HHIQCD at YITP
5th March (Thu.)

U(1) axial anomaly with chiral fermion at finite temperature

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Osaka University (Ph.D student)
→ ??? (Looking for a post)

(Related talk by G.Cossu today 17:00-)

Reference:

A. Tomiya, G. Cossu, H. Fukaya, S. Hashimoto and J. Noaki, arXiv:1412.7306 [hep-lat].

Full paper is in preparation:

PURPOSE:

We would like to judge the $U(1)_A$ symmetry is restored above the QCD critical temperature or not

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KEYWORD : Chiral symmetry

1.Introduction

Sym. of Nf=2 QCD (m=0)

$$\mathcal{L} : SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$$

$$\int d^4x \bar{\psi}(\not{D})\psi \quad \psi(x) = \begin{pmatrix} u(x) \\ d(x) \end{pmatrix}$$

At zero temperature…

SU(2) chiral symmetry is spontaneously broken

<u>$U(1)_A$ Sym. is violated by the anomaly</u>

Anomaly = Sym. Violation by Quantum Eff.

Nf=2 massless QCD has chiral symmetries

$$Z=\int \mathcal{D}A_\mu \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left[-\int d^4x\ \bar{\psi}(\not{D})\psi\right] \quad \psi=\begin{pmatrix} u \\ d \end{pmatrix}$$

1. K. Fujikawa and H. Suzuki, Path Integrals and Quantum Anomalies (Oxford University Press, Oxford, 2004)

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$$\begin{cases} \psi \to \psi' = e^{i\gamma_5\theta}\psi \\ \bar{\psi} \to \bar{\psi}' = \bar{\psi}e^{i\gamma_5\theta} \end{cases}$$

←U(1)$_A$Trf(Chiral trf)

Rotate all R/L handed quark opposite dir.

$$\gamma_5 \not{D} = -\not{D}\gamma_5$$

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Action is inv., Measure is not invariant.

U(1)A Anomaly, Chiral anomaly

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U(1)A Anomaly, Chiral anomaly

$$\mathcal{D}\bar{\psi}\mathcal{D}\psi = \mathcal{D}\bar{\psi}'\mathcal{D}\psi' e^{\Gamma}, \quad \Gamma \sim i\int d^4x[\mathrm{tr}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}]$$

1. K. Fujikawa and H. Suzuki, Path Integrals and Quantum Anomalies (Oxford University Press, Oxford, 2004)

Chiral symmetry breaking in QCD (N$_f$=2, m$_{ud}$=0)

$T=0$

$$\underbrace{SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}}_{\text{SSB}} \times U(1)_{\mathrm{V}} \times \underbrace{U(1)_{\mathrm{A}}}_{\text{Anomaly}}$$

$$\longrightarrow SU(2)_{\mathrm{V}} \times U(1)_{\mathrm{V}}$$ Residual symmetry

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$$\longrightarrow SU(2)_\mathrm{V}\times U(1)_\mathrm{V}$$ Residual symmetry

$T>T_c$

$$SU(2)_\mathrm{V}\longrightarrow SU(2)_\mathrm{L}\times SU(2)_\mathrm{R}$$ Restored

$$U(1)_\mathrm{A}\longrightarrow$$ **??**

Chiral symmetry breaking in QCD (N_f=2, m_{ud}=0)

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$T > T_c$

$$SU(2)_{\mathrm{V}} \longrightarrow SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}$$ Restored

$$U(1)_{\mathrm{A}} \longrightarrow$$ **??**

What happens on the anomaly above the Tc?

U(1)$_A$ may be restored...

Two supporting evidences

1. Finite temperature = Theories on L^3x(1/T)

   T=∞ ←→ Theory in D=3 (No anomaly)

   Anomaly disappears at infinite temperature

   (Anomaly could be disappeared at finite temperature?)

2. Cohen's argument (1996, Next page)

   SU(2) chiral symmetry restoration may be related to U(1)$_A$ restoration

Cohen: SU(2) & U(1)$_A$ may be restored

(1996)

Fact : T>Tc , when m→0, chiral symmetry is restored <=> $\langle\bar{\psi}\psi\rangle = 0$

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Cohen：

If we assume $\langle\bar{\psi}\psi\rangle = 0$, gap in the Dirac spectrum, it may lead $\chi_{U(1)_A} = 0$

Order parameter of SU(2) chiral sym.

Order parameter of U(1)A Sym. (Later)

※Additional condition is needed

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Spectral rep :

$$\langle\bar{\psi}\psi\rangle = \lim_{m\to 0}\int_0^\infty d\lambda\, \rho(\lambda)\frac{2m}{\lambda^2+m^2}$$

Banks-Casher rel.

$$\lim_{m\to 0}\chi_{U(1)_A} = \lim_{m\to 0}\int_0^\infty d\lambda\, \rho(\lambda)\frac{4m^2}{(\lambda^2+m^2)^2}$$

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→ Let's check this by Lattice QCD !

Previous studies (DW type) are controversial !

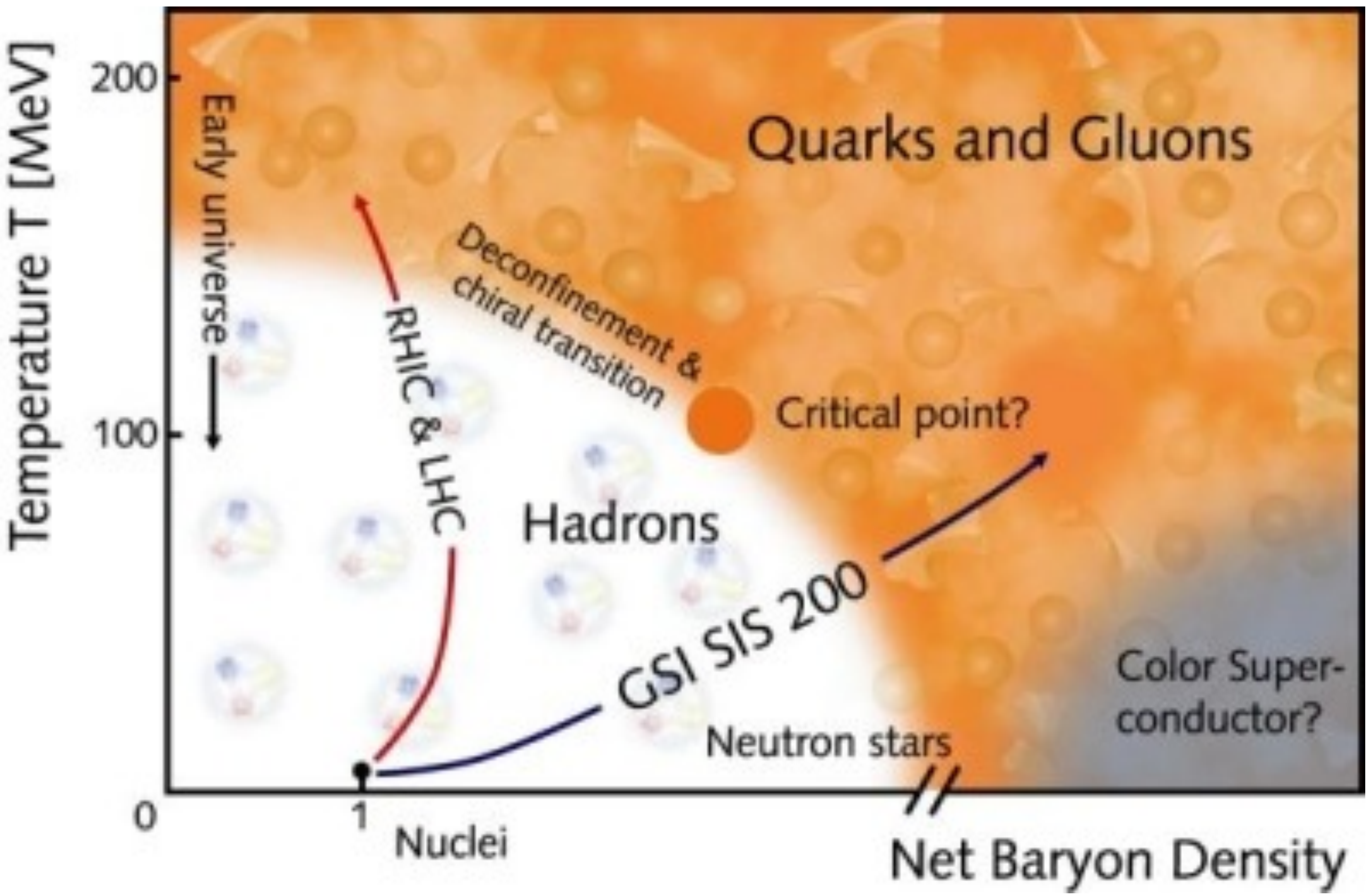
Group	Fermion	Size	Gap in the spectrum	$U_A(1)$ Correlator	$U(1)_A$
JLQCD (2013)	**Overlap (Top. fixed)**	2 fm	**Gap**	Degenerate	**Restored**
TWQCD (2013)	Optimal domain-wall	3 fm	**No gap**	Degenerate	**Restored ?**
LLNL/RBC (2013)	**Domain-wall**	2, 4 fm	**Peak**	No degeneracy	**Violated**

What makes such difference?

Fermion, Volumes or Topology ?

By the way

If $U(1)_A$ symmetry is restored ?



Ref: http://personal.kent.edu/~mstrick6/

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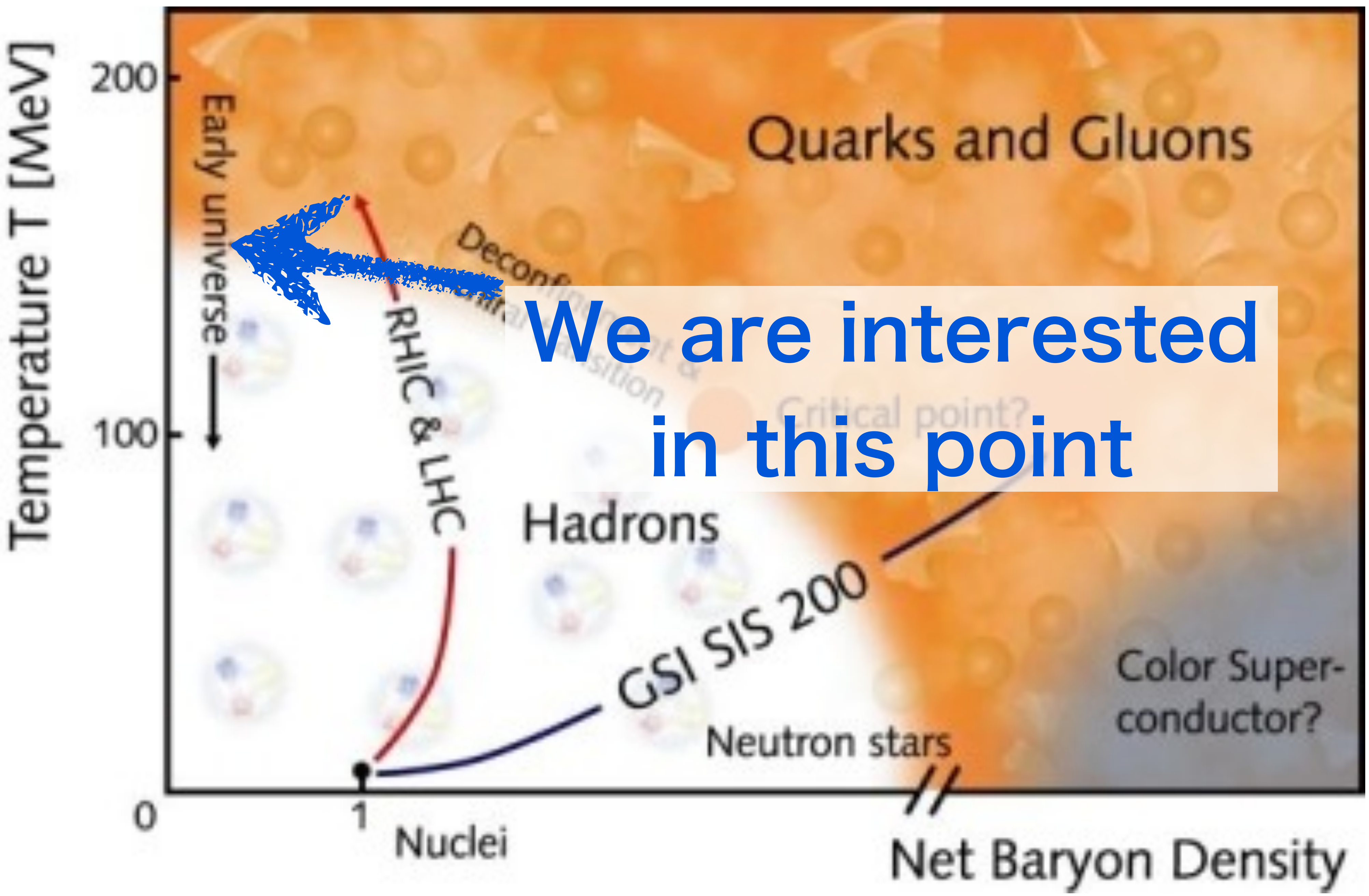
If U(1)A symmetry is restored ?



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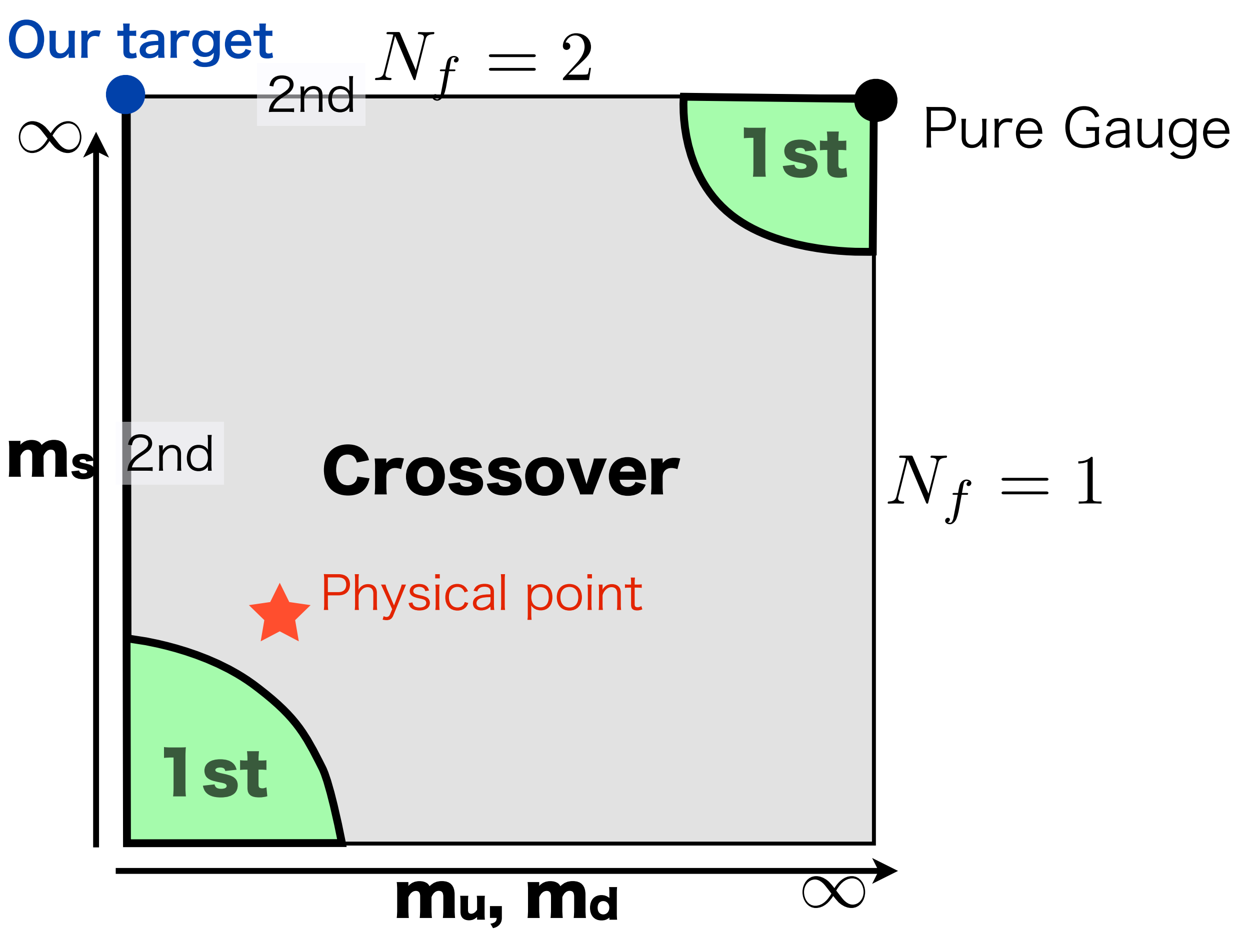

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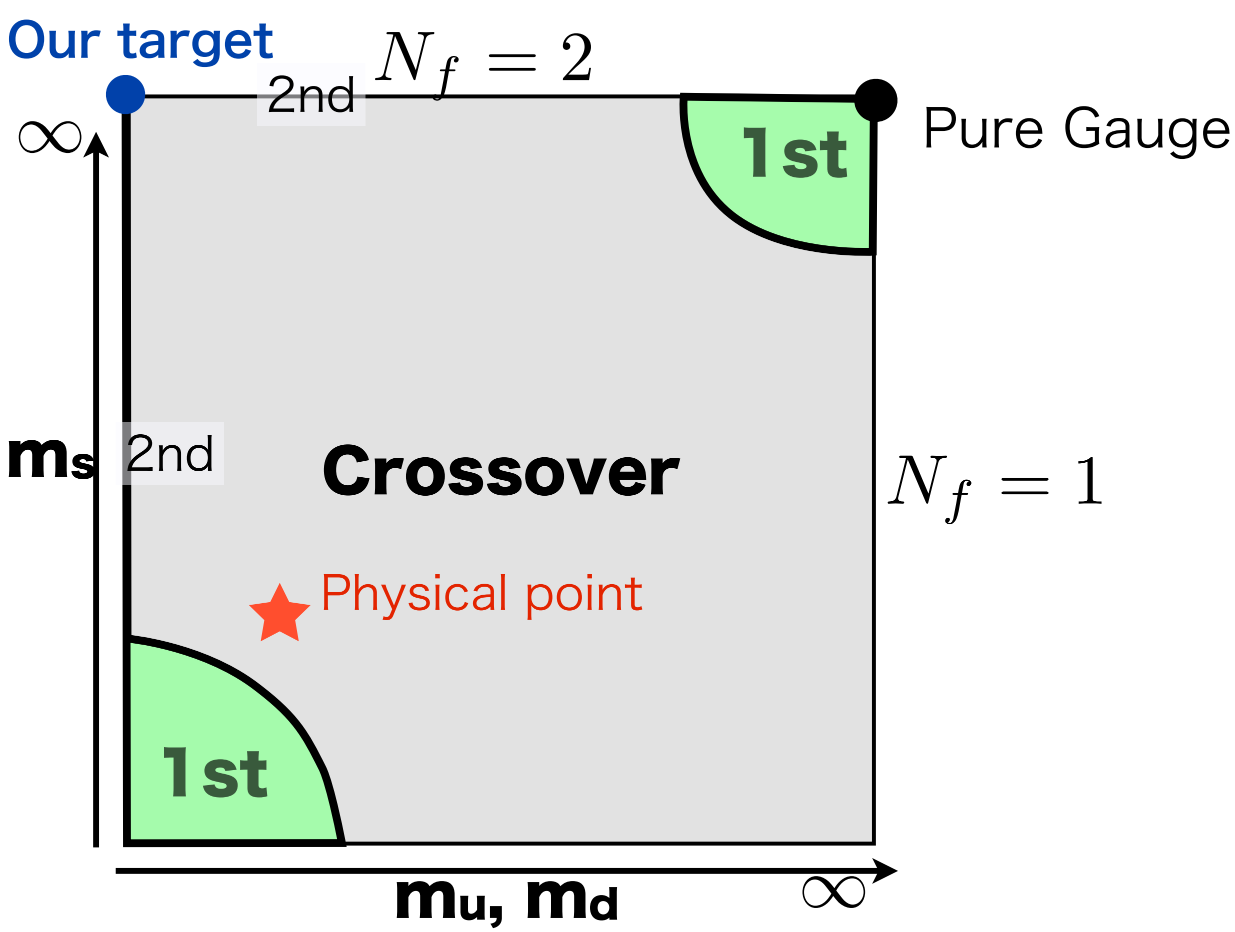
If U(1)A symmetry is restored ?

The order of the transition may be changed :

2nd→1st (Pisarski&Wilczek1983)



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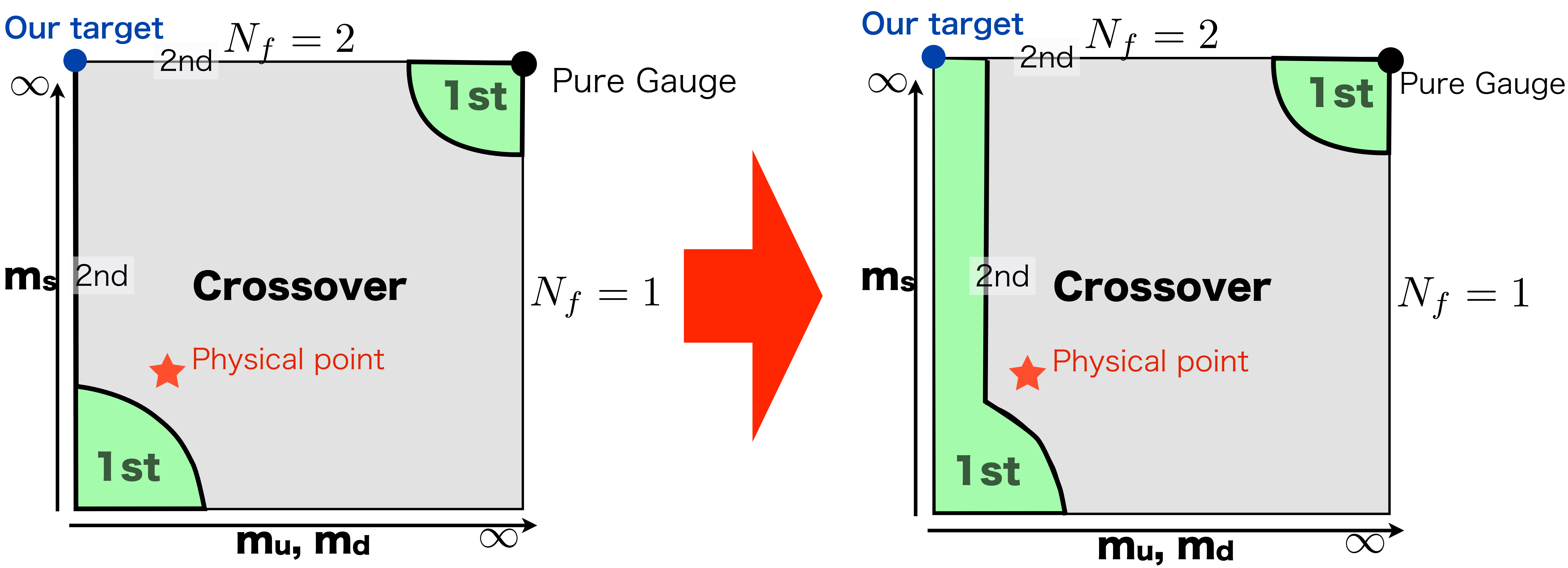


Possibly realized...

Table of contents

1. Introduction
2. $U(1)_A$ symmetry and Correlators
3. The Dirac spectrum
4. Previous studies
5. Our setup
6. Our results
7. Summary

2.U(1)$_A$Sym. & Correlators

Sym. of QCD<=>Degeneracy

$$
\begin{tikzcd}
\langle \pi(x)\pi(0)\rangle \arrow[rr, leftrightarrow, "SU(2)_L \times SU(2)_R"] \arrow[d, leftrightarrow, "U(1)_A"'] & & \langle \sigma(x)\sigma(0)\rangle \arrow[d, leftrightarrow, "U(1)_A"] \\
\langle \delta(x)\delta(0)\rangle \arrow[rr, leftrightarrow, "SU(2)_L \times SU(2)_R"] & & \langle \eta(x)\eta(0)\rangle
\end{tikzcd}
$$

$$\pi(x) = i\bar{\psi}(x)\gamma_5\tau\psi(x) \qquad \sigma(x) = \bar{\psi}(x)\psi(x)$$

$$\delta(x) = \bar{\psi}(x)\tau\psi(x) \qquad \eta(x) = i\bar{\psi}(x)\gamma_5\psi(x)$$

Degeneracy of these channels

<=>There are symmetries

Sym. of QCD<=>Degeneracy

$$\begin{array}{ccc}
\langle \pi(x)\pi(0)\rangle & \xleftrightarrow{SU(2)_L \times SU(2)_R} & \langle \sigma(x)\sigma(0)\rangle \\
{\scriptstyle U(1)_A}\updownarrow & & \updownarrow{\scriptstyle U(1)_A} \\
\langle \delta(x)\delta(0)\rangle & \xleftrightarrow{SU(2)_L \times SU(2)_R} & \langle \eta(x)\eta(0)\rangle
\end{array}$$

$$\chi_{U(1)_A} \equiv \int d^4x[\langle \pi(x)\pi(0)\rangle - \langle \delta(x)\delta(0)\rangle]$$

Order parameter
of U(1)A

If this quantity 0 at V →∞, m→0,
$U(1)_A$ symmetry is restored.

Let's check Cohen's argument

$$\langle\bar{\psi}\psi\rangle = \lim_{m\to 0}\int_0^\infty d\lambda\,\rho(\lambda)\frac{2m}{\lambda^2+m^2}$$

Order parameter of SU(2) Chiral symmetry

$$\lim_{m\to 0}\chi_{U(1)_A} = \lim_{m\to 0}\int_0^\infty d\lambda\,\rho(\lambda)\frac{4m^2}{(\lambda^2+m^2)^2}$$

Order parameter U(1)A symmetry

<u>What happens T>Tc</u>

Checking U(1)$_A$ Sym. restoration at T>T$_c$ ....

- Directly measure $\chi_{U(1)_A}$
- Measure $\rho(\lambda)$ → Has a gap?

3.Dirac spectrum

$$\langle \bar{\psi}\psi \rangle = \lim_{m\to 0}\int_0^\infty d\lambda\, \underline{\rho(\lambda)}\, \frac{2m}{\lambda^2+m^2}$$

$\rho(\lambda)$:Dirac spectrum

=Spectral density of the Dirac operator

$$(\gamma_5 \underline{D\!\!\!/})\psi_j = \lambda_j \psi_j$$

$$= \gamma^\mu(\partial_\mu + A_\mu)$$

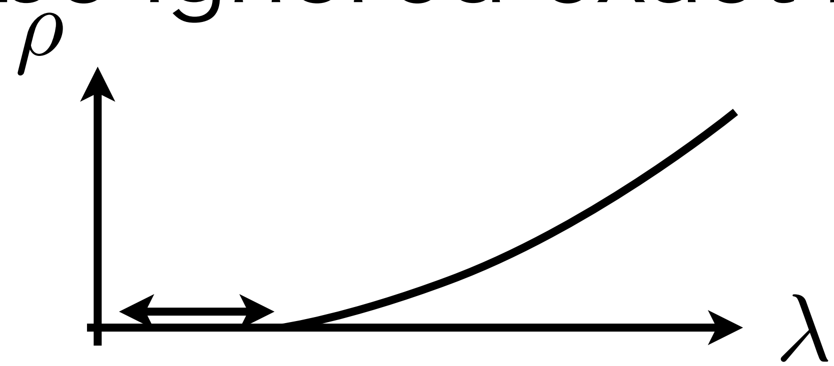
ρ(λ) : Distribution of λ

→This reflects symmetry of quarks with the gauge field!

Argument by Cohen(1996)

If there is a gap in the Dirac spectrum

(and can be ignored exact zero-modes)



U(1)$_A$ violation

$$\lim_{m\to 0}\int d^4x[\langle\pi(x)\pi(0)\rangle - \langle\delta(x)\delta(0)\rangle] = \lim_{m\to 0}\int_0^\infty d\lambda \frac{4m^2\rho(\lambda)}{(m^2+\lambda^2)^2}$$

$$= 0$$

Cf : Aoki-Fukaya-Taniguchi (2012)

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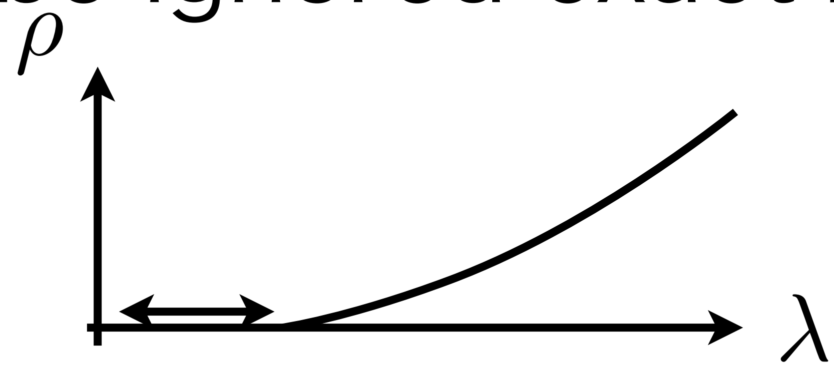

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$$= 0$$

Cf : Aoki-Fukaya-Taniguchi (2012)

λ〜0 modes are important

4.Previous studies

Previous studies (DW type) are controversial !

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Chiral symmetry on the Lat.
=Ginsparg-Wilson rel.

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$S = \int d^4x \; \bar{\psi} \not{D} \psi$ is chiral symmetric <=>

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Lat. : $D\gamma_5 + \gamma_5 D = 2aD\gamma_5 D$

Ginsparg-Wilson relation

a: lattice spacing

Chiral sym. on the Lat.

Lat. : $D\gamma_5 + \gamma_5 D = 2aD\gamma_5 D$

Ginsparg-Wilson relation

a: lattice spacing

D (satisfies above) : 2 good things

1. Action has exact chiral symmetry

$$\begin{cases} \psi \to \psi' = e^{i\gamma_5(1-aD)\theta}\psi \\ \bar{\psi} \to \bar{\psi}' = \bar{\psi}e^{i\gamma_5\theta} \end{cases}$$

→ The action has full SU(2) and $U(1)_A$ symmetries

2. $U(1)_A$ sym. is violated by the quantization as same as the continuum theory.

The overlap fermion satisfies the Ginsparg-Wilson relation exactly !

$$D\gamma_5 + \gamma_5 D = 2aD\gamma_5 D$$

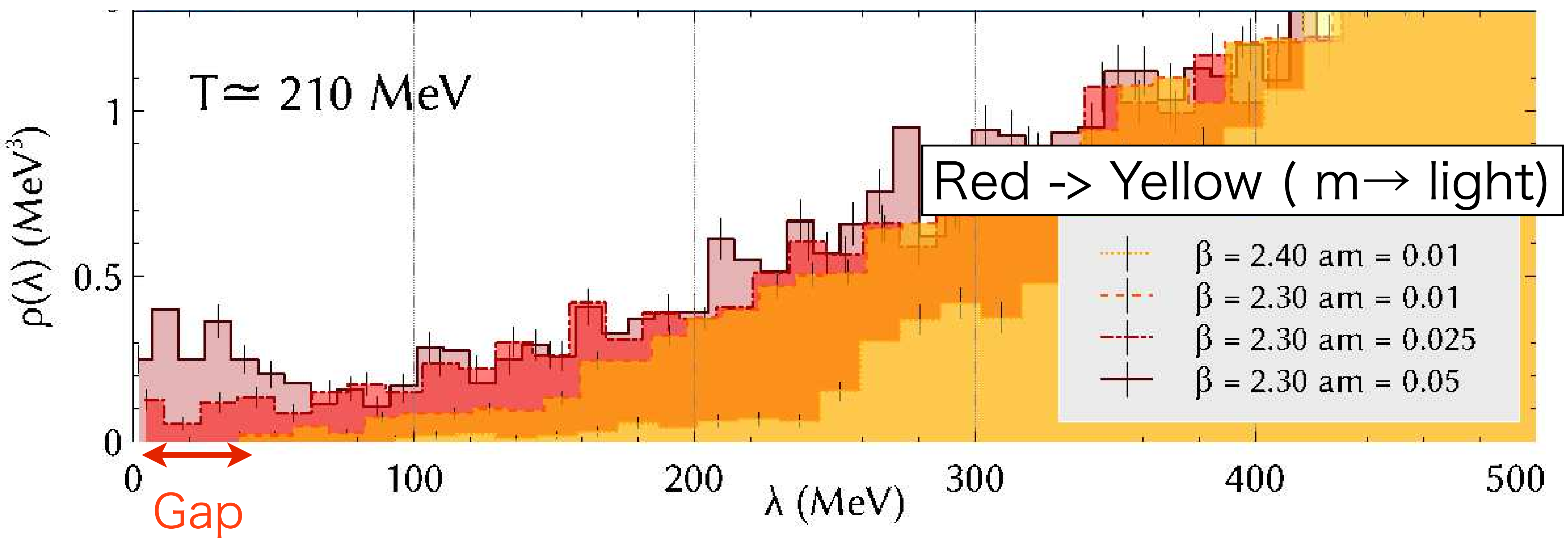
$$D_{\text{ov}} = \frac{1+m}{2} - \frac{1-m}{2}\gamma_5 \underline{\text{sgn}(H_T)}$$

H_T is a hertimitan Dirac operator

○ Exact chiral symmetry on the lattice

× Bad for the numerical simulation because of the sign function (needs special care).

Previous result by JLQCD(2013)



Simulation with the overlap (Exactly chiral)

Volume : L=2 fm & fixing topology

Finite temperature simulation!

→ As a result, U(1) is restored above the Tc

Objections: Do finite volume affect near zero modes ?

, Does topology-fixing change the physics?

In this work, we change our set-up and Check U(1) restoration

By the way

Ideal simulation?

Ideal simulation

- Overlap action:
  Large volume,
  several volume
- Without topology
  fixing term

By the way

Ideal simulation?

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Our set-up

- Overlap like action →reweighting to OV
- Large volume、several volume
- Without topology fixing term

5. Our set-up

Mobius Domain-wall fermion
= Better approximation of the overlap fermion

The overlap (Exactly chiral , used in JLQCD2013):

$$D_{\mathrm{ov}} = \frac{1+m}{2} - \frac{1-m}{2}\gamma_5 \mathrm{sgn}(H_T)$$

H_T is a hermitian Dirac op.

Domain-wall fermion (used in RBC/LLNL)
1 digit precision chiral symmetry

$$\tanh\left[L_s \tanh^{-1}(H_T)\right]$$

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Domain-wall fermion (used in RBC/LLNL)
1 digit precision chiral symmetry

$$\tanh\left[L_s \tanh^{-1}(H_T)\right]$$

Mobius Domain-wall fermion
(This work)
3 digit chiral symmetry

Edwards-Heller (2000)

$$\tanh\left[L_s \tanh^{-1}(2H_T)\right]$$

→ Better approximation of the OV

(Still it violates the Ginsparg-Wilson rel.)

Lattice set up

Gauge action:tree level Symanzik
Fermion :Mobius DW(b=2, c=1, Scaled Shamir + Tanh)
w/ Stout smearing(3)
code :Irolro++(G. Cossu et al.)
Resource :BG/Q(KEK)

	$L^3 \times L_t$	β	$m_{\rm ud}$(MeV)	L_s	$m_{\rm res}$(MeV)	Temp.(MeV)
L=2 fm	$16^3 \times 8$	4.07	30	12	2.5	180
	$16^3 \times 8$	4.07	15^*	12	2.4	180
	$16^3 \times 8$	4.07	3.0	24	1.4	180
	$16^3 \times 8$	4.10	32	12	1.2	200
	$16^3 \times 8$	4.10	16^*	12	1.2	200
	$16^3 \times 8$	4.10	3.2	24	0.8	200
L=4 fm	$32^3 \times 8$	4.07	3.0	24	5^{**}	180
	$32^3 \times 8$	4.10	32	12	1.7	200
	$32^3 \times 8$	4.10	16	24	1.7	200
	$32^3 \times 8$	4.10	3.2	24	0.7	200

Table 1: Our lattice set-up. Those with $m^*_{\rm ud}$ are obtained by the stochastic reweighting of the Dirac operator determinant from the ensemble with the higher quark mass. Residual mass with ** is estimated by weighted average of g_i with some threshold.

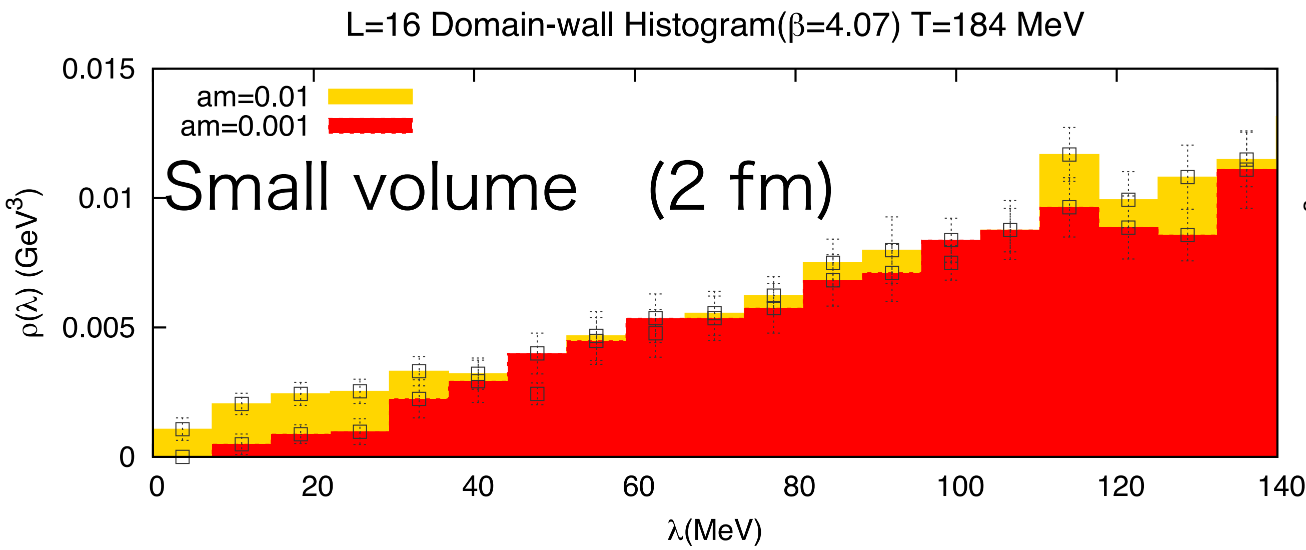
$m_{\rm res}$: Scale of violation of Ginsparg-Wilson relation

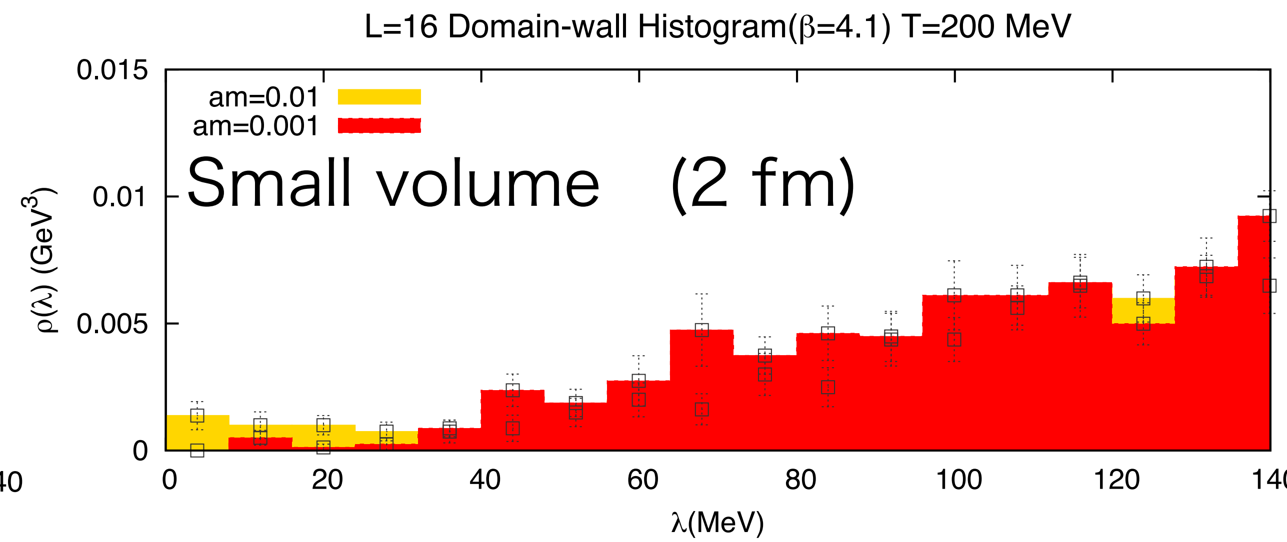
6. Our results

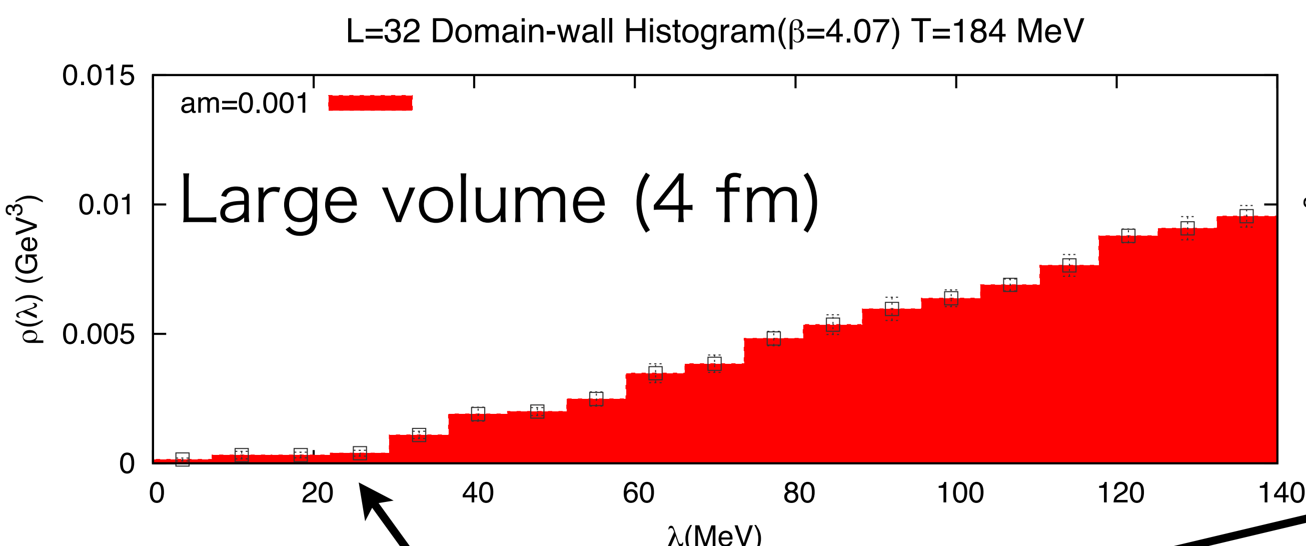
Result of the Mobius DW

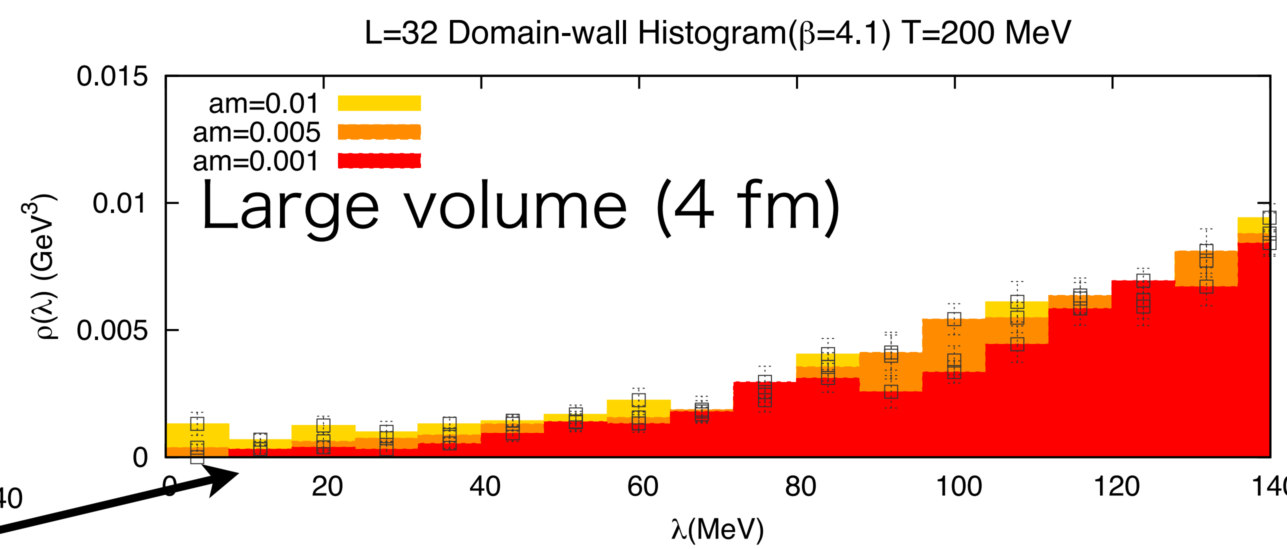
T~Tc

T>Tc









No clear gap in the spectrum

Even for the lightest mass (Red bar).

→$U_A(1)$ looks violated?

3.Histogram for DW

No clear gap in the domain-wall spectrum
in large volume system
→$U(1)_A$ looks violated.

Same result as the previous study by LLNL/RBC 2013
What was wrong with previous JLQCD?

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Ginsparg-Wilson violation in DW ?

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Ginsparg-Wilson violation in DW ?

Ginsparg-Wilson violation for each mode

$S = \int d^4x \, \bar{\psi} D \psi$ is chiral symmetric <=>

Lattice : $D\gamma_5 + \gamma_5 D = 2aD\gamma_5 D$

Eigen-function of D↓

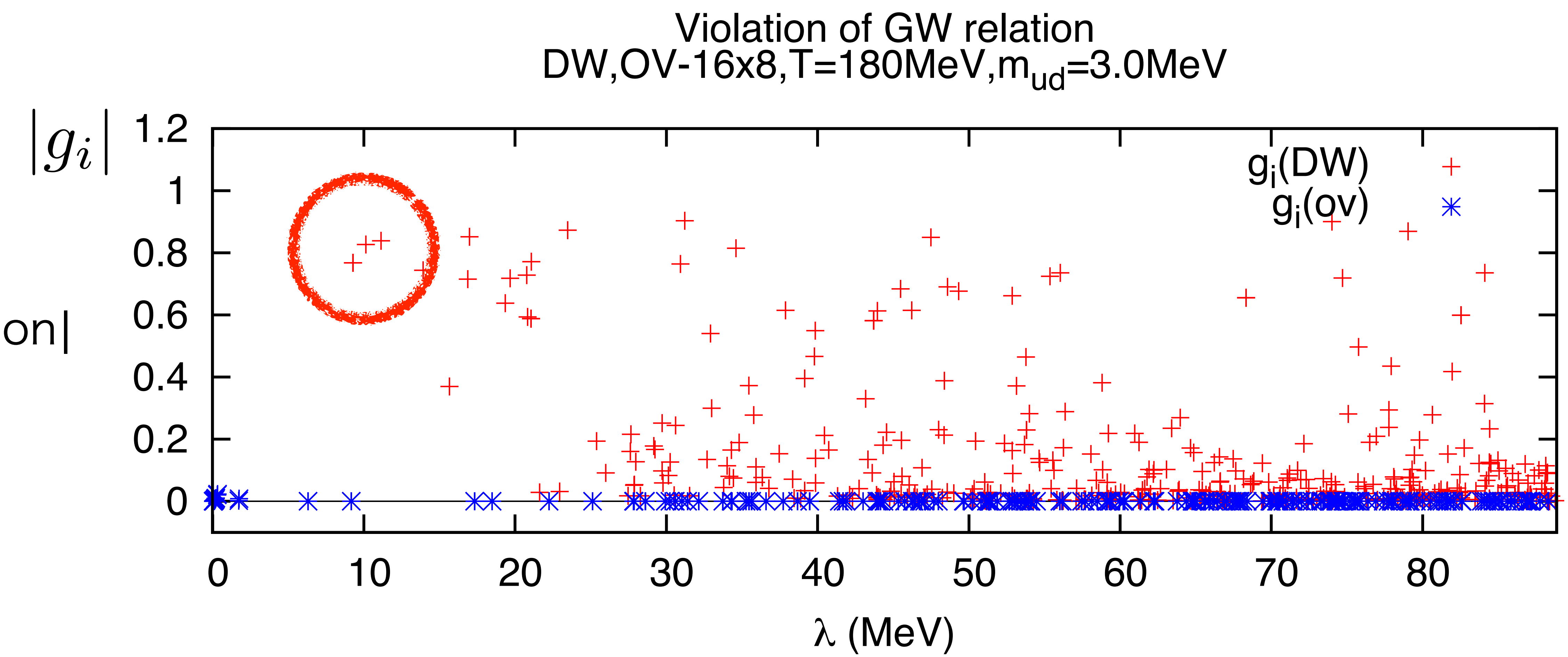
$$g_i \propto \psi_i^\dagger \gamma_5 [D\gamma_5 + \gamma_5 D - 2aD\gamma_5 D]\psi_i$$

g_i =0 for the chiral fermion

What happens on the Mobius domain-wall ?

Ginsparg-Wilson relation is violated even for improved domain-wall fermion

|GW violation|



Near zero modes important for the issue...

Observation

Eigenmodes of improved domain-wall :

Ginsparg-Wilson is violated

Observation

Eigenmodes of improved domain-wall :

Ginsparg-Wilson is violated

Using reweighting technique,

we switch the fermion determinant to the OV one

(toward the ideal simulation...)

(skip)

Re-weighting tech. enables us to change another fermion determinant

$$\langle \mathcal{O} \rangle_{\rm Overlap} \propto \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu \; \mathcal{O} \; e^{-S_{\rm gauge}} e^{-\bar{\psi}[D_{\rm OV}]\psi}$$

$$= \int \mathcal{D}A_\mu \; \mathcal{O} \; e^{-S_{\rm gauge}} {\rm Det}[D^2_{\rm OV}]$$

$$= \int \mathcal{D}A_\mu \; \mathcal{O} \; e^{-S_{\rm gauge}} {\rm Det}[D^2_{\rm OV}] \frac{{\rm Det}[D^2_{\rm DW}]}{{\rm Det}[D^2_{\rm DW}]}$$

$$= \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu \; \mathcal{O}R \; e^{-S_{\rm gauge}} e^{-\bar{\psi}[D_{\rm DW}]\psi}$$

$$\propto \langle \mathcal{O}R \rangle_{\rm Domain\ Wall}$$

$$R = \frac{{\rm Det}[D^2_{\rm OV}]}{{\rm Det}[D^2_{\rm DW}]}$$

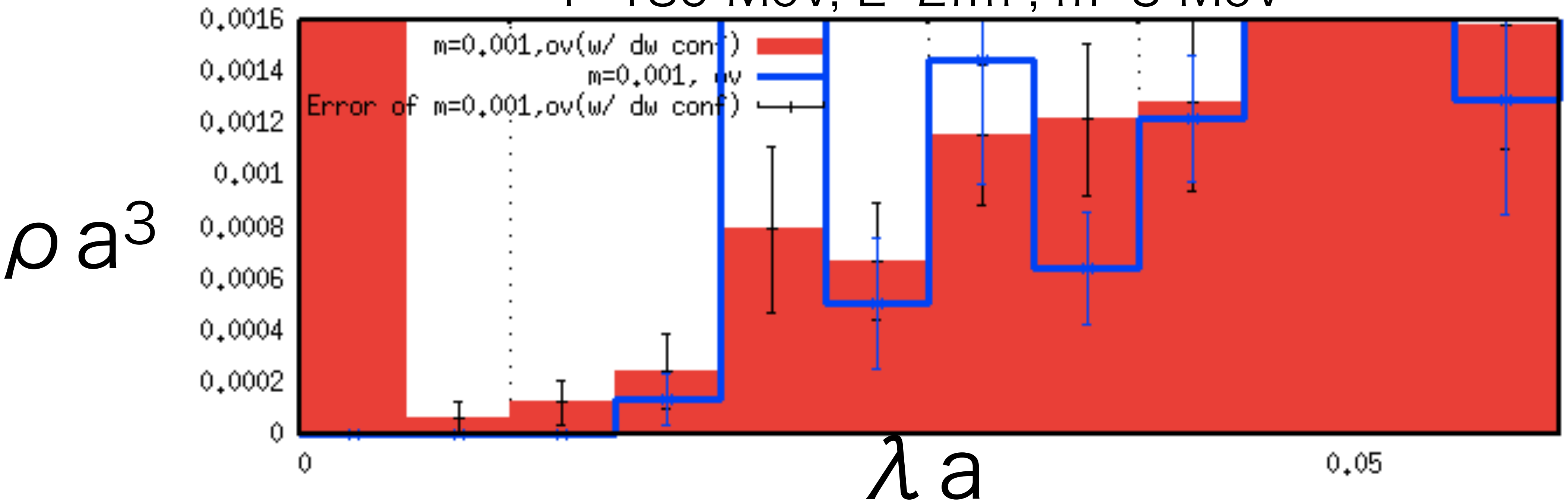
Multiplying R and taking average, we obtain the result with the overlap determinant

Reweighting suppress near zero-modes!

Spectral density for the overlap Dirac operator

with different fermion determinant (OV or DW)

T=180 MeV, L=2fm , m=3 MeV



Red: Partially quenched = w/ det of DW

Blue: Reweighted =w/ det of OV

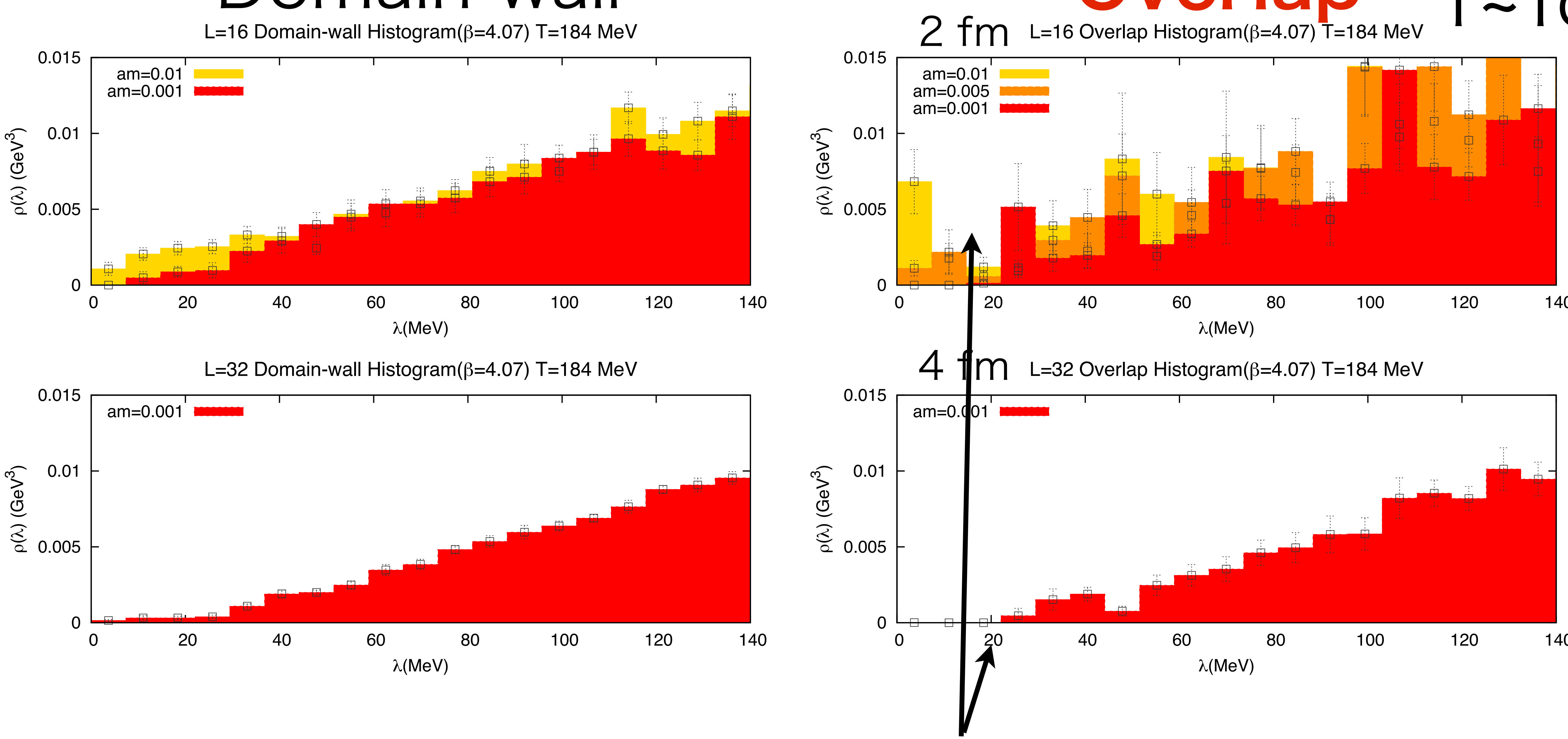
Cf: Microscopic Origin of $U_A(1)$ Symmetry Violation in the High Temperature Phase of QCD - Dick, Viktor et al. arXiv:1502.06190 [hep-lat]

DW/OV Dirac spectrum has different shape

Domain-wall

Overlap

T~Tc



<u>The overlap spectrums have gaps on the spectrum(λ~20MeV)</u>

7.Summary

1.

1.1. Thanks to the reweighting, we can perform the simulation with the overlap in large volume without topology fixing

1.2. We find, the ov and DW spectrum have different shape

1.3. We find Ginsparg-Wilson violation for DW each eigenmodes

1.4. We find gaps in the spectrum for OV: T=180 MeV, T=200MeV , 2 fm and 4 fm.

2. We are going to,

2.1. find a gap in the spectrum for finer lattice

2.2. do quantitative evaluation for the gap (Check vol. scaling)

2.3. check consistency with the measurement for $\chi_{U(1)}$

Reference:

A. Tomiya, G. Cossu, H. Fukaya, S. Hashimoto and J. Noaki, arXiv:1412.7306 [hep-lat].

Full paper is in preparation: