Influence of threshold effects induced by heavy flavor meson rescattering

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Outline

- **Motivation**

- **Coupled-channel effects and threshold enhancement phenomena**
  - Dipion transitions $\text{e}^+\text{e}^- \rightarrow J/\psi \pi \pi, \psi' \pi \pi, h_c \pi \pi$
  - Zb production, hunting for some XYZ particles
  - Anomalous threshold singularity

- **Summary**
Motivation

- Observations of XYZ particles;
- Discrepancy between conventional quenched quark model predictions and experimental data;
- Coupled-channel effects will largely affect the mass and decay properties of heavy quarkonia;
Coupled-channel effects

✓ E. Eichten et al, PRD17,3090(1978), PRD73,104014(2006);
✓ M.R. Pennington, D.J. Wilson, PRD76,077502(2008);
✓ T. Barnes, E.S. Swanson, PRC77,055206(2008);
✓ B.Q. Li, C.Meng, K.T. Chao, PRD80,014012(2009);
✓ F.K. Guo et al; PRD83,034013(2011);
✓ ......
Unconventional states in heavy quarkonium region

arXiv:1411.7738
### Status of searches for new states

Table 11. Status of searches for the new states in $B$ decays, for several final states $f$, updated with respect to Drenska et al.$^{78}$ Final states where each exotic state was observed (S: “seen”) or excluded (NS: “not seen”) are indicated; F is reserved for final states which have been searched and not seen, but are forbidden by quantum numbers not known at the time of the analysis. A final state is marked as NP (“not performed”) if the analysis has not been performed in a given mass range and with MF (“missing fit”) if the spectra are published but a fit to a given state has not been performed. Finally “—” indicates that the known quantum numbers or available energy forbid the decay; and “hard” that an analysis is experimentally too challenging. As explained in Sec. 3.6, we consider a state $Y(3915)$ decaying into $J/\psi \omega$, seen both in $B$ decays and in $\gamma \gamma$ fusion, and a state $X(3940)$ seen in double charmonium production and decaying into $D\bar{D}^*$. “Vectors” indicates the $1^{--}$ states discovered via ISR not explicitly mentioned in the table.

<table>
<thead>
<tr>
<th>State</th>
<th>$J^{PC}$</th>
<th>$\psi\pi\pi$</th>
<th>$\psi\omega$</th>
<th>$\psi\gamma$</th>
<th>$\psi\phi$</th>
<th>$\psi'\pi\pi$</th>
<th>$\psi'\omega$</th>
<th>$\chi_c\gamma$</th>
<th>$p\bar{p}$</th>
<th>$\Lambda_c\bar{\Lambda}_c$</th>
<th>$D\bar{D}$</th>
<th>$D\bar{D}^*$</th>
<th>$D^<em>\bar{D}^</em>$</th>
<th>$D_s^{(<em>)}\bar{D}_s^{(</em>)}$</th>
<th>$\gamma\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X(3872)$</td>
<td>$1^{++}$</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>F</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>NS</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>F</td>
</tr>
<tr>
<td>$Y(3915)$</td>
<td>$0^{++}$</td>
<td>MF</td>
<td>S</td>
<td>NS</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>MF</td>
<td>F</td>
<td>MF</td>
<td>—</td>
<td>MF</td>
<td>NP</td>
<td>NP</td>
<td>NP</td>
</tr>
<tr>
<td>$Z(3930)$</td>
<td>$2^{++}$</td>
<td>MF</td>
<td>MF</td>
<td>NS</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>MF</td>
<td>—</td>
<td>MF</td>
<td>—</td>
<td>MF</td>
<td>MF</td>
<td>MF</td>
<td>NP</td>
</tr>
<tr>
<td>$Y(4140)$</td>
<td>$0^{++}$</td>
<td>MF</td>
<td>MF</td>
<td>NP</td>
<td>S</td>
<td>—</td>
<td>NP</td>
<td>—</td>
<td>—</td>
<td>MF</td>
<td>—</td>
<td>MF</td>
<td>NP</td>
<td>NP</td>
<td>NP</td>
</tr>
<tr>
<td>$X(4160)$</td>
<td>$0^{++}$</td>
<td>MF</td>
<td>MF</td>
<td>NP</td>
<td>MF</td>
<td>—</td>
<td>NP</td>
<td>—</td>
<td>—</td>
<td>MF</td>
<td>—</td>
<td>NF</td>
<td>NP</td>
<td>NP</td>
<td>NP</td>
</tr>
<tr>
<td>$X(4350)$</td>
<td>$1^{--}$</td>
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<td>—</td>
<td>—</td>
<td>MF</td>
<td>NP</td>
<td>—</td>
<td>MF</td>
<td>—</td>
<td>NP</td>
<td>—</td>
<td>NF</td>
<td>NP</td>
<td>NP</td>
<td>NP</td>
</tr>
<tr>
<td>$Y(4260)$</td>
<td>$1^{--}$</td>
<td>MF</td>
<td>—</td>
<td>—</td>
<td>MF</td>
<td>NP</td>
<td>—</td>
<td>MF</td>
<td>—</td>
<td>NP</td>
<td>—</td>
<td>NF</td>
<td>NP</td>
<td>NP</td>
<td>NP</td>
</tr>
<tr>
<td>vectors</td>
<td>$1^{--}$</td>
<td>MF</td>
<td>—</td>
<td>—</td>
<td>MF</td>
<td>NP</td>
<td>—</td>
<td>MF</td>
<td>—</td>
<td>NP</td>
<td>—</td>
<td>NF</td>
<td>NP</td>
<td>NP</td>
<td>NP</td>
</tr>
<tr>
<td>$Y(4660)$</td>
<td>$1^{--}$</td>
<td>NP</td>
<td>—</td>
<td>—</td>
<td>MF</td>
<td>NP</td>
<td>—</td>
<td>MF</td>
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<td>—</td>
<td>NF</td>
<td>NP</td>
<td>NP</td>
<td>NP</td>
</tr>
</tbody>
</table>

Table 12. Status of searches for the new states in ISR production for several final states $f$, updated with respect to Drenska et al.$^{78}$ In this table we consider the $Y(4630)$ decaying into $\Lambda_c\bar{\Lambda}_c$ and the $Y(4660)$ decaying into $\psi'\pi\pi$ to be the same state. The meaning of the symbols is explained in the caption of Table 11.
Y(4260)

BABAR 2005

CLEO 2006

Belle, 2007

observed in J/ψππ
Belle 2008 $e^+e^- \rightarrow \bar{D}D$

Belle, PRD79,092001; sum of $e^+e^- \rightarrow$ open charm

Dip at $Y_{4260}$!

$\chi^2$/d.o.f=1.08

R-value Scan, BES, 2007
Theoretical explanation

✓ Hybrid  S.L. Zhu; Close&Page; Kou & Pene

✓ Tetraquark  Ebert et al; Maiani et al

✓ Baryonium of \( \Lambda_c \) anti-\( \Lambda_c \)  C.F. Qiao

✓ \( \chi_{c0}\rho \), \( \chi_{c1}\omega \) molecule, sizeable coupling with \( \chi_{c0}\omega \)  Liu et al; Yuan et al; Dai et al

✓ Interference of other charmonium  Chen et al

✓ \( D_1D, D_0D^* \) molecular state  Close et al; Kalashnikova et al; G.J. Ding

✓ .......
Coupled-channel effects with P-wave states involved

✓ Combinations of S- and P-wave charmed mesons are very close to some conventional higher charmonia (ψ(4160), ψ(4415)) and Y(4260), Y(4360), Z(4430) ……);

✓ The coupling with the parity-odd charmonia could be S-wave, supposed to be strong;

<table>
<thead>
<tr>
<th></th>
<th>$D_0D^*$</th>
<th>$D_1'D$</th>
<th>$D_1'D^*$</th>
<th>$D_1D$</th>
<th>$D_1D^*$</th>
<th>$D_2D$</th>
<th>$D_2D^*$</th>
<th>$D_{s0}D_{s}^*$</th>
<th>$D_{s1}D_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold [MeV]</td>
<td>4325</td>
<td>4292</td>
<td>4434</td>
<td>4286</td>
<td>4428</td>
<td>4327</td>
<td>4470</td>
<td>4430</td>
<td>4424</td>
</tr>
</tbody>
</table>

Connections between coupled channel effects and XYZ?
Model based on HHChPT

Doublets with light degrees of freedom $j^P=1/2^-,1/2^+,3/2^+$

\[
H_a &= \frac{1 + \gamma^5}{2} \left[ D^*_a \gamma^\mu - D_a \gamma_5 \right], \\
S_a &= \frac{1 + \gamma^5}{2} \left[ D'^\mu_{1a} \gamma^\mu \gamma_5 - D^*_0 a \right], \\
T^\mu_a &= \frac{1 + \gamma^5}{2} \left\{ D^\mu\nu_{2a} \gamma^\nu \\
&- \sqrt{\frac{3}{2}} D_{1a\nu} \gamma_5 \left[ g^\mu\nu - \frac{1}{3} \gamma^\nu (\gamma^\mu - \nu^\mu) \right] \right\},
\]

HQSS allowed coupling  
(LDOF will also be conserved)

<table>
<thead>
<tr>
<th></th>
<th>HH</th>
<th>SH</th>
<th>TH</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi(nS)$</td>
<td>P-wave</td>
<td>S-wave</td>
<td>D-wave</td>
</tr>
<tr>
<td>$\psi(nD)$</td>
<td>P-wave</td>
<td>D-wave</td>
<td>S-wave</td>
</tr>
</tbody>
</table>
Leading Order Effective Lagrangian

According to HHChPT power counting

\[ \mathcal{L}_1 = \frac{g_T}{\sqrt{2}} < J^{\mu \nu} \bar{H}_a^\dagger \gamma_\nu \bar{T}_{a\mu} - J^{\mu \nu} \bar{T}_{a\mu}^\dagger \gamma_\nu \bar{H}_a > \\
+ ig_H < J^{\mu \nu} \bar{H}_a^\dagger \gamma_\mu \leftrightarrow \partial_\nu \bar{H}_a > \\
+ g_S < J \bar{S}_a^\dagger \bar{H}_a + J \bar{H}_a^\dagger \bar{S}_a > \\
+ C_S < J \bar{H}_b^\dagger \gamma_\mu \gamma_5 \bar{H}_a \mathcal{A}_{ba}^\mu > \\
+ iC_P < J^{\mu \nu} \bar{H}_b^\dagger \sigma_{\mu \nu} \gamma_5 \bar{H}_a \mathcal{A}_{ba}^\nu > + h.c., \\
\mathcal{L}_2 = \frac{i}{\Lambda_\chi} \hbar' < \bar{H}_a T_b^{\mu \nu} \gamma_5 (D_\mu \mathcal{A}_\nu + D_\nu \mathcal{A}_\mu)_{ba} > \\
+ ih < \bar{H}_a S_b \gamma_\mu \gamma_5 \mathcal{A}_{ba}^\mu > + ig < H_b \gamma_\mu \gamma_5 \mathcal{A}_{ba}^\mu \bar{H}_a > . \\
\]

\[ J = \frac{1 + \sqrt{y}}{2} [\psi(nS)^\mu \gamma_\mu] \frac{1 - \sqrt{y}}{2}, \]

\[ J^{\mu \nu} = \frac{1 + \sqrt{y}}{2} \left\{ \psi(nD)\alpha \left[ \frac{1}{2} \sqrt{\frac{3}{5}} [(\gamma^\mu - v^\mu)g^{\alpha \nu} + (\gamma^\nu - v^\nu)g^{\alpha \mu}] - \sqrt{\frac{1}{15}} (g^{\mu \nu} - v^\mu v^\nu) \gamma^\alpha \right] \right\} \frac{1 - \sqrt{y}}{2} \]
Dipion Transitions

\[
\begin{align*}
(A) & : \text{Tree} & (B) & : \text{THH} \\
(C) & : \text{HHH} & (D) & : \text{SHH}
\end{align*}
\]

**\(\psi(4160)\) as the input \(\psi(nD)\):**

✓ Widely accepted as a conventional \(^2^3D_1\) charmonia
✓ Couple to TH via S-wave, respect HQSS
✓ Close to \(Y(4260)\)

\[M=4153\pm3\text{ MeV}, \Gamma=103\pm8\text{ MeV} \quad \text{PDG averaged}\]
\[M=4191.7\pm6.5\text{ MeV}, \Gamma=71.8\pm12.3\text{ MeV} \quad \text{BES, PLB660,315(2008)}\]
\[M=4193\pm7\text{ MeV}, \Gamma=79\pm14\text{ MeV} \quad \text{X.H. Mo et al, PRD82,077501(2010)}\]

\[
\Gamma_{ee}=0.83\pm0.07\text{ KeV}, \quad \text{not small}
\]
THH Loop

Triangle singularity (TS) may occur under special kinematic configurations

(B) : $THH$

I) $\{D_1 D \ [D^*]\}$,

II) $\{D_1 D^* \ [D^*]\}$,

III) $\{D_2 D^* \ [D]\}$,

IV) $\{D_2 D^* \ [D^*]\}$,

In the heavy quark limit

$\mathcal{M}^I : \mathcal{M}^{II} : \mathcal{M}^{III} : \mathcal{M}^{IV} = 1 : \frac{1}{2} : -\frac{1}{5} : \frac{3}{10}$. 
Triangle Singularity

Largely isospin violation in $\eta(1405/1475) \to 3\pi$
$Br \sim 10\%$, [BESIII, PRL108, 182001 (2012)]


References:
Landshoff and Treiman, Nuovo Cimento 19, 1249 (1961)
Eden et al., <<The Analytic S-Matrix>>, 1966
$e^+ e^- \rightarrow J/\psi \pi\pi$ via $\psi(4160)$ and THH loops
$e^+e^- \rightarrow \psi'\pi\pi$ via $\psi(4160)$ and THH loops

- Results are sensitive to kinematics
- Same dynamics, different kinematics
- Direct prediction to check the mechanism

BarBar, PRL98,212001(2007)
$e^+ e^- \rightarrow h_c \pi\pi$ via $\psi(4160)$ and THH loops

C.Z. Yuan, arXiv:1312.6399

Belle, PRL111, 242001(2013)
Comparison with Molecule Ansatz

✓ Similar points: kinematics, singularities of rescattering loops
✓ Different points:
  • Incorporates the $D_1D, D_1D^*, D_2D^*$ combinations in a single Lagrangian with the relative phase and coupling strength fixed in the heavy quark limit;
  • No matter whether the molecular state exist or not, it seems to be natural to suppose the coupled channel effects should exist physically

$Y(4260)$ as molecular state:
✓ $D_1D, D_0D^*$ molecular state, F. Close et al, PRL102,242003(2009), PRD81,074033(2010); Kalashnikova and Nefediev, PRD77,054025(2008);
✓ Potential model, G.J. Ding PRD79,014001(2009)
✓ ......
No obvious cusp around 3.9GeV, inconsistent with CLEOc result

Broad width of $D_0$ and $D_1'$ will lower the amplitude and smooth the cusps
Zb Production

Cusps at BB* and B*B* thresholds \leftrightarrow Production of Zb(10610) and Zb(10650)
Hunting For Partners of Y(4160) & Y(4274)

Isospin violation processes

Cusps at $D_sD_s^*$ and $D_s^*D_s^*$ thresholds ($C=-1$)

Production of partners of Y(4160) and Y(4274) (observed in J/$\psi\phi$, $C=+1$)
Anomalous Threshold Singularity

✓ Singularity in the complex space

Landau Equation

\[ I_3 = \prod_{i=1}^{3} \int_0^1 da_k \frac{\delta(1 - \sum_k a_k)}{D - i\epsilon} \]

\[ D = \sum_{i,j} a_i a_j Y_{ij}, \quad Y_{ij} = \frac{1}{2} \left[ m_i^2 + m_j^2 - (q_{i-1} - q_{j-1})^2 \right] \]

Necessary conditions

\[ D = 0, \]  \[ \text{either } a_j = 0 \text{ or } \frac{\partial D}{\partial a_j} = 0. \]

The position of the singularity is obtained by solving

\[ \det Y_{ij} = 0 \]
Anomalous Threshold Singularity

\[ s_1 = p_1^2, \quad s_2 = p_2^2, \quad s_3 = p_3^2 \]

Normal threshold
\[ s_{2n} = (m_1 + m_3)^2 \]

Anomalous threshold
\[ s_2^\pm = (m_1 + m_3)^2 + \frac{1}{2m_2^2} [2m_2m_3(m_1^2 + m_2^2 - s_3) - 4m_2^2m_1m_3 \pm \lambda^{1/2}(s_1, m_2^2, m_3^2) \lambda^{1/2}(s_3, m_1^2, m_2^2)] \]
Anomalous Threshold Singularity

Single dispersion relation

\[ \Gamma(s_1, s_2, s_3) = \frac{1}{\pi} \int_0^\infty \frac{d s'_2}{(m_1 + m_3)^2} \frac{d s'_2}{s'_2 - s_2 - i \epsilon} \sigma_2(s_1, s'_2, s_3) \]

\[ \sigma_2 = \sigma_+ - \sigma_- \]

\[ \sigma_{\pm}(s_1, s_2, s_3) = \frac{1}{16 \pi \lambda^{1/2}(s_1, s_2, s_3)} \log[-s_2(s_1 + s_3 - s_2 + m_1^2 + m_3^2 - 2m_2^2) - (s_1 - s_3)(m_1^2 - m_3^2) \pm \lambda^{1/2}(s_1, s_2, s_3) \lambda^{1/2}(s_2, m_1^2, m_3^2)] \]

Brach points of the log function is at \( s_{2}^{\pm} \)

Work in the kinematical region

\[ s_1 \leq (m_2 + m_3)^2, \quad s_3 \leq (m_2 - m_1)^2 \]
Anomalous Threshold Singularity

Double dispersion relation

\[ \Gamma(s_1, s_2, s_3) = \int_{(m_2 + m_3)^2}^{\infty} \frac{ds'_1}{\pi(s'_1 - s_1 - i\epsilon)} \int_{s_2^-}^{s_2^+} \frac{ds'_2}{\pi(s'_2 - s_2 - i\epsilon)} \Delta(s'_1, s'_2, s_3) \]

\[ + 2 \int_{s_1^0}^{\infty} \frac{ds'_1}{\pi(s'_1 - s_1 - i\epsilon)} \int_{s_2^0}^{s_2^L} \frac{ds'_2}{\pi(s'_2 - s_2 - i\epsilon)} \Delta(s'_1, s'_2, s_3) \]

Non-Landau type contribution

Double spectral function can be obtained according to Cutkosky rule

\[ \Delta(s_1, s_2, s_3) = \frac{1}{16\lambda^{1/2}(s_1, s_2, s_3)} \]
Anomalous Threshold Singularity

When

\[ s_1 = (m_2 + m_3)^2 \]

\[ s_2^\pm = (m_1 + m_3)^2 + \left[ \frac{m_3}{m_2} [(m_2 - m_1)^2 - s_3] \right] \]

How to amplify the discrepancy between normal and anomalous threshold?

If the discrepancy is larger, it could be used to distinguish the cusp effects and molecular states.
Anomalous Threshold Singularity

\[ \Gamma(K^*) = 0 \]

\[ D_s(2860) \rightarrow DK^*[\pi] \rightarrow DK\pi \]
Anomalous Threshold Singularity

For investigating \( J/\psi\pi \) interaction
Summary

- The lineshape behavior of the cross sections and distributions for the dipion transitions are studied. Coupled-channel effects may largely affect the threshold behavior, especially that induced by the couplings between D-wave charmonia and TH charmed mesons, taking into account these leading order S-wave couplings will respect HQSS.

- Some interesting cusps are obtained, which may have some underlying connections with the XYZ states observed around the TH and HH thresholds.

- Kinematics plays a crucial role in generating the cusps.

- Anomalous threshold singularity would be used to discriminate coupled-channel effects and genuine resonances.
Thanks!