Charmed Tetraquarks from Lattice QCD

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HAL QCD (Hadrons to Atomic nuclei from Lattice QCD)

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**Spectrum of charmonium(-like) system**

Quark potential models well describe mass spectra below open charm threshold

- Barnes, Godfrey, Swanson, PRD 72 (2005).

**“NEW”** charmonium-like (X, Y, Z) states:
- not within quark model spectrum
- candidates of exotic hadrons

**“Other”** exotic candidates (expected from quark models):
- doubly charmed tetra-quark, but experimentally not observed so far

**Our target: tetra-quark channels**

**“Tetraquark”** Tcc (ccu\textsuperscript{bar}d\textsuperscript{bar}) is manifest 4-quark channel

**“Charged”** charmonium-like states (cc\textsuperscript{bar} + π\textsuperscript{+/-}) require at least 4 quarks
Tcc bound state

\[ g \propto \gamma^\mu \frac{\lambda^a}{2} \approx \bar{c}\gamma^\mu \frac{\lambda^a}{2} \, cA^a \Rightarrow \text{color magnetic + color electric forces} \]

\[ \text{magnetic interactions} \ll \text{electric interactions} \]

\[ \Rightarrow \text{magnetic gluon coupling is suppressed by O}(1/m_c) \]

\[ \checkmark \text{Heavy quark spin symmetry} \]

- **Color magnetic interaction** is enhanced in light-quark sector

\[ V_{ij}^{\text{CMI}} \propto -\frac{(\vec{\lambda}(i) \cdot \vec{\lambda}(j))(\vec{\sigma}(i) \cdot \vec{\sigma}(j))}{M_i M_j} \]

- **Color-spin matrix elements**:

  \[ \langle v_{ij} \rangle = -\langle(\vec{\lambda}(i) \cdot \vec{\lambda}(j))(\vec{\sigma}(i) \cdot \vec{\sigma}(j))\rangle \]

<table>
<thead>
<tr>
<th>(&lt;v_{ij}&gt;)</th>
<th>C=1</th>
<th>C=8</th>
<th>C=3</th>
<th>C=6^{\text{bar}}</th>
</tr>
</thead>
<tbody>
<tr>
<td>S=0</td>
<td>-16</td>
<td>2</td>
<td>-8</td>
<td>4</td>
</tr>
<tr>
<td>S=1</td>
<td>16/3</td>
<td>-2/3</td>
<td>8/3</td>
<td>-4/3</td>
</tr>
</tbody>
</table>

\[ \Rightarrow I=0 \, [ud]-\text{diquark correlation (good diquark)} \Rightarrow \text{Tcc bound state?} \]
Charmonium-like $Z_c(3900)$

- **observed in $\pi^{\pm}/J/\Psi$ invariant mass (confirmed by CLOE-c)** *(Xiao et al., PLB727 (2013).)*
- $J^P = 1^+$ seems most probable *(BESIII Coll., PRL112 (2014).)*

★ **Structure of $Z_c(3900)$**

- Tetra-quark? *Maiani et al. (2013).*
- $D^{\text{bar}}D^*$ molecule? *Nieves et al. (2011) + many others*
- $c\bar{c} +$ meson cloud? *Voloshin (2008).*
- pole + meson cloud? *Wang et al. (2013).*
- cusp? *Chen et al. (2013), Swanson (2014).*

$D^{\text{bar}}D^* = 3872$ ★ $Z_c(3900)$

$\Delta = 640$

$\pi J/\Psi = 3232$

energy spectrum consistent with scattering


coupled-channel scattering from LQCD

Chen et al. (2013), Swanson (2014).
Contents

- Introduction
- HAL QCD method to define (coupled-channel) potentials
- Tcc in $I(J^P)=0,1(1^+)$ channels [DD* single-channel]
- Zc(3900) in $I(J^P)=1(1^+)$ [$\pi J/\Psi-\rho\eta c-D\overline{D}D^*$ coupled-channel]
- Summary

\[ DD^* = 3872 \]
\[ D\overline{D}D^* = 3872 \]
\[ Zc(3900) \]
\[ \pi J/\Psi = 3232 \]
\[ \Delta = 640 \]
Two identical methods for scattering

- **Lüscher's finite size formula**
  Interaction energy --> phase shift


- **NBS wave function**

- **Energy-independent potential**

  \[ k \cot \delta(k) = \frac{1}{a} - \frac{1}{2} r e k^2 + \ldots \]

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- **Guaranteed to be the same**

  Kurth et al., JHEP 1312 (2013) 015.

- **Scattering parameters**

  Aoki, Hatsuda, Ishii, PPTP123, 89 (2010).

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Two identical methods for scattering

- **Lüscher's finite size formula**
  Interaction energy $\rightarrow$ phase shift
  \[ k \cot \delta(k) = \frac{1}{a} - \frac{1}{2} r_k k^2 + \ldots \]

- **NBS wave function**

- **Energy-independent potential**

- **Guaranteed to be the same**
  Kurth et al., JHEP 1312 (2013) 015.

- **Scattering parameters**

- **LQCD potentials can be applied to...**
  properties of hadrons & nuclei, construction of EOS, etc.

\[ V(r) = 45 \text{[MeV]} \rightarrow 0 \text{[MeV]} \]

- E ~ 0 MeV

- E ~ 45 MeV

Aoki, Hatsuda, Ishii, PPTP123, 89 (2010).
Resonance from LQCD

T-matrix in formal scattering theory (N/D method)

\[ T^{-1}(\sqrt{s}) = V^{-1} + \frac{1}{2\pi} \int_{s}^{\infty} ds' \frac{\rho(s')}{s' - s} \]

Interaction part is not determined within scattering theory

\[ \Rightarrow \text{interactions faithful to phase shift in QCD} \]

Analyticity of T-matrix is uniquely determined

**Bound states (physical sheet, 1st)**
- binding energy --> T-matrix pole position
- coupling --> residue of pole

**Resonance/Virtual states (unphysical sheet, 2nd)**
- Analytic continuation of T-matrix
- resonance energy --> T-matrix pole position
- coupling --> (complex) residue of pole?
“Potentials” in QCD

Hadron 4pt functions & **Nambu-Bethe-Salpeter (NBS)** wave function

\[
\psi^{ab}(\vec{r}, \tau) = \sum_{\vec{x}} \langle 0 | \phi_1^a(\vec{x} + \vec{r}, \tau) \phi_2^a(\vec{x}, \tau) \mathcal{J}^{b\dagger}(\tau = 0) | 0 \rangle \\
= \sum_n A_n^b \exp \left[-W_n \tau\right] \sqrt{Z_1^a} \sqrt{Z_2^a} \psi_n^a(\vec{r})
\]

- Helmholtz eq. of NBS wave func.

\[
(\nabla^2 + (\vec{k}^a)^2) \psi_W^a(\vec{r}) = 0 \quad (|\vec{r}| > R)
\]

\[
\psi_W^{(l)}(r) \sim \frac{e^{i\delta_l(k)}}{kr} \sin(kr + \delta_l(k) - l\pi/2)
\]

- NBS wave func. in QFT ~ wave func. in Q.M.

- Coupled-channel potential matrix (faithful to phase shifts)

\[
(\nabla^2 + (\vec{k}^a)^2) \psi_n^a(\vec{r}) = 2\mu^a \sum_b \int d\vec{r}' U^{ab}(\vec{r}, \vec{r}') \psi_n^b(\vec{r}')
\]

- Coupled-channel potentials are energy-independent (non-local in general)

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HAL QCD method

✓ Definition of energy-independent coupled-channel potentials:

\[
\psi_n(\vec{r}) = \langle 0 | \phi_1^a(\vec{r} + \vec{x}) \phi_2^b(\vec{x}) | W_n; JP \rangle
\]

\[
(\nabla^2 + (\vec{k}^a)^2) \psi_n^a(\vec{r}) = 2\mu \sum_b \int d\vec{r}' U^{ab}(\vec{r}, \vec{r}') \psi_n^b(\vec{r}')
\]

✓ Since energy-independent potential can produce all scattering states, single-state saturations in simulations is not required

✓ Extract energy-independent potential from time-dependent Schrödinger-type eq.

\[
R^{ab}(\vec{r}, \tau) \equiv \psi^{ab}(\vec{r}, \tau) \frac{e^{(m_1^a + m_2^a)\tau}}{\sqrt{Z_1^a} \sqrt{Z_2^a}}
\]

\[
\delta = \frac{m_1^a - m_2^a}{m_1^a + m_2^a} \quad \Delta^{ae} = \frac{e^{(m_1^a + m_2^a)\tau}}{e^{(m_1^c + m_2^c)\tau}}
\]

\[
\left[ -\partial_\tau + \nabla^2/2\mu^a + \partial_\tau^2/8\mu^a + O(\delta^2) \right] R^{ab}(\vec{r}, \tau) = \sum_c \int d\vec{r}' \Delta^{ae} U^{ac}(\vec{r}, \vec{r}') R^{cb}(\vec{r}', \tau)
\]

✓ Velocity expansion:

\[
U(\vec{r}, \vec{r}') = V(\vec{r}, \nabla) \delta(\vec{r} - \vec{r}')
\]

\[
V(\vec{r}, \nabla) = V_C(\vec{r}) + \vec{L} \cdot \vec{S} V_{LS}(\vec{r}) + O(\nabla^2)
\]

✓ Calculate observable: phase shift, binding energy, pole position, ...
Tcc in $I(J^P) = 0, 1(1^+)$

Asymptotic states: $DD^*(s$-wave)
**Lattice QCD Setup**

**N_f=2+1 full QCD configurations generated by PACS-CS Coll.**

- Iwasaki gauge & $O(a)$-improved Wilson quark actions
- $a=0.0907(13)\ \text{fm} \rightarrow L\sim 2.9\ \text{fm} \ (32^3 \times 64)$

**Light meson mass [conf.1, conf.2, conf.3] (MeV)**
- $M_{\pi}=699(1), 572(2), 411(2) \ [\text{PDG:135} \ (\pi^0)]$
- $M_K=787(1), 714(1), 635(2) \ [\text{PDG:498} \ (K^0)]$

**Tsukuba-type Relativistic Heavy Quark (RHQ) action for charm quark**

- **Charmed meson mass [conf.1, conf.2, conf.3] (MeV)**
  - $M_{\eta_c}=3024(1), 3005(1), 2988(2) \ [\text{PDG:2981}]$
  - $M_{J/\Psi}=3142(1), 3118(1), 3097(2) \ [\text{PDG:3097}]$
  - $M_D=1999(1), 1946(1), 1902(3) \ [\text{PDG:1865} \ (D^0)]$
  - $M_{D^*}=2159(4), 2099(6), 2048(12) \ [\text{PDG:2007} \ (D^{*0})]$
S-wave DD* in I=1: “bad” diquark

- Repulsive s-wave potentials of DD*
- Weak quark mass dependence
- It is unlikely to form bound state even at physical point

Y. Ikeda et al. (HAL QCD), PLB729, 85 (2014).
S-wave DD* in I=0: “good” diquark

- **Attractive** S-wave potentials
- Attraction increases, as $m_q$ decreases
- Check whether bound $T_{cc}$ exist or not $\rightarrow$ phase shift analysis

Y. Ikeda et al. (HAL QCD), PLB729, 85 (2014).
S-wave phase shifts: $T_{cc}$ in $I=0$

- Attraction is not sufficiently strong to generate bound state
- Rapid increase at threshold of DD* phase shift --> effect of virtual state?

solve Schrödinger equation --> phase shifts

Y. Ikeda et al. (HAL QCD), PLB729, 85 (2014).

➡ examine pole position
Pole search w/ LQCD potential at $m_\pi = 410$ MeV

Virtual pole on the DD* unphysical energy plane ➔ threshold cusp of the amplitude ➔ rapid increase of scattering phase shift
$Zc(3900)$ in $I^G(J^P)=1^+(1^+)$
Lattice QCD setup

- $N_f=2+1$ full QCD configurations (PACS-CS) w/ $L=2.9\text{fm}$
  
  
  - Tsukuba-type RHQ action for charm quark


S. Aoki et al., PTP109, 383 (2003)

Y. Namekawa et al., PRD84, 074505 (2011)

- Thresholds in $I^GJ^P=1^+1^+$ channel

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<th>Light meson mass (MeV)</th>
<th>Charmed meson mass (MeV)</th>
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<td>$M_{\pi}=411(2)$ [PDG:135 ($\pi^0$)]</td>
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</tr>
<tr>
<td>$M_{\rho}=895(14)$ [PDG:775]</td>
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</tr>
</tbody>
</table>

| $M_{\eta_c}=2988(2)$ [PDG:2981] | $M_{\eta_c}=2988(2)$ [PDG:2981] |
| $M_{J/\Psi}=3097(2)$ [PDG:3097] | $M_{J/\Psi}=3097(2)$ [PDG:3097] |
| $M_D=1902(3)$ [PDG:1865 ($D^0$)] | $M_D=1902(3)$ [PDG:1865 ($D^0$)] |
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- **Light meson mass (MeV)**
  - $M_{\pi}=411(2)$ [PDG:135 ($\pi^0$)]
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  - $M_{J/\Psi}=3097(2)$ [PDG:3097]
  - $M_D=1902(3)$ [PDG:1865 ($D^0$)]
  - $M_{D^*}=2048(12)$ [PDG:2007 ($D^{*0}$)]

**LQCD simulation**

- $D^{\text{bar}}D^*=3951$
- $D^{\text{bar}}D^*=3872$
- $\rho\eta_c=3883$
- $\pi\Psi'=3821$
- $\pi\pi\eta_c=3256$
- $\pi J/\Psi=3508$
- $\pi J/\Psi=3232$

**Physical thresholds**

- $M_{\pi\Psi'} > M_{D^{\text{bar}}D^*}$ due to heavy pion mass
- $\rho\rightarrow\pi\pi$ decay not allowed in our setup

- S-wave $\pi J/\Psi - \rho\eta_c - D^{\text{bar}}D^*$ coupled-channel analysis is performed
Potential matrix \((\pi J/\psi - \rho\eta_c - D^\text{bar}D^*)\)

- All diagonal potentials are weak

\[ V_{\pi J/\psi-\pi J/\psi} \]

\[ V_{\rho\eta_c-\rho\eta_c} \]

\[ V_{D^\text{bar}D^*-D^\text{bar}D^*} \]

\[ \text{Weak} \rightarrow \text{no bound } D^\text{bar}D^* \]
Potential matrix \((\pi J/\Psi - \rho\eta_c - D^{\text{bar}}D^*)\)

- **Weak charm spin-flip potential**
- **Heavy quark spin symmetry**

(charm quark spin-flip amplitude is suppressed)
Potential matrix \((\pi J/\Psi - \rho \eta_c - D^{\text{bar}}D^*)\)

- Strong off-diagonal \(D^{\text{bar}}D^*\) potentials
- \(\checkmark\) strong charm-quark-exchange interactions
Potential matrix \((\pi J/\Psi - \rho\eta_c - D^{\text{bar}}D^*)\)

![Graphs and diagrams showing potential matrix entries for different particle interactions.](image)
Invariant mass spectra of $\pi J/\Psi$ & $D^{\text{bar}}D^*$

- $\pi J/\Psi$ invariant mass ($m_\pi=410\text{MeV}$)
- $D^{\text{bar}}D^*$ invariant mass ($m_\pi=410\text{MeV}$)

✓ enhancement near $D^{\text{bar}}D^*$ threshold due to large $\pi J/\Psi$-$D^{\text{bar}}D^*$ coupling

- peak in $\pi J/\Psi$ invariant mass
- enhancement (cusp?) in $D^{\text{bar}}D^*$ invariant mass
LQCD results & EXP. results

- **πJ/Ψ invariant mass (m_π=410MeV)**
  - ![Graph](image1)

- **D^0D^* invariant mass (m_π=410MeV)**
  - ![Graph](image2)

- **e^+e^- \rightarrow π(πJ/Ψ) @ 4.26GeV**
  - ![Graph](image3)

- **e^+e^- \rightarrow π^+/− (D^0D^*)^+/−**
  - ![Graph](image4)

✓ We observe similar line shapes of πJ/Ψ & D^0D^* inv. mass
Pole search ($\pi J/\Psi :2nd$, $\rho \eta_c :2nd$, $D^{\text{bar}}D^* :2nd$)

✓ Poles on the most adjacent complex energy plane for $Z_c(3900)$ are found
✓ How do these poles contribute to enhancement in $T$-matrix?
**T-matrix of \( \pi J/\Psi \) & \( \bar{D}D^* \)**

- calculate residues of T-matrix in each channel

\[
S(k) = 1 + 2iT(k)
\]

- \( \pi J/\Psi-\pi J/\Psi \) T-matrix (\( m_\pi = 410 \text{MeV} \))

- \( \bar{D}D^*-\bar{D}D^* \) T-matrix (\( m_\pi = 410 \text{MeV} \))

✓ sizable pole contributions (especially in \( \pi J/\Psi \) channel)

\( S(k) = 1 + 2iT(k) \)
Quark mass dependence

- $\pi J/\Psi$ invariant mass
- $D^{\text{bar}}D^*$ invariant mass

- enhancement near $D^{\text{bar}}D^*$ threshold due to large $\pi J/\Psi-D^{\text{bar}}D^*$ coupling
- No $m_q$ dependence on qualitative behaviors of line shapes
Summary

Applications of HAL QCD method to tetra-quarks, Tcc & Zc(3900)

- **Tcc search on the lattice @ m_\pi = 410-700 MeV**
  - Tcc is not bound for m_\pi > 400 MeV (T_{bb} is already bound)
  - sizable correlation of diquarks is found
    - l=0 good diquark channel: attractive
    - l=1 bad diquark channel: repulsive

- **Zc(3900) in I^G(J^P)=1^+(1^+) channel on the lattice @ m_\pi = 410 MeV**
  - Large channel coupling between \pi J/\Psi and D^{bar}D^* is a key
  - Heavy quark spin symmetry is seen in c.c. potentials
    - Zc(3900) is neither simple D^{bar}D^* molecule nor J/\Psi + \pi-cloud
    - shadow poles on complex energy plane are found (w/ relatively large width)

- **Physical point simulation is the next step**