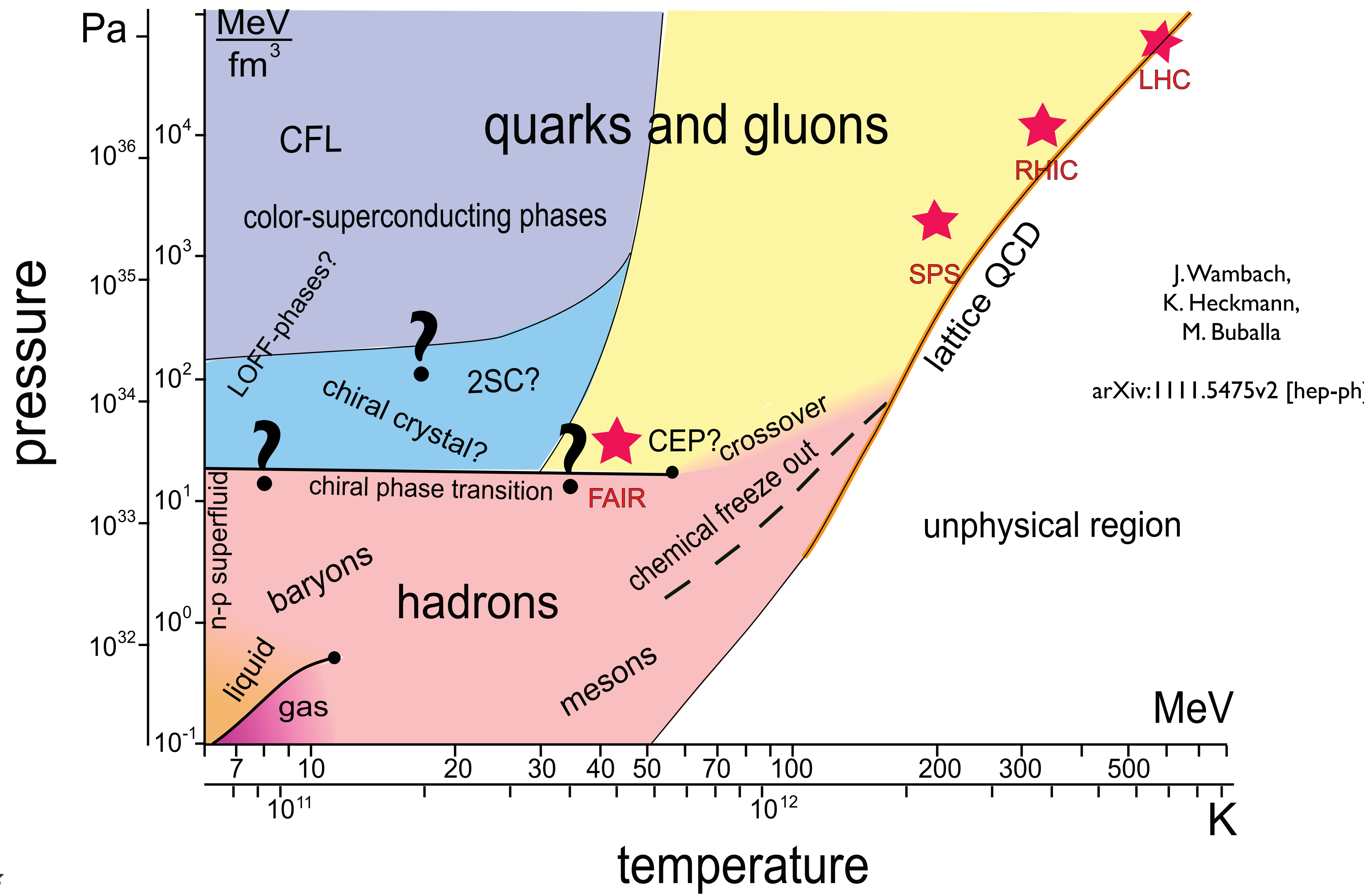
CHIRAL EFFECTIVE FIELD THEORIES
and
PHASES of QCD

Wolfram Weise

ECT* Trento and Technische Universität München

- Introductory glance at the **QCD phase diagram**
- **Chiral models and EFT** of the **nuclear equation of state**
- Beyond mean field:
  fluctuations and **Functional Renormalisation Group**
- Symmetric and asymmetric **nuclear matter**
- **Neutron matter** and **neutron stars**
- Density & temperature dependence of **chiral order parameter**

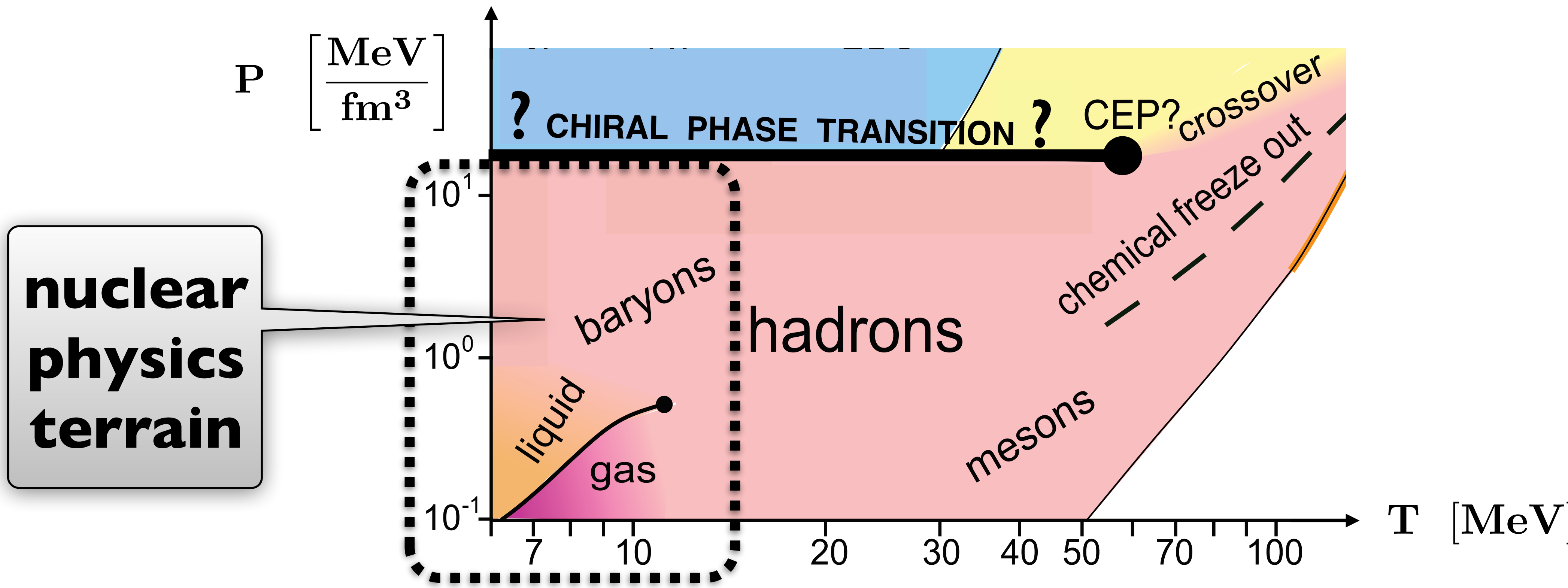
PHASES and STRUCTURES of QCD
- facts and visions -



CHIRAL RESTORATION

from **Nambu-Goldstone** to
Wigner-Weyl Realisation of **Chiral Symmetry**

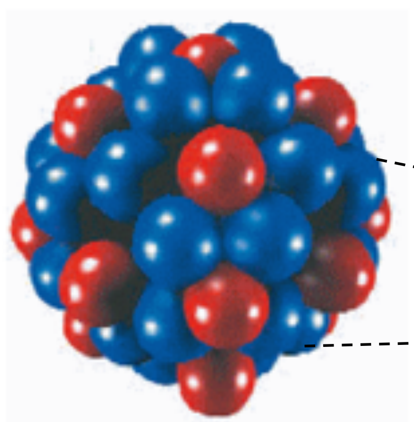
- **PHASE TRANSITION** or smooth **CROSSOVER ?**

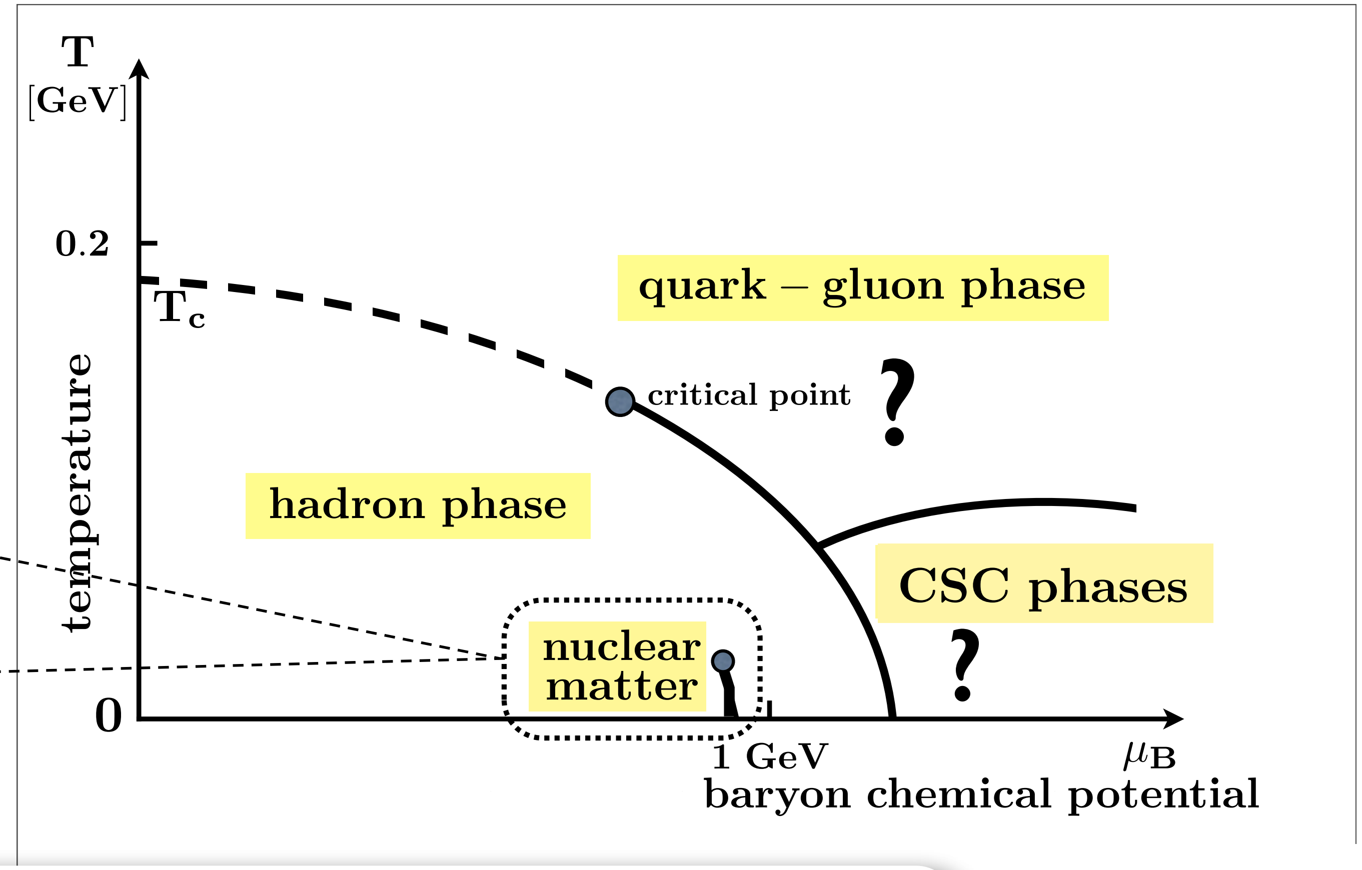


- **Chiral 1st order phase transition** incl. **critical point** based on chiral quark models (NJL, PNJL, quark-meson models, …)
- These models do not respect **nuclear physics** constraints
- Needed: systematic (EFT) approach to **nuclear** thermodynamics

NUCLEAR MATTER and QCD PHASES

nuclei





Scales in N = Z nuclear matter

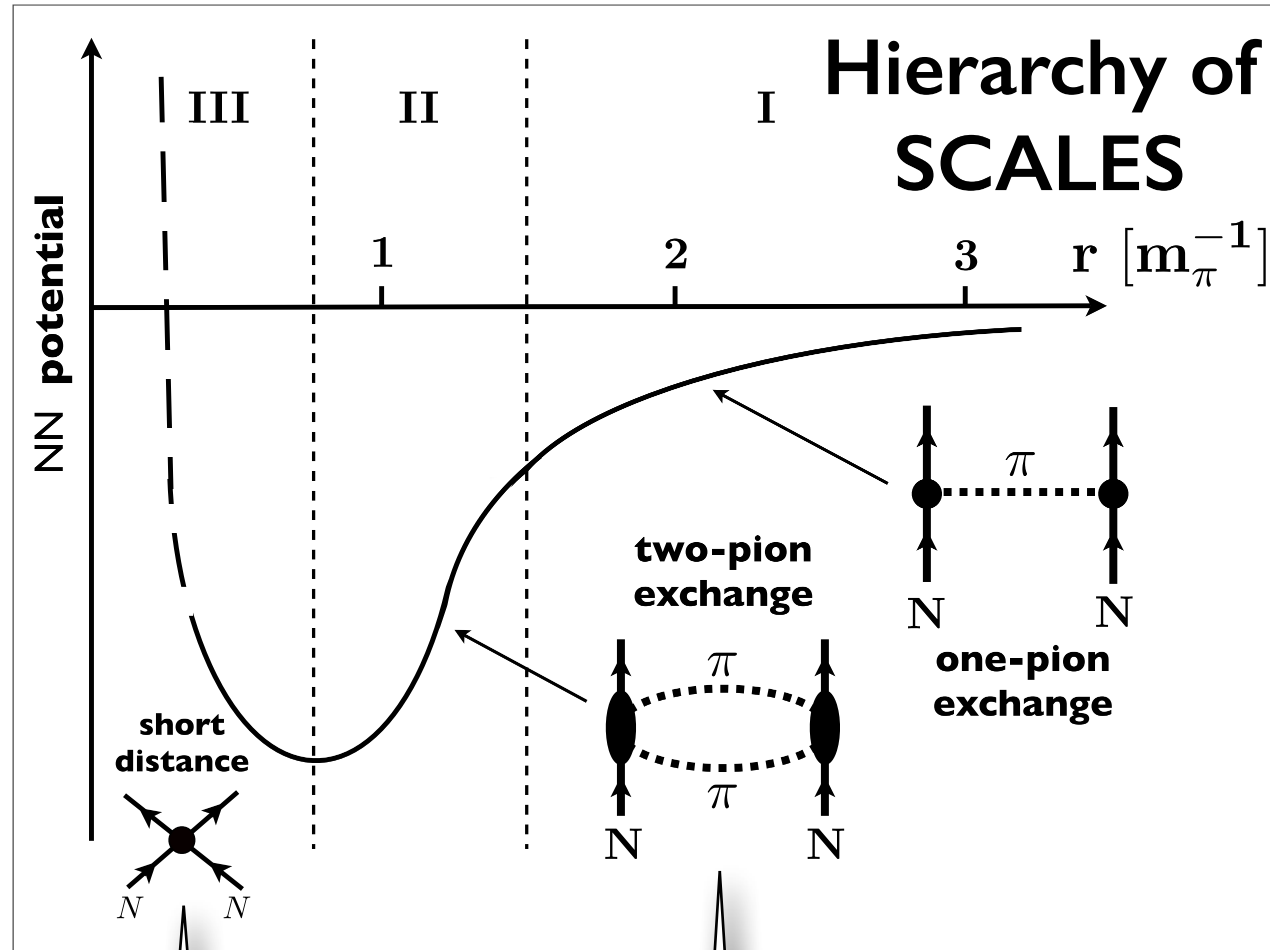
- momentum scale:
  Fermi momentum $k_F \simeq 1.4 \text{ fm}^{-1} \sim 2m_\pi$
- NN distance: $d_{NN} \simeq 1.8 \text{ fm} \simeq 1.3 \ m_\pi^{-1}$
- energy per nucleon: $E/A \simeq -16 \text{ MeV}$
- compression modulus: $K = (260 \pm 30) \text{ MeV} \sim 2m_\pi$

Nuclear Forces

contemporary approaches:

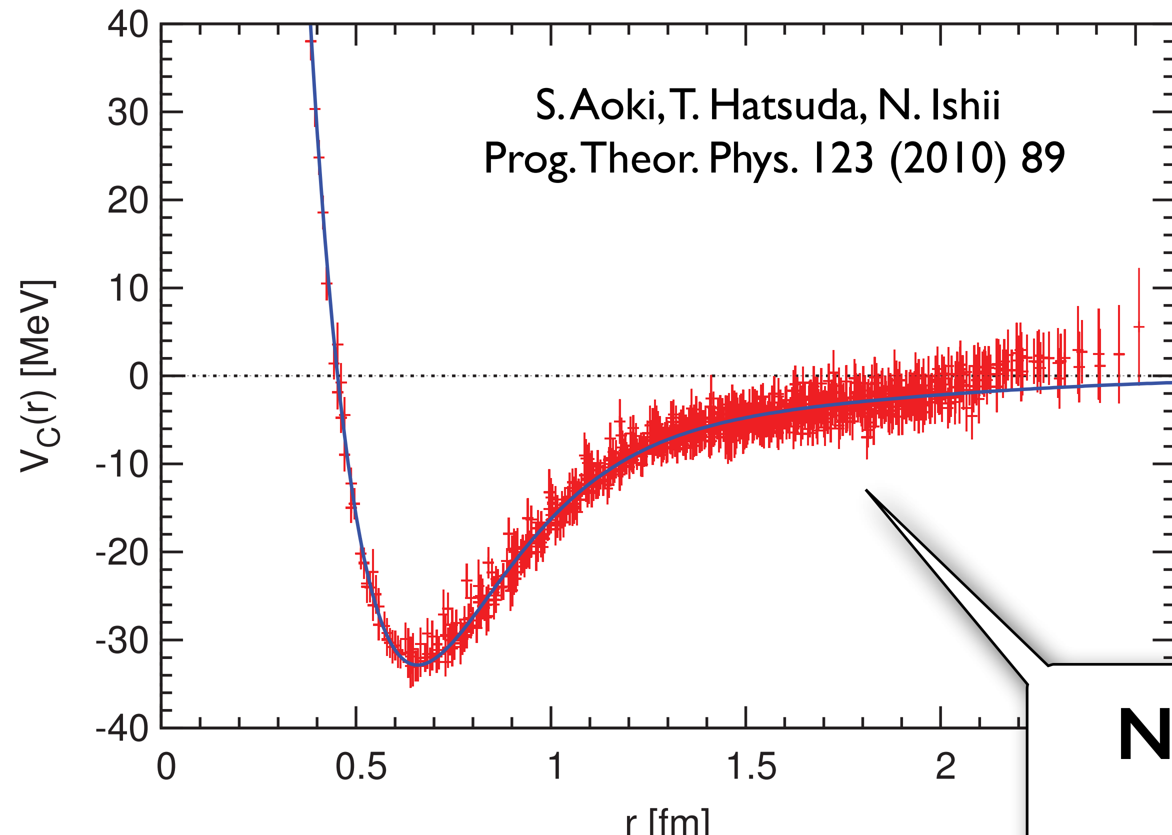
Chiral Effective Field Theory
&
Lattice QCD

Early history: M. Taketani et al. (1951)



contact terms

explicit treatment of two-pion exchange



NN Central Potential from Lattice QCD

PIONS, NUCLEONS and **NUCLEI** in the context of **LOW-ENERGY QCD**

- **CONFINEMENT** of quarks and gluons in hadrons
- Spontaneously broken **CHIRAL SYMMETRY**
- **LOW-ENERGY QCD** with light (u,d) quarks: **E**ffective **F**ield **T**heory of (weakly) interacting **Nambu-Goldstone Bosons** (pions)

- **Chiral EFT** represents QCD at energy/momentum scales

$$\mathbf{Q << 4\pi\, f_\pi \sim 1\, GeV}$$

- **Strategies** at the interface between **QCD and nuclear physics** :

In-medium **Chiral Perturbation Theory** based on **non-linear sigma model** (with inclusion of nucleons)

expansion of free energy density in powers of Fermi momentum

Chiral Nucleon-Meson model based on **linear sigma model**

non-perturbative Renormalization Group approach

PART I:

In-medium Chiral Perturbation Theory
and the nuclear many-body problem

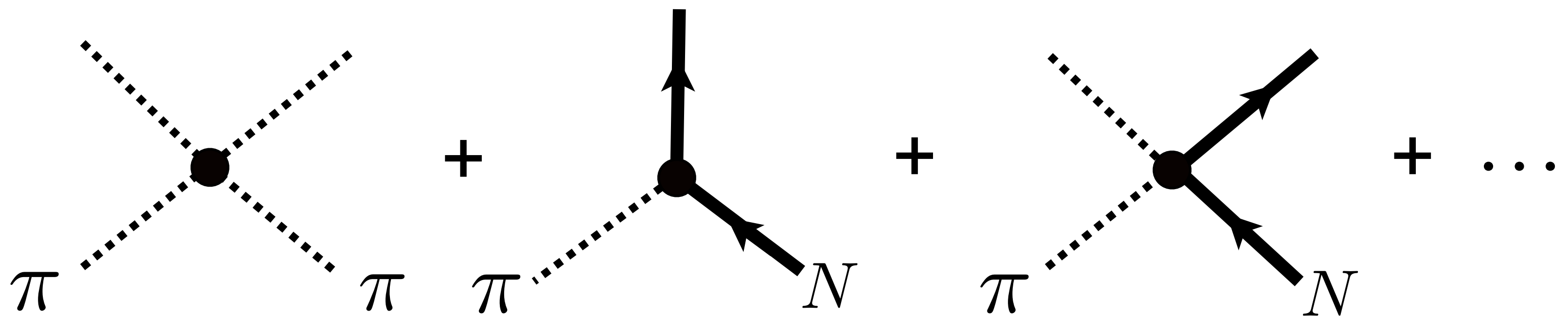
CHIRAL EFFECTIVE FIELD THEORY

- Interacting systems of
  PIONS (light / fast) and **NUCLEONS** (heavy / slow):

$$\mathcal{L}_{eff} = \mathcal{L}_\pi(U, \partial U) + \mathcal{L}_N(\Psi_N, U, \ldots)$$

$$U(x) = exp[i\tau_a \pi_a(x)/f_\pi]$$

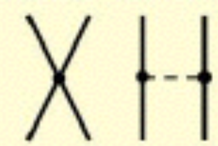
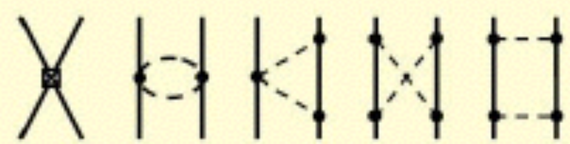
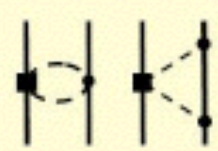
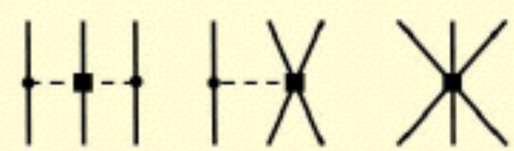
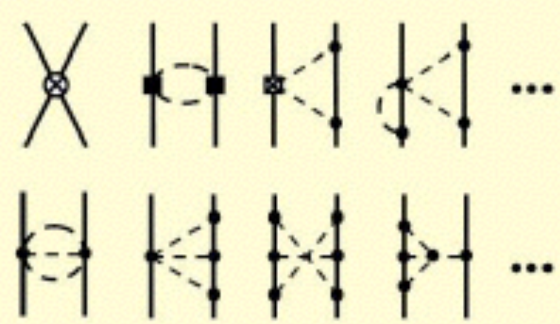
- Construction of Effective Lagrangian: **Symmetries**



short distance dynamics: **contact terms**

NUCLEAR INTERACTIONS from CHIRAL EFFECTIVE FIELD THEORY

Weinberg Bedaque & van Kolck Bernard, Epelbaum, Kaiser, Meißner; ...

		Two-nucleon force	Three-nucleon force	Four-nucleon force
$\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$	LO		—	—
$\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$	NLO		—	—
$\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$	N²LO			—
$\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$	N³LO			

- **Systematically organized HIERARCHY**

IN-MEDIUM CHIRAL PERTURBATION THEORY

- Small scales: energy, momentum, $\mathbf{m}_\pi$, $\mathbf{k_F} << 4\pi \mathbf{f}_\pi \sim 1\,\mathbf{GeV}$

- **Loop expansion** of (In-Medium) Chiral Perturbation Theory

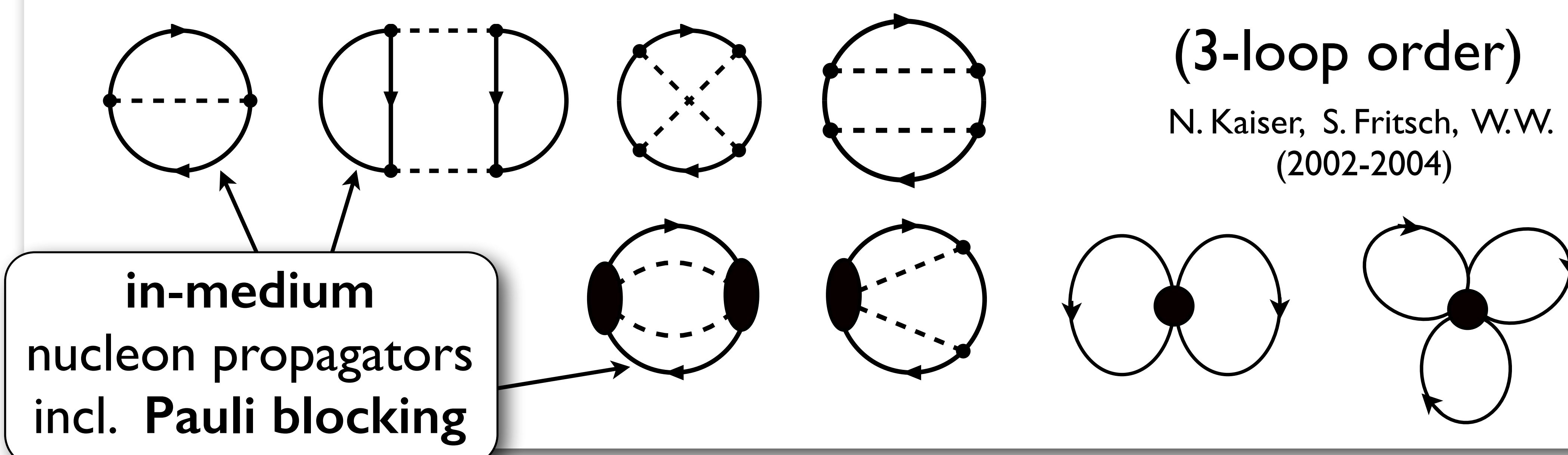
  ↕

  Systematic expansion of **ENERGY DENSITY** $\mathcal{E}(\mathbf{k_F})$ in **powers of Fermi momentum** [modulo functions $\mathbf{f_n}(\mathbf{k_F}/\mathbf{m}_\pi)$]

- Nuclear **thermodynamics**: compute **free energy density**

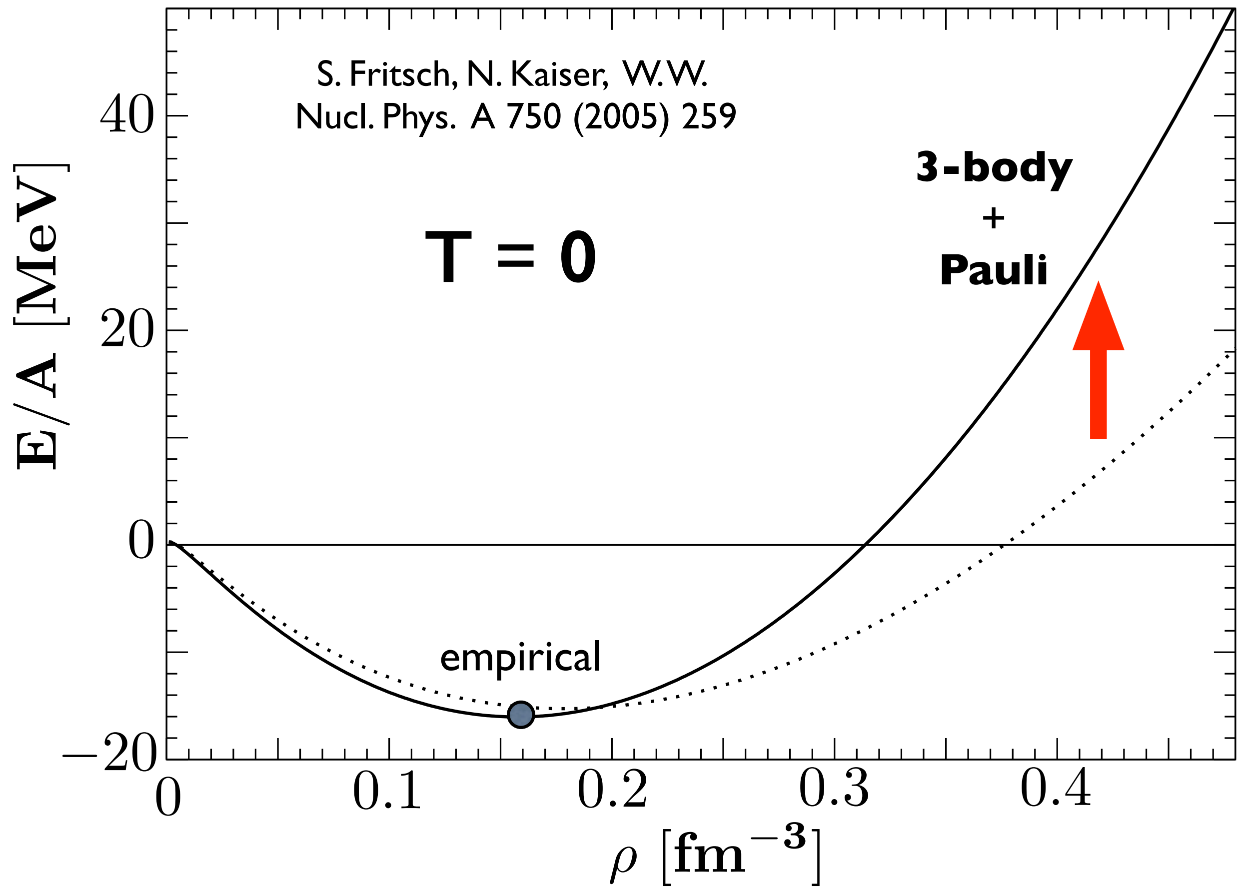
  (3-loop order)

  N. Kaiser, S. Fritsch, W.W. (2002-2004)



NUCLEAR MATTER

- **In-medium ChPT**
  3-loop (π, N, Δ)
- **Input** parameters:
  few contact terms
- basically:
  analytic calculation



- **Output:**
  - Binding & saturation
    $\mathbf{E_0/A = -16\,MeV}$
    $\rho_0 = \mathbf{0.16\,fm^{-3}}$
    $\mathbf{K = 290\,MeV}$
  - Asymmetry energy:
    $\mathbf{A(k_F^0) = 34\,MeV}$

- Realistic (complex, momentum dependent) single-particle potential
  ... satisfying Hugenholtz - van Hove and Luttinger theorems (!)
- Fermi Liquid Theory:
  Quasiparticle interaction and Landau parameters
- Nuclear Energy Density Functional and finite nuclei

J.W. Holt, N. Kaiser, W.W.
Nucl. Phys. A 870 (2011) 1,
Nucl. Phys. A 876 (2012) 61,
Phys. Rev. C 87 (2013) 014338

C. Wellenhofer, J.W. Holt,
N. Kaiser, W.W.
Phys. Rev. C 89 (2014) 064009

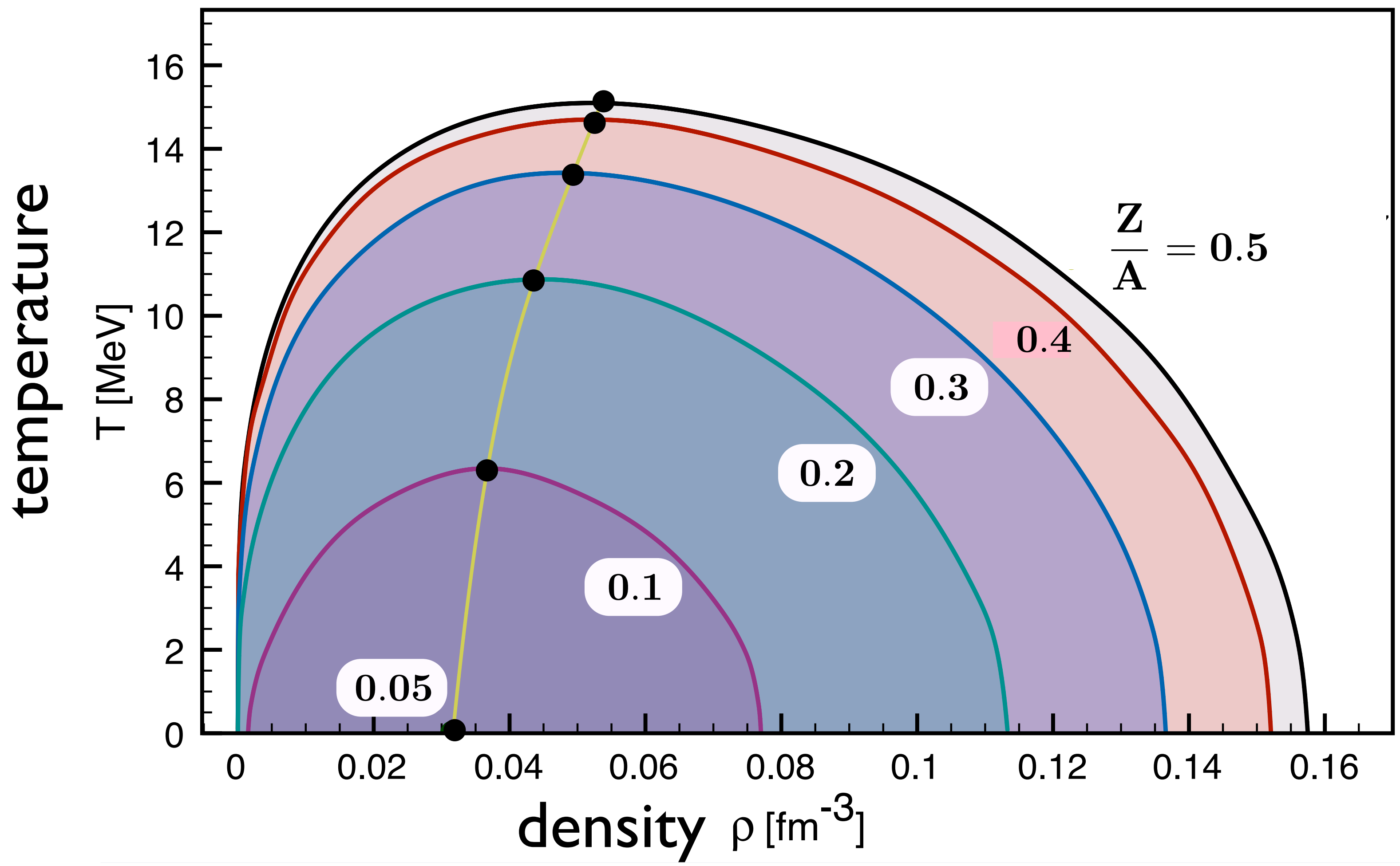
Recent reviews:
J.W. Holt, N. Kaiser, W.W. Prog. Part. Nucl. Phys. 73 (2013) 35
J.W. Holt, M. Rho, W.W. arXiv:1411.6681, to appear in Phys. Reports

CHIRAL THERMODYNAMICS: PHASE DIAGRAM of NUCLEAR MATTER

Nuclear liquid - gas phase transition:

- Trajectory of **CRITICAL POINT** for **asymmetric matter**

as function of proton fraction Z/A

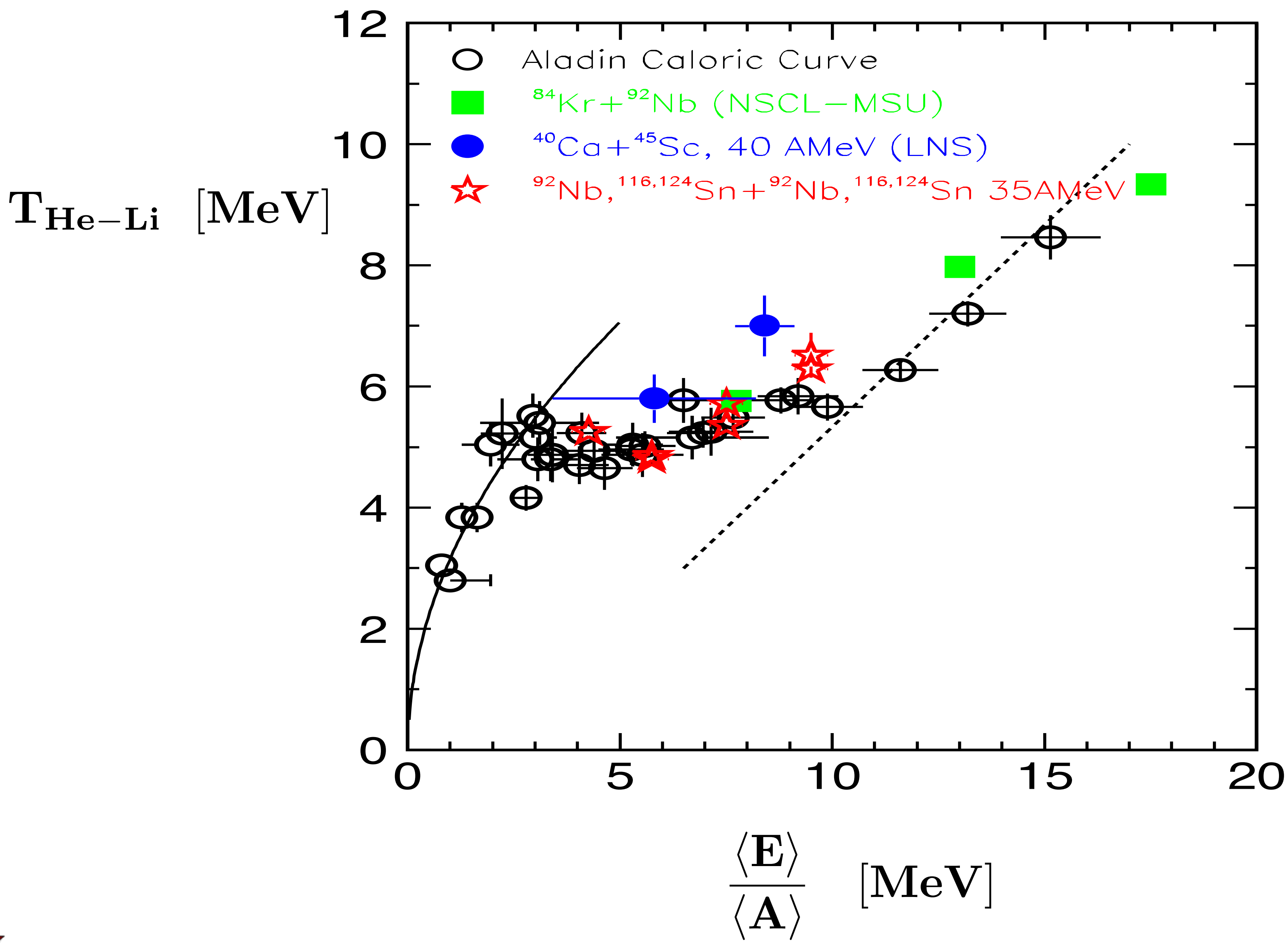


S. Fiorilla, N. Kaiser, W.W. Nucl. Phys. A880 (2012) 65

. . . determined almost completely by
isospin dependent (one- and two-) pion exchange dynamics

NUCLEAR LIQUID-GAS TRANSITION

from multifragmentation measurements in heavy-ion collisions



J. Pochodzalla et al.
Phys. Rev. Lett. 75 (1995) 1040

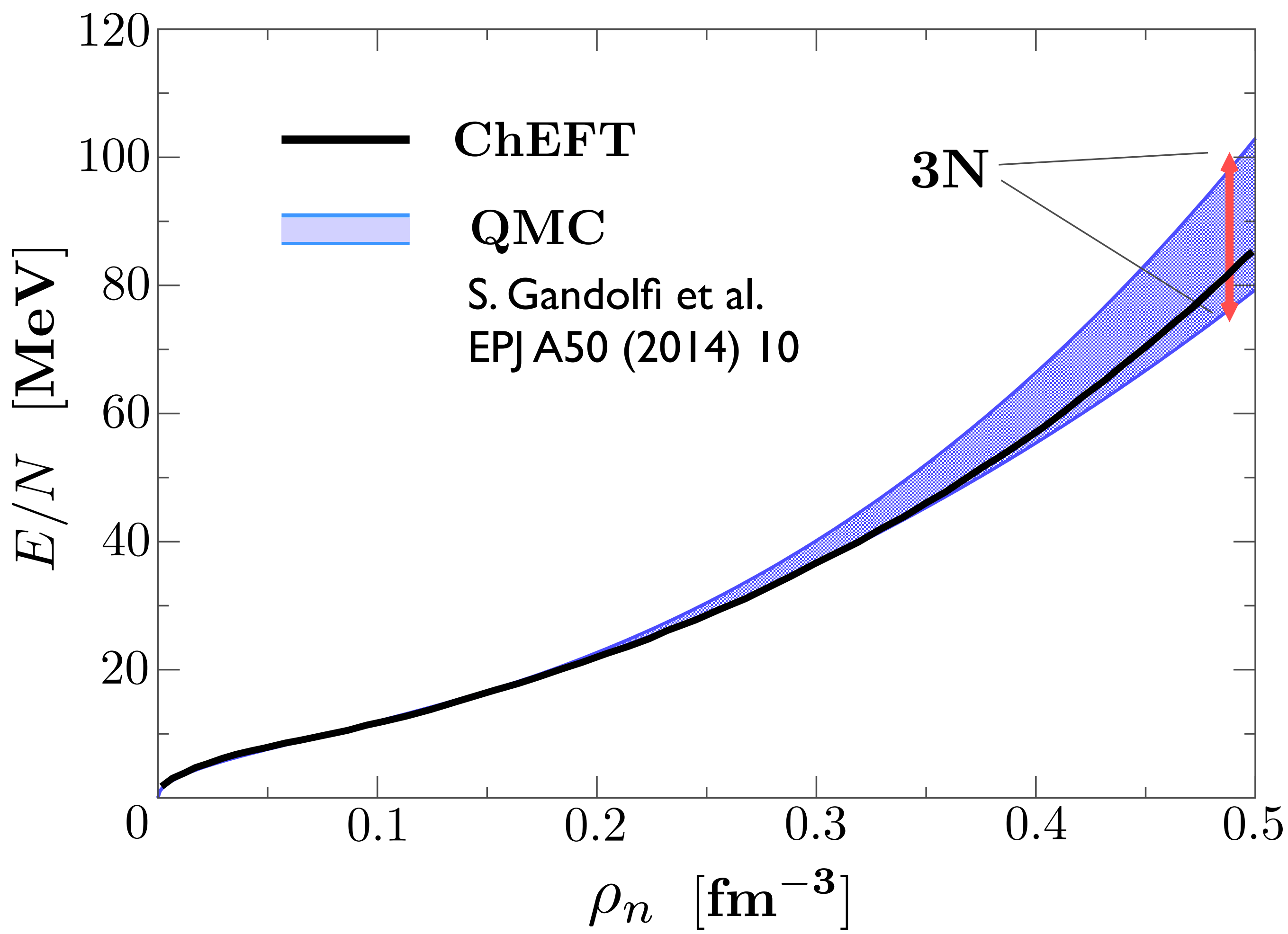
M. D'Agostino et al.
Nucl. Phys. A 749 (2005) 55

ECT*

Technische Universität München

NEUTRON MATTER

- In-medium chiral effective field theory (3-loop) with resummation of short distance contact terms (large nn scattering length, $a_s = 19\ \mathrm{fm}$)



- Neutron matter behaves almost (but not quite) like a unitary Fermi gas

- Bertsch parameter

$$\xi = \frac{\bar{E}}{E_{\mathrm{Fermi\,gas}}} \simeq 0.5$$

J.W. Holt, N. Kaiser, W.W.
Phys. Rev. C 87 (2013) 014338

- **agreement with sophisticated many-body calculations**

(e.g. recent **Q**uantum **M**onte **C**arlo computations)

PART II:

Chiral Nucleon-Meson Model
and
Functional Renormalization Group

Mesons, Nucleons, Nuclear Matter and Functional Renormalization Group

- **Chiral nucleon - meson model** $\psi = (\psi_p, \psi_n)^T$

$$\mathcal{L} = \bar{\psi} i\gamma_\mu \partial^\mu \psi + \frac{1}{2}\partial_\mu \sigma\, \partial^\mu \sigma + \frac{1}{2}\partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi}$$
$$- \bar{\psi}\Big[g(\sigma + i\gamma_5\, \boldsymbol{\tau}\cdot\boldsymbol{\pi}) + \gamma_\mu(g_\omega\, \omega^\mu + g_\rho \boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu)\Big]\psi$$
$$- \frac{1}{4}F^{(\omega)}_{\mu\nu}F^{(\omega)\mu\nu} - \frac{1}{4}\boldsymbol{F}^{(\rho)}_{\mu\nu}\cdot\boldsymbol{F}^{(\rho)\mu\nu}$$
$$+ \frac{1}{2}m_V^2\,(\omega_\mu\omega^\mu + \boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu) - \mathcal{U}(\sigma, \boldsymbol{\pi})$$

- **Effective potential** constructed to reproduce standard nuclear thermodynamics around equilibrium

- **Mean field** calculations

  S. Floerchinger, Ch. Wetterich : Nucl. Phys. A 890-891 (2012) 11

- **Mesonic** and **nucleonic particle-hole** fluctuations treated non-perturbatively using **FRG**

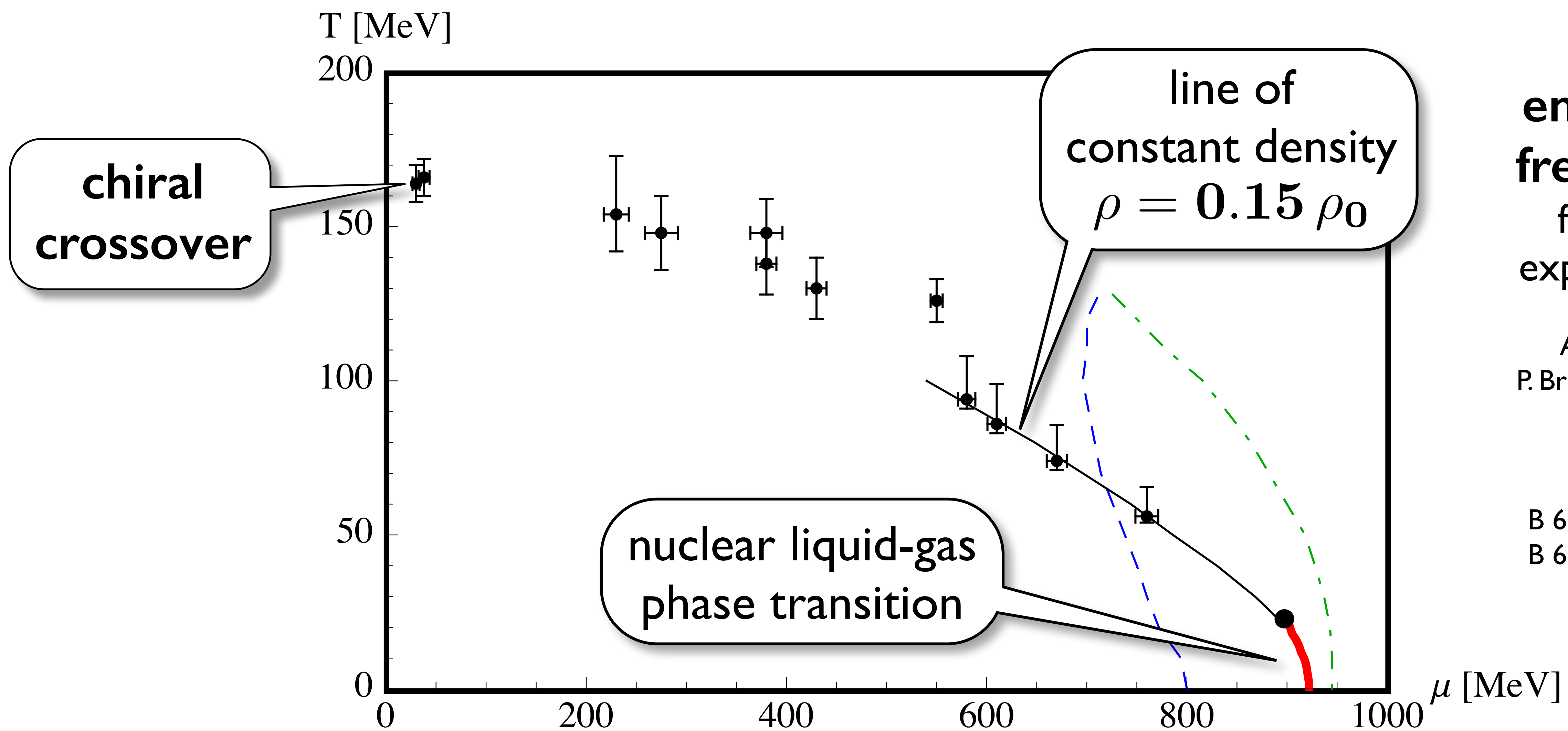
  M. Drews, T. Hell, B. Klein, W.W. Phys. Rev. D88 (2013) 096011

  M. Drews, W.W. Phys. Lett. B738 (2014) 187 arXiv:1412.7838, Phys. Rev. C (in print)

CHEMICAL FREEZE-OUT

S. Floerchinger, Ch. Wetterich : Nucl. Phys. A 890-891 (2012) 11

- **Chiral nucleon - meson model** in mean-field approximation



empirical freeze-out from HI experiments

A. Andronic, P. Braun-Munzinger, J. Stachel

Phys. Lett. B 673 (2009) 142 B 678 (2009) 516

- **Chemical freeze-out in baryonic matter at T < 100 MeV is not associated with (chiral) phase transition or rapid crossover**

Fixing the input: some comments

- **Potential** $\mathcal{U}(\sigma, \boldsymbol{\pi}) = \mathcal{U}_0(\chi) - m_\pi^2 f_\pi(\sigma - f_\pi)$

  chiral invariant part
  parametrized in powers of $\chi = \frac{1}{2}(\sigma^2 + \boldsymbol{\pi}^2)$

  explicit chiral
  symmetry breaking

- **Scalar ("sigma") field**
  has mean-field (chiral **order parameter**) and fluctuating pieces.
  σ mass: **NOT** to be confused with pole in I = 0 s-wave pion-pion T matrix.

  Nucleon mass: $m_N^2 = 2g\,\chi$ ... in vacuum: $m_N = g\,f_\pi$

- **Vector fields** encode short-distance NN dynamics,
  self-consistently determined background mean fields (non-fluctuating)
  (**NOT** to be identified with physical ω and ρ mesons)

  Effective chemical potentials $\mu_{n,p}^{\text{eff}} = \mu_{n,p} - g_\omega\,\omega_0 \pm g_\rho\,\rho_0^3$

  Relevant quantities: $G_\rho = \dfrac{g_\rho^2}{m_V^2}\,,\; G_\omega = \dfrac{g_\omega^2}{m_V^2}$ $\longleftrightarrow$ **contact terms** in **ChEFT**

- **Parameters:** 2 coefficients in $\mathcal{U}_0\,,\;\; \mathbf{m_\sigma \simeq 0.8\,GeV}\,,\;\; \mathbf{G_\rho \sim G_\omega/4 \simeq 1\,fm^2}$
  determined by nuclear matter properties and symmetry energy

Chiral nucleon - meson model beyond mean-field
- Renormalization Group strategies -

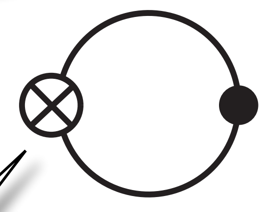
M. Drews, T. Hell, B. Klein, W.W. Phys. Rev. D 88 (2013) 096011

- **Fluctuations:** Wetterich's RG flow equations

C. Wetterich:
Phys. Lett. B 301 (1993) 90

effective action

$$k \frac{\partial \Gamma_k}{\partial k} =$$



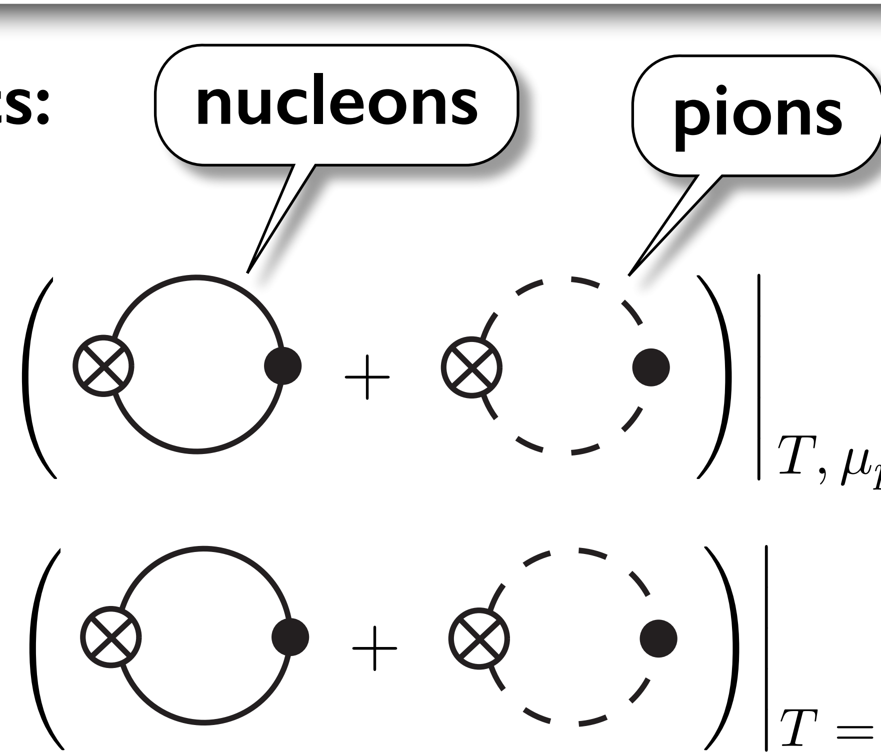
$$= \frac{1}{2} \mathrm{Tr} \frac{k \frac{\partial R_k}{\partial k}}{\Gamma_k^{(2)} + R_k}$$

full propagator

regulator: $R_k(\boldsymbol{p}^2) = (k^2 - \boldsymbol{p}^2)\,\theta(k^2 - \boldsymbol{p}^2)$

- **Thermodynamics:**

$$k\,\partial_k \bar{\Gamma}_k(T,\mu) =$$



$\Big|_{T,\,\mu_p,\,\mu_n}$

$\Big|_{T=0,\,\mu=\mu_0\,(=m_N - E_0/A)}$

included:

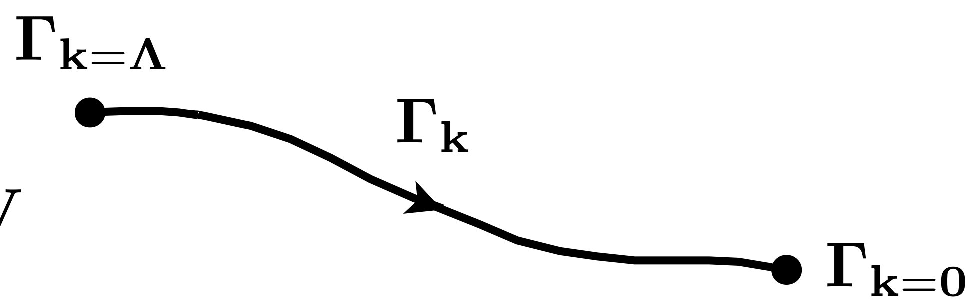
multi-pion exchange processes

nucleon-hole excitations

multi-nucleon correlations

Flow equations in practice

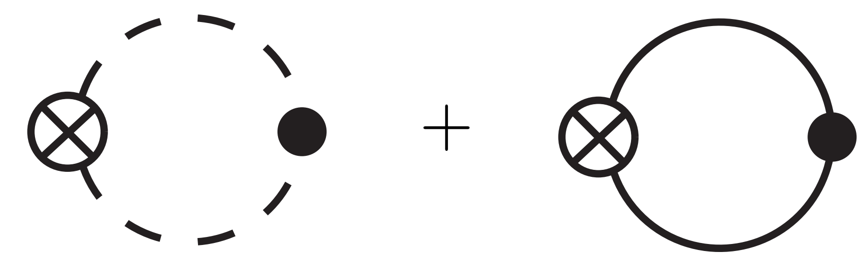
UV scale:
$\Lambda = 1.4\,\mathrm{GeV}$



"full" effective action

effective potential $U_{k=0}$

$$k\frac{\partial U_k}{\partial k}(T,\mu_p,\mu_n,\chi,\omega_0,\rho_0^3) =$$



$$= \frac{k^5}{12\pi^2}\left\{\frac{1+2n_B(E_\sigma)}{E_\sigma} + \frac{3\left[1+2n_B(E_\pi)\right]}{E_\pi} - 4\sum_{i=n,p}\frac{1 - n_F(E_N - \mu_{i,\mathrm{eff}})}{E_N}\right\}$$

$$E_\pi^2 = k^2 + U_k'(\chi)\,,\;\; E_\sigma^2 = k^2 + U_k'(\chi) + 2\chi\, U_k''(\chi)\,,$$

$$U_k'(\chi) = \frac{\partial U_k(\chi)}{\partial\chi}\,, \quad E_N^2 = k^2 + 2g^2\chi\,,$$

$$\mu_{n,p}^{\mathrm{eff}}(k) = \mu_{n,p} - g_\omega\,\omega_0(k) \pm g_\rho\,\rho_0^3(k)\,,$$

$$n_B(E) = \frac{1}{e^{E/T}-1}\,,$$

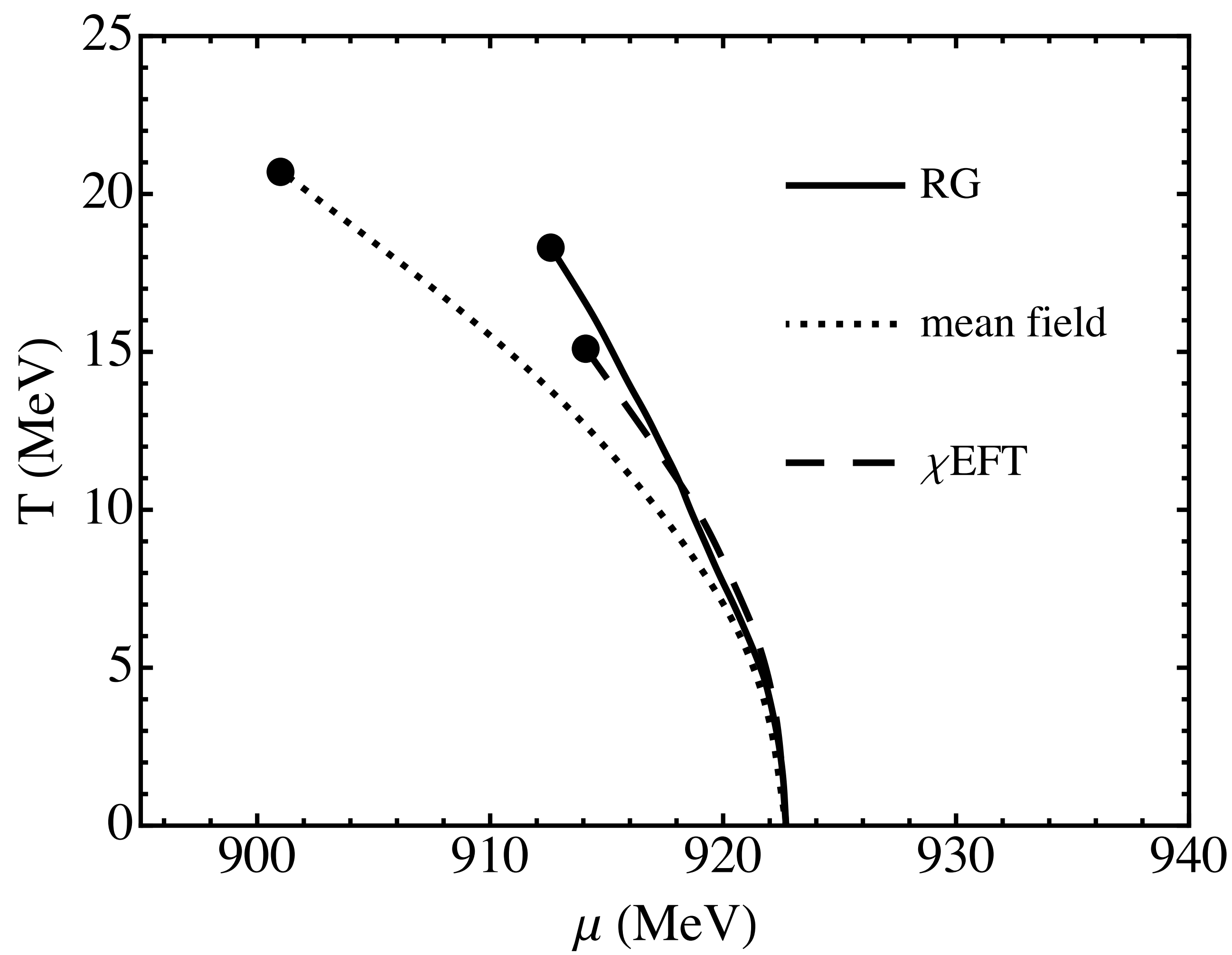
$$n_F(E) = \frac{1}{e^{E/T}+1}\,.$$

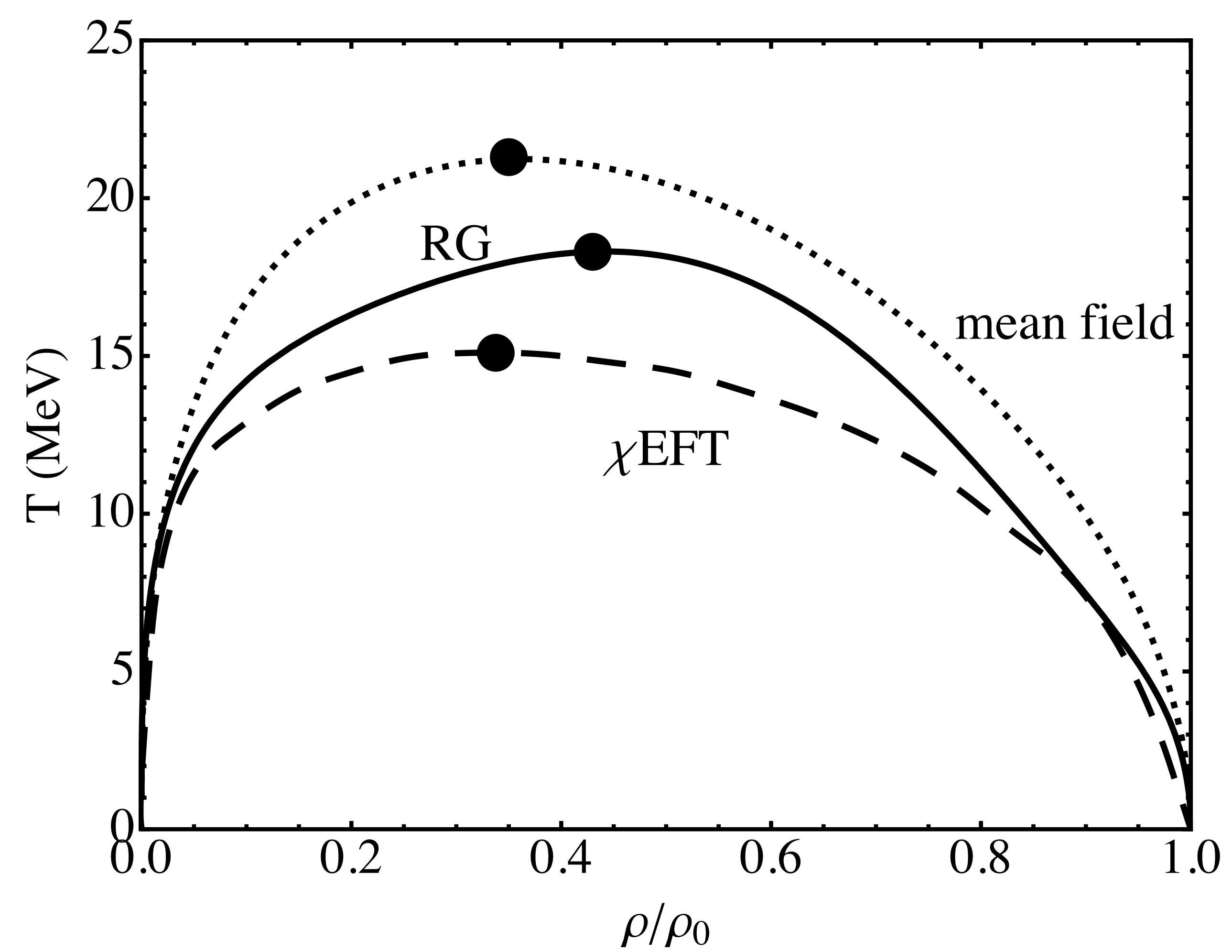
...plus vector field equations, then full system of equations solved on a grid.

Results : Liquid - Gas Transition

- symmetric nuclear matter -

M. Drews, T. Hell, B. Klein, W. W.
Phys. Rev. D 88 (2013) 096011

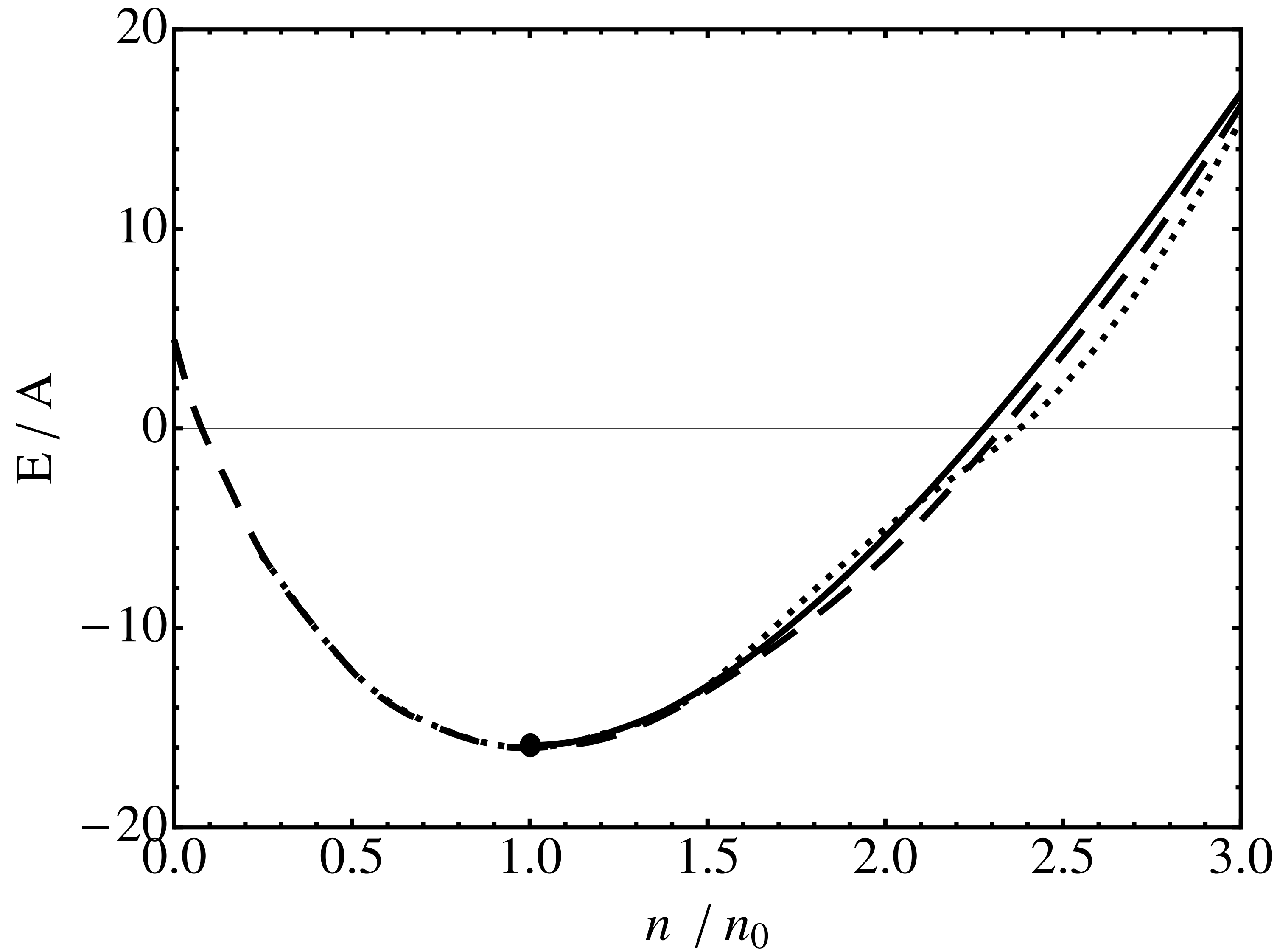




- close correspondence between **(perturbative) in-medium ChEFT** and **(non-perturbative) FRG** results

Symmetric nuclear matter in the chiral FRG approach

energy per nucleon at T = 0

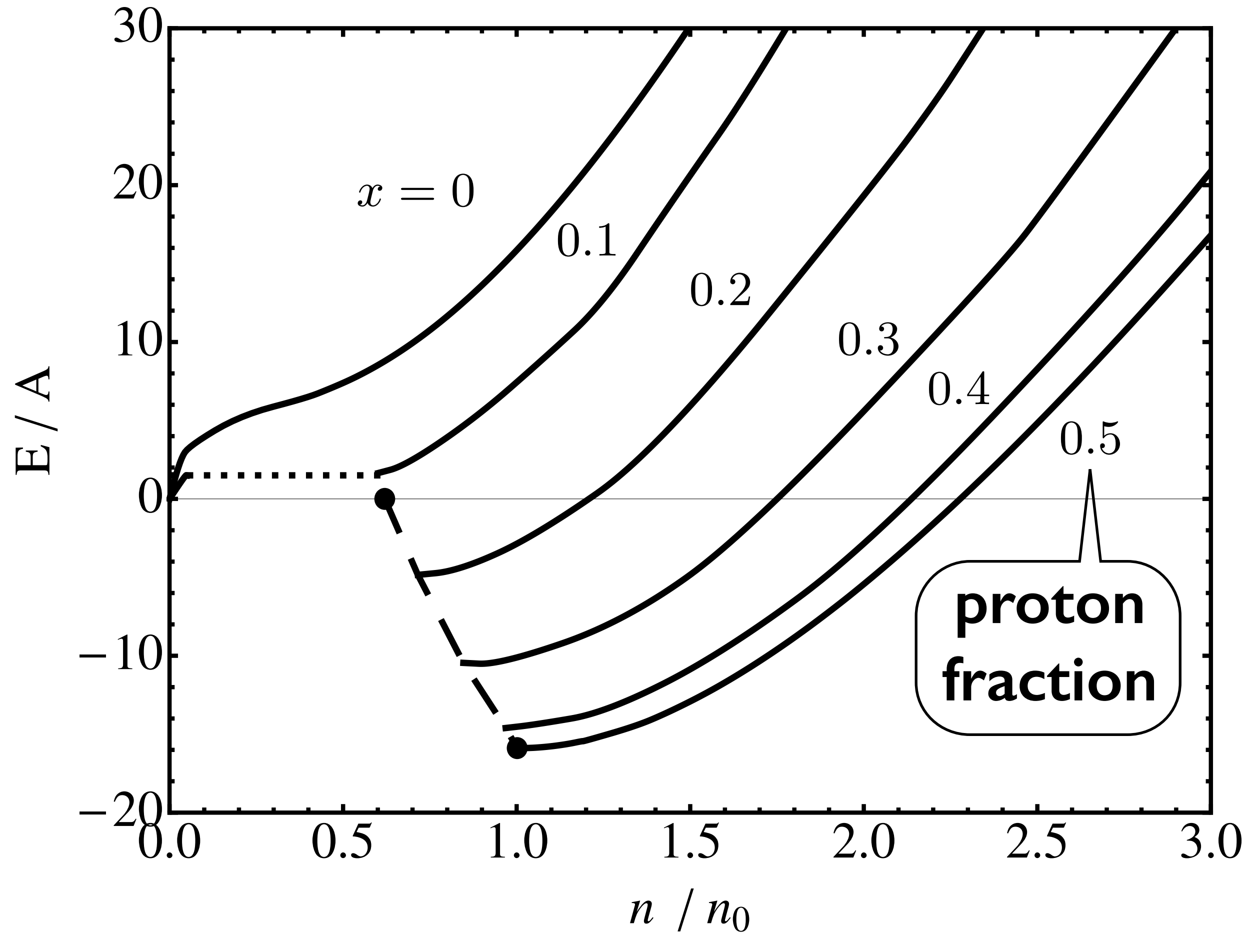


M. Drews, T. Hell, B. Klein, W.W.
Phys. Rev. D 88 (2013) 096011

- **FRG-Nucleon-Meson-Model** (solid curve) in comparison with advanced many-body (variational and QMC) computations

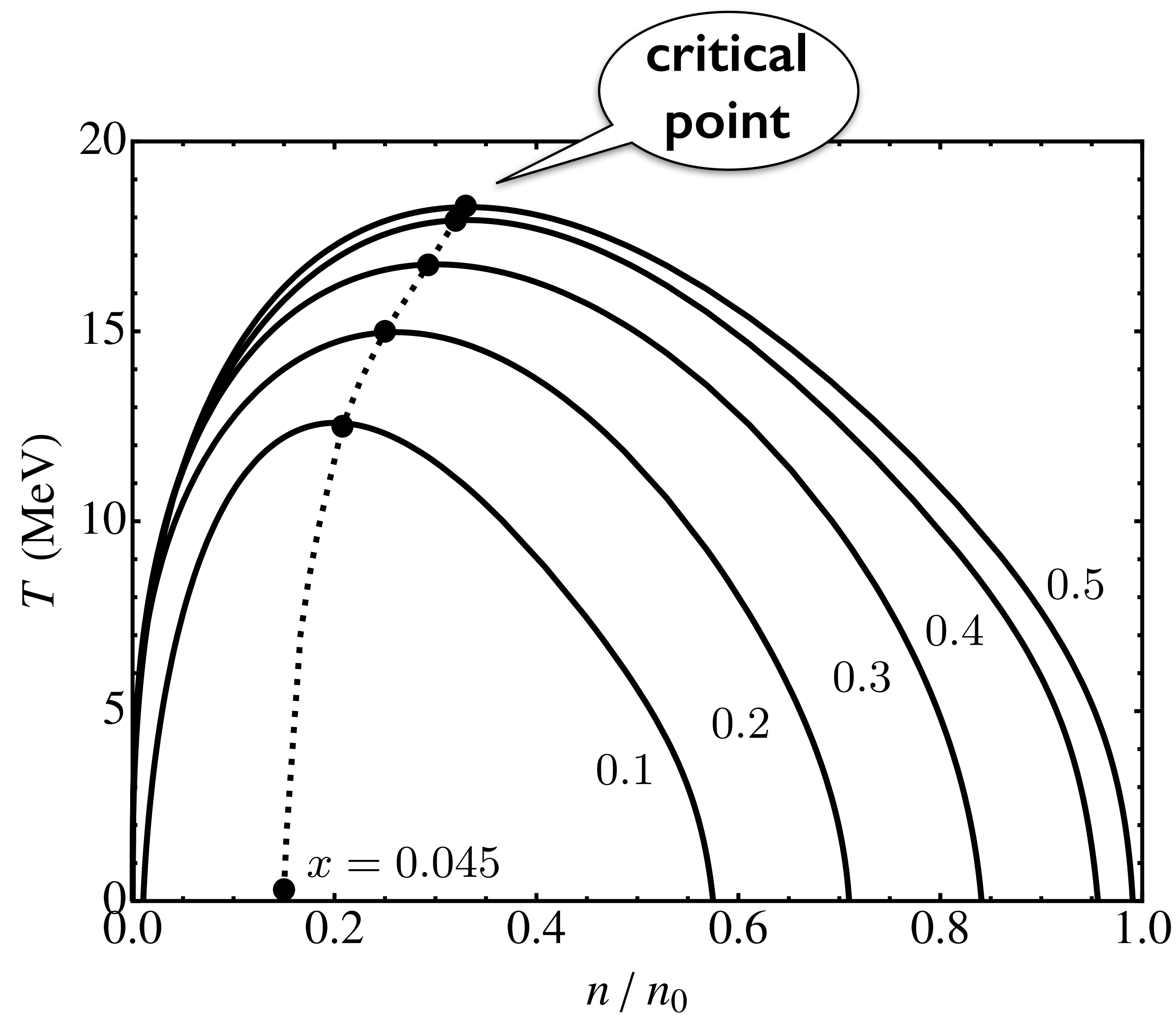
Asymmetric nuclear matter
in the **chiral FRG approach**

M. Drews, W.W. Phys. Lett. B738 (2014) 187 arXiv:1412.7838, Phys. Rev. C (in print)



Liquid-gas phase transition:

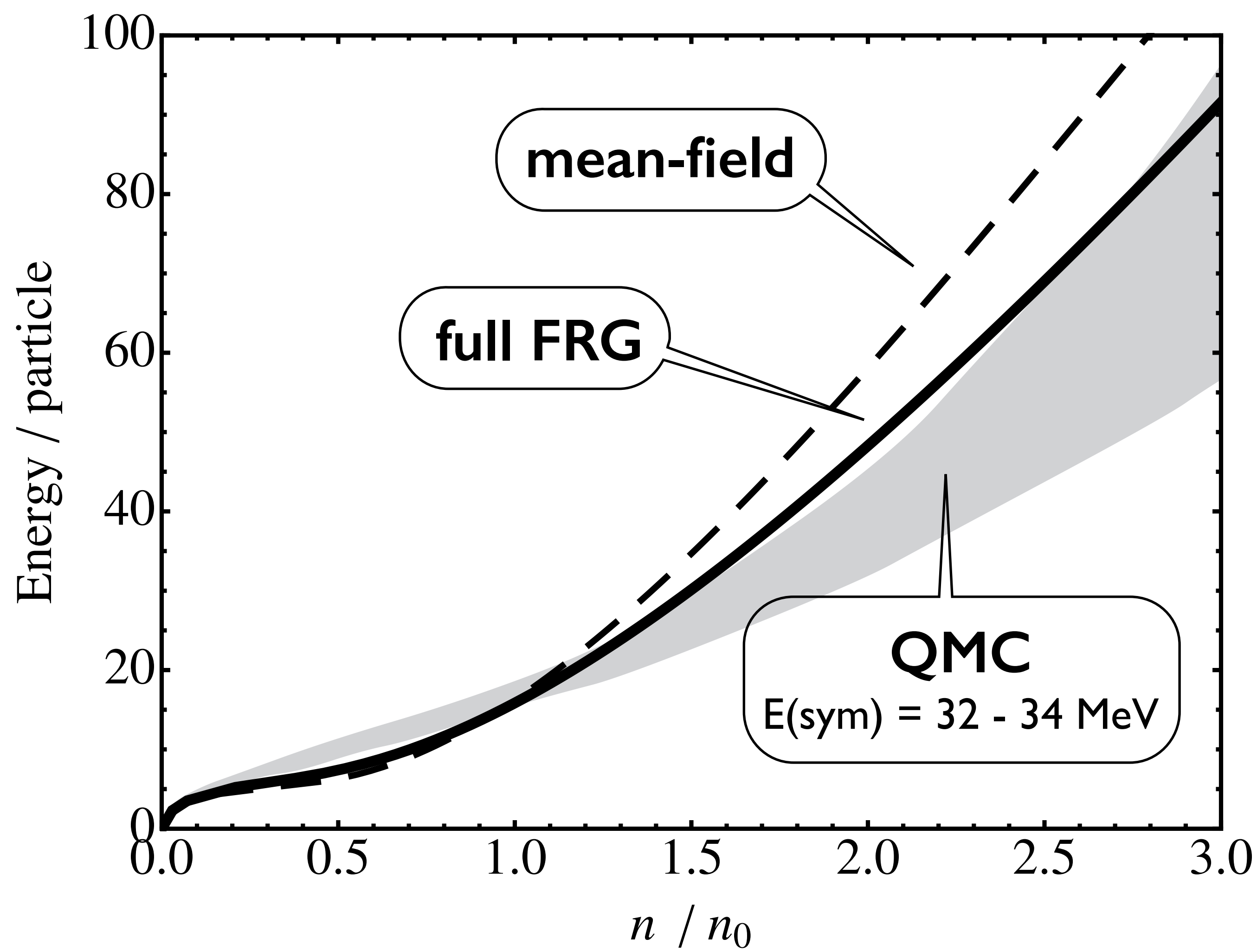
evolution of coexistence regions from symmetric to asymmetric nuclear matter



- **FRG results** (non-perturbative) are remarkably similar to (perturbative) in-medium **Chiral EFT calculations**

Neutron matter in the **chiral FRG approach**

M. Drews, W.W.
Phys. Lett. B738 (2014) 187



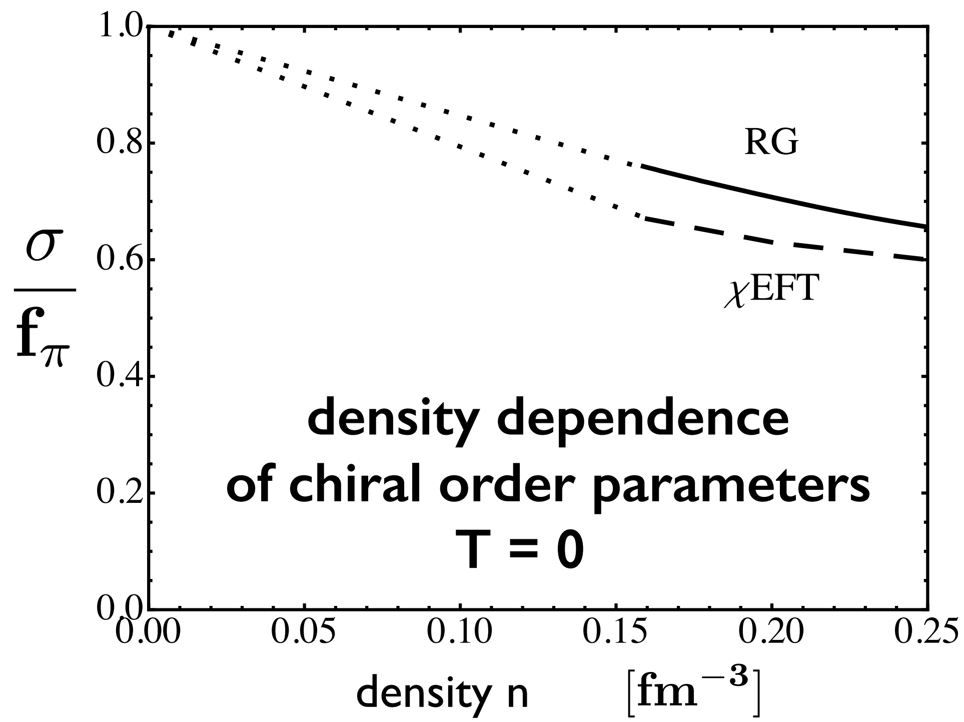
Quantum Monte Carlo computations:

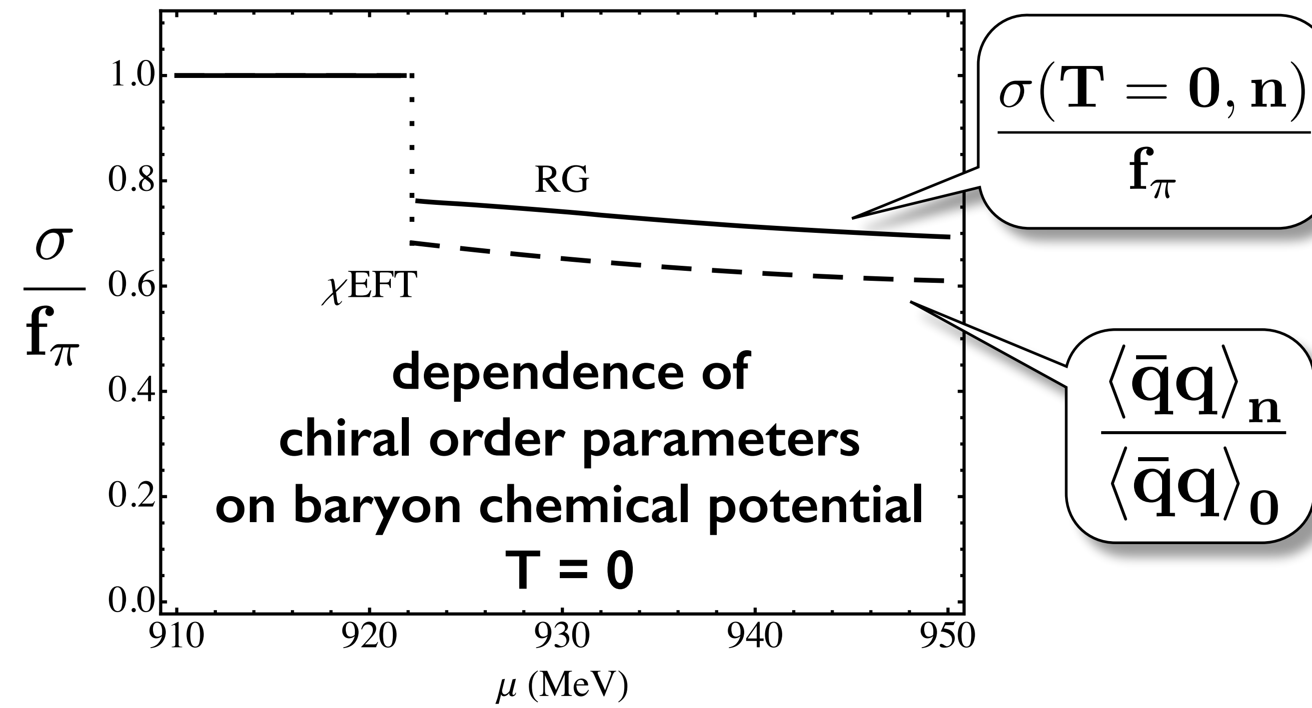
S. Gandolfi et al.
EPJ A50 (2014) 10

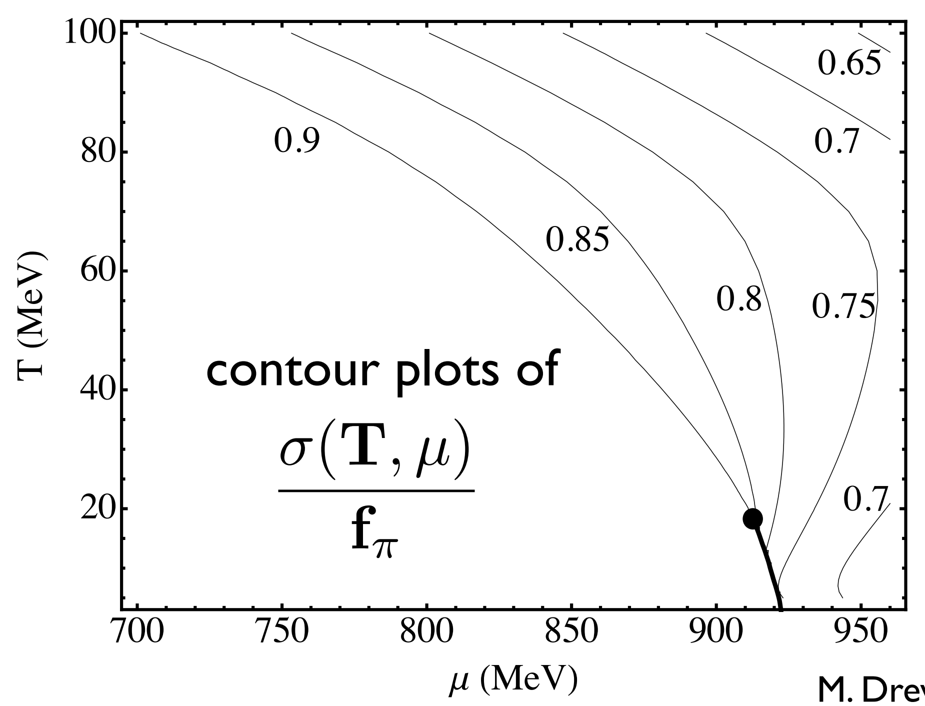
- Coupling strength of isovector-vector field / contact term fixed by symmetry energy E(sym) = 32 MeV

Chiral Order Parameters

- Comparison of **chiral effective field theory** and **NM-FRG** results



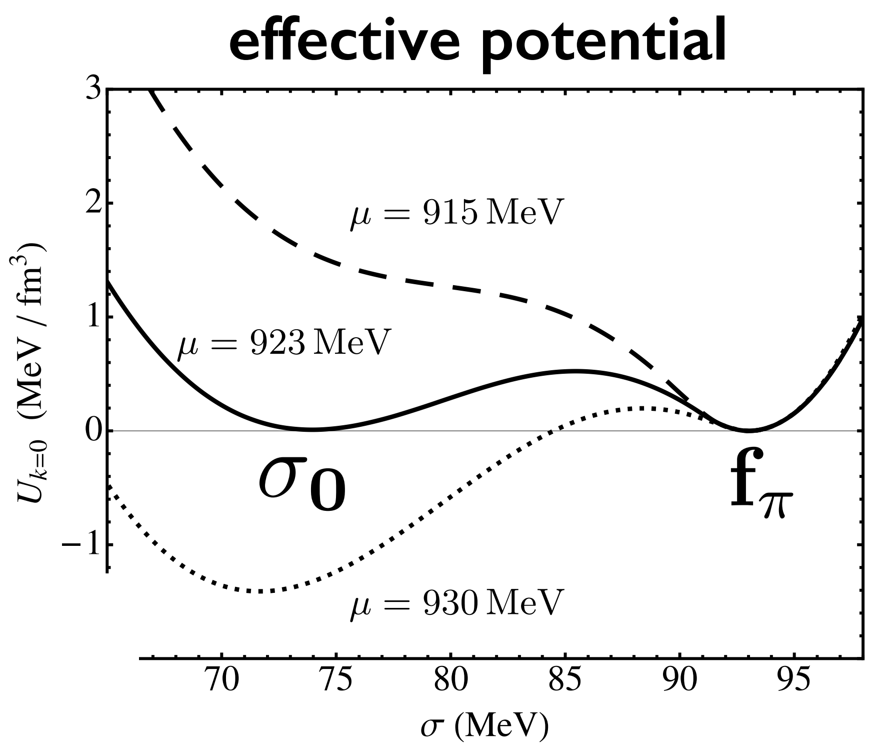




- **No** tendency towards **chiral phase transition** for baryon chemical potentials $\mu \lesssim 1\ \mathbf{GeV}$ and temperatures $\mathbf{T} \lesssim 100\ \mathbf{MeV}$

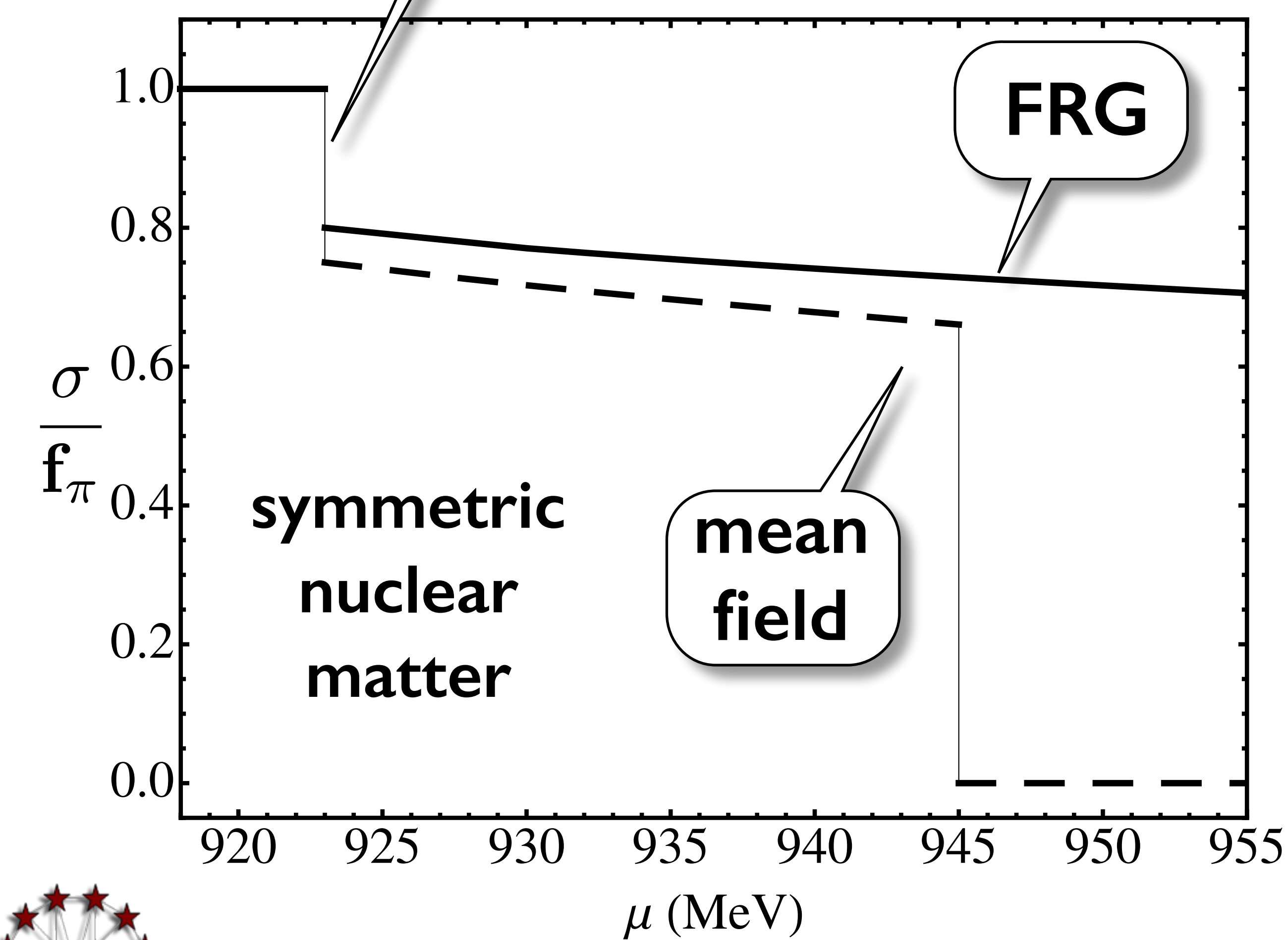
M. Drews, T. Hell, B. Klein, W.W.
Phys. Rev. D 88 (2013) 096011

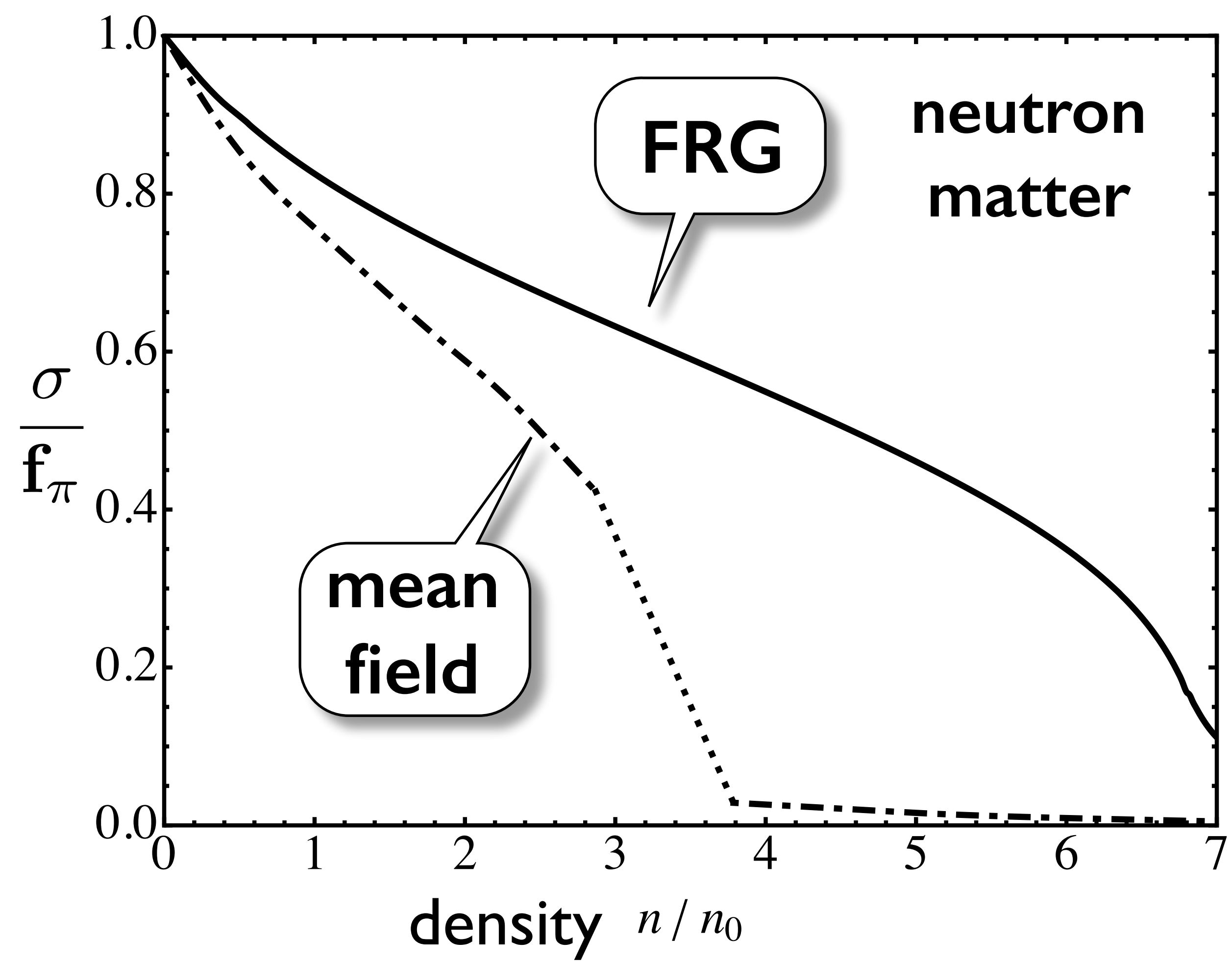
Chiral Order Parameter



important role of **fluctuations**
beyond mean-field approximation:

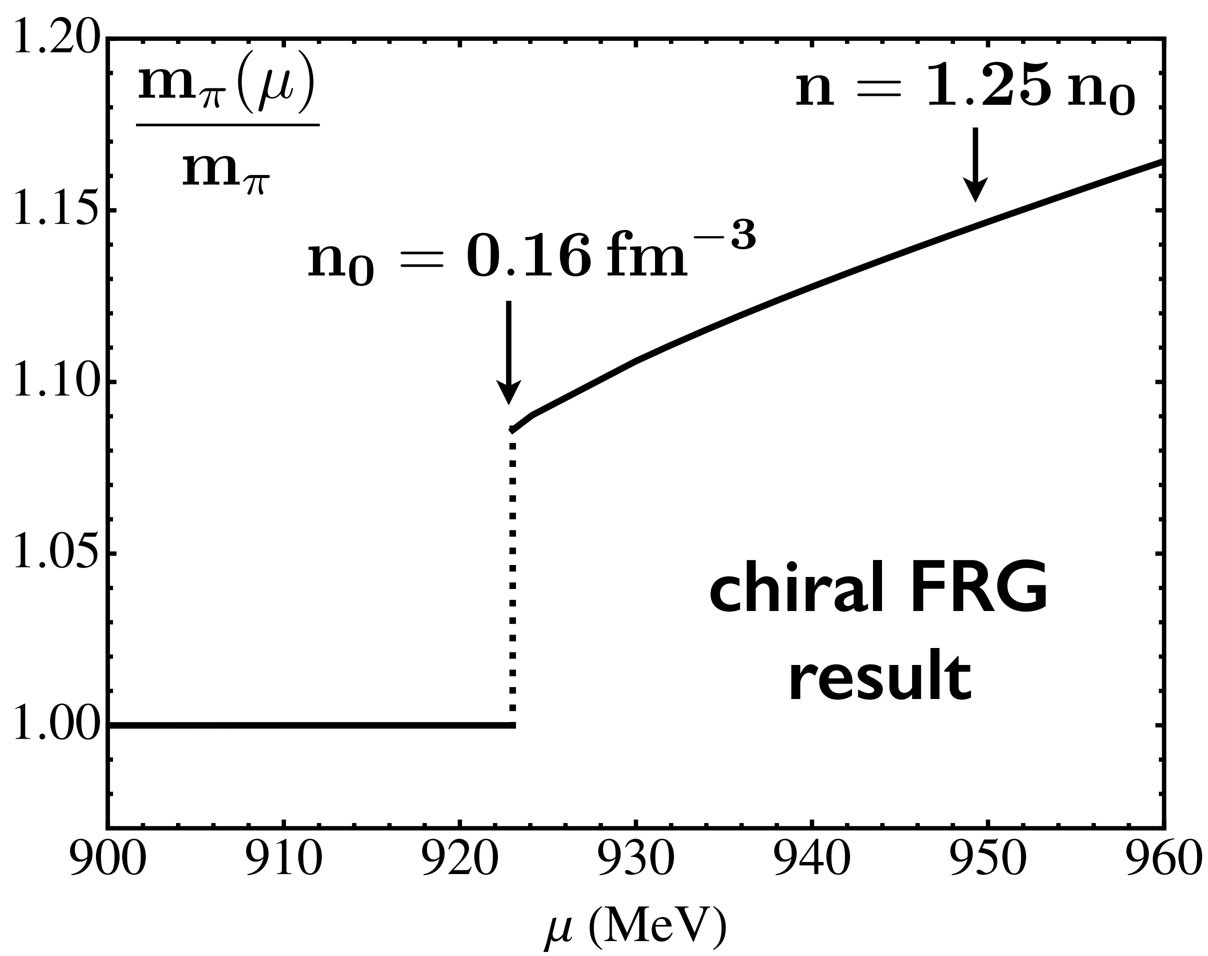
DISAPPEARANCE of
first-order chiral phase transition

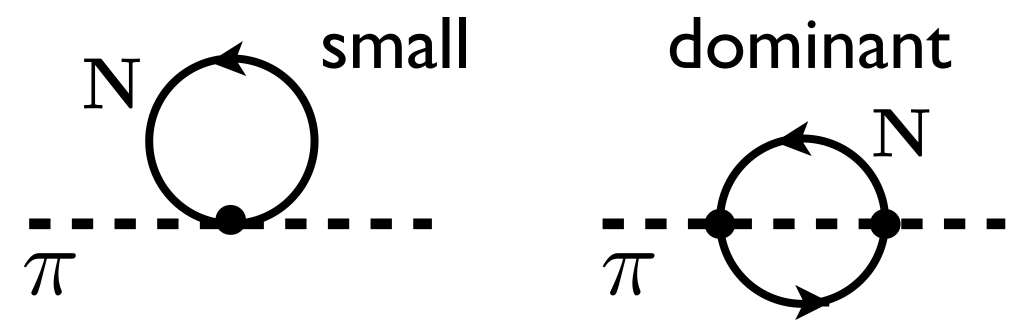




In-medium pion mass

- Contact with phenomenology :
  compare with **s-wave pion-nuclear optical potential** from **pionic atoms**





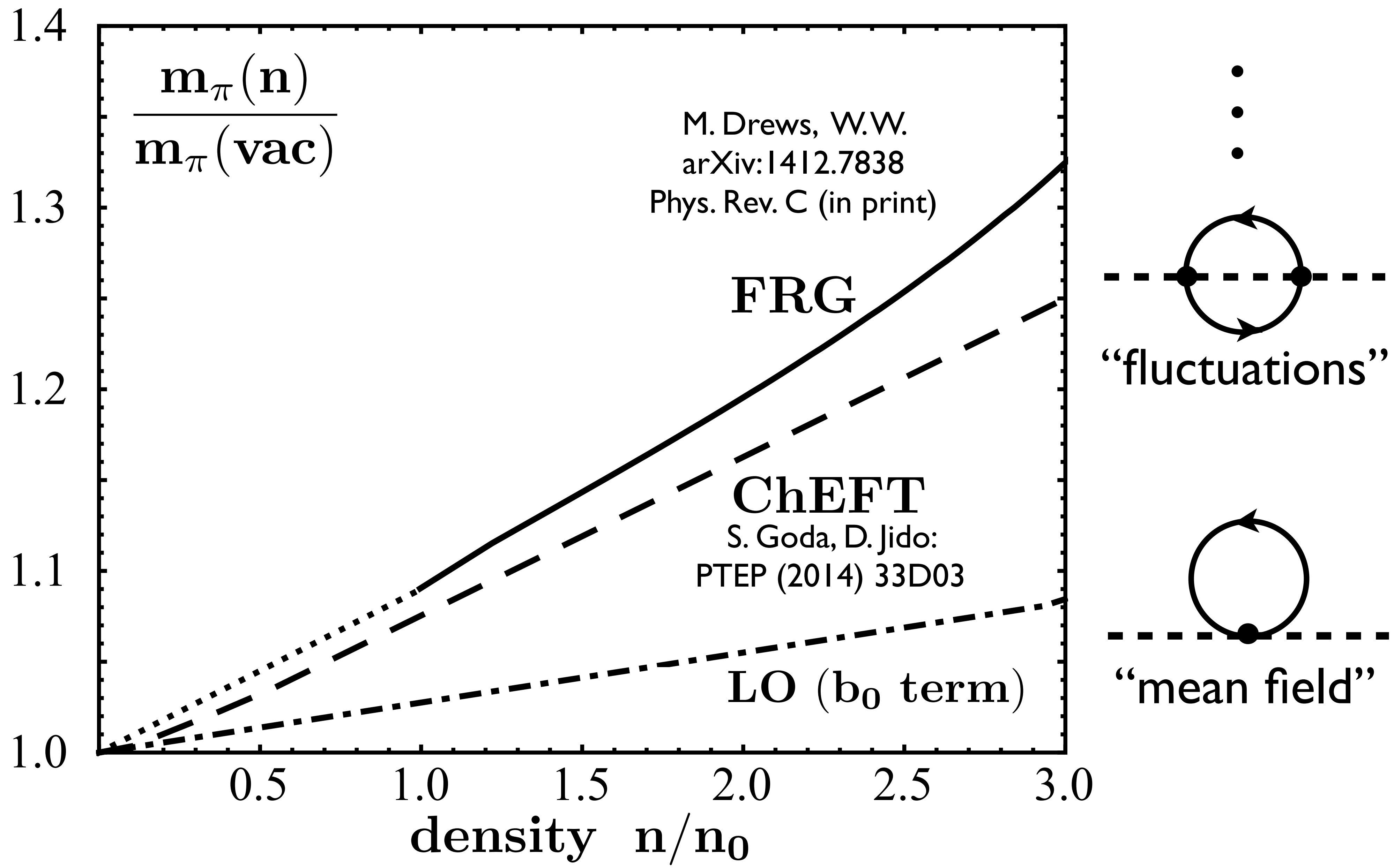
$$U(n) = -\frac{2\pi}{m_\pi}\left[b_0 - (b_0^2 + 2b_1^2)\left\langle \frac{1}{r} \right\rangle\right] \cdot n$$

$$\frac{m_\pi(n)}{m_\pi} \simeq 1 + \frac{U(n)}{m_\pi} \simeq 1.1\, \frac{n}{n_0}$$

- Good agreement of **FRG** calculation with empirical **in-medium pion mass shift,** both in sign and magnitude

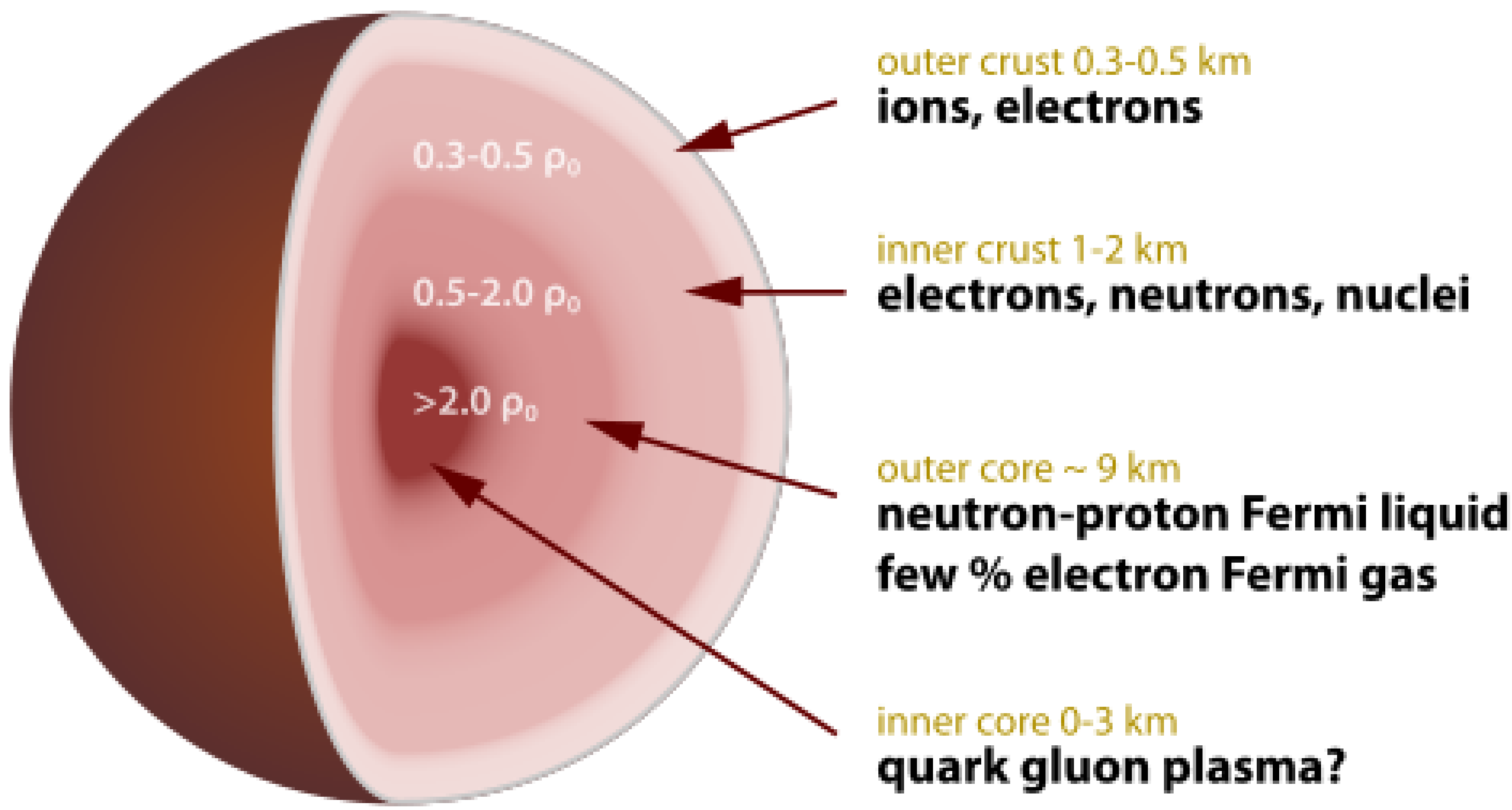
In-medium pion mass (contd.)

- **Non-perturbative FRG** result in comparison with in-medium **Chiral Pertubation Theory**



PART III:

Short digression on Neutron Stars

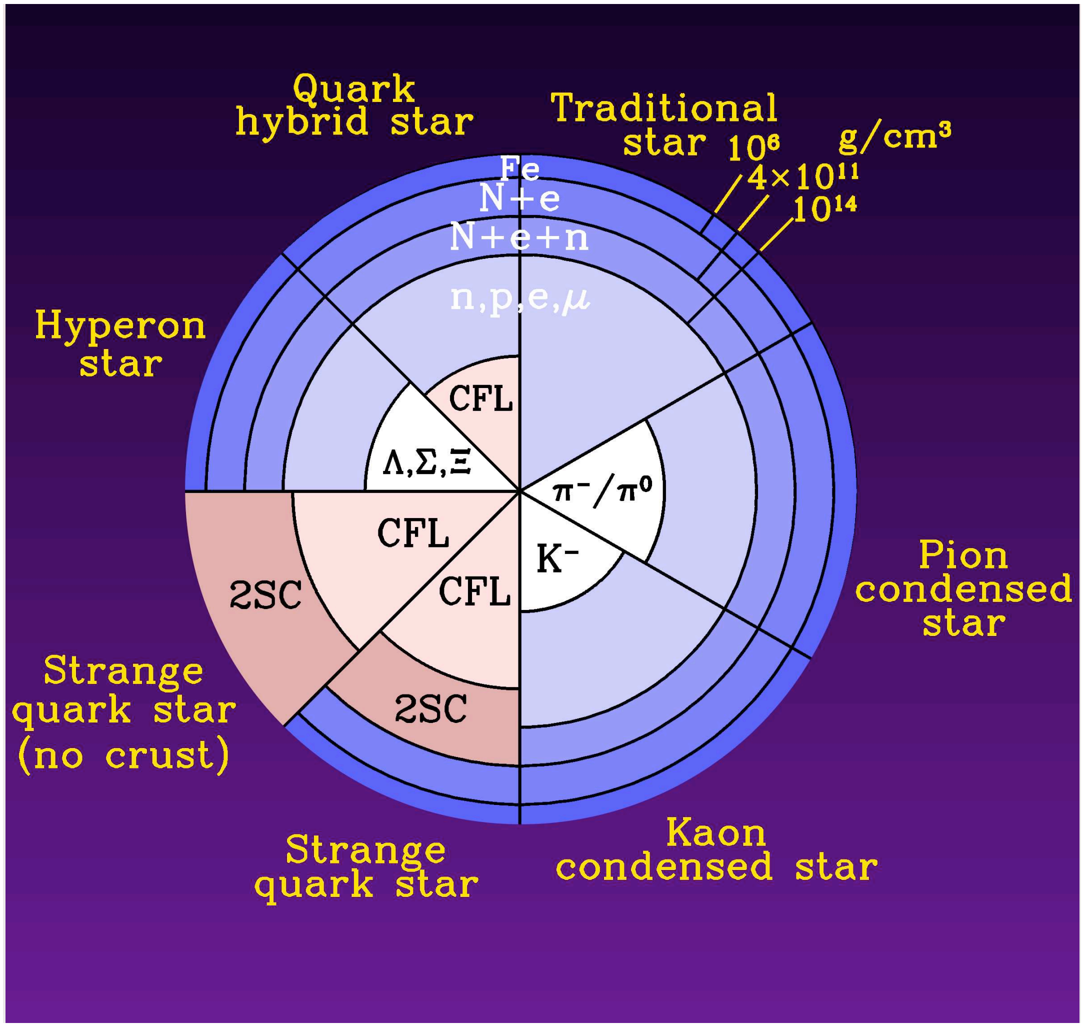


ECT*

Technische Universität München

NEUTRON STARS and the EQUATION OF STATE of DENSE BARYONIC MATTER

J. Lattimer, M. Prakash: Astrophys. J. 550 (2001) 426
Phys. Reports 442 (2007) 109



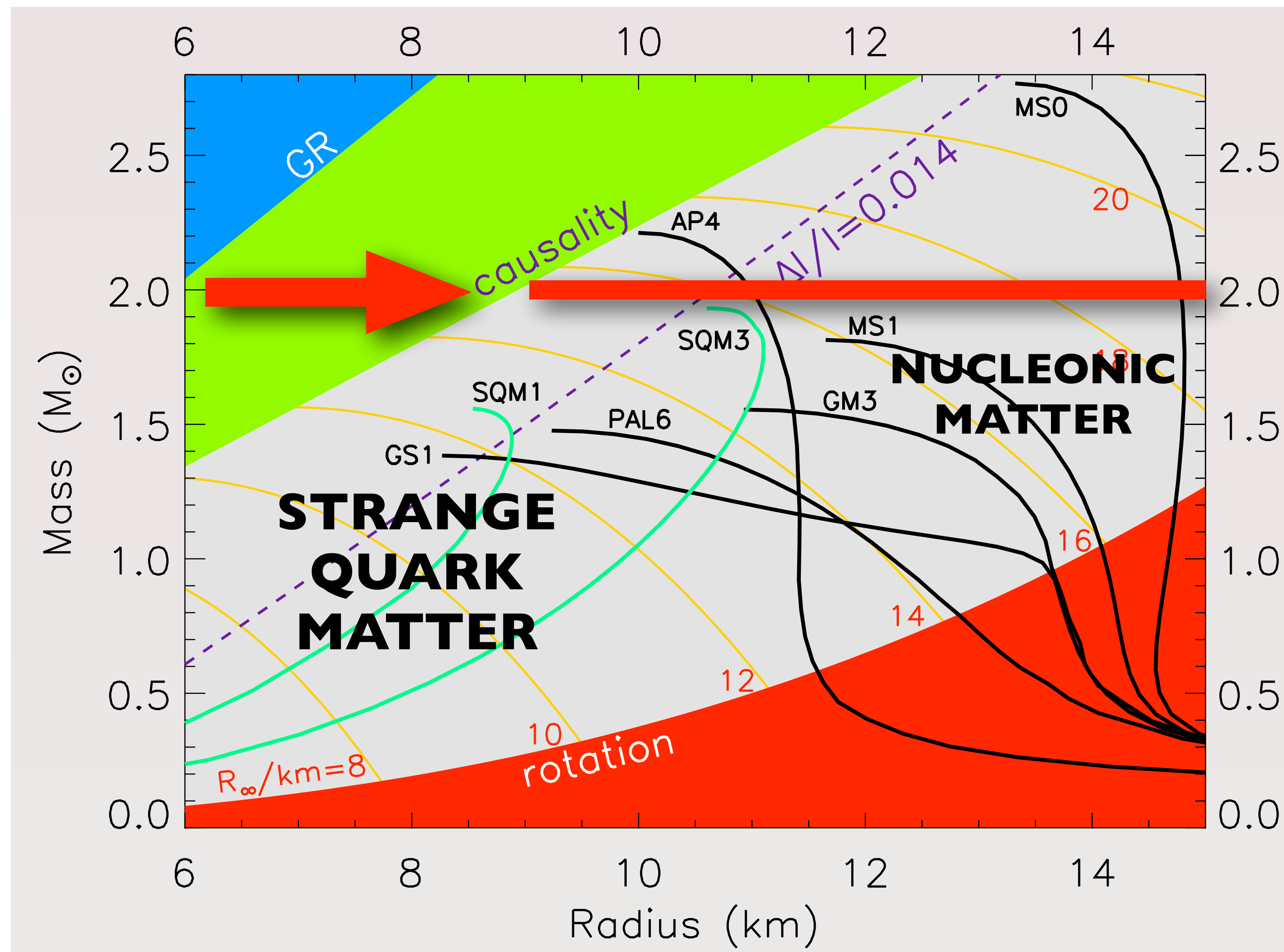
Neutron Star Scenarios

Tolman-Oppenheimer-Volkov Equations

$$\frac{dP}{dr} = -\frac{G}{c^2}\frac{(M + 4\pi P r^3)(\mathcal{E} + P)}{r(r - GM/c^2)}$$

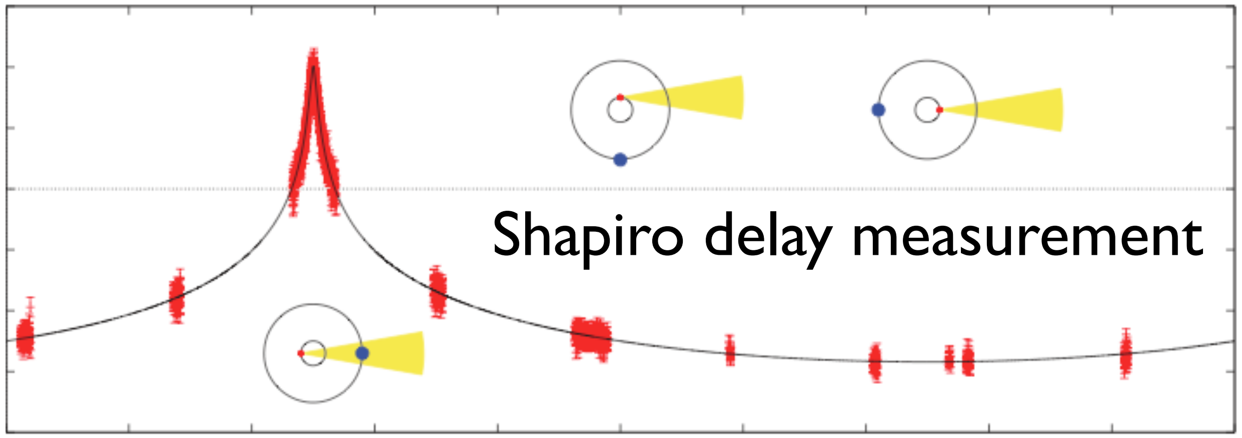
$$\frac{dM}{dr} = 4\pi r^2 \frac{\mathcal{E}}{c^2}$$

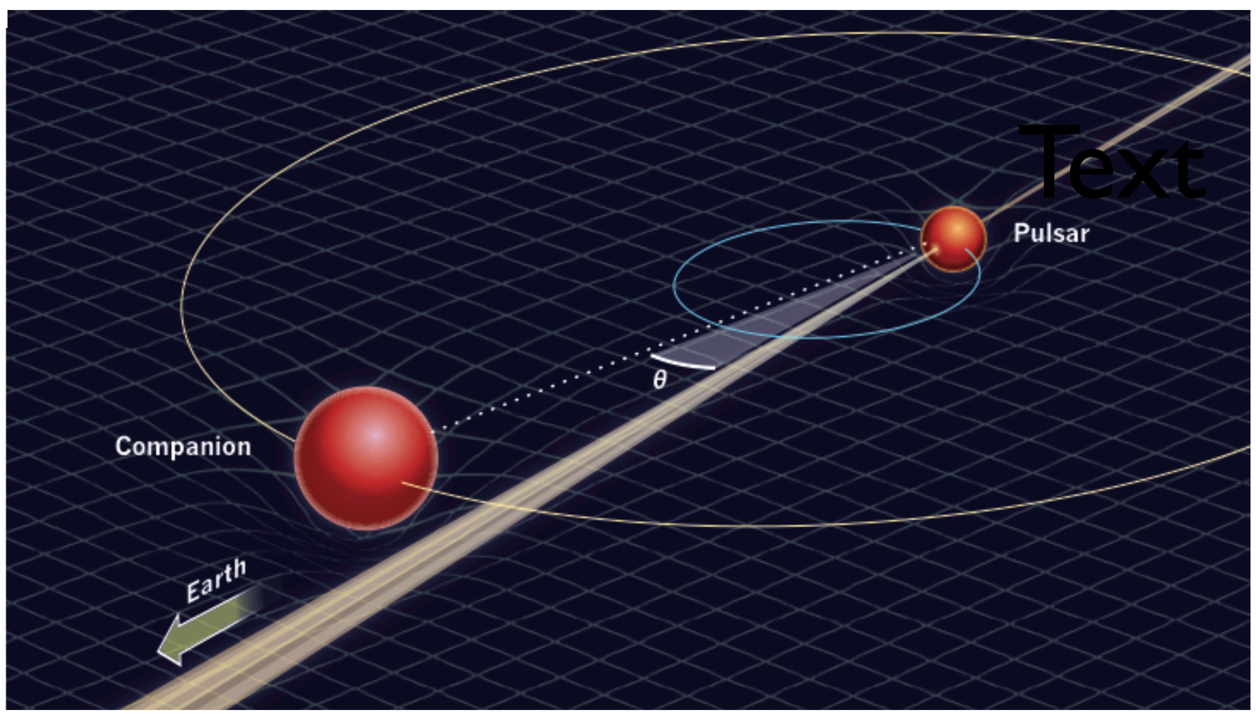
- Mass-Radius Relation



New constraints from NEUTRON STARS

P.B. Demorest et al.
Nature 467 (2010) 1081

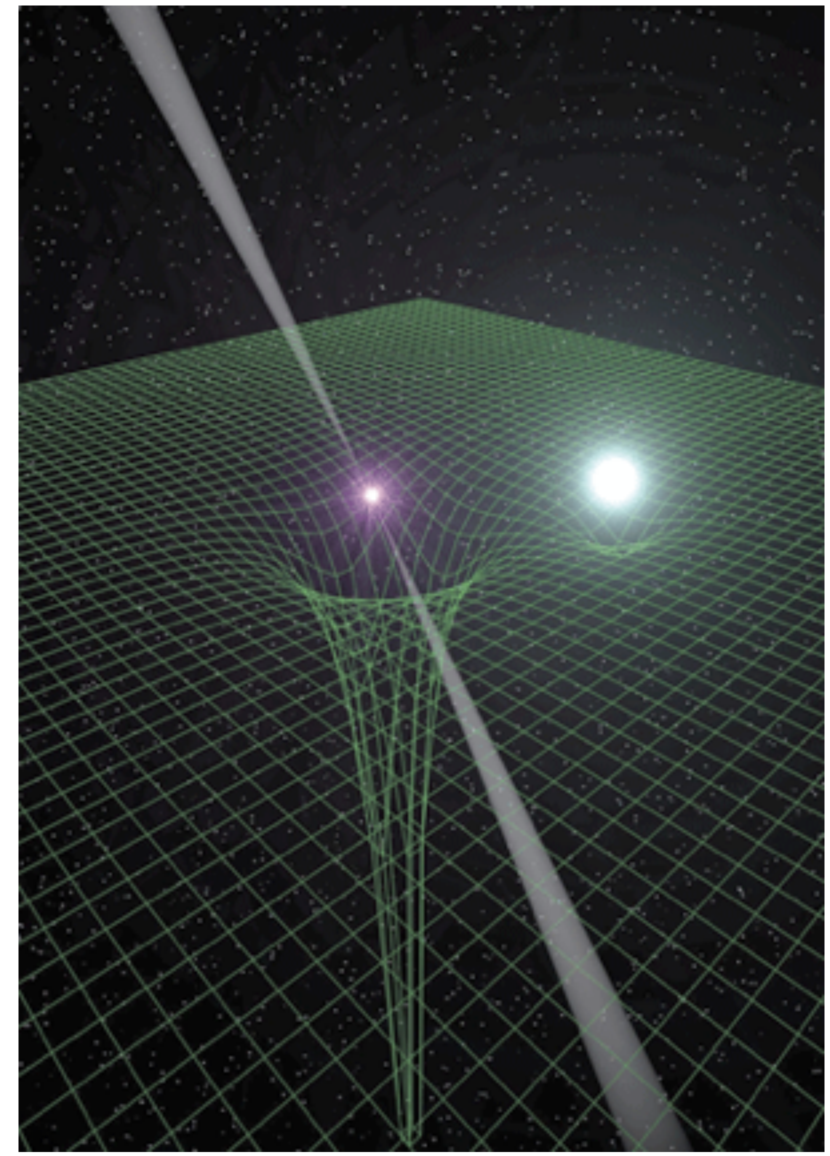




PSR J1614+2230

$$\mathrm{M} = 1.97 \pm 0.04\ \mathrm{M}_\odot$$

J. Antoniadis et al.
Science 340 (2013) 6131



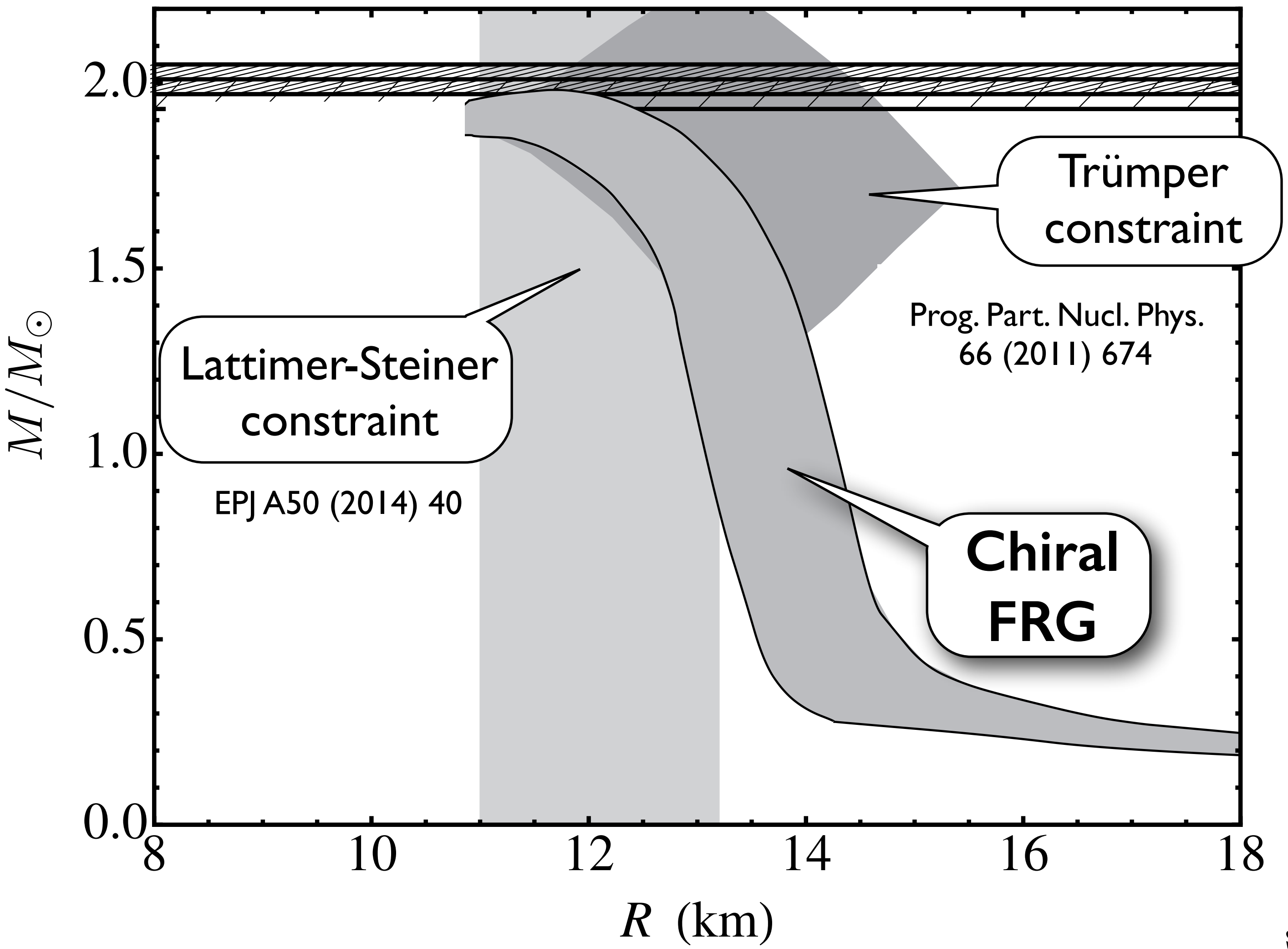
PSR J0348+0432

$$\mathrm{M} = 2.01 \pm 0.04\ \mathrm{M}_\odot$$

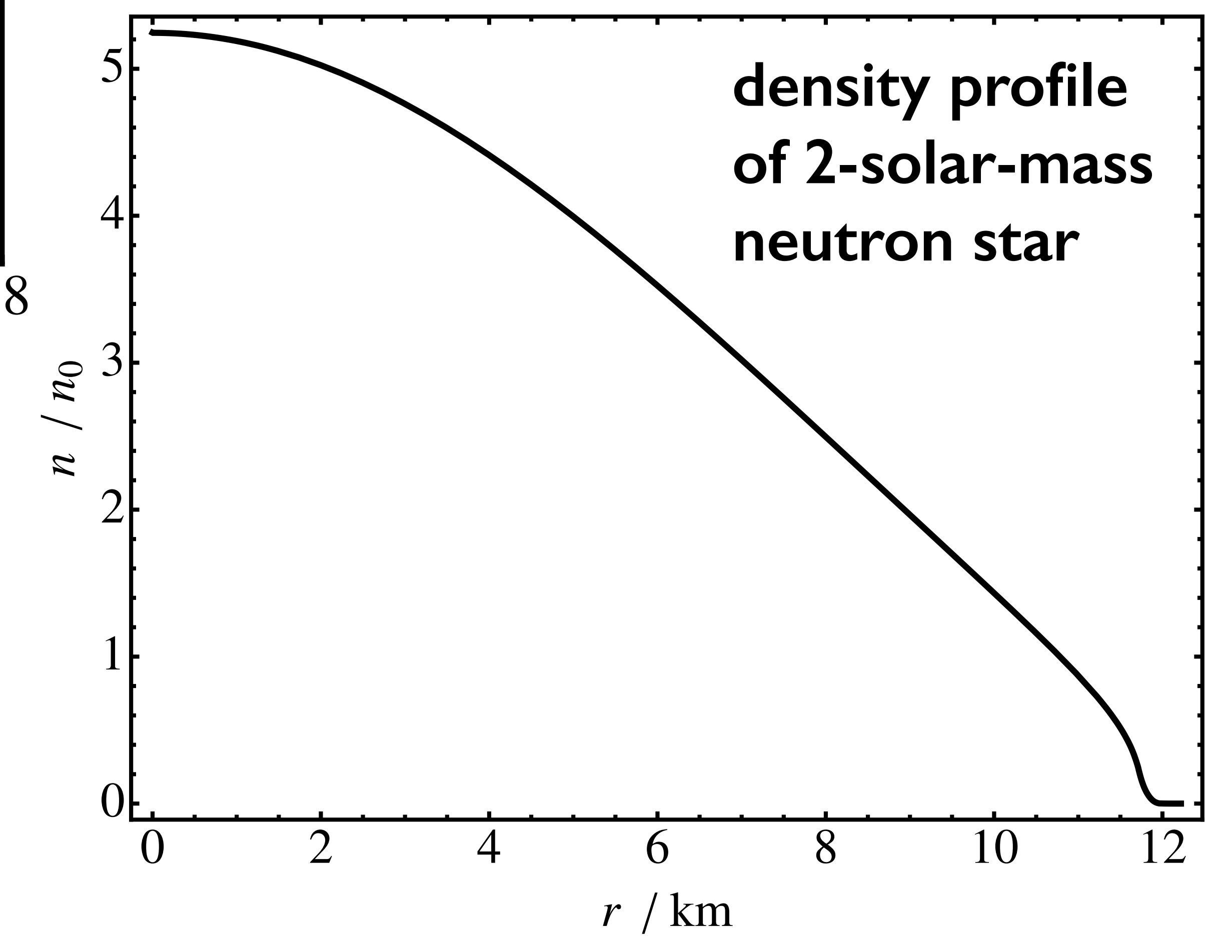
ECT*

Technische Universität München

NEUTRON STAR MATTER from **Chiral** Nucleon-Meson Model and **FRG**



M. Drews, W.W. arXiv:1412.7838
Phys. Rev. C (2015), in print



- **Neutron matter plus proton admixture** (beta equilibrium)
- **Symmetry energies 30 - 37 MeV**

- **No ultrahigh densities in the neutron star core**

CONCLUSIONS

- **Functional Renormalization Group** provides non-perturbative approach to **Nuclear Chiral Thermodynamics** from symmetric to asymmetric nuclear matter and neutron (star) matter
  - **Fluctuations beyond mean field** include important multi-pion exchange mechanisms and low-energy nucleonic particle-hole excitations
  - 1st order phase transition: Fermi liquid $\leftrightarrow$ interacting Fermi gas
- **No** indication of **first-order chiral phase transition**
  - **Fluctuations work against early restoration of chiral symmetry**
- **New** constraints from **neutron stars** for the **equation-of-state** of **dense & cold baryonic matter**:
  - Mass - radius relation: **stiff equation of state** required ! **No** ultrahigh densities $(\varrho_{\mathbf{max}} \sim 5\,\varrho_0)$
  - **Conventional** (nucleon-meson, "non-exotic") **EoS** meets constraints (issue of **strangeness**: suppression of hyperons in neutron stars ?)

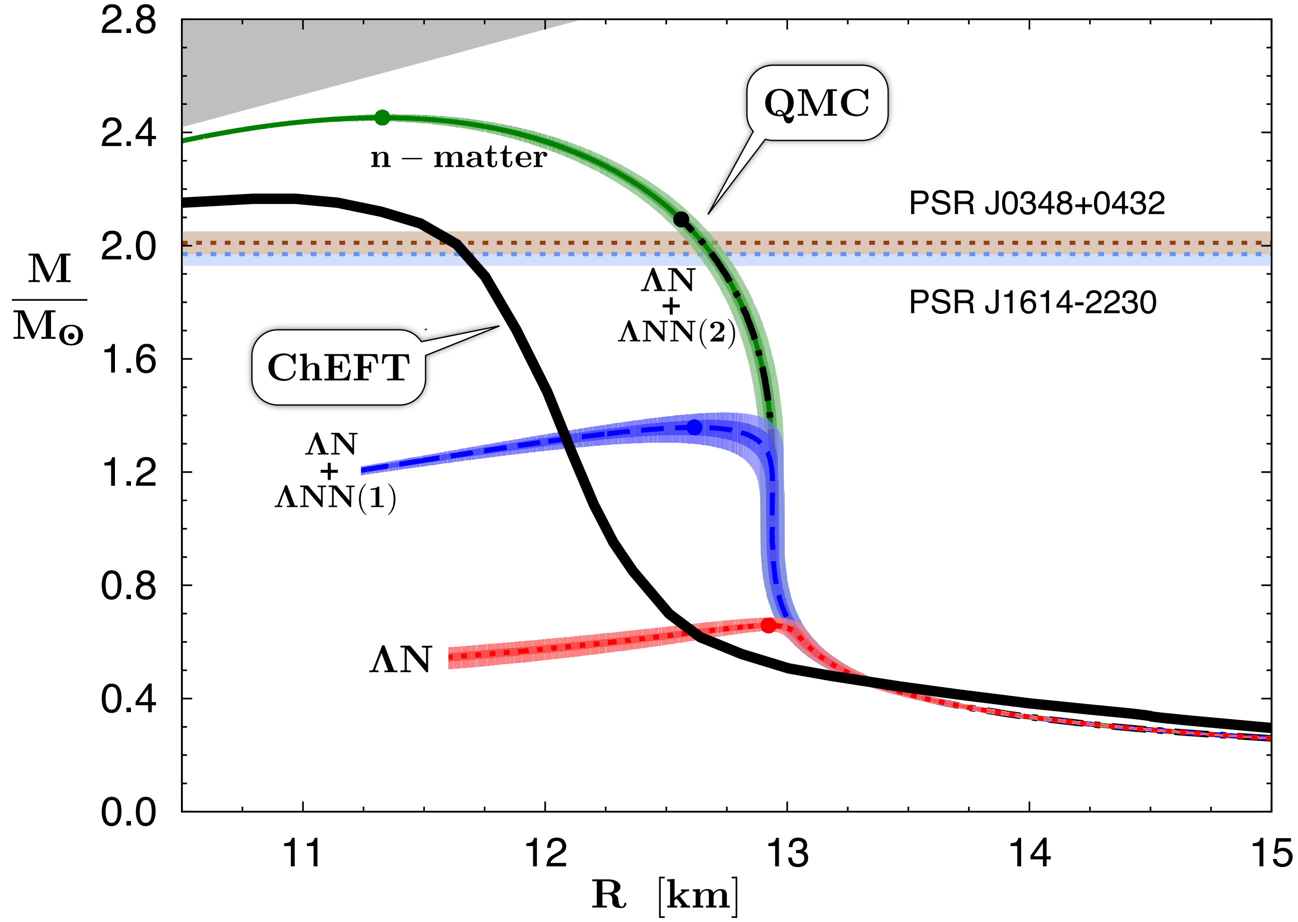
Appendix :
NEUTRON STAR MATTER including **HYPERONS**

New **Q**uantum **M**onte **C**arlo **calculations** using phenomenological hyperon-nucleon and hyperon-NN **three-body** interactions constrained by **hypernuclei**

Mass - radius relation of neutron stars

ChEFT calculations (“conventional” n-star matter):

T. Hell, W.W.
PRC90 (2014) 045801

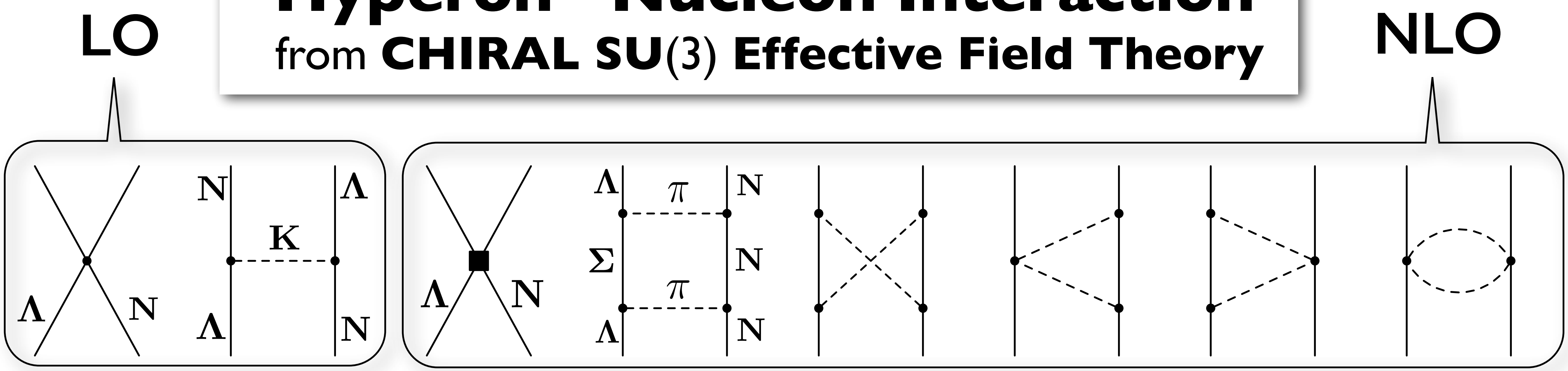


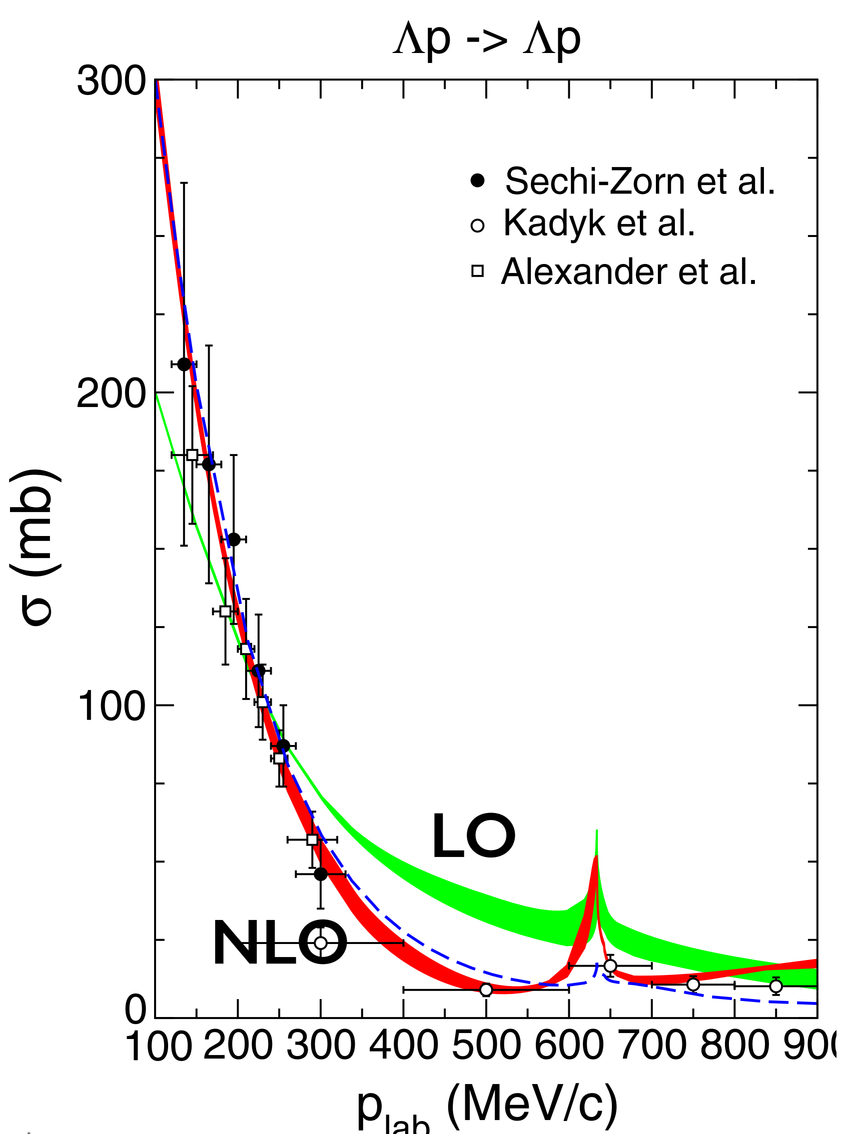
QMC computations (hyper-neutron matter):

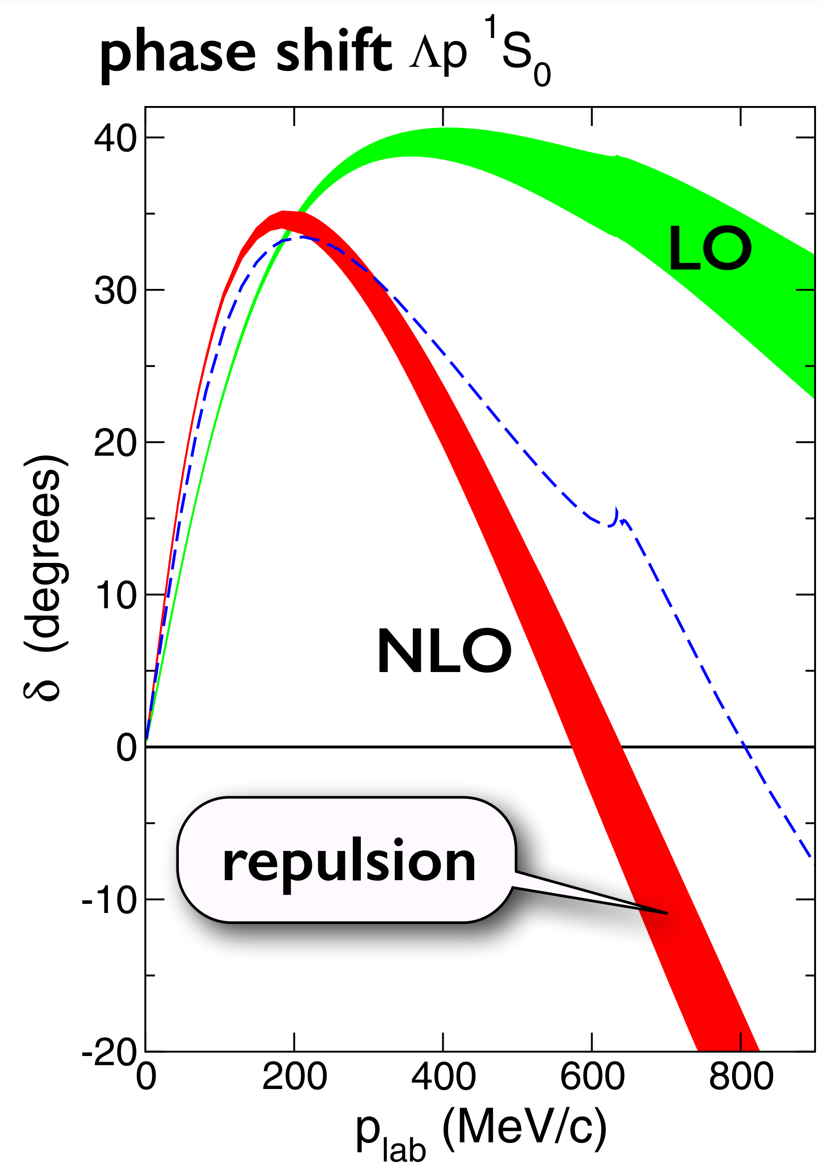
D. Lonardoni,
A. Lovato,
S. Gandolfi,
F. Pederiva
arXiv:1407.4448

with inclusion of hyperons: EoS **too soft** to support 2-solar-mass star unless strong short-range **repulsion** in YN and / or YNN interactions

Hyperon - Nucleon Interaction
from **CHIRAL SU(3) Effective Field Theory**





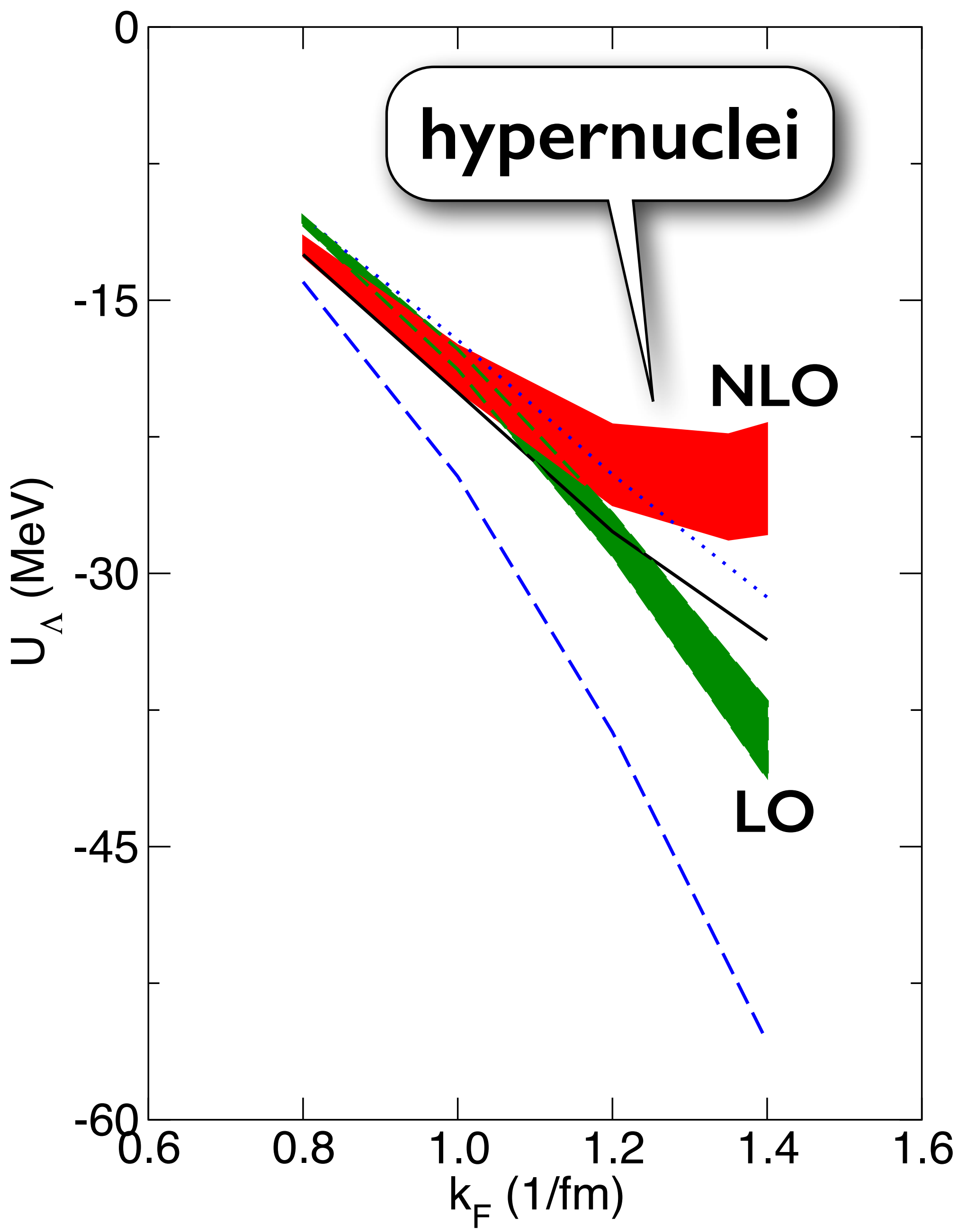


- **moderate attraction** at low momenta
  - ➜ relevant for **hypernuclei**
- **strong repulsion** at higher momenta
  - ➜ relevant for **dense baryonic matter**

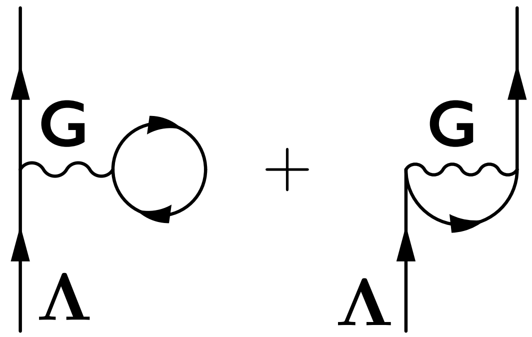
J. Haidenbauer, S. Petschauer, N. Kaiser,
U.-G. Meißner, A. Nogga, W.W.
Nucl. Phys. A 915 (2013) 24

Density dependence of Λ single particle potential

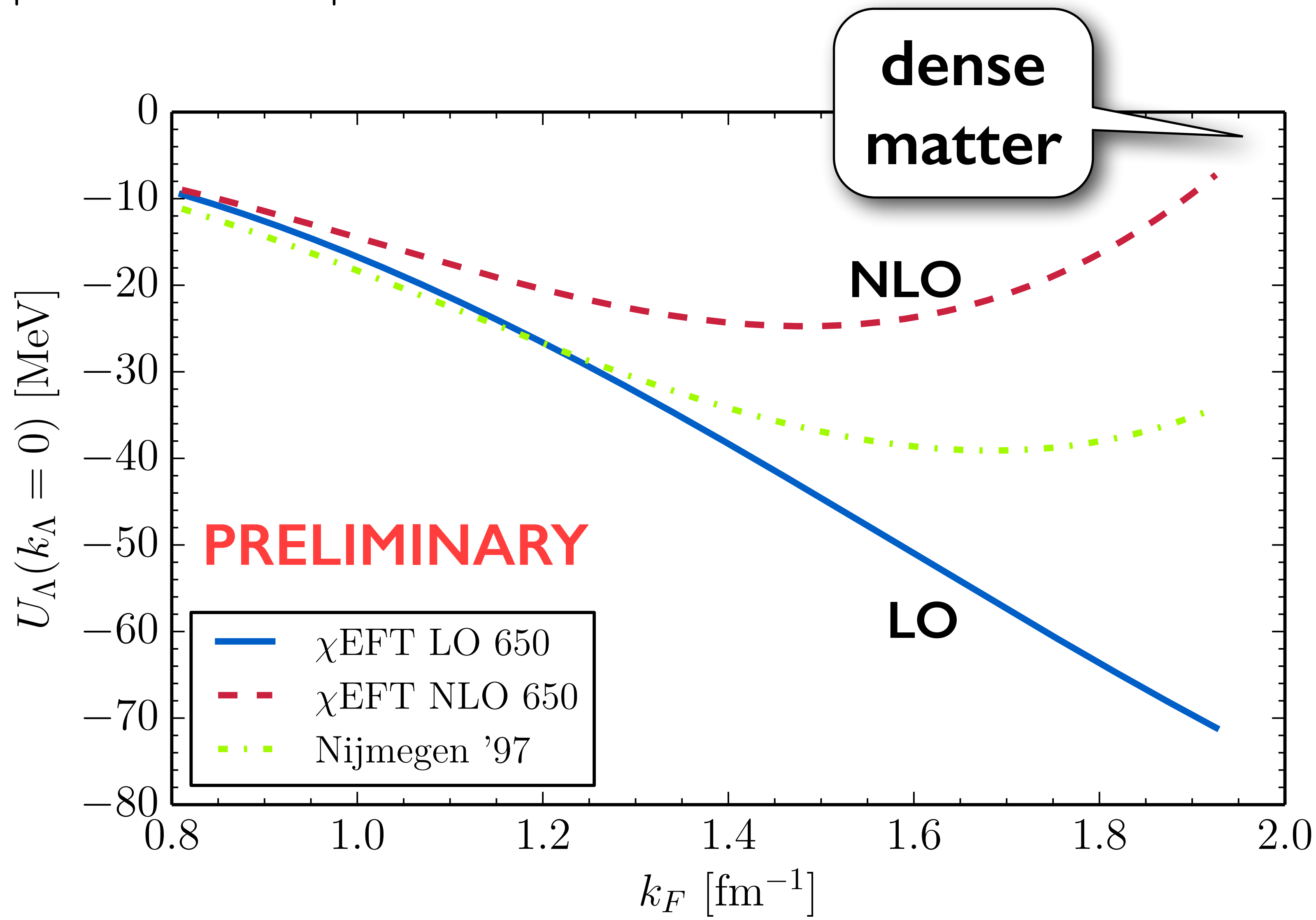
Brueckner calculations
using chiral SU(3) interaction



J. Haidenbauer, U.-G. Meißner,
Nucl. Phys. A 936 (2015) 29



$$G(\omega) = V + V \frac{Q}{e(\omega) + \mathrm{i}\epsilon} G(\omega)$$



S. Petschauer et al. (2015)