable pole contributions (especially in πJ/Ψ channel)

Quark mass dependence

- **πJ/Ψ invariant mass**



- **$D^{bar}D^*$ invariant mass**



✤ **enhancement near $D^{bar}D^*$ threshold due to large πJ/Ψ-$D^{bar}D^*$ coupling**

✤ **No m_q dependence on qualitative behaviors of line shapes**

Summary

✤ Applications of HAL QCD method to tetra-quarks, Tcc & Zc(3900)

- T_{cc} search on the lattice@m_π=410--700MeV

➡ T_{cc} is not bound for m_π>400MeV (T_{bb} is already bound)

➡ sizable correlation of diquarks is found

- I=0 good diquark channel : attractive
- I=1 bad diquark channel : repulsive

- Zc(3900) in $I^G(J^P)=1^+(1^{+})$ channel on the lattice@m_π=410MeV

➡ Large channel coupling between πJ/Ψ and $D^{bar}D^*$ is a key

➡ Heavy quark spin symmetry is seen in c.c. potentials

- Zc(3900) is neither simple $D^{bar}D^*$ molecule nor J/Ψ + π-cloud
- shadow poles on complex energy plane are found (w/ relatively large width)

✤ Physical point simulation is the next step