Λ-Λ Interaction from Relativistic Heavy Ion Collisions

Kenji Morita
(YITP, Kyoto)

in Collaboration with
Akira Ohnishi (YITP)
Takenori Furumoto (Ichinoseki National Col.)

Outline

Goal : $\Lambda\Lambda$ interaction from HIC

1. Introduction
2. Effect of $\Lambda\Lambda$ interaction (w/ simplest source)
3. +Collective Expansion
   1. Spherical source
   2. Bjorken and transverse expansion
4. Feed-down Contribution
5. Residual Correlation
6. Summary
Role of $\Lambda\Lambda$ Interaction

Possible Emergence of Hyperons in NS core

To understand EoS, Information on Hyperon-Hyperon Interaction is indispensable

$H$-dibaryon (uuddss)?

$\Lambda-\Lambda$ bound state due to strong attraction?

Resonance?
RHIC Can Tell Us about $\Lambda\Lambda$ interaction

Au+Au 200AGeV

$$\frac{dN_\Lambda}{dy} \simeq 0.6 - 13 \times 10^8\text{ events}$$

(60-80%) (0-5%)

Initial state
Energy Stopping
Hard Collisions
Hydrodynamic
Evolution
Hadron Freezeout

$C(Q = p_1 - p_2) = \frac{N_2(p_1, p_2)}{N(p_1)N(p_2)}$

$V(r)$

$p_1$

$p_2$

Hadronization
\(~5\text{ fm/c, }\sim 160\text{MeV}\)

\text{Time}
**ΛΛ Correlation in HIC**

Independent (Chaotic) emission

(←Thermal Source)

Identical particle correlation from quantum statistics (HBT effect)

\[ C(Q) : \text{effective source size} \]

ΛΛ Interaction : No Coulomb!

Affects \( C(Q) \) when effective range \( r_{\text{eff}} \) is comparable with \( R \)

Different results for repulsive and attractive interaction

\[ \sim \frac{1}{R} \]
**Approach : Thermal Source + \( V_{LL} \)**

**Formula** (Gong et al., ‘91)

\[
C_2(Q, K) = \frac{\int d^4x_1 d^4x_2 S(x_1, K)S(x_2, K)|\Psi_{12}(Q, x_1 - x_2 - (t_2 - t_1)K/m)|^2}{\int d^4x_1 d^4x_2 S(x_1, k_1)S(x_1, k_2)}
\]

- **Emission source func.**
  - Thermal source model (Mimic hydro)
    - Static Spherically Symmetric
    - Spherically Symmetric + Hubble Flow
    - Cylindrically Symmetric + Boost-invariance + Transverse flow

- **\( \Lambda\Lambda \) relative S-wave func.**
  - Various potentials (via 2 or 3 range Gaussian Fit)
    - Meson exchange models (Nijmegen model D, F, Soft Core89/97, ESC08)
    - Phenomenological (Ehime)
    - Quark model (fss2)
    - Fit to \( \Lambda \Lambda ^6 \text{He} \)(Nagara) Filikhin-Gal (FG)
    - Hiyama et al. (HKMYY)
Potential, Wave func., / Correlation

Correlation Function for the Static Source

$$C_{\text{stat}}(Q) = 1 - \frac{1}{2} e^{-Q^2 R^2} + \frac{1}{4 \sqrt{\pi R^3}} \int_0^\infty dr \ r^2 e^{-\frac{r^2}{4R^2}} \left[ \left| \chi_Q(r) \right|^2 - \left| j_0 \left( Qr/2 \right) \right|^2 \right]$$

HBT (size R)

Interaction: deviation from free w.f.

Input $V_{\Delta\Delta}$
Expanding Source Model

\[ S(x, k) \propto m_T \cosh(y - Y_L) n_F(u \cdot k / T) \exp \left[ -\frac{(\tau - \tau_0)}{2(\Delta \tau)^2} - \frac{x^2 + y^2}{2R^2} \right] \]

\[ \sim \exp \left[ -\frac{\gamma_T M_T}{T} \cosh(y - Y_L) \right] \exp \left[ \frac{\gamma_T k_T \cos \phi}{T} \right] \]

Give a finite longitudinal extent

\[ R_L \sim \tau_0 \sqrt{\frac{T}{M_T}} \]

Width of \( C(Q) \) (\( Q = |p_1 - p_2| \)) : effective 3d size

Large \( R_L \) : narrow width

Transverse flow: fit to \( p_t \) distribution of \( \Lambda \) (STAR ’12)
Illustration: Spherical Source

Expansion makes the apparent source size smaller

Attraction in C(Q) becomes stronger
Effect of $\eta_f$ is rather small: longitudinal expansion dominates $C(Q)$.

Behavior at small $Q$ is different from the static source!
Combined effects from flow and $V_{\Lambda\Lambda}$

Effect of $\eta_f$ is absorbed into change of $R_{opt}$

Longitudinal expansion gives another type of “best fit”

$fss2$, Optimized (min. $\chi^2$ against $R$)

- $\eta_f=0.0$, Spherical, $R=1.2\text{ fm}$
- $\eta_f=0.33$, Spherical, $R=1.6\text{ fm}$
- $\eta_f=0.0$, Cylindrical, $R=0.6\text{ fm}$
- $\eta_f=0.33$, Cylindrical, $R=0.7\text{ fm}$

STAR 0-80%
$V_{\Lambda\Lambda}$ from Expanding Source Model

Same $V_{\Lambda\Lambda}$ are chosen with smaller $R$!
Feed-Down Contribution

Assume $\Xi$ is not included

- DCA $< 0.4$ cm from the primary vertex

$\Sigma^0 \to \Lambda + \gamma$

$C(Q) \to 1 + \left(\frac{\Lambda^{\text{dir}}}{\Lambda^{\text{tot}}}\right)^2 (C(Q) - 1)$

$(0.67)^2$

consistent w/ thermal model

Using $\Sigma^0/\Lambda = 0.278$ (p+Be data) and $\Xi/\Lambda = 0.85$ (RHIC), $\Lambda^{\text{dir}}/\Lambda^{\text{tot}} = 0.67$

(0.52 if including $\Xi$)
Feed-Down Effect

Sensitivity at low Q is reduced

Unphysically small size is preferred; due to the long tail in C(Q) which cannot be included in the present framework.
Long Tail : Residual Correlation?

Assuming a short-range correlation

\[ C(Q) \rightarrow C(Q) + a_{\text{res}} e^{-r_{\text{res}}^2 Q^2} \]

- Search for min \( \chi^2 \) in \((a_{\text{res}}, r_{\text{res}})\) for each \( R \)

- \( \chi^2/N_{\text{dof}} \approx 1 \) for shown \( R \) range

- Small \( R \) : Large \( r_{\text{res}} > R \)

Low Q behavior \( \leftrightarrow \) \( \Lambda \Lambda \Lambda \) Interaction
An Example for Agreement

Cylindrical, $+C_{res}(Q)$
\[ \lambda = (0.67)^2 \]

Smaller difference than error-bars
FG and fss2 have the almost same $a_0$

\[ \text{fss2, R=2.5fm} \]
\[ \text{ESC08, R=2.5fm} \]
\[ \text{FG, R=2.5fm} \]
\[ \text{HKMY, R=2.5fm} \]
\[ \text{STAR 0-80\%} \]
Constraints on $a_0$ and $r_{\text{eff}}$

$1/a_0 < -0.8 \, [\text{fm}^{-1}]$

No constraint on $r_{\text{eff}}$

$\chi^2 / N_{\text{dof}} < 1$
Summary and Outlook

- RHIC has potential to determine $\Lambda \Lambda$ interaction
- Expansion effect modifies low Q behavior
- Ideal measurements (i.e., decay contribution is subtracted) will give strong constraints
- Feed-down effect reduces resolving power
- Long-tail in STAR data needs to be subtracted and its origin needs to be understood
- Scattering length $1/a_0 < -0.8 \text{ fm}^{-1}$
Backup
Analysis by STAR Coll.

Fit w/ Lednicky-Lyuboshitz model

Data: long tail (→ Small source size)

Introduce “residual correlation” for a better fit to data

\[
\chi^2/N_{dof} = 0.56
\]

\[
a_0 = -1.10 \pm 0.37^{+0.68}_{-0.08} \text{ fm}
\]

\[
r_{\text{eff}} = 8.52 \pm 2.56^{+2.09}_{-0.74} \text{ fm}
\]

\[
C_{\text{fit}}(Q) = N \left[ 1 + \lambda \left\{ -\frac{1}{2} e^{-r_0^2 Q^2} + \chi(a_0, r_{\text{eff}}) \right\} + a_{\text{res}} \exp(-r_{\text{res}}^2 Q^2) \right]
\]

HBT (size \(r_0\))

Interaction

Residual correlation

1.006

0.18

2.96 fm

-0.044

0.43 fm
<table>
<thead>
<tr>
<th>Model</th>
<th>$a_0$ (fm)</th>
<th>$r_{\text{eff}}$ (fm)</th>
<th>$\mu_1$ (fm)</th>
<th>$V_1$ (MeV)</th>
<th>$\mu_2$ (fm)</th>
<th>$V_2$ (MeV)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>ND46</td>
<td>4.621</td>
<td>1.300</td>
<td>1.0</td>
<td>-144.89</td>
<td>0.45</td>
<td>127.87</td>
<td>[18] $r_c = 0.46$ fm</td>
</tr>
<tr>
<td>ND48</td>
<td>14.394</td>
<td>1.633</td>
<td>1.0</td>
<td>-150.83</td>
<td>0.45</td>
<td>355.09</td>
<td>[18] $r_c = 0.48$ fm</td>
</tr>
<tr>
<td>ND50</td>
<td>-10.629</td>
<td>2.042</td>
<td>1.0</td>
<td>-151.54</td>
<td>0.45</td>
<td>587.21</td>
<td>[18] $r_c = 0.50$ fm</td>
</tr>
<tr>
<td>ND52</td>
<td>-3.483</td>
<td>2.592</td>
<td>1.0</td>
<td>-150.29</td>
<td>0.45</td>
<td>840.55</td>
<td>[18] $r_c = 0.52$ fm</td>
</tr>
<tr>
<td>ND54</td>
<td>-1.893</td>
<td>3.389</td>
<td>1.0</td>
<td>-147.65</td>
<td>0.45</td>
<td>1114.72</td>
<td>[18] $r_c = 0.54$ fm</td>
</tr>
<tr>
<td>ND56</td>
<td>-1.179</td>
<td>4.656</td>
<td>1.0</td>
<td>-144.26</td>
<td>0.45</td>
<td>1413.75</td>
<td>[18] $r_c = 0.56$ fm</td>
</tr>
<tr>
<td>ND58</td>
<td>-0.764</td>
<td>6.863</td>
<td>1.0</td>
<td>-137.74</td>
<td>0.45</td>
<td>1666.78</td>
<td>[18] $r_c = 0.58$ fm</td>
</tr>
<tr>
<td>NF42</td>
<td>3.659</td>
<td>0.975</td>
<td>0.6</td>
<td>-878.97</td>
<td>0.45</td>
<td>1048.58</td>
<td>[19] $r_c = 0.42$ fm</td>
</tr>
<tr>
<td>NF44</td>
<td>23.956</td>
<td>1.258</td>
<td>0.6</td>
<td>-1066.98</td>
<td>0.45</td>
<td>1646.65</td>
<td>[19] $r_c = 0.44$ fm</td>
</tr>
<tr>
<td>NF46</td>
<td>-3.960</td>
<td>1.721</td>
<td>0.6</td>
<td>-1327.26</td>
<td>0.45</td>
<td>2561.56</td>
<td>[19] $r_c = 0.46$ fm</td>
</tr>
<tr>
<td>NF48</td>
<td>-1.511</td>
<td>2.549</td>
<td>0.6</td>
<td>-1647.40</td>
<td>0.45</td>
<td>3888.96</td>
<td>[19] $r_c = 0.48$ fm</td>
</tr>
<tr>
<td>NF50</td>
<td>-0.772</td>
<td>4.271</td>
<td>0.6</td>
<td>-2007.35</td>
<td>0.45</td>
<td>5678.97</td>
<td>[19] $r_c = 0.50$ fm</td>
</tr>
<tr>
<td>NF52</td>
<td>-0.406</td>
<td>8.828</td>
<td>0.6</td>
<td>-2276.73</td>
<td>0.45</td>
<td>7415.56</td>
<td>[19] $r_c = 0.52$ fm</td>
</tr>
<tr>
<td>NSC89-1020</td>
<td>-0.250</td>
<td>7.200</td>
<td>1.0</td>
<td>-22.89</td>
<td>0.45</td>
<td>67.45</td>
<td>[20] $m_{\text{cut}} = 1020$ MeV</td>
</tr>
<tr>
<td>NSC89-920</td>
<td>-2.100</td>
<td>1.900</td>
<td>0.6</td>
<td>-1080.35</td>
<td>0.45</td>
<td>2039.54</td>
<td>[20] $m_{\text{cut}} = 920$ MeV</td>
</tr>
<tr>
<td>NSC89-820</td>
<td>-1.110</td>
<td>3.200</td>
<td>0.6</td>
<td>-1904.41</td>
<td>0.45</td>
<td>4996.93</td>
<td>[20] $m_{\text{cut}} = 820$ MeV</td>
</tr>
<tr>
<td>NSC97a</td>
<td>-0.329</td>
<td>12.370</td>
<td>1.0</td>
<td>-69.45</td>
<td>0.45</td>
<td>653.86</td>
<td>[21]</td>
</tr>
<tr>
<td>NSC97b</td>
<td>-0.397</td>
<td>10.360</td>
<td>1.0</td>
<td>-78.42</td>
<td>0.45</td>
<td>741.76</td>
<td>[21]</td>
</tr>
<tr>
<td>NSC97c</td>
<td>-0.476</td>
<td>9.130</td>
<td>1.0</td>
<td>-91.80</td>
<td>0.45</td>
<td>914.67</td>
<td>[21]</td>
</tr>
<tr>
<td>NSC97d</td>
<td>-0.401</td>
<td>1.150</td>
<td>0.4</td>
<td>-445.77</td>
<td>0.30</td>
<td>373.64</td>
<td>[21]</td>
</tr>
<tr>
<td>NSC97e</td>
<td>-0.501</td>
<td>9.840</td>
<td>1.0</td>
<td>-110.45</td>
<td>0.45</td>
<td>1309.55</td>
<td>[21]</td>
</tr>
<tr>
<td>NSC97f</td>
<td>-0.350</td>
<td>16.330</td>
<td>1.0</td>
<td>-106.53</td>
<td>0.45</td>
<td>1469.33</td>
<td>[21]</td>
</tr>
<tr>
<td>Ehime</td>
<td>-4.21</td>
<td>2.41</td>
<td>1.0</td>
<td>-146.6</td>
<td>0.45</td>
<td>720.9</td>
<td>[23]</td>
</tr>
<tr>
<td>fss2</td>
<td>-0.81</td>
<td>3.99</td>
<td>0.92</td>
<td>-103.9</td>
<td>0.41</td>
<td>658.2</td>
<td>[25]</td>
</tr>
<tr>
<td>ESC08</td>
<td>-0.97</td>
<td>3.86</td>
<td>0.80</td>
<td>-293.66</td>
<td>0.45</td>
<td>1429.27</td>
<td>[22]</td>
</tr>
</tbody>
</table>

TABLE II: $\Lambda\Lambda$ potentials from Nagara event. The scattering length ($a_0$) and effective range ($r_{\text{eff}}$) are fitted using a three-range gaussian potential, $V_{\Lambda\Lambda}(r) = V_1 \exp(-r^2/\mu_1^2) + V_2 \exp(-r^2/\mu_2^2) + V_3 \exp(-r^2/\mu_3^2)$.
\[ N = 1.006 \pm 0.001 \]
\[ \lambda = 0.18 \pm 0.05^{+0.12}_{-0.06} \]
\[ a_0 = -1.10 \pm 0.37^{+0.68}_{-0.08} \]
\[ r_{\text{eff}} = 8.52 \pm 2.56^{+2.09}_{-0.74} \]
\[ r_0 = 2.96 \pm 0.38^{+0.96}_{-0.02} \]
\[ a_{\text{res}} = -0.044 \pm 0.004^{+0.048}_{-0.009} \]
\[ r_{\text{res}} = 0.43 \pm 0.04^{+0.43}_{-0.03} \]
\[ \chi^2 / N_{\text{dof}} = 0.56 \]
Kenji Morita (YITP, Kyoto)

Results from the Static Source

Larger variation among potentials than data error-bars

Size: determined from min. $\chi^2$

Strong attraction is reflected onto $C(Q)$
Collectivity Deforms Source Function

\[ C_2(Q, K) = \frac{\int d^4x_1 d^4x_2 S(x_1, K) S(x_2, K) |\Psi_{12}(Q, x_1 - x_2 - (t_2 - t_1)K/m)|^2}{\int d^4x_1 d^4x_2 S(x_1, k_1) S(x_1, k_2)} \]

Influence on the best-fit potentials?

C(Q) is fairly sensitive to interaction
Scattering Length and Effective Range

$-1.8 < 1/a_0 < -0.8 \text{ [fm}^{-1}]$

$3.5 < r_{\text{eff}} < 7 \text{ [fm]}$

$\Sigma^0$ feed-down affects this?