Effective Temperature of Non-equilibrium Steady States in AdS/CFT Correspondence

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Refs.

S. N. and H. Ooguri, PRD88 (2013) 126003.

H. Hoshino and S. N. PRD91 (2015) 026009.

We employ the natural unit: k_B=c=$\hbar$=1.

My personal view on quark-hadron physics

- QCD: physics of quarks, hadrons,……
- "Condensed matter physics"
  - Phase diagram, etc. : statistical physics
  - Many people are working on/interested in non-equilibrium physics.

What is the difference from condensed matter physics?

Relativistic system.

It is "high-energy condensed matter physics,"
or "relativistic condensed matter physics."

Relativistic systems out of equilibrium

Are there any non-equilibrium phenomena that are peculiar to/vivid in relativistic systems of particles?

I would like to bring some results that can be a starting point for further discussion.

My method: AdS/CFT correspondence

Going to analyze non-equilibrium physics in terms of general relativity.

Fundamental question

in statistical physics

What is temperature?

We have many answers.

Definitions of temperature

$$P \propto e^{-E/T},\;\; t_E \approx t_E + 1/T$$ Statistical distributions

$$dE = TdS$$ Thermodynamics

$$D = T\mu$$ Fluctuation-dissipation relation

Diffusion const.

Two sides of the same coin.

We have another definition of temperature:

$$\xi^a \nabla_a \xi^b \Big|_{\text{Horizon}} = 2\pi T \xi^b \Big|_{\text{Horizon}}$$ Hawking temperature

Killing vector

Photo taken from http://www.iqs.com/iqs-blog/common-ground-quality-management-risk-management/

One side of the coin: general relativity

Gravitational force is coming from the curved spacetime.

Einstein-Hilbert action:

Curvature:~combination of second derivatives of the metric

$$S = \frac{c^3}{16\pi G_N}\int d^{d+1}x\sqrt{-g}\left(R-2\Lambda\right)+S_{\text{star}}$$

Cosmological constant

Metric: defines unit length in the geometry

Einstein's equation:

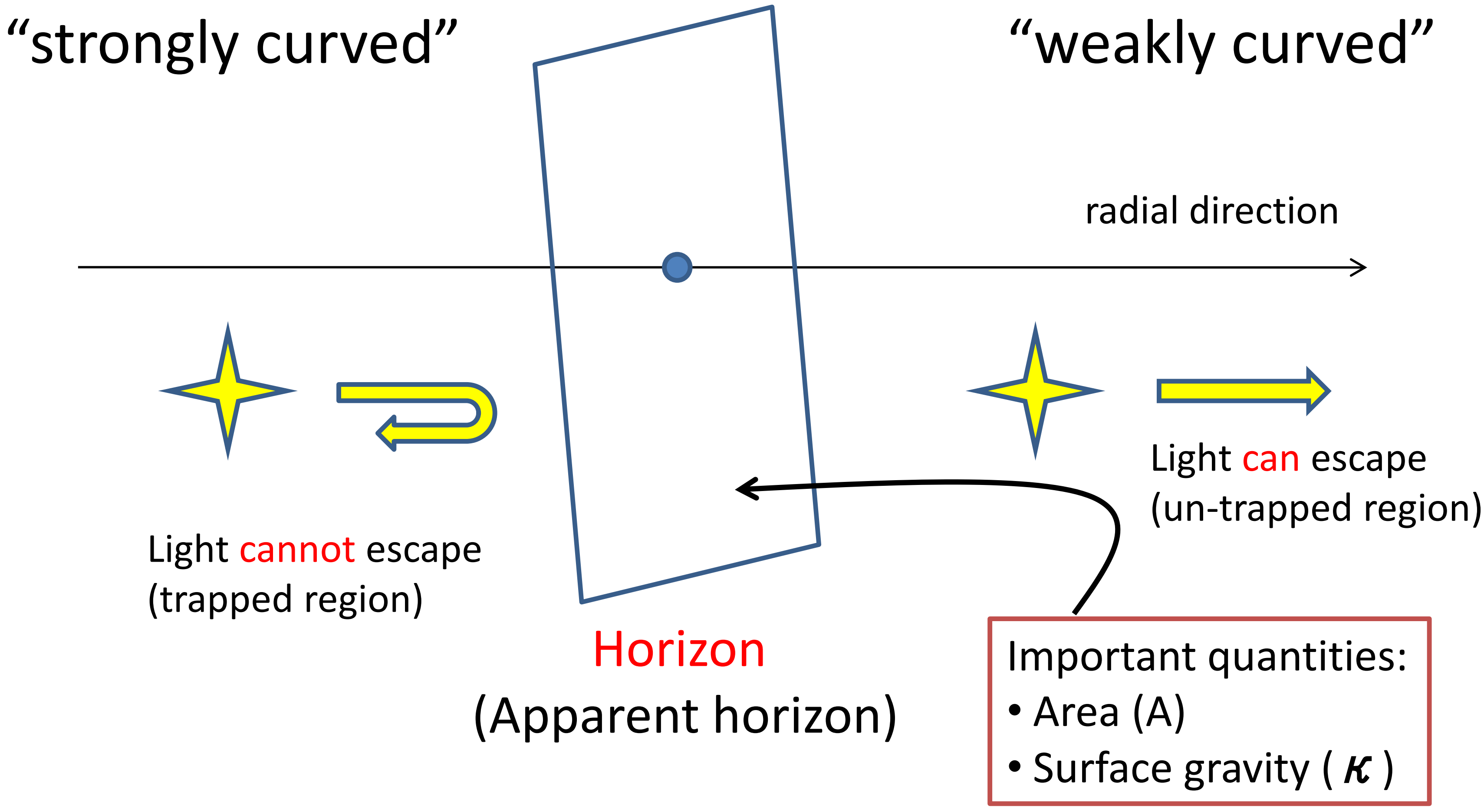
$$R_{\mu\nu}-\frac{1}{2}R\,g_{\mu\nu}+\Lambda g_{\mu\nu}=\frac{8\pi G_N}{c^4}T_{\mu\nu}^{(\text{star})}$$

2nd-order differential equation

Picture:http://www.faculty.iu-bremen.de/course/fall02/c210101/students/BlackHoles/

Black hole

A solution to the Einstein's equation.



Black hole mimics thermodynamics

Black holes obey:

Area of the horizon

$$dM = T_H dS_{BH} \qquad S_{BH} = \frac{k_B c^3}{\hbar G_N}\frac{A}{4}$$

Mass of the black hole

Hawking temperature

Newton's constant

This resembles of the first-law of thermodynamics

$$dE = TdS$$

This is not only an analogy. A black hole radiates a black-body radiation with Hawking temperature: we can assign a temperature to a black hole.

Hawking, S. W. (1974). "Black hole explosions?". *Nature* **248** (5443): 30.

Temperature in black hole?

Black hole thermodynamics

Natural unit: $k_B = c = \hbar = 1$.

	Thermodynamics	Black hole
0-th law	T= const. at the equilibrium.	κ is constant in the static solution.
1st law	$dE = T\,dS$	$dM = [\kappa/(8\pi G_N)]\,dA$
2nd law	Entropy never decreases.	The area of horizon (A) never decreases.
3rd law	We cannot reach T=0 in any physical process.	κ cannot reach zero in any physical process.

κ : surface gravity (the gravitational acceleration at the horizon of the black hole)

G_N: Newton's constant, M: mass of the black hole

A: area of the horizon

κ and A mimic T and S, respectively.

$$T = \frac{\kappa}{2\pi}, \quad S = \frac{A}{4G_N}$$

$T_H = \kappa/2\pi$, by Hawking.

This does not seem to be just a coincidence.

An answer from string theory:

AdS/CFT correspondence

[Maldacena, 1997]

A quantum field theory of gauge particles

A conjecture, but no contradiction has been established so far.

Equivalent

A classical theory of gravity on a curved geometry in higher-dimensions.

When this correspondence was discovered, the curved spacetime was AdS and the gauge theory was CFT.

However, non-AdS/non-CFT version has been also proposed.

An answer from string theory:

AdS/CFT correspondence

[J. Maldacena, 1997]

A quantum field theory of gauge particles $\Leftrightarrow$ A classical theory of gravity on a curved geometry

Finite temperature

Many-body system of gauge particles at temperature T $\Leftrightarrow$ Black hole geometry (typically on AdS_5) at temperature T

[E. Witten, 1998]

Mystery in gravity

Many-body system of gauge particles at temperature T

⟺

Black hole geometry (typically on AdS) at temperature T

- AdS/CFT is a correspondence at the level of microscopic theory of gauge particles.
- What we have done in the gravity side was just solving the differential equation.

This is a solution to the Einstein's equation (2nd-order differential equation).

Who did the coarse graining to reach the thermodynamics?

Benefit in gravity

Many-body system
of gauge particles
at temperature T

⟺

Black hole geometry
(typically on AdS)
at temperature T

- AdS/CFT is a correspondence at the level of microscopic theory of gauge particles.

This is a solution to the Einstein's equation (2nd-order differential equation).

- What we have done in the gravity side was just solving the differential equation.

Somehow, the coarse graining is "automatic."

Realization of non-equilibrium systems

- Usually, we begin with an equilibrium system.
- Then, we drive the system into non-equilibrium by acting an external force.

In the gravity dual:

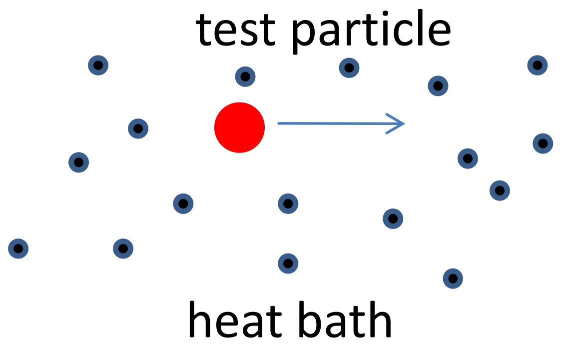
- We have already prepared an equilibrium system in terms of black hole.
- The external force corresponds to a particular boundary condition in Einstein's equation.

We can solve the equation, and the solution describes the non-equilibrium state.

Our system to consider

Langevin system

A test particle immersed in
a heat bath is driven by
a constant external force.



The friction produces dissipation:
the system is driven to out of equilibrium.

The heat bath: gluons in the deconfinement phase at T.

The test particle: a heavy quark.

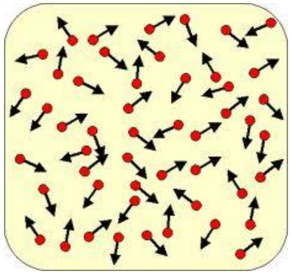
Strategy

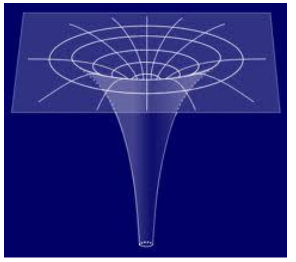
A strongly-interacting quantum gauge theory

AdS/CFT

equivalent

A classical gravity (general relativity) on a curved spacetime of one dimension higher.



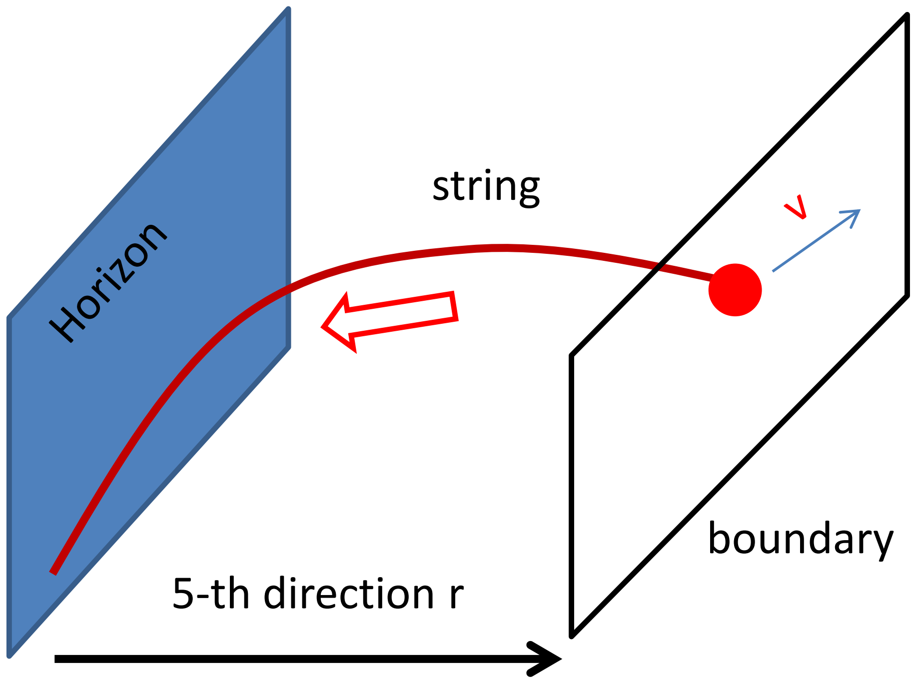


Heat bath (gluons) → Black Hole

Test particle (quark) in the heat bath → A string in the black hole geometry

Picture of many-body system is taken from internet.
Picture of black hole is taken from https://www.kahaku.go.jp/exhibitions/vm/resource/tenmon/space/theory/theory06.html

A cartoon in the gravity dual



[Gubser, 2006]
[Herzog et al., 2006]

Energy-momentum tensor of string

$T^0{}_r$ = energy flow into the black hole in unit time: dissipation

= Work in unit time by the force acting on the test particle

$$f = \left.\frac{\partial L}{\partial(\partial_r x)}\right|_{\text{boundary}} \neq 0 \quad \text{at} \quad v \neq 0.$$

[Gubser, 2006]
[Herzog et al., 2006]

Computation of drag force

[Gubser, 2006], [Herzog et al., 2006]

$$L_{\text{string}} = -(\text{tension})\sqrt{-\det\left(\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}\right)}$$

$$X(t,r) = vt + x(r)$$

$$\partial_r \frac{\partial L}{\partial(\partial_r x)} = 0 \Rightarrow \frac{\partial L}{\partial(\partial_r x)} = f$$

$$(\partial_r x)^2 = f^2 \frac{g_{rr}}{-g_{tt}g_{rr}} \frac{(-g_{tt}) - g_{xx}v^2}{(-g_{tt})g_{xx} - f^2}$$

Right-hand-side can be negative.

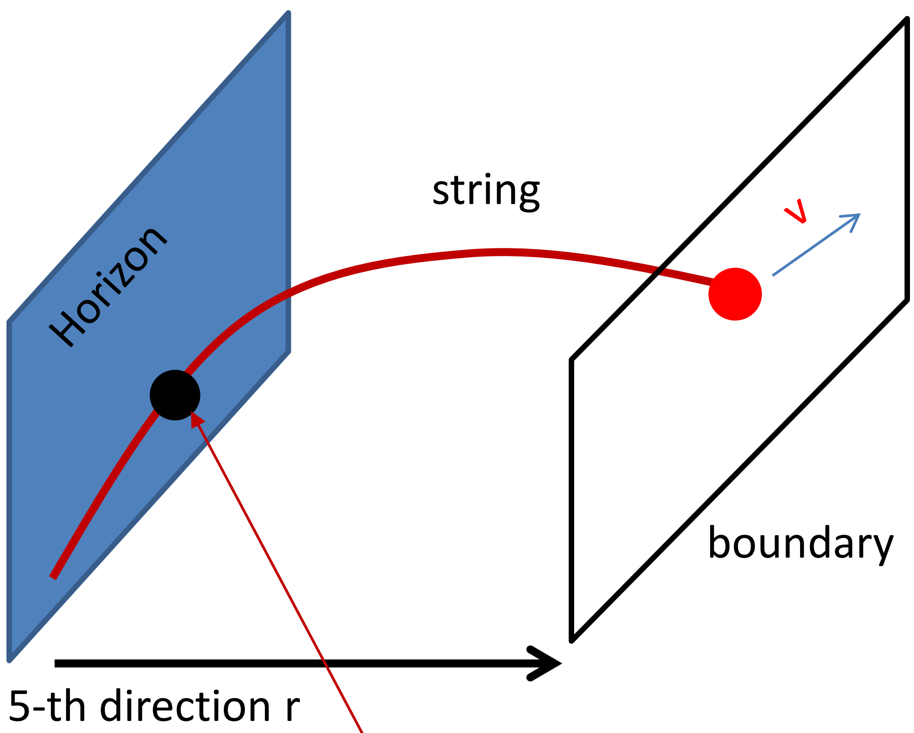
Let us define a point r=r_* by $(-g_{tt}) - g_{xx}v^2\Big|_{r_*} = 0$.

$$(-g_{tt})g_{xx} - f^2\Big|_{r_*} = 0$$

If f satisfies this, $\partial_r x$ can be real.

f is given as a function of v.

A special point



There is a special point ($r=r_*$).

What is this special point?

“Special point” $r=r_*$

It is a “horizon” on the string worldsheet seen by the small fluctuations.

(See also [Gubser 2008, Kim-Shock-Tarrio 2011, Sonner-Green 2012])

We call it “effective horizon.”

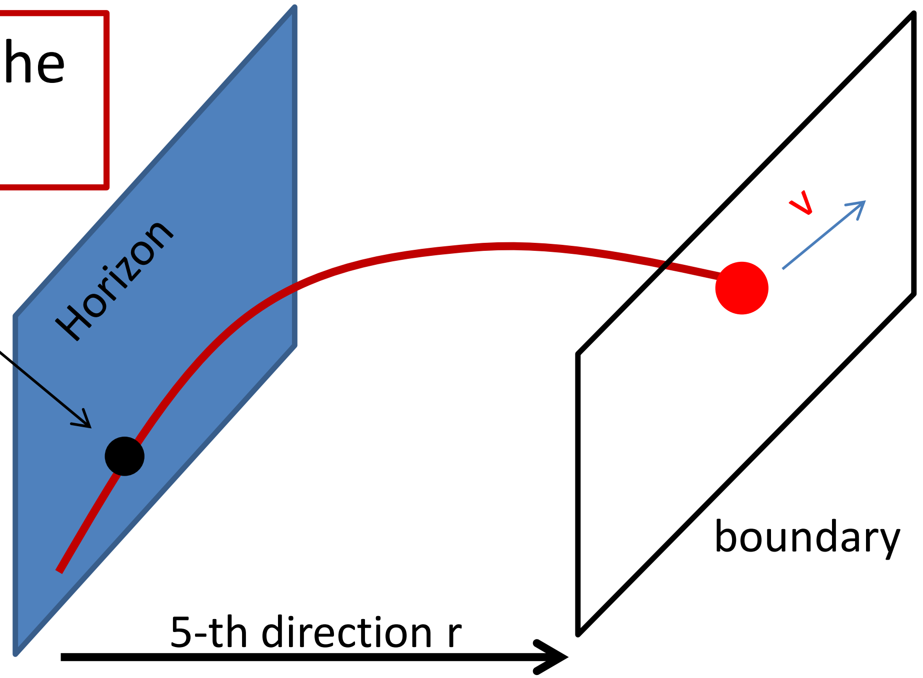
How to see it?

- We find a curved string in the presence of external force.
- What is the “metric” that governs the small fluctuation of the string?

Langevin system

See also, [Gubser, 2008]

Horizon at r=r$_*$ on the induced geometry.



Equation of motion for small fluctuation δX of the string:

$$\partial_a\left(\sqrt{-\tilde{g}}\,\tilde{g}^{ab}\partial_b\,\delta X^{\mu}\right)=0,$$

$$\tilde{g}_{ab}=\partial_a X^{\mu}\partial_a X^{\nu} g_{\mu\nu}$$

Klein-Gordon equation on a curved spacetime given by the induced metric.

Now we have two temperatures

Black hole horizon gives the temperature of the heat bath.

The effective horizon on the string gives a different "Hawking temperature" that governs the fluctuations of the test particle.

We call this effective temperature T_{eff} of NESS.

> If the system is driven to NESS,
> $r_H < r_*$ at the order of v^2 .

Two temperatures appear only in the non-linear regime.

Computations of effective temperature

[S. N. and H. Ooguri, PRD88 (2013) 126003]

We have computed T_{eff} for wide range of models.

- Heat bath:

  Near-horizon geometry of Dp-brane solution at T.

  [Itzhaki-Maldacena-Sonnenschein-Yankielowitcz, 1998]

$$ds^2 = r^{\frac{7-p}{2}}\left[-\left(1-{r_0^{7-p}}/{r^{7-p}}\right)dt^2 + d\vec{x}^2\right] + \frac{dr^2}{r^{\frac{7-p}{2}}\left(1-{r_0^{7-p}}/{r^{7-p}}\right)} + r^{\frac{p-3}{2}} d\Omega_{8-p}^2$$

- Test particle: probe D(q+1+n)-brane or F1 string

  wrapped on n-sphere

- Charged particles: probe D(q+1+n)-brane

Behavior of effective temperature

Can never been understood as a Lorentz factor.

Beyond the linear-response regime

$$T_{\rm eff} = \left(1-v^2\right)^{\frac{1}{7-p}}\left(1+Kv^2\right)^{\frac{1}{2}} T = T + \frac{1}{2}\left(K - \frac{2}{7-p}\right) v^2 T + O(v^4)$$

$$c_0 = \tfrac{4\pi}{7-p}, \quad K = \frac{1}{2}\left(q+3-p+\frac{p-3}{7-p}n\right)$$

This factor can be negative!

$T_{\rm eff} < T$ can be realized.

For a conformally invariant model (N=4 susy YM), K=0:

[Gubser, 2008]

$$T_{\rm eff} = T/\sqrt{\gamma} < \mathrm{T} \qquad \gamma = 1/\sqrt{1-v^2}$$

The temperature seen by the fluctuation can be made smaller by driving the system into out of equilibrium.

Is T_{eff}<T allowed?

It is not forbidden.

Some examples of smaller effective temperature:

[K. Sasaki and S. Amari, J. Phys. Soc. Jpn. 74, 2226 (2005)]

[Also, private communication with S. Sasa]

Is it OK with the second law?

No contradiction.

- The system is an open system.
- The second law of thermodynamics applies to a closed system.
- The definition of "non-eq." entropy (beyond the linear response regime) is not clear.

What is the physical meaning of T_{eff}?

Fluctuation of string ⟺ Fluctuation of external force acting on the test particle

Computations of correlation functions of fluctuations in the gravity dual is governed by the ingoing-wave boundary condition at the effective horizon.

$$\int dt \left\langle \delta f(t) \delta f(0) \right\rangle \Big|_{\mathrm{v} \neq 0} = 2T_{\mathrm{eff}} \left. \frac{\mathrm{Im}\, G^{R}(\omega)}{-\omega} \right|_{\substack{\omega \to 0, \\ \mathrm{v} \neq 0}}$$

fluctuation dissipation

See also, [Gursoy et al.,2010]

The fluctuation-dissipation relation at NESS is characterized by the effective temperature (at least for our systems).

What is temperature?

Definitions of equilibrium temperature:

$$P \propto e^{-E/T}, \;\; t_E \approx t_E + 1/T$$

Distributions

$$dE = TdS$$

Thermodynamics

$$D = T\mu$$

diffusion const. mobility

Fluctuation-dissipation relation

We have another definition of temperature:

$$\xi^a \nabla_a \xi^b \Big|_{\text{Horizon}} = 2\pi T \xi^b \Big|_{\text{Horizon}}$$

Killing vector

Hawking temperature

What is temperature?

Definitions of effective temperature:

$$P \propto e^{-E/T},\ \ t_E \approx t_E + 1/T$$ Distributions

$$dE = TdS$$ Thermodynamics

$$D = T_{\mathrm{eff}} \frac{\partial v}{\partial f}$$

diffusion const. differential mobility

Fluctuation-dissipation relation

They give the same temperature.

$$\xi^a \nabla_a \xi^b \Big|_{\mathrm{Eff.\,Horizon}} = 2\pi T_{\mathrm{eff}}\, \xi^b \Big|_{\mathrm{Eff.\,Horizon}}$$

Killing vector

Hawking temperature

What is temperature?

Definitions of effective temperature:

$$P \propto e^{-E/T}, \quad \cancel{t_E \approx t_E + 1/T}$$

Distributions

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What is temperature?

Definitions of effective temperature:

$$P \propto e^{-E/T_{\text{eff}}}, \cancel{t_E \approx t_E + 1/T}$$

Distributions of small fluctuations

$$\boxed{dE = TdS}$$

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They give the same temperature.

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Killing vector

Hawking temperature

Associated thermodynamics?

$$dE \overset{??}{=} T_{\mathrm{eff}}\, dS$$

It is highly nontrivial.

Hawking radiation (Hawking temperature) is more general than the thermodynamics of black hole.

Hawking radiation:
It occurs as far as the "Klein-Gordon equation" of fluctuation has the same form as that in the black hole.

Thermodynamics of black hole:
We need the Einstein's equation. It relies on the theory of gravity.

Example of "non-gravity" Hawking radiation

Sonic black hole in liquid helium.

Fast Sonic horizon where the flow velocity exceeds the velocity of sound. Slow

- The sound cannot escape from inside the "horizon".
- It is expected that the sonic horizon radiates a "Hawking radiation" of sound at the "Hawking temperature".

[W. G. Unrhu, PRL51(1981)1351]

However, any "thermodynamics" associated with the Hawking temperature of sound has not been established so far.

[See for example, M. Visser, gr-qc/9712016]

Summary

For some examples of non-equilibrium steady states:

- There exists two temperatures in the non-linear regime.
- The effective temperature appears in terms of the Hawking temperature at the effective horizon.
- It agrees with the coefficient in the generalized fluctuation-dissipation relation in NESS.
- $T_{eff} < T$ can happen for some cases.

Some more hint for non-equilibrium physics?

Any relevance in QGP physics?

For a conformally invariant model (N=4 susy YM), K=0:

[Gubser, 2008]

$$T_{\rm eff} = T/\sqrt{\gamma} < \mathrm{T} \qquad \gamma = 1/\sqrt{1-v^2/c^2}$$

"Relativistic" effect

$$T_{\rm eff} = \left(1-v^2/c^2\right)^{\frac{1}{7-p}}\left(1+K\,v^2/c^2\right)^{\frac{1}{2}} T = T + T\,\frac{1}{2}\left(K-\frac{2}{7-p}\right)v^2/c^2 + O\left(v^4/c^4\right)$$

Another scale in general.

I cannot say that the test particle has to be relativistic.

But, the effect will be vivid if the particle is relativistic.

$T_{\rm eff}\sim 0$ if $v\sim c$, (at least for this model).