Hidden charm and bottom molecular states

M. Albaladejo, F.-K. Guo, C. Hidalgo-Duque,
A. Ozpineci, M. Pavón Valderrama and J.N.

- arXiv:1404.1776: Eur. Phys. J. C **74**, 2885 (2014)
- arXiv:1303.6608: Phys. Rev. D **88**, 054007 (2013)
- arXiv:1210.5431: Phys. Rev. D **87**, 076006 (2013)
- arXiv:1204.2790: Phys. Rev. D **86**, 056004 (2012)
- arXiv:1305.4052: Phys. Rev. D **88**, 054014 (2013)

Isospin violation in the $X(3872)$ $[J^{PC} = 1^{++}$ **(LHCb)**] decays into $J/\Psi\,\omega$ and $J/\Psi\,\rho$

$$\mathcal{B}_X = \frac{\Gamma(X(3872) \to J/\Psi\, \overbrace{\pi^+\pi^-}^{\rho})}{\Gamma(X(3872) \to J/\Psi\, \underbrace{\pi^+\pi^-\pi^0}_{\omega})} = 1.3 \pm 0.5$$

Belle Collab. [PRD84 (2011) 052004]

- $X(3872)$ is not a purely $I = 0$ state **or**
- Transition operator violates isospin

or both...

Isospin violation in the $X(3872)$ [$J^{PC} = 1^{++}$ **(LHCb)**] into $J/\Psi\,\omega$ and $J/\Psi\,\rho$

$$\mathcal{B}_X = \frac{\Gamma(X(3872) \to J/\Psi\, \overbrace{\pi^+\pi^-}^{\rho}\,)}{\Gamma(X(3872) \to J/\Psi\, \underbrace{\pi^+\pi^-\pi^0}_{\omega}\,)} = 1.3 \pm 0.5$$

Belle Collab. [PRD84 (2011) 052004]

- $X(3872)$ **is not a purely** $I = 0$ **state** or
- Transition operator violates isospin

or both...

The $X(3872)$, with a mass of $m_X = 3871.69 \pm 0.17\,\mathrm{MeV}$, **is extremely close to the $D^{0*}\bar{D}^0$ threshold** ($m_{D^0} + m_{D^{*0}} = 3871.80 \pm 0.12\,\mathrm{MeV}$) $\Rightarrow$ **molecule?**

$X(3872) \sim D^0\bar{D}^{*0}$ & $D^+\bar{D}^{*-}$ **bound state ($D\bar{D}^*-hc$ with C-parity=+)**

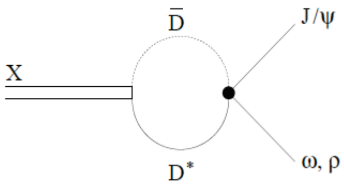
For such a small binding energy, the mass difference **(8 MeV)** between neutral and charged channels plays a role ($m_{D^+} + m_{D^{*-}} = 3879.87 \pm 0.12\,\mathrm{MeV}$) and the **mass term breaks isospin invariance!**

Isospin violations in X(3872) decays…

Because of the mass differences $D^0 \longleftrightarrow D^-$ and $D^{*0} \longleftrightarrow D^{*-}$ the kinetic energy (including the mass terms) does not conmute with isospin, the X(3872) is an admixture of isospin I=0 and I=1 (r is the $D\bar{D}^*$ relative distance in the molecule X)

$$\psi_{X(3872)} = \frac{\varphi_{neu}(r)+\varphi_{ch}(r)}{\sqrt{2}}|I=0\rangle + \frac{\varphi_{neu}(r)-\varphi_{ch}(r)}{\sqrt{2}}|I=1\rangle$$

neu: $D^0 \bar{D}^{*0}$ + cc and ch: $D^+ D^{*-}$ + cc *(neutral and charged thresholds differ in 8 MeV)*



Assuming a contact interaction in the

I=0 channel….

$$\frac{B(X\to J/\psi\rho)}{B(X\to J/\psi\omega)} \propto \left|\frac{\varphi_{neu}(0)-\varphi_{ch}(0)}{\varphi_{neu}(0)+\varphi_{ch}(0)}\right|^2 \sim \mathbf{0.7}$$

D. Gammermann, E. Oset Phys. Rev. D80 (2009) 014003; D. Gammermann, J. Nieves, E. Oset and E. Ruiz-Arriola, Phys. Rev. D81 (2010) 014029

this ratio can be qualitatively understood !

$X(3872) \sim D^0\bar{D}^{*0}$ & $D^+\bar{D}^{*-}$ **bound state** ($D\bar{D}^*-hc$ **with C-parity=+**)

HQSS is an appropriate tool to study $c\bar{c}$ XYZ resonances. Though approximate, its predictions arise from QCD in the heavy quark limit and they are more robust than those deduced in other schemes, which starting point is SU(4) flavour symmetry and some arbitrary pattern of its breaking.

HQSS predicts that all types of spin interactions vanish for infinitely massive quarks: the **dynamics is unchanged under arbitrary transformations in the spin of the heavy quark (Q).** The spin-dependent interactions are proportional to the chromomagnetic moment of the heavy quark, hence are of the order of $1/m_Q$.

To describe the **molecular states of heavy mesons we need an effective Lagrangian that describes the strong interactions of the heavy mesons and antimesons.** We use matrix field $H^{(Q)}$ $[H^{(\bar{Q})}]$ to describe the combined isospin doublet of pseudoscalar heavy-meson [antimeson] $P_a^{(Q)} = (P^0, P^+)$ $[P_a^{(\bar{Q})} = (\bar{P}^0, P^-)]$ fields and their vector HQSS partners $P_a^{*(Q)}$ $[P_a^{*(\bar{Q})}]$

$$
\begin{aligned}
H_a^{(Q)} &= \frac{1+\not{v}}{2}\left(P_{a\mu}^{*(Q)}\gamma^\mu - P_a^{(Q)}\gamma_5\right), \qquad v\cdot P_a^{*(Q)} = 0 \\
H^{(\bar{Q})a} &= \left(P_\mu^{*(\bar{Q})a}\gamma^\mu - P^{(\bar{Q})a}\gamma_5\right)\frac{1-\not{v}}{2}, \qquad v\cdot P^{*(\bar{Q})a} = 0
\end{aligned}
$$

H^c $[H^{\bar{c}}]$ **annihilates** D $[\bar{D}]$ **and** D^* $[\bar{D}^*]$ **mesons with definite velocity** v**.**

The field $H_a^{(Q)}$ $[H^{(\bar{Q})a}]$ transforms as a $(2,\bar{2})$ $[(\bar{2},2)]$ under the **heavy spin** $\otimes$ SU$(2)_V$ isospin symmetry

$$H_a^{(Q)} \to \mathbf{S}\left(H^{(Q)}U^\dagger\right)_a, \qquad H^{(\bar{Q})a} \to \left(UH^{(\bar{Q})}\right)^a \mathbf{S}^\dagger$$

The hermitian conjugate fields are defined by,

$$\bar{H}^{(Q)a} = \gamma^0 H_a^{(Q)\dagger}\gamma^0, \qquad \bar{H}_a^{(\bar{Q})} = \gamma^0 \bar{H}^{(\bar{Q})a\dagger}\gamma^0$$

and transform as

$$\bar{H}^{(Q)a} \to \left(U\bar{H}^{(Q)}\right)^a \mathbf{S}^\dagger, \qquad \bar{H}^{(\bar{Q})}{}_a \to \mathbf{S}\left(\bar{H}^{(\bar{Q})}U^\dagger\right)^a$$

Moreover, $\mathcal{C}P_a^{(Q)}\mathcal{C}^{-1} = P^{(\bar{Q})a}$ and $\mathcal{C}P_{a\mu}^{*(Q)}\mathcal{C}^{-1} = -P_\mu^{*(\bar{Q})a} \Rightarrow$

$$\mathcal{C}H_a^{(Q)}\mathcal{C}^{-1} = c\,H^{(\bar{Q})aT}\,c^{-1}, \qquad \mathcal{C}\bar{H}^{(Q)a}\mathcal{C}^{-1} = c\,\bar{H}_a^{(\bar{Q})T}\,c^{-1}$$

with c the Dirac space charge conjugation matrix $c\gamma_\mu c^{-1} = -\gamma_\mu^T$.

At LO in the EFT expansion, the **four meson local interaction Lagrangian consistent with HQSS** (T. Alfiky et al., PLB640 238)

$$
\begin{aligned}
\mathcal{L}_{4H} &= \mathbf{C_a}\,\mathrm{Tr}\left[\bar{H}^{(Q)i}H_i^{(Q)}\gamma_\mu\right]\mathrm{Tr}\left[H^{(\bar{Q})i}\bar{H}_i^{(\bar{Q})}\gamma^\mu\right] \\
&+ \mathbf{C_a^\tau}\,\mathrm{Tr}\left[\bar{H}^{(Q)j}H_i^{(Q)}\gamma_\mu\right]\mathrm{Tr}\left[H^{(\bar{Q})n}\bar{H}_m^{(\bar{Q})}\gamma^\mu\right]\vec{\tau}_{\mathbf{j}}^{\,\mathbf{i}}\vec{\tau}_{\mathbf{n}}^{\,\mathbf{m}} \\
&+ \mathbf{C_b}\,\mathrm{Tr}\left[\bar{H}^{(Q)i}H_i^{(Q)}\gamma_\mu\gamma_5\right]\mathrm{Tr}\left[H^{(\bar{Q})i}\bar{H}_i^{(\bar{Q})}\gamma^\mu\right] \\
&+ \mathbf{C_b^\tau}\,\mathrm{Tr}\left[\bar{H}^{(Q)j}H_i^{(Q)}\gamma_\mu\right]\mathrm{Tr}\left[H^{(\bar{Q})n}\bar{H}_m^{(\bar{Q})}\gamma^\mu\gamma_5\right]\vec{\tau}_{\mathbf{j}}^{\,\mathbf{i}}\vec{\tau}_{\mathbf{n}}^{\,\mathbf{m}}
\end{aligned}
$$

For each isospin channel (1 or $\vec{\tau}\cdot\vec{\tau}$), we have only two independent constants, which are associated to the two possible spins (0 or 1) of the light quarks in a $\mathrm{H}\bar{\mathrm{H}}$ system!

Trivial extension to SU(3) light flavor symmetry (still four LEC's) !

$$|D\,\bar{D}; J=0,T\rangle = \frac{\sqrt{3}}{2}|1_{c\bar{c}},1_{\bar{l}l}; J=0,T\rangle + \frac{1}{2}|0_{c\bar{c}},0_{\bar{l}l}; J=0,T\rangle$$

$$|D^*\,\bar{D}^*; J=0,T\rangle = \frac{1}{2}|1_{c\bar{c}},1_{\bar{l}l}; J=0,T\rangle - \frac{\sqrt{3}}{2}|0_{c\bar{c}},0_{\bar{l}l}; J=0,T\rangle$$

$$|D\,\bar{D}^*; J=1,T\rangle = \frac{1}{2}|1_{c\bar{c}},0_{\bar{l}l}; J=1,T\rangle - \frac{1}{2}|0_{c\bar{c}},1_{\bar{l}l}; J=1,T\rangle - \frac{1}{\sqrt{2}}|1_{c\bar{c}},1_{\bar{l}l}; J=1,T\rangle$$

$$|D^*\,\bar{D}; J=1,T\rangle = \frac{1}{2}|1_{c\bar{c}},0_{\bar{l}l}; J=1,T\rangle - \frac{1}{2}|0_{c\bar{c}},1_{\bar{l}l}; J=1,T\rangle + \frac{1}{\sqrt{2}}|1_{c\bar{c}},1_{\bar{l}l}; J=1,T\rangle$$

$$|D^*\,\bar{D}^*; J=1,T\rangle = -\frac{1}{\sqrt{2}}|0_{c\bar{c}},1_{\bar{l}l}; J=1,T\rangle - \frac{1}{\sqrt{2}}|1_{c\bar{c}},0_{\bar{l}l}; J=1,T\rangle$$

$$|D^*\,\bar{D}^*; J=2,T\rangle = -|1_{c\bar{c}},1_{\bar{l}l}; J=2,T\rangle$$

T **and** $\mathcal{L}=0_{\bar{l}l},1_{\bar{l}l}$ **isospin and total spin of the light dof!**

HQSS $\Rightarrow \langle S'_{c\bar c}, \mathcal{L}'; J, T | H_{\rm QCD} | S_{c\bar c}, \mathcal{L}; J, T\rangle = \delta_{S'_{c\bar c} S_{c\bar c}} \delta_{\mathcal{L}'\mathcal{L}} \times O^T_{\mathcal{L}}$

$$\begin{aligned}
\mathcal{B}(J^{PC}=0^{++}) &= \left\{|D\bar D\rangle, |D^*\bar D^*\rangle\right\}, \quad \mathcal{B}(J^{PC}=2^{++}) = |D^*\bar D^*\rangle \\
\mathcal{B}(J^{PC}=1^{+-}) &= \left\{\frac{1}{\sqrt{2}}\left(|D\bar D^*\rangle + |D^*\bar D\rangle\right), |D^*\bar D^*\rangle\right\} \\
\mathcal{B}(J^{PC}=1^{++}) &= \left\{\frac{1}{\sqrt{2}}\left(|D\bar D^*\rangle - |D^*\bar D\rangle\right)\right\}
\end{aligned}$$

For each isospin channel T $\left[O^T_{\mathcal{L}=0} = C_{Ta} - 3\,C_{Tb}, O^T_{\mathcal{L}=1} = C_{Ta} + C_{Tb}\right]$**,**
with $C_{T(a,b)} = C_{a,b} + (2\vec{T}^{\,2} - 3) C^{\tau}_{a,b}$ from $\mathcal{L}_{4H}$

$$V_C(0^{++}) = \begin{pmatrix} C_{Ta} & \sqrt{3}\,C_{Tb} \\ \sqrt{3}\,C_{Tb} & C_{Ta} - 2\,C_{Tb} \end{pmatrix}, \; V_C(1^{+-}) = \begin{pmatrix} C_{Ta} - C_{Tb} & 2\,C_{Tb} \\ 2\,C_{Tb} & C_{Ta} - C_{Tb} \end{pmatrix}$$

$$V_C(1^{++}) \; = \; C_{Ta} + C_{Tb}, \qquad V_C(2^{++}) = C_{Ta} + C_{Tb}$$

HQSS $\Rightarrow \langle S'_{c\bar c}, \mathcal{L}'; J, T | H_{\rm QCD} | S_{c\bar c}, \mathcal{L}; J, T\rangle = \delta_{S'_{c\bar c} S_{c\bar c}} \delta_{\mathcal{L}'\mathcal{L}} \times O^T_{\mathcal{L}}$

$$\begin{aligned}
\mathcal{B}(J^{PC}=0^{++}) &= \left\{|D\bar D\rangle, |D^*\bar D^*\rangle\right\}, \quad \mathcal{B}(J^{PC}=2^{++}) = |D^*\bar D^*\rangle \\
\mathcal{B}(J^{PC}=1^{+-}) &= \left\{\frac{1}{\sqrt{2}}\left(|D\bar D^*\rangle + |D^*\bar D\rangle\right), |D^*\bar D^*\rangle\right\} \\
\mathcal{B}(J^{PC}=1^{++}) &= \left\{\frac{1}{\sqrt{2}}\left(|D\bar D^*\rangle - |D^*\bar D\rangle\right)\right\}
\end{aligned}$$

For each isospin channel T $\left[O^T_{\mathcal{L}=0} = C_{Ta} - 3\,C_{Tb}, O^T_{\mathcal{L}=1} = C_{Ta} + C_{Tb}\right]$, with $C_{T(a,b)} = C_{a,b} + (2\vec{T}^{\,2} - 3) C^{\tau}_{a,b}$ from $\mathcal{L}_{4H}$

$$V_C(0^{++}) = \begin{pmatrix} C_{Ta} & \sqrt{3}\,C_{Tb} \\ \sqrt{3}\,C_{Tb} & C_{Ta} - 2\,C_{Tb} \end{pmatrix}, \; V_C(1^{+-}) = \begin{pmatrix} C_{Ta} - C_{Tb} & 2\,C_{Tb} \\ 2\,C_{Tb} & C_{Ta} - C_{Tb} \end{pmatrix}$$

$$\mathbf{V_C(1^{++}) \; = \; C_{Ta} + C_{Tb}}, \qquad \mathbf{V_C(2^{++}) = C_{Ta} + C_{Tb}}$$

... existence of a heavy quark spin symmetry partner of the $X(3872)$, with $J^{PC} = 2^{++}$, $X(4012)$ around the $D^*\bar{D}^*$ threshold!

$X(3872)$ **mass and the branching fraction**

$$\mathcal{B}_X = \frac{\Gamma(X(3872) \to J/\Psi \overbrace{\pi^+\pi^-}^{\rho})}{\Gamma(X(3872) \to J/\Psi \underbrace{\pi^+\pi^-\pi^0}_{\omega})} = 1.3 \pm 0.5$$

fix $(C_{0a} + C_{0b})$ and $(C_{1a} + C_{1b})$, $J^{PC} = 1^{++}$ counter-terms in the $I = 0$ and $I = 1$ sectors (solve a regularized LSE & poles in the FRS).

Solve the LSE in $^{2S+1}L_J$ partial waves (momentum space),

$$\mathbf{T^{S'S}_{JL'L}(E;p',p)} = V^{S'S}_{JL'L}(p',p) + \sum_{L'',S''}\int_0^{+\infty}\frac{dq q^2 4\pi}{(2\pi)^3}\frac{V^{S'S''}_{JL'L''}(p',q)\mathbf{T^{S''S}_{JL''L}(E;q,p)}}{E-q^2/2\mu_{12}-M_1-M_2+i\epsilon}$$

- UV behavior: a cut-off is needed to regularize the EFT potential
  - $V(p',p)\to V(p',p)f(\frac{p'}{\Lambda})\,f(\frac{p}{\Lambda})$, f.i. $f(x)=e^{-x^2}$ (Gaussian)
- Bound/resonant states: <u>poles in the FRS or SRS</u> of the $T-$matrix
- Contact interaction $\mathcal{L}_{4H}\Rightarrow \underbrace{\delta_{L0}\delta_{L'0}}_{S-\text{wave}}\delta_{JS}\delta_{JS'}$ (no mixing of different partial waves).
- RGE:

$$\frac{1}{C_0(\Lambda)}\sim\frac{\mu}{2\pi}\left(\gamma_B-\frac{2}{\pi}\Lambda\right),\quad \gamma_B=\sqrt{-2\mu E_B}$$

<u>**LO**</u>**:** $\mathcal{L}_{4H}$, counter–terms $C's\sim\mathcal{O}(Q^{-1})$ in the EFT expansion (Q is a soft scale) since for shallow bound states

$$\mathcal{O}(V)=\mathcal{O}(VGV),\quad G=\frac{1}{E-H_0}\sim\int\frac{d^3q}{E-M_1-M_2-\vec{q}^{\,2}/2\mu_{12}}\sim\mathcal{O}(Q)$$

Isospin violations in X(3872) decays…

Because of the mass differences $D^0 \longleftrightarrow D^-$ and $D^{*0} \longleftrightarrow D^{*-}$ the kinetic energy (including the mass terms) does not conmute with isospin, the X(3872) is an admixture of isospin I=0 and I=1 (r is the $D\bar{D}^*$ relative distance in the molecule X)

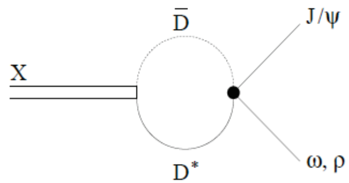
$$\psi_{X(3872)} = \frac{\varphi_{neu}(r)+\varphi_{ch}(r)}{\sqrt{2}}|I=0\rangle + \frac{\varphi_{neu}(r)-\varphi_{ch}(r)}{\sqrt{2}}|I=1\rangle$$

neu: $D^0 \bar{D}^{*0}$ + cc and ch: $D^+ D^{*-}$ + cc *(neutral and charged thresholds differ in 8 MeV)*

Assuming a contact interaction in the

I=0 channel….

$$\frac{B(X \to J/\psi\rho)}{B(X \to J/\psi\omega)} \propto \left|\frac{\varphi_{neu}(0)-\varphi_{ch}(0)}{\varphi_{neu}(0)+\varphi_{ch}(0)}\right|^2 \sim \mathbf{0.7}$$



D. Gammermann, E. Oset Phys. Rev. D80 (2009) 014003; D. Gammermann, J. Nieves, E. Oset and E. Ruiz-Arriola, Phys. Rev. D81 (2010) 014029

this ratio can be qualitatively understood !

$X(3872) \sim D^0\bar{D}^{*0}$ & $D^+\bar{D}^{*-}$ **bound state** ($D\bar{D}^* - hc$ **with C-parity=+**)

... existence of a heavy quark spin symmetry partner of the $X(3872)$, with $J^{PC}=2^{++}$, $X(4012)$ around the $D^*\bar{D}^*$ threshold!

$X(3872)$ mass and the branching fraction

$$\mathcal{B}_X = \frac{\Gamma(X(3872)\to J/\Psi\, \overbrace{\pi^+\pi^-}^{\rho}\,)}{\Gamma(X(3872)\to J/\Psi\, \underbrace{\pi^+\pi^-\pi^0}_{\omega}\,)} = 1.3 \pm 0.5$$

fix $(C_{0a}+C_{0b})$ and $(C_{1a}+C_{1b})$, $J^{PC}=1^{++}$ counter-terms in the $I=0$ and $I=1$ sectors (solve a regularized LSE & poles in the FRS).

Heavy Flavor symmetry: $Z_b(10610)$ & $Z_b(10650)$ $(I=1,\ J^{PC}=1^{+-})$ have been already discussed by Voloshin (PRD84 031502) and Mehen and Powell (PRD84 114013) in the case of $B^{(*)}\bar{B}^{(*)}$ molecules.

HQSS $\Rightarrow \langle S'_{c\bar{c}}, \mathcal{L}'; J, T | H_{\rm QCD} | S_{c\bar{c}}, \mathcal{L}; J, T \rangle = \delta_{S'_{c\bar{c}} S_{c\bar{c}}} \delta_{\mathcal{L}'\mathcal{L}} \times O^T_{\mathcal{L}}$

$$
\begin{aligned}
\mathcal{B}(J^{PC}=0^{++}) &= \left\{ |D\bar{D}\rangle, |D^*\bar{D}^*\rangle \right\}, \quad \mathcal{B}(J^{PC}=2^{++}) = |D^*\bar{D}^*\rangle \\
\mathcal{B}(J^{PC}=1^{+-}) &= \left\{ \frac{1}{\sqrt{2}} \left(|D\bar{D}^*\rangle + |D^*\bar{D}\rangle \right), |D^*\bar{D}^*\rangle \right\} \\
\mathcal{B}(J^{PC}=1^{++}) &= \left\{ \frac{1}{\sqrt{2}} \left(|D\bar{D}^*\rangle - |D^*\bar{D}\rangle \right) \right\}
\end{aligned}
$$

For each isospin channel T $\left[O^T_{\mathcal{L}=0} = C_{Ta} - 3\, C_{Tb}, O^T_{\mathcal{L}=1} = C_{Ta} + C_{Tb} \right]$,
with $C_{T(a,b)} = C_{a,b} + (2\vec{T}^{\,2} - 3) C^{\tau}_{a,b}$ from $\mathcal{L}_{4H}$

$$
V_C(0^{++}) = \begin{pmatrix} C_{Ta} & \sqrt{3}\, C_{Tb} \\ \sqrt{3}\, C_{Tb} & C_{Ta} - 2\, C_{Tb} \end{pmatrix}, \quad \mathbf{V_C(1^{+-})} = \begin{pmatrix} C_{Ta} - C_{Tb} & 2\, C_{Tb} \\ 2\, C_{Tb} & C_{Ta} - C_{Tb} \end{pmatrix}
$$

$$
V_C(1^{++}) = C_{Ta} + C_{Tb}, \qquad V_C(2^{++}) = C_{Ta} + C_{Tb}
$$

... existence of a heavy quark spin symmetry partner of the $X(3872)$, with $J^{PC} = 2^{++}$, $X(4012)$ around the $D^*\bar{D}^*$ threshold!

$X(3872)$ **mass and the branching fraction**

$$\mathcal{B}_X = \frac{\Gamma(X(3872) \to J/\Psi\, \overbrace{\pi^+\pi^-}^{\rho})}{\Gamma(X(3872) \to J/\Psi\, \underbrace{\pi^+\pi^-\pi^0}_{\omega})} = 1.3 \pm 0.5$$

fix $(C_{0a} + C_{0b})$ and $(C_{1a} + C_{1b})$, $J^{PC} = 1^{++}$ counter-terms in the $I = 0$ and $I = 1$ sectors (solve a regularized LSE & poles in the FRS).

Heavy Flavor symmetry: $Z_b(10610)$ & $Z_b(10650)$ ($I = 1$, $J^{PC} = 1^{+-}$) $B\bar{B}^*$ **and** $B^*\bar{B}^*$ **molecules** $\to C_{1b}$.

We predict

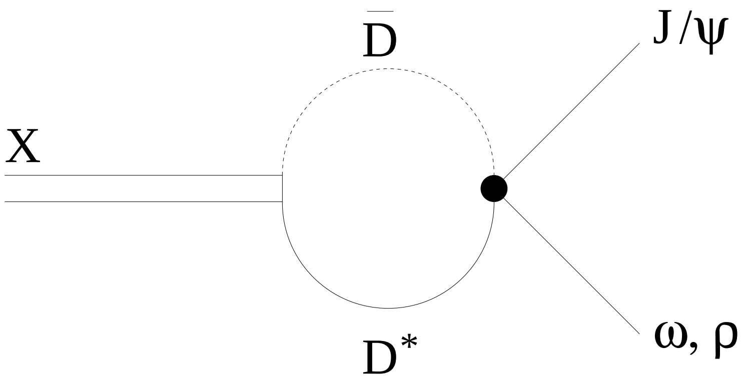
$(Q\bar{Q}'l\bar{l}')$ $\mathcal{O}(1/m_Q)$

V_C	$I(J^{PC})$	States	Thresholds	Masses ($\Lambda = 0.5$ GeV)	Measurements
C_X	$0(1^{++})$	$\frac{1}{\sqrt{2}}(D\bar{D}^* - D^*\bar{D})$	3875.87	**3871.68 (input)**	3871.68 ± 0.17 PDG [$X(3872)$]
	$0(2^{++})$	$D^*\bar{D}^*$	4017.3	4012^{+4}_{-5}	**?**
	$0(1^{++})$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})$	10604.4	10580^{+9}_{-8}	**?**
	$0(2^{++})$	$B^*\bar{B}^*$	10650.2	10626^{+8}_{-9}	**?**
	$0(2^{+})$	D^*B^*	7333.7	7322^{+6}_{-7}	**?**
C_Z	$1(1^{+-})$	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})$	10604.4	10602.4 ± 2.0 **(input)**	10607.2 ± 2.0 Belle [$Z_b(10610)$]
	$1(1^{+-})$	$B^*\bar{B}^*$	10650.2	10648.1 ± 2.1	10652.2 ± 1.5 Belle [$Z_b(10650)$]
	$1(1^{+-})$	$\frac{1}{\sqrt{2}}(D\bar{D}^* + D^*\bar{D})$	3875.87	3871^{+4}_{-12} (V)	$3899.0 \pm 3.6 \pm 4.9$ BESIII [$Z_c(3900)$]
					$3894.5 \pm 6.6 \pm 4.5$ Belle
					$3886 \pm 4 \pm 2$ CLEO-c
	$1(1^{+-})$	$D^*\bar{D}^*$	4017.3	4013^{+4}_{-11} (V)	$4026.3 \pm 2.6 \pm 3.7$ BESIII [$Z_c(4020)$]
	$1(1^{+})$	D^*B^*	7333.7	$7333.6^{\dagger}_{-4.2}$ (V)	**?**

and using also the heavy antiquark-diquark symmetry $\Rightarrow$ triply heavy pentaquarks

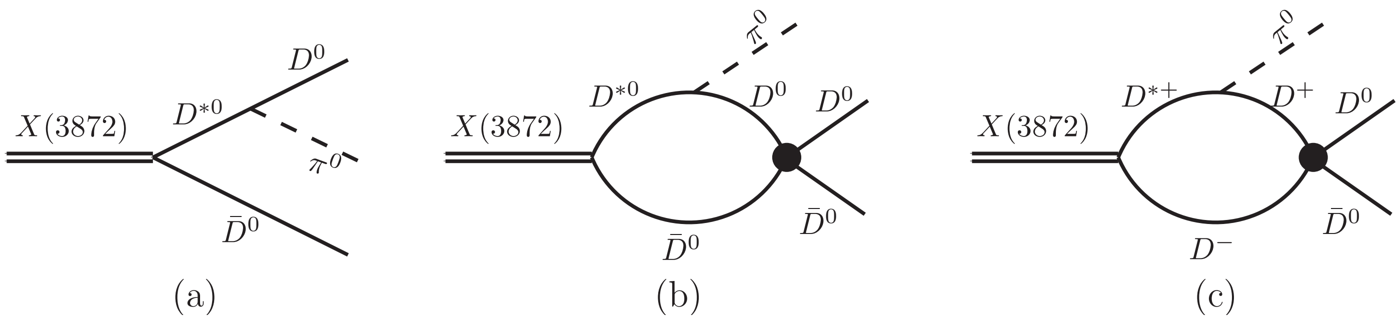
$$(QQl)Q\bar{l}' \sim (``\bar{Q}"l)Q\bar{l}' \quad (QQQl\bar{l}') \quad \mathcal{O}(1/m_Q \cdot v)$$

State	$I(J^P)$	$V^{\rm LO}$	Thresholds	Mass ($\Lambda = 0.5$ GeV)	Mass ($\Lambda = 1$ GeV)
$\Xi^*_{cc} D^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	5715	$(M_{\rm th} - 10)^{+10}_{-15}$	$(M_{\rm th} - 19)^{\dagger}_{-44}$
$\Xi^*_{cc} \bar{B}^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	9031	$(M_{\rm th} - 21)^{+16}_{-19}$	$(M_{\rm th} - 53)^{+45}_{-59}$
$\Xi^*_{bb} D^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	12160	$(M_{\rm th} - 15)^{+9}_{-11}$	$(M_{\rm th} - 35)^{+25}_{-31}$
$\Xi^*_{bb} \bar{B}^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	15476	$(M_{\rm th} - 29)^{+12}_{-13}$	$(M_{\rm th} - 83)^{+38}_{-40}$
$\Xi'_{bc} D^*$	$0(\frac{3}{2}^-)$	$C_{0a} + C_{0b}$	8967	$(M_{\rm th} - 14)^{+11}_{-13}$	$(M_{\rm th} - 30)^{+27}_{-40}$
$\Xi'_{bc} \bar{B}^*$	$0(\frac{3}{2}^-)$	$C_{0a} + C_{0b}$	12283	$(M_{\rm th} - 27)^{+15}_{-16}$	$(M_{\rm th} - 74)^{+45}_{-51}$
$\Xi^*_{bc} D^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	9005	$(M_{\rm th} - 14)^{+11}_{-13}$	$(M_{\rm th} - 30)^{+27}_{-40}$
$\Xi^*_{bc} \bar{B}^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	12321	$(M_{\rm th} - 27)^{+15}_{-16}$	$(M_{\rm th} - 74)^{+46}_{-51}$
$\Xi_{bb} \bar{B}$	$1(\frac{1}{2}^-)$	C_{1a}	15406	$(M_{\rm th} - 0.3)^{\dagger}_{-2.5}$	$(M_{\rm th} - 12)^{+11}_{-15}$
$\Xi_{bb} \bar{B}^*$	$1(\frac{1}{2}^-)$	$C_{1a} + \frac{2}{3}\, C_{1b}$	15452	$(M_{\rm th} - 0.9)[{\rm V}]^{\rm N/A}_{\dagger\dagger}$	$(M_{\rm th} - 16)^{+14}_{-17}$
$\Xi_{bb} \bar{B}^*$	$1(\frac{3}{2}^-)$	$C_{1a} - \frac{1}{3}\, C_{1b}$	15452	$(M_{\rm th} - 1.2)^{\dagger}_{-2.9}$	$(M_{\rm th} - 10)^{+9}_{-13}$
$\Xi^*_{bb} \bar{B}$	$1(\frac{3}{2}^-)$	C_{1a}	15430	$(M_{\rm th} - 0.3)^{\dagger}_{-2.4}$	$(M_{\rm th} - 12)^{+11}_{-13}$
$\Xi^*_{bb} \bar{B}^*$	$1(\frac{1}{2}^-)$	$C_{1a} - \frac{5}{3}\, C_{1b}$	15476	$(M_{\rm th} - 8)^{+8}_{-7}$	$(M_{\rm th} - 5)^{\dagger}_{-8}$
$\Xi^*_{bb} \bar{B}^*$	$1(\frac{3}{2}^-)$	$C_{1a} - \frac{2}{3}\, C_{1b}$	15476	$(M_{\rm th} - 2.5)^{\dagger}_{-3.6}$	$(M_{\rm th} - 9)^{+9}_{-11}$
$\Xi^*_{bb} \bar{B}^*$	$1(\frac{5}{2}^-)$	$C_{1a} + C_{1b}$	15476	$(M_{\rm th} - 4.3)[{\rm V}]^{\rm N/A}_{+3.3}$	$(M_{\rm th} - 18)^{+17}_{-19}$



sensitive only to $D\bar{D}^*$ short distances!

$$\int d^3p\Psi(\vec{p}) \sim \Psi(\vec{0})$$



$\boxed{X(3872) \to D^0\bar{D}^0\pi^0}$

$\sim \Psi(\vec{p}_{D^0})$

sensitive also to $D\bar{D}^*$ long-distance dynamics!

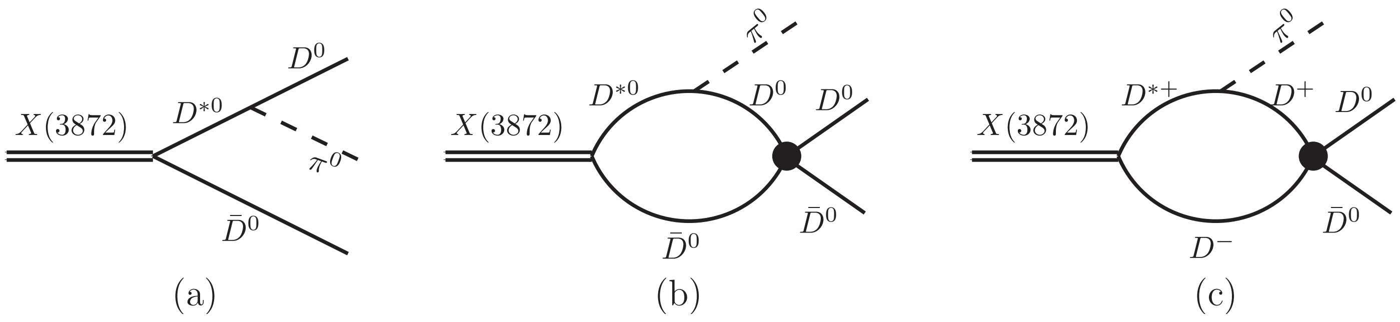
$g_0[X(3872)\bar{D}^0D^{*0}]$

$g_c[X(3872)D^-D^{*+}]$

$T[D^0\bar{D}^0 \to D^0\bar{D}^0]$

$T[D^+D^- \to D^0\bar{D}^0]$

$\boxed{X(3872) \to D^0 \bar{D}^0 \pi^0}$



$\sim \Psi(\vec{p}_{D^0})$

sensitive also to $D\bar{D}^*$

long-distance dynamics!

$g_0[X(3872)\bar{D}^0 D^{*0}]$

$g_c[X(3872)D^- D^{*+}]$

$T[D^0\bar{D}^0 \to D^0\bar{D}^0]$

$T[D^+D^- \to D^0\bar{D}^0]$

$$T_{\rm tree} = -2i\frac{g\;\overbrace{\mathbf{g_0^X}}^{\textbf{residue}}}{f_\pi}\sqrt{M_X}M_{D^{*0}}M_{D^0}\vec{\epsilon}_X\cdot\vec{p}_\pi\left(\frac{1}{p_{12}^2-M_{D^{*0}}^2}+\frac{1}{p_{13}^2-M_{D^{*0}}^2}\right),$$

$$d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32M_X^3}\overline{|T|}^2 dm_{12}^2 dm_{23}^2$$

$$\boldsymbol{\Gamma(X(3872) \to D^0\bar{D}^0\pi^0)_{\rm tree}} = \underbrace{44.0^{+2.4}_{-7.2}}_{\boldsymbol{\Lambda=0.5}\ \text{GeV}}\ \Big(\underbrace{42.0^{+3.6}_{-7.3}}_{\boldsymbol{\Lambda=1}\ \text{GeV}}\Big)\ \text{keV}$$

$$T^{(0)}_{\rm loop} = -16i\frac{g\mathbf{g_0^X}}{f_\pi}\sqrt{M_X}M_{D^{*0}}M^3_{D^0}\,\vec{\epsilon}_X\cdot\vec{p_\pi}\;\mathbf{T_{00\to 00}(m_{23})}\,I(M_{D^{*0}},M_{D^0},M_{D^0},\vec{p}_\pi),$$

where $T_{00\to 00}$ **is the** T**-matrix element for the** $D^0\bar{D}^0 \to D^0\bar{D}^0$ process, and the three-point loop function is defined as

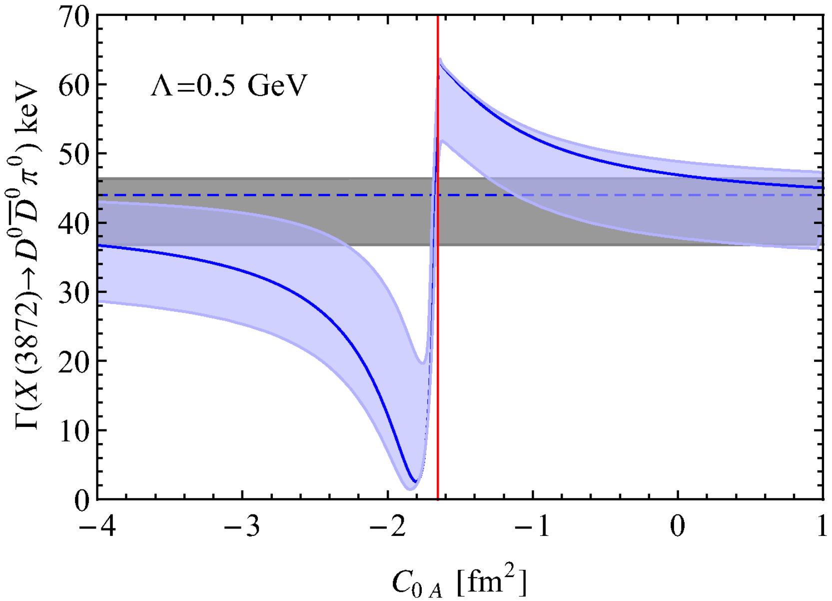
$$I\left(M_{1,2,3},\vec{p}_\pi\right) = i\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2-M_1^2+i\varepsilon}\frac{1}{(P-q)^2-M_2^2+i\varepsilon}\frac{1}{(q-p_\pi)^2-M_3^2+i\varepsilon},$$

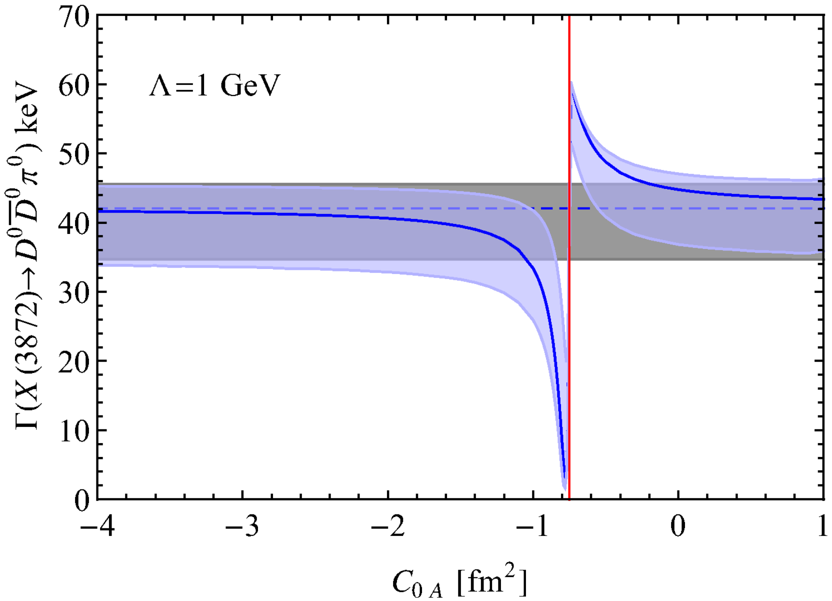
with $P^\mu = (M_X,\vec{0})$ in the rest frame of the $X(3872)$. **This loop integral is convergent !**

$$\begin{aligned} T^{(c)}_{\rm loop} &= 16i\frac{g\mathbf{g_c^X}}{f_\pi}\sqrt{M_X}M_{D^{*0}}M_{D^0}M^2_{D^\pm}\,\vec{\epsilon}_X\cdot\vec{p_\pi}\;\mathbf{T_{+-\to 00}(m_{23})} \\ &\times\; I(M_{D^{*\pm}},M_{D^\pm},M_{D^\pm},\vec{p}_\pi), \end{aligned}$$

where $T_{+-\to 00}$ **is the** T**-matrix element for the** $D^+D^- \to D^0\bar{D}^0$ process.

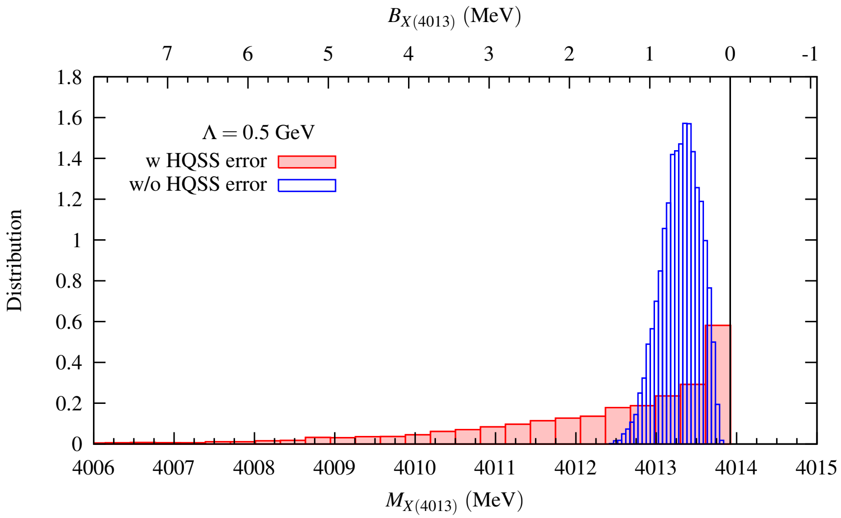
$D^0\bar{D}^0 \to D^0\bar{D}^0$ and $D^+D^- \to D^0\bar{D}^0$ FSI contributions...





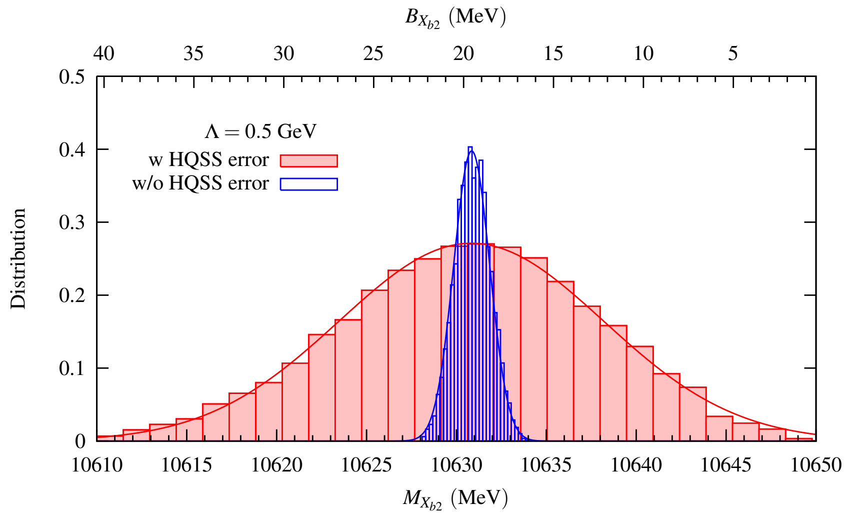
These contributions depend on the four $\boxed{C_{0a}}$, C_{0b}, C_{1a} and C_{1b} counter-terms! C_{0b}, C_{1a} and C_{1b} counter-terms are determined by the $X(3872)$, $Z_b(10610)$ and $Z_b(10650)$ resonances.

For some values of C_{0a} appears a $D\bar{D}$ bound state close to threshold....



HQSS predicts that the s-wave $D^*\bar{D}^*$ interaction in the 2^{++} sector is, up to corrections suppressed by the charm quark mass, identical to that in the $X(3872)$ sector (1^{++}) and given by ($C_{0X} = C_{0a} + C_{0b}$, $C_{1X} = C_{1a} + C_{1b}$)

$$V_{2^{++}} = \frac{1}{2}\begin{pmatrix} C_{0X} + C_{1X} & C_{0X} - C_{1X} \\ C_{0X} - C_{1X} & C_{0X} + C_{1X} \end{pmatrix} + \mathcal{O}(q/m_c)$$



$$M_{X_2} - M_{X(3872)} \approx M_{D^*} - M_D \approx 140 \text{ MeV}$$

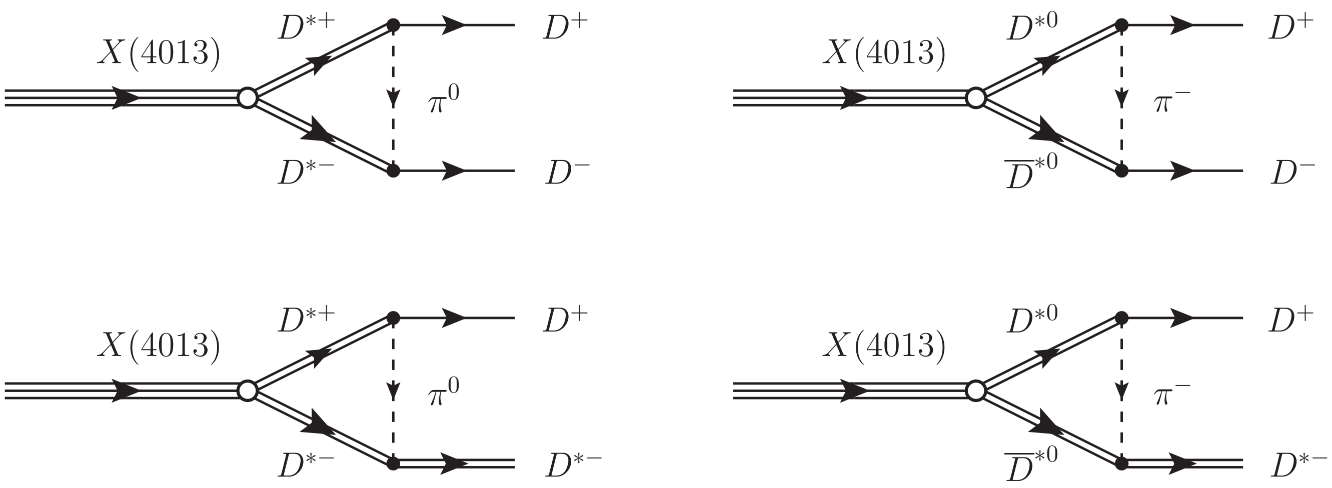
$$M_{X_{b2}} - M_{X_b} \approx M_{B^*} - M_B \approx 46 \text{ MeV}$$

... states with 2^{++} quantum numbers exist as well as spin partners of the 1^{++} states in the spectra of the conventional heavy quarkonia and tetraquarks. However, the mass splittings would only be accidentally the same as the fine splitting between the vector and pseudoscalar heavy-light mesons.

JLAB LQCD group found a 2^{++} state $M = 4025 \pm 12$ MeV, with $m_\pi \simeq 400$ (JHEP 1207 (2012) 126).

X_2 hadronic decays (D−wave decays)

	without pion FF		with pion FF	
	$\Lambda = 0.5$ GeV	$\Lambda = 1$ GeV	$\Lambda = 0.5$ GeV	$\Lambda = 1$ GeV
$\Gamma(X_2 \to D^+ D^-)$ [MeV]	$3.3^{+3.4}_{-1.4}$	$7.3^{+7.9}_{-2.1}$	$0.5^{+0.5}_{-0.2}$	$0.8^{+0.7}_{-0.2}$
$\Gamma(X_2 \to D^0 \bar{D}^0)$ [MeV]	$2.7^{+3.1}_{-1.2}$	$5.7^{+7.8}_{-1.8}$	$0.4^{+0.5}_{-0.2}$	$0.6^{+0.7}_{-0.2}$
$\Gamma(X_2 \to D^+ D^{*-})$ [MeV]	$2.4^{+2.1}_{-1.0}$	$4.4^{+3.1}_{-1.2}$	$0.7^{+0.6}_{-0.3}$	$1.0^{+0.5}_{-0.2}$
$\Gamma(X_2 \to D^0 \bar{D}^{*0})$ [MeV]	$2.0^{+2.1}_{-0.9}$	$3.5^{+3.5}_{-1.0}$	$0.5^{+0.6}_{-0.2}$	$0.7^{+0.5}_{-0.2}$



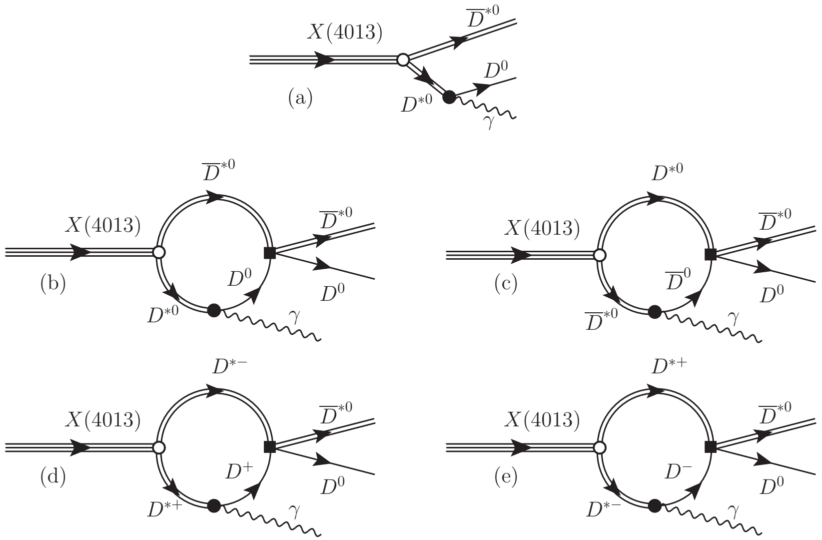
$$\Gamma(X_2 \to D\bar{D}) = 1.2 \pm \underbrace{0.3}_{\text{sys}\,(\Lambda)}\, {}^{+1.3}_{-0.4}\ \text{MeV}$$

$$\Gamma(X_2 \to D\bar{D}^*) + \Gamma(X_2 \to D^*\bar{D}) = 2.9 \pm \underbrace{0.5}_{\text{sys}\,(\Lambda)}\, {}^{+2.0}_{-1.0}\ \text{MeV}.$$

X_{b2} hadronic decays (D−wave decays)

	without pion FF		with pion FF	
	$\Lambda = 0.5$ GeV	$\Lambda = 1$ GeV	$\Lambda = 0.5$ GeV	$\Lambda = 1$ GeV
$\Gamma(X_{b2} \to B\bar{B})$ **[MeV]**	$26.0^{+1.0}_{-3.3}$	8^{+15}_{-7}	$4.4^{+0.1}_{-0.4}$	$0.7^{+1.4}_{-0.6}$
$\Gamma(X_{b2} \to B\bar{B}^*)$ **[MeV]**	$7.1^{+3.4}_{-3.7}$	–	$2.0^{+0.9}_{-1.0}$	–

X_2 & X_{b2} radiative decays (S−wave decays)



The coupling of the photon to the s-wave heavy mesons contains two contributions: magnetic couplings to the light and heavy quarks (Amundson et al., PLB 296 (1992) 415). Both terms are needed to understand the observed electromagnetic small width of the D^{*+} relative to the D^{*0}

$$\Gamma(D^{*0} \to D^0\gamma) = \frac{\alpha}{3}\frac{m_{D^0}}{m_{D^{*0}}}\left(\beta_1 + \frac{2}{3m_c}\right)^2 E_\gamma^3,$$

$$\Gamma(D^{*+} \to D^+\gamma) = \frac{\alpha}{3}\frac{m_{D^+}}{m_{D^{*+}}}\left(\beta_2 + \frac{2}{3m_c}\right)^2 E_\gamma^3$$

$$\beta_1 = \frac{2}{3}\beta - \frac{g^2 m_K}{8\pi f_K^2} - \frac{g^2 m_\pi}{8\pi f_\pi^2}, \ \beta_2 = -\frac{1}{3}\beta + \frac{g^2 m_\pi}{8\pi f_\pi^2}$$

$$\Gamma(D^{*0} \to D^0\gamma) = 22.7 \pm 2.6 \text{ keV}$$

$$\Gamma(D^{*+} \to D^+\gamma) = 1.33 \pm 0.33 \text{ keV}$$

$$-\mathbf{i}\mathcal{T}(\lambda,\lambda_*,\lambda_\gamma)^{\rm FSI\,(b+c)}_{\mathbf{D^0\bar D^{*0}}\gamma} = -g_0^{X_2}\sqrt{4\pi\alpha}N_\gamma\left(\beta_1+\frac{2}{3m_c}\right)\epsilon_{ijm}\epsilon^{jn}(\lambda)\epsilon^{*n}(\lambda_*)\epsilon^{*i}_\gamma(\lambda_\gamma)p^m_\gamma$$

$$\times\left\{4m_Dm_{D^*}\widehat{\mathbf{T}}_{\mathbf{00\to 00}}(\mathbf{m_{23}})\right\}J(m_{D^{*0}},m_{D^{*0}},m_{D^0},\vec p_\gamma)$$

$$-\mathbf{i}\mathcal{T}(\lambda,\lambda_*,\lambda_\gamma)^{\rm FSI\,(d+e)}_{\mathbf{D^0\bar D^{*0}}\gamma} = -g_c^{X_2}\sqrt{4\pi\alpha}N_\gamma\left(\beta_2+\frac{2}{3m_c}\right)\epsilon_{ijm}\epsilon^{jn}(\lambda)\epsilon^{*n}(\lambda_*)\epsilon^{*i}_\gamma(\lambda_\gamma)p^m_\gamma$$

$$\times\left\{4m_Dm_{D^*}\widehat{\mathbf{T}}_{+-\to\mathbf{00}}(\mathbf{m_{23}})\right\}J(m_{D^{*+}},m_{D^{*+}},m_{D^+},\vec p_\gamma)$$

$$-\mathbf{i}\mathcal{T}(\lambda,\lambda_*,\lambda_\gamma)^{\rm FSI}_{\mathbf{D^+\bar D^{*-}}\gamma} = -\sqrt{4\pi\alpha}N_\gamma\epsilon_{ijm}\epsilon^{jn}(\lambda)\epsilon^{*n}(\lambda_*)\epsilon^{*i}_\gamma(\lambda_\gamma)p^m_\gamma 4m_Dm_{D^*}$$

$$\times\left\{g_c^{X_2}\left(\beta_2+\frac{2}{3m_c}\right)\left[\widehat{\mathbf{T}}_{+-\to+-}(\mathbf{m_{23}})J(m_{D^{*+}},m_{D^{*+}},m_{D^+},\vec p_\gamma)\right]\right.$$

$$\left.+\; g_0^{X_2}\left(\beta_1+\frac{2}{3m_c}\right)\left[\widehat{\mathbf{T}}_{\mathbf{00}\to+-}(\mathbf{m_{23}})J(m_{D^{*0}},m_{D^{*0}},m_{D^0},\vec p_\gamma)\right]\right\}$$

$\widehat{T}_{00\to 00}=T_{D^0\bar D^{*0}\to D^0\bar D^{*0}}+T_{D^{*0}\bar D^0\to D^0\bar D^{*0}}$, $\widehat{T}_{+-\to 00}=T_{D^+D^{*-}\to D^0\bar D^{*0}}+T_{D^{*+}D^-\to D^0\bar D^{*0}}$, $\widehat{T}_{+-\to+-}=T_{D^+D^{*-}\to D^+D^{*-}}+T_{D^{*+}D^-\to D^+D^{*-}}$, $\widehat{T}_{00\to+-}=T_{D^0\bar D^{*0}\to D^+D^{*-}}+T_{D^{*0}\bar D^0\to D^+D^{*-}}=\widehat{T}_{+-\to 00}$.

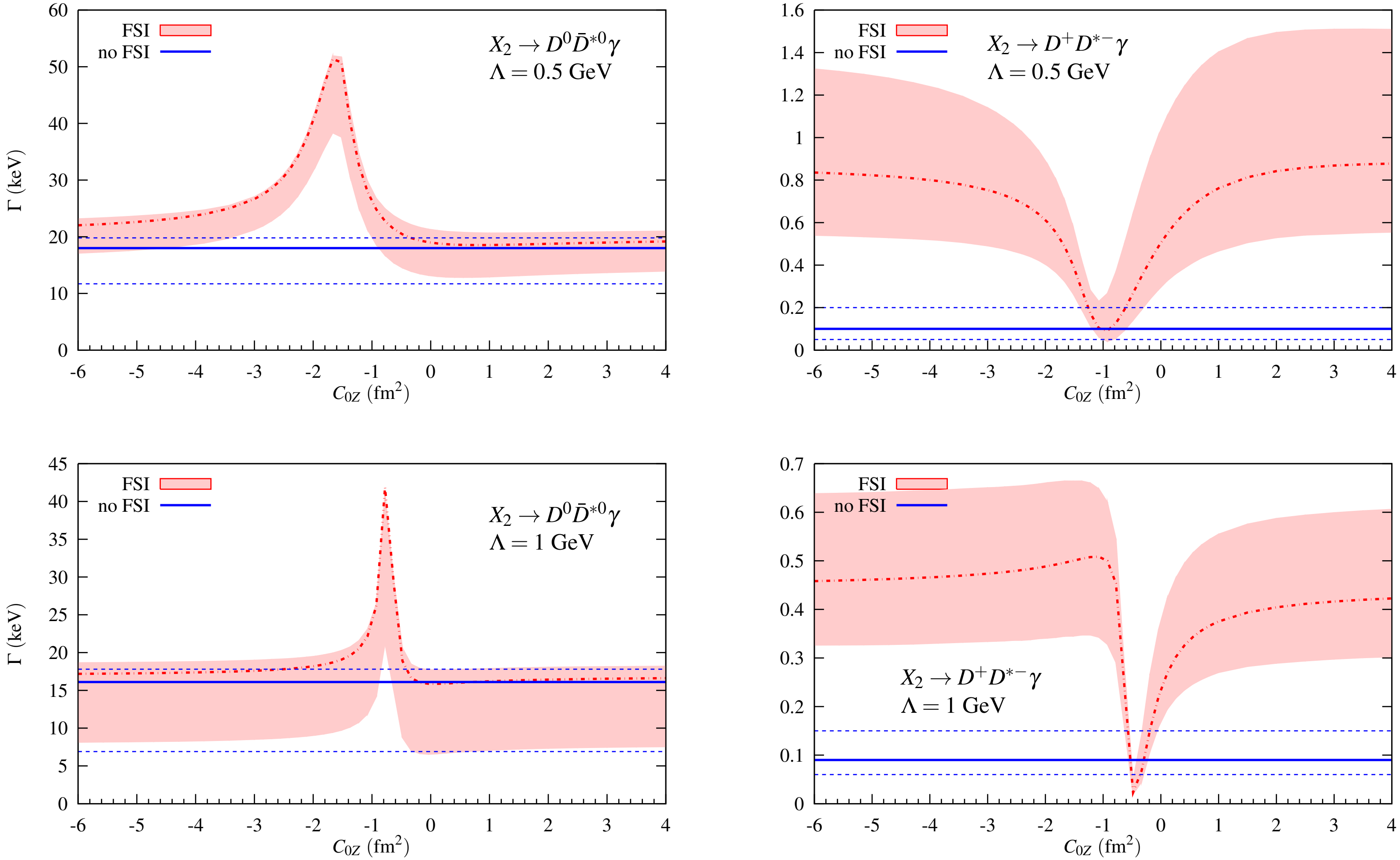
Thanks to the conservation of $C-$parity, the FSI corrections will depend only on the $C=-1$ terms, C_{0Z} and C_{1Z}, of the interaction,

$$\begin{pmatrix} \widehat{T}_{00\to 00} & \widehat{T}_{+-\to 00} \\ \widehat{T}_{00\to +-} & \widehat{T}_{+-\to +-} \end{pmatrix}^{-1} = \begin{pmatrix} \frac{C_{0Z}+C_{1Z}}{2} & \frac{C_{0Z}-C_{1Z}}{2} \\ \frac{C_{0Z}-C_{1Z}}{2} & \frac{C_{0Z}+C_{1Z}}{2} \end{pmatrix}^{-1} - \begin{pmatrix} G_{D^0\bar{D}^{*0}} & 0 \\ 0 & G_{D^+\bar{D}^{*-}} \end{pmatrix}$$

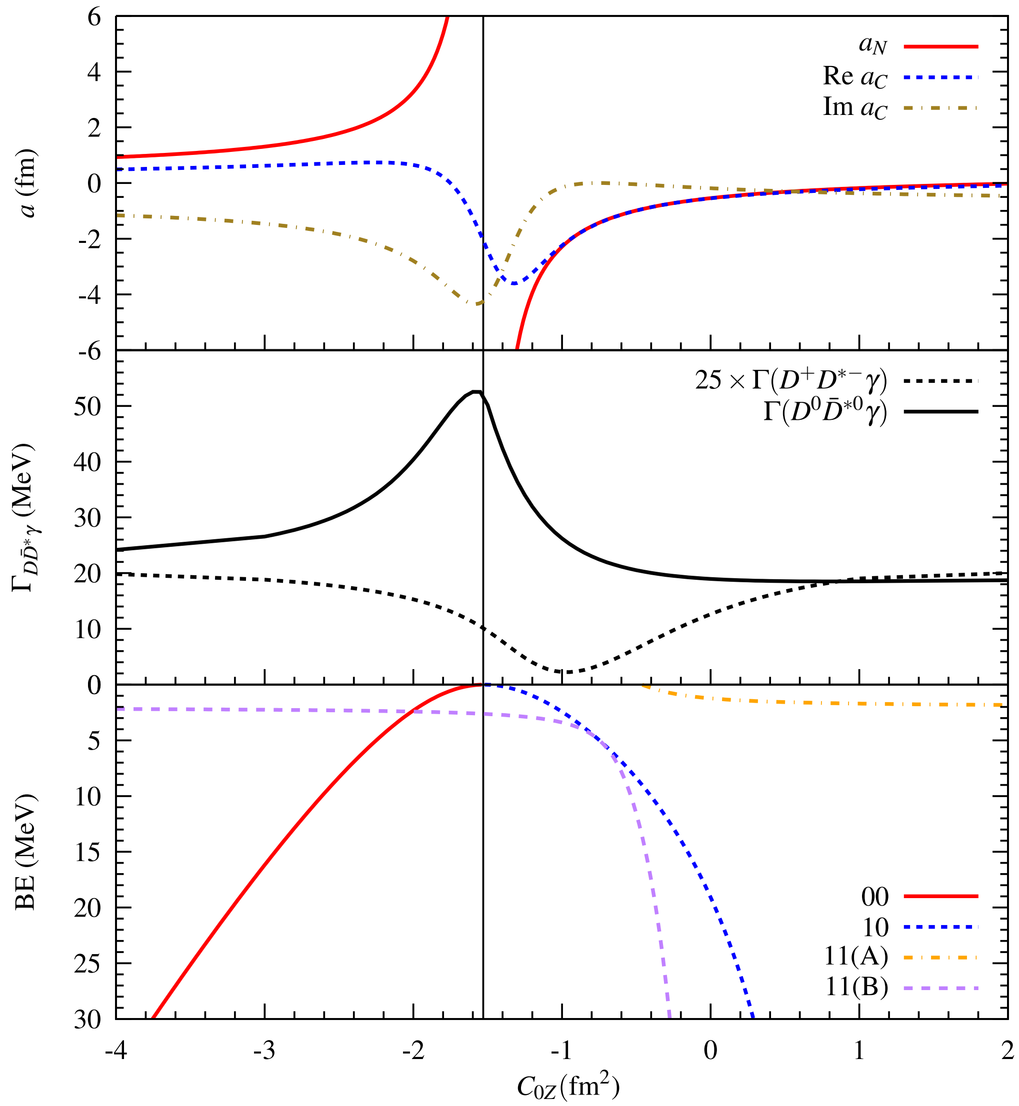
with $C_{IZ} = C_{IA} - C_{IB}$.

Heavy Flavor symmetry: $Z_b(10610)$ & $Z_b(10650)$ **(**$I=1$**,** $J^{PC}=1^{+-}$**)** $B\bar{B}^*$ **and** $B^*\bar{B}^*$ **molecules** $\to C_{1Z}$**,**

$$C_{1Z} \equiv C_{1A} - C_{1B} = -0.75^{+0.10}_{-0.17}\ (-0.30^{+0.02}_{-0.04})\,\mathrm{fm}^2$$



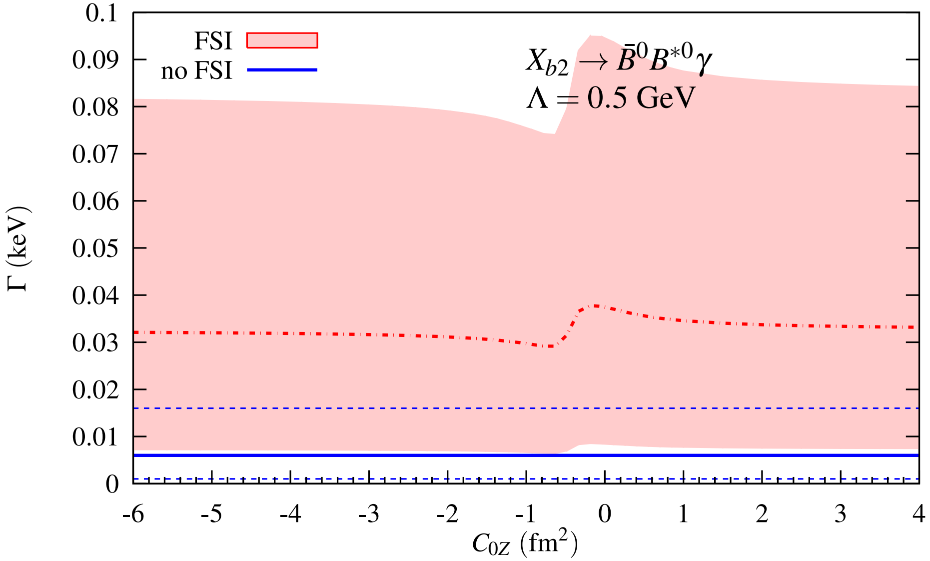
FSI
no FSI
$X_2 \to D^0\bar{D}^{*0}\gamma$
$\Lambda = 0.5$ GeV
$X_2 \to D^+D^{*-}\gamma$
$\Lambda = 0.5$ GeV
$X_2 \to D^0\bar{D}^{*0}\gamma$
$\Lambda = 1$ GeV
$X_2 \to D^+D^{*-}\gamma$
$\Lambda = 1$ GeV
Γ (keV)
C_{0Z} (fm^2)

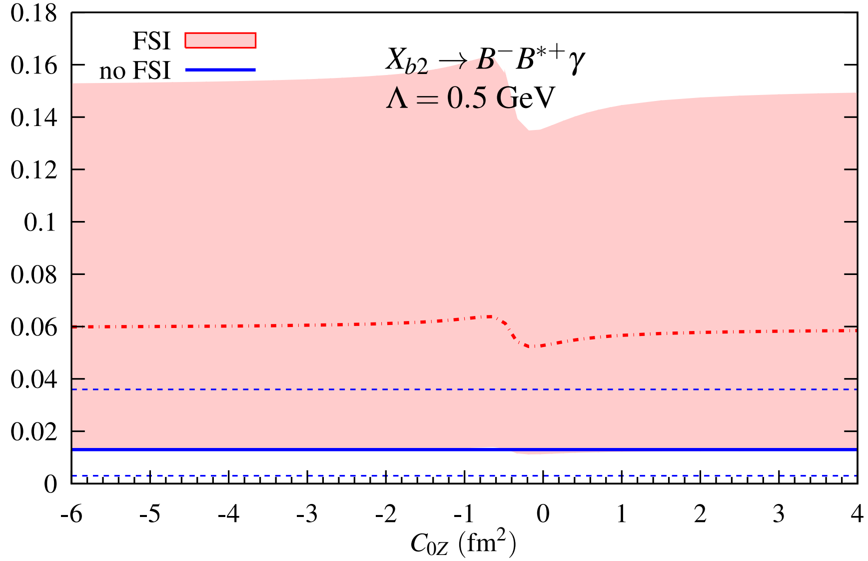


FSI effects turn out to be important, and for some values of C_{0Z}, they dominate the decay width. The maximum effects of the FSI mechanisms approximately occur for values of C_{0Z} which give rise to an isoscalar 1^{+-} $D\bar{D}^*$ bound or virtual state close to threshold.

FSI corrections are always important in the $D^+D^{*-}\gamma$ channel. This is because the tree level amplitude involves only the $D^{*\pm}D^{\pm}\gamma$ magnetic coupling, while FSI brings in the neutral magnetic coupling, which is much larger than the former one.

Dependence of the $X_{b2} \to \bar{B}^0 B^{*0}\gamma$ and $X_{b2} \to B^- B^{*+}\gamma$ partial decay widths on the low-energy constant C_{0Z}





FSI corrections are quite important and larger than the tree level estimates because of the the $T^{I=1}_{C=-1}$ bound state ($Z_b(10610)$), placed almost at threshold that greatly enhances the loop mechanisms

CONCLUSIONS

- Heavy quark symmetries for heavy light meson-antimeson systems in a contact-range EFT have been investigated. In addition to HQSS, HFS and HADS can be used to predict new heavy meson molecules and triply heavy pentaquarks.

- In the SU(3) light flavor limit, the leading order Lagrangian respecting heavy quark spin symmetry contains four independent counter-terms. Neglecting $1/m_Q$ corrections, three of these low energy constants can be determined by theorizing a molecular description of the $X(3872)$ and $Z_b(10610)$ states. Thus, we can **predict new hadronic molecules, in particular the isovector charmonium partners of the $Z_b(10610)$ and the $Z_b(10650)$ states.**

- We studied the $X(3872) \to D^0 \bar{D}^0 \pi^0$ decay using an **EFT based on the hadronic molecule assumption for the $X(3872)$.**

- This decay is unique in the sense that it is sensitive to the long-distance structure of the $X(3872)$ as well as the strength of the S-wave interaction between the D and $\bar{D}$.

- If there was a near threshold pole in the $D\bar{D}$ system, the partial decay width can be very different from the result neglecting the FSI effects. This decay may be used to measure the so far unknown counter-term C_{0a}.

- A future measurement of the $d\Gamma/d|\vec{p}_{D^0}|$ distribution might provide valuable information on the $X(3872)$ wave function at the fixed momentum $\Psi(\vec{p}_{D^0})$

- **We have also studied the hadronic and radiative decays of a molecular $P^*\bar{P}^*$ $J^{PC}=2^{++}$ state in the charm (X_2) and bottom sectors using an EFT approach.**

- **The hadron d-wave $X_2 \to D\bar{D}$ and $X_2 \to D\bar{D}^*$ decays are driven via OPE and we find widths of the order of few MeV. The analysis runs in parallel in the bottom sector, where we also find widths of the order of the MeV.**

- The radiative $X_2 \to D\bar{D}^*\gamma$ and $X_{b2} \to \bar{B}B^*\gamma$ decay widths are small of the order of keV's (eV's) in the charm (bottom) sectors. **They are affected by large $D\bar{D}^*$ or $B\bar{B}^*$ FSI mechanisms because they are enhanced by the presence of the isovector $Z_c(3900)$ and $Z_b(10610)$ resonances located near the $D^0\bar{D}^{*0}$ and $\bar{B}B^*$ thresholds, respectively. In the charm sector, FSI corrections turn out to be also sensitive to the negative $C-$parity isoscalar $D\bar{D}^*$ interaction (C_{0Z}).**

BACK UP MATERIAL

What can radiative decays of the $X(3872)$ teach us about its nature? by Feng-Kun Guo, C. Hanhart, Yu.S. Kalashnikova, Ulf-G. Meißner, A.V. Nefediev **(PLB742 (2015) 394)**

"... We show that radiative decays of the $X(3872)$ into $\gamma J/\Psi$ and $\gamma\Psi'$ are not sensitive to the long-range structure of the $X(3872)$. In particular, contrary to earlier claims, we argue that the experimentally determined ratio is not in conflict with a wave function of the $X(3872)$ that is dominated by the $D\bar{D}^*$ hadronic molecular component."

EFT framework for the description of heavy meson-antimeson molecules $(T=0)$:

- LO: $\mathcal{L}_{4H}$ (contact range interaction)

$$V_C(0^{++}) = \begin{pmatrix} C_{0a} & \sqrt{3}\,C_{0b} \\ \sqrt{3}\,C_{0b} & C_{0a} - 2\,C_{0b} \end{pmatrix}, \; V_C(1^{+-}) = \begin{pmatrix} C_{0a} - C_{0b} & 2\,C_{0b} \\ 2\,C_{0b} & C_{0a} - C_{0b} \end{pmatrix}$$

$$V_C(1^{++}) \;=\; C_{0a} + C_{0b}, \qquad V_C(2^{++}) = C_{0a} + C_{0b}$$

EFT framework for the description of heavy meson-antimeson molecules ($T = 0$):

- LO: $\mathcal{L}_{4H}$ (contact range interaction)

$$V_C(0^{++}) = \begin{pmatrix} C_{0a} & \mathbf{0} \\ \mathbf{0} & C_{0a} - 2\,C_{0b} \end{pmatrix}, \; V_C(1^{+-}) = \begin{pmatrix} C_{0a} - C_{0b} & \mathbf{0} \\ \mathbf{0} & C_{0a} - C_{0b} \end{pmatrix}$$

$$V_C(1^{++}) \;=\; C_{0a} + C_{0b}, \qquad V_C(2^{++}) = C_{0a} + C_{0b}$$

- pion exchanges and particle coupled channel effects to be sub-leading corrections.

EFT framework for the description of heavy meson-antimeson molecules ($T = 0$):

- LO: $\mathcal{L}_{4H}$ (contact range interaction)

$$V_C(0^{++}) = \begin{pmatrix} C_{0a} & \mathbf{0} \\ \mathbf{0} & C_{0a} - 2\,C_{0b} \end{pmatrix},\ V_C(1^{+-}) = \begin{pmatrix} C_{0a} - C_{0b} & \mathbf{0} \\ \mathbf{0} & C_{0a} - C_{0b} \end{pmatrix}$$

$$V_C(1^{++}) \;=\; C_{0a} + C_{0b}, \qquad V_C(2^{++}) = C_{0a} + C_{0b}$$

- pion exchanges and particle coupled channel effects to be sub-leading corrections.

- **Counter–terms ?**

NLO: One Pion Exchange

$H_a^{(Q)}$ $[H^{(\bar Q)a}]$ transforms as a $(2,\bar 2)$ $[(\bar 2, 2)]$ under the heavy spin $\otimes$**SU(2)$_\mathbf{V}$ isospin symmetry**

$$H_a^{(Q)} \to S\left(H^{(Q)}\mathbf{U}^\dagger\right)_a, \qquad H^{(\bar Q)a} \to \left(\mathbf{U}H^{(\bar Q)}\right)^a S^\dagger$$

$$\mathcal{L}_{\pi HH} = -\frac{g}{\sqrt{2}f_\pi}\left(\mathrm{Tr}\left[\bar H^{(Q)j}H_i^{(Q)}\gamma_\mu\gamma_5\right] + \mathrm{Tr}\left[H^{(\bar Q)j}\bar H_i^{(\bar Q)}\gamma^\mu\gamma_5\right]\right)(\vec\tau\partial_\mu\vec\phi)^i_j + \cdots$$

$$\mathbf{V^{OPE}_{H\bar H}(\vec q\,)} \sim \frac{\mathbf{g^2}}{\mathbf{2f_\pi^2}}\frac{(\vec{\mathbf{a}}\cdot\vec{\mathbf{q}}\,)(\vec{\mathbf{b}}\cdot\vec{\mathbf{q}}\,)}{\vec{\mathbf{q}}^{\,2}+\mathbf{m_\pi^2}} \sim \mathcal{O}(\mathbf{Q^0})$$

$g \simeq 0.6$ is the axial coupling between the heavy meson and the pion , $f_\pi \simeq 132\,\mathrm{MeV}$, and $\vec q$ the momentum exchanged by the heavy meson and antimeson. Besides, $\vec a$ and $\vec b$ are the polarization operators.

NLO: One Pion Exchange $\sim \mathcal{O}(Q^0) \Rightarrow$ **distinctive feature of the OPE potential can mix different $^{2S+1}L_J$ partial waves.**

J^{PC}	$\mathrm{H\bar{H}}$	$^{2S+1}L_J$	E ($\Lambda = 0.5$ GeV)	E ($\Lambda = 1$ GeV)	Exp
0^{++}	$D\bar{D}$	$^1S_0$	3708 (3706 $\pm$ 10)	3720 (3712^{+13}_{-17})	–
1^{++}	$D^*\bar{D}$	$^3S_1$-$^3D_1$	Input	Input	3872
1^{+-}	$D^*\bar{D}$	$^3S_1$-$^3D_1$	3816 (3814 $\pm$ 17)	3823 (3819^{+24}_{-27})	–
0^{++}	$D^*\bar{D}^*$	$^1S_0$-$^5D_2$	Input	Input	3917
1^{+-}	$D^*\bar{D}^*$	$^3S_1$-$^3D_1$	3954 (3953 $\pm$ 17)	3958 (3956^{+25}_{-28})	3942
2^{++}	$D^*\bar{D}^*$	$^1D_2$-$^5S_2$-$^5D_2$-$^5G_2$	4015 (4012 $\pm$ 3)	4014 (4012^{+4}_{-9})	–

Predicted masses (in MeV) of the $X(3872)$ **HQSS partners when the OPE potential is included.** We display results for two different values of the Gaussian cutoff. Now, we find $C_{0a} = -3.46$ fm^2 and $C_{0b} = 1.98$ fm^2, and $C_{0a} = -0.98$ fm^2 and $C_{0b} = 0.69$ fm^2, for $\Lambda = 0.5$ and 1 GeV, respectively. **OPE $\delta(r)$ contribution can be absorbed within the contact range piece $C_{0b} \to C_{0b} - \frac{g^2}{2f_\pi^2}$.**

Binding energies of the states are stable with respect to the iteration of the OPE potential: agreement with the *a priori* EFT expectations! [PRD 86, 056004 (2012)]

NNLO: Particle Coupled Channels Momentum scale **(hard)** associated with the coupled channels is

$$\Lambda_C(0^{++}) = \sqrt{2\mu\,(2\Delta_Q)} \sim 750\,\text{MeV}, \quad \Lambda_C(1^{+-}) = \sqrt{2\mu\,\Delta_Q} \sim 520\,\text{MeV}$$

with $\Delta_Q \sim (M_{D^*} - M_D) \Rightarrow G \sim \mathcal{O}(Q^3) \Rightarrow VGV \sim \mathcal{O}(Q^1)$.

- In contrast to the OPE corrections, the **LO counter-term structure stemming from HQSS is not expected to be able to absorb the kind of divergences associated with the coupled channel calculations.**
- One should **add new counter-terms at $\mathcal{O}(Q)$ to soften the regularization scheme dependence** and make the EFT *renormalizable* again. Higher orders will introduce new unknown constants that cannot be fixed at the moment owing to the scarce experimental data available.

J^{PC}	$\mathrm{H}\bar{\mathrm{H}}$	$E - i\Gamma/2$ ($\Lambda = 0.5$ GeV)	$E - i\Gamma/2$ ($\Lambda = 1$ GeV)	Exp
0^{++}	$D\bar{D}$, $D^*\bar{D}^*$	3690 (3706 $\pm$ 10)	3694 (3712^{+13}_{-17})	–
1^{++}	$D^*\bar{D}$	Input	Input	3872
1^{+-}	$D^*\bar{D}$, $D^*\bar{D}^*$	3782 (3814 $\pm$ 17)	3782 (3819^{+24}_{-27})	–
0^{++}	$D\bar{D}$, $D^*\bar{D}^*$	$3939 - \frac{i}{2}\,12$ (3917)	$3937 - \frac{i}{2}\,31$(3917)	$3917 \pm 3 - \frac{i}{2}\,28^{+10}_{-9}$
1^{+-}	$D^*\bar{D}$, $D^*\bar{D}^*$	$3984 - \frac{i}{2}\,17$ (3953 $\pm$ 17)	$3982 - \frac{i}{2}\,29$ (3956^{+25}_{-28})	$3942 \pm 9 - \frac{i}{2}\,37^{+27}_{-17}$
2^{++}	$D^*\bar{D}^*$	4012 (4012 $\pm$ 3)	4012 (4012^{+4}_{-9})	–

Masses and widths (in MeV) of the $X(3872)$ **HQSS partners when coupled channels effects are included. The contact terms are adjusted to reproduce the** $X(3872)$ **and** $X(3915)$ **masses neglecting coupled channel effects**. OPE interactions are not included.

$$|\Delta E_B| \simeq |E_B| \left(\frac{\gamma_B}{\Lambda_C}\right)^2 \sim (30 - 40)\ \text{MeV}$$

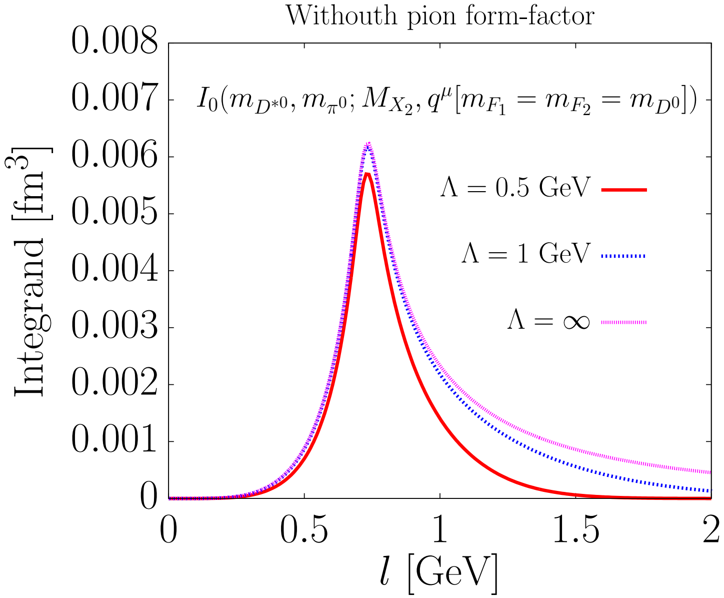
[PRD 86, 056004 (2012)]

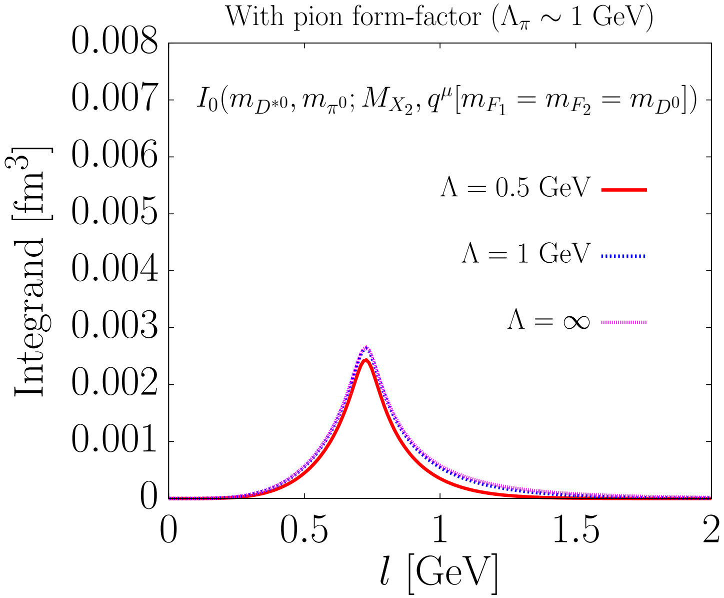
NNLO: Particle Coupled Channels[PRD 86, 056004 (2012)]

J^{PC}	$\mathrm{H\bar{H}}$	$E - i\Gamma/2$ ($\Lambda = 0.5$ GeV)	$E - i\Gamma/2$ ($\Lambda = 1$ GeV)	Exp
0^{++}	$D\bar{D}$, $D^*\bar{D}^*$	3658 (3706 $\pm$ 10)	3669 (3712^{+13}_{-17})	–
1^{++}	$D^*\bar{D}$	Input	Input	3872
1^{+-}	$D^*\bar{D}$, $D^*\bar{D}^*$	3730 (3814 $\pm$ 17)	3739 (3819^{+24}_{-27})	–
0^{++}	$D\bar{D}$, $D^*\bar{D}^*$	$3917 - \frac{i}{2}\,23$ (3917)	$3917 - \frac{i}{2}\,50$ (3917)	$3917 \pm 3 - \frac{i}{2}\,28^{+10}_{-9}$
1^{+-}	$D^*\bar{D}$, $D^*\bar{D}^*$	$3979 - \frac{i}{2}\,24$ (3953 $\pm$ 17)	$3979 - \frac{i}{2}\,39$ (3956^{+25}_{-28})	$3942 \pm 9 - \frac{i}{2}\,37^{+27}_{-17}$
2^{++}	$D^*\bar{D}^*$	4012 (4012 $\pm$ 3)	4012(4012^{+4}_{-9})	–

Masses and widths (in MeV) of the $X(3872)$ **HQSS partners when coupled channels effects are included. The contact terms are adjusted to reproduce the $X(3872)$ and $X(3915)$ masses, while OPE effects are neglected.** We find $C_{0a} = -4.16$ fm^2 and $C_{0b} = 2.21$ fm^2, and $C_{0a} = -1.14$ fm^2 and $C_{0b} = 0.35$ fm^2, for $\Lambda = 0.5$ and 1 GeV, respectively.

- In contrast to the OPE corrections, the **LO counter-term structure stemming from HQSS is not expected to be able to absorb the kind of divergences associated with the coupled channel calculations.**
- One should **add new counter-terms at $\mathcal{O}(Q)$ to soften the regularization scheme dependence** and make the EFT *renormalizable* again (problem: it might occur a transition from a power counting in which Λ_C is a hard scale to a different one in which it is a soft scale). Higher orders will introduce new unknown constants that cannot be fixed at the moment owing to the scarce experimental data available.


Withouth pion form-factor
$I_0(m_{D^{*0}}, m_{\pi^0}; M_{X_2}, q^\mu[m_{F_1} = m_{F_2} = m_{D^0}])$
$\Lambda = 0.5$ GeV
$\Lambda = 1$ GeV
$\Lambda = \infty$
Integrand [fm^3]
l [GeV]
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.5
1
1.5
2


With pion form-factor ($\Lambda_\pi \sim 1$ GeV)
$I_0(m_{D^{*0}}, m_{\pi^0}; M_{X_2}, q^\mu[m_{F_1} = m_{F_2} = m_{D^0}])$
$\Lambda = 0.5$ GeV
$\Lambda = 1$ GeV
$\Lambda = \infty$
Integrand [fm^3]
l [GeV]
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.5
1
1.5
2