宇宙現象を用いた究極理論探査 (II)

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講義計画

第2章 究極理論

2.1 究極理論の候補

基礎理論として何を使うか

ボトムアップロジック

- Standard model ⇒ GUT: gauge-sector unification
  - hypercharge structure, α-unification, neutrino mass
  - Baryon asymmetry, strong CP(Peccei-Quinn symmetry)
- GUT ⇒ SGUT: boson-fermion correspondence

  Dark matter, Λ problem, hierarchy problem
- SGUT ⇒ Sugra GUT: inclusion of gravity
  - Primordial inflation, flat inflaton potential
- Sugra GUT ⇒ HD Sugra GUT: matter sector unification
  - Generation repetition, CKM/neutrino mixing, CP violation
- HD Sugra GUT ⇒ Superstring/M theory
  - Consistency as a quantum theory, finite control parameters
  - No Λ freedom (M-theory)

Candidates of the Ultimate Theory

- Mysteries of SM
- Inflation in the early Universe
- Dark Energy

	SST	Riem² Gravity	Horava Gravity	Loop QG
UV completion of Einstein gravity	Perturtatively finite	Renomalisable with ghosts	Renomalisable? Einstein at IR?	Not a sensible theory yet
Any info on physics beyond SM?	Unified theory of all interactions	NA	Violation of Lorentz inv. No other info on the matter sector	NA

At present, superstring theory and its extension is the only viable candidate of the ultimate theory.

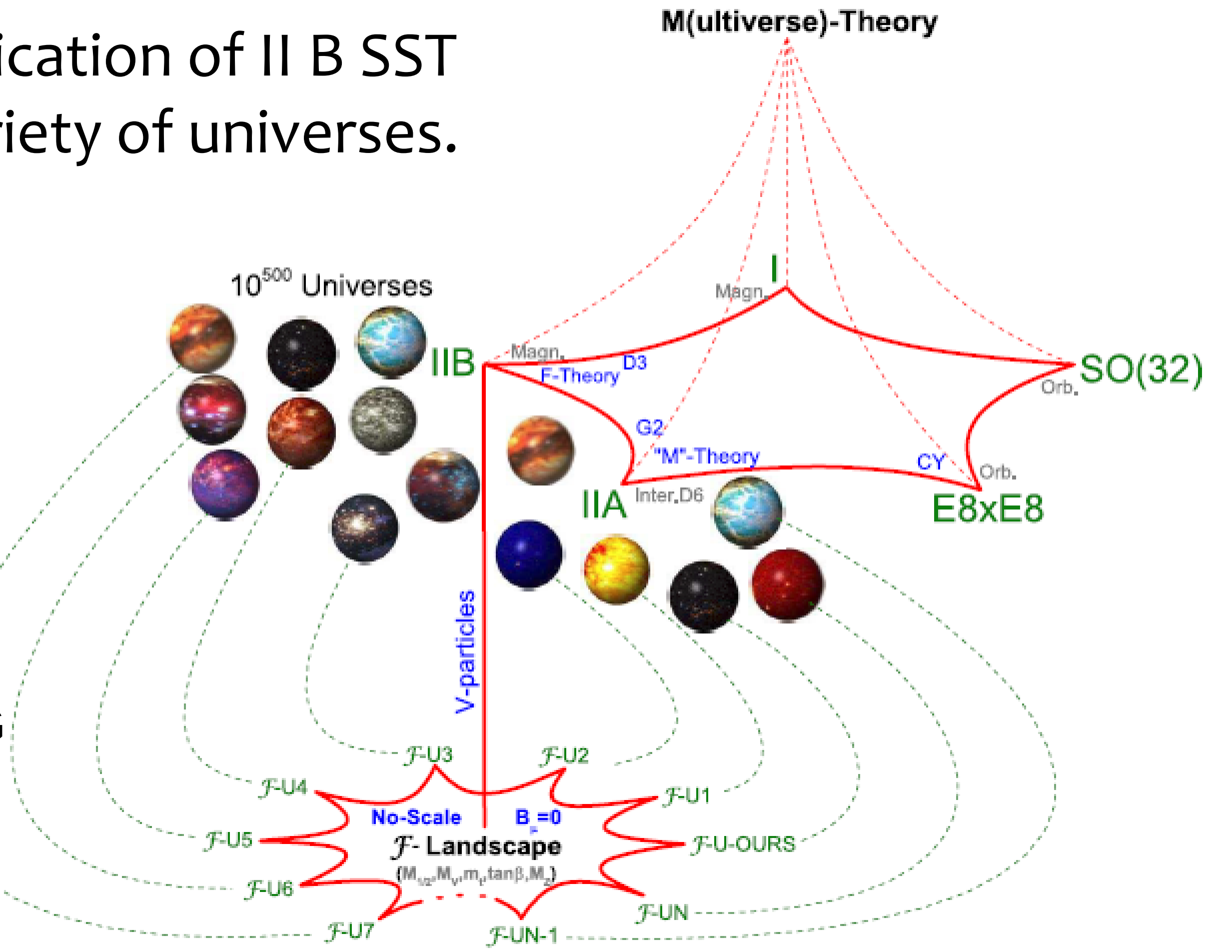
Does SST actually describe our Universe?

Landscape Problem

- The flux compactification of II B SST predicts a huge variety of universes.

- Intersecting D-brane model in IIA SST gives 10^{15} MSSM-like models.

  [Gmeiner F, Honecker G JHEP 09 (2007) 128]



Li T, Maxin JA, Nanopoulos DV, Walker JW: arXiv: 1111.0236

Our Universe is not in this landscape?

- On the particle physics side:
  - "Not a single string based model has yet been found which satisfies all known constraints."
  - [Heckman JJ: arXiv:1001.0577]
- On the cosmology side:
  - "A typical analysis collects 'ingredients' that are understood to varying degrees in isolation, and assembles them in a single compactification with suitable cosmological properties … in which the mutual interactions are neglected."
  - [Burgess CP, McAllister L: CQG28(2011)204002]

We need more info !!

Windows to the Ultimate Theory

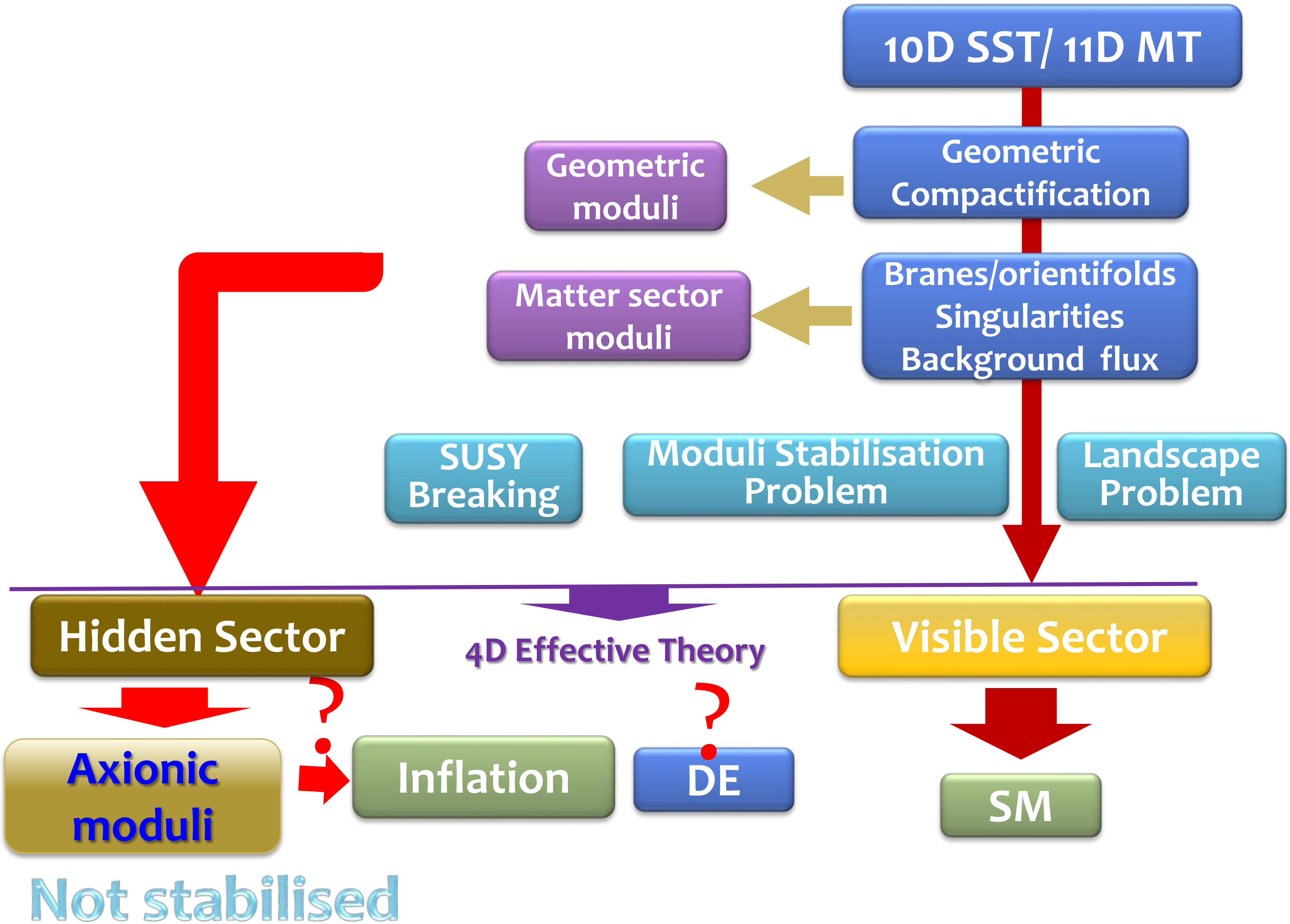
From High Energy

- Collider experiments (LHC, ILC)
- INFLATION
- Cosmological relics (DM, Baryon#, GW, cosmic string,…)

From Low Energy

- Change of the fundamental constants on cosmological scales
- New forces in submm ranges
- AXION COSMOPHYSICS

Find phenomena characteristic to string theory!!



２．２　超弦理論と超重力理論

平坦な時空上の超弦理論の分類

閉弦のみの理論

- I型（16 susy)
  - ヘテロ型　$E_8 \times E_8/Z_2$ 理論 $\Rightarrow$ 10次元 Type I sugra + $E_8 \times E_8$-SYM
  - ヘテロ型　SO(32)理論 $\Rightarrow$ 10次元 Type I sugra + SO(32)-SYM
- II型（32 susy)
  - IIA型理論 $\Rightarrow$ 10次元type IIA sugra
  - IB型理論 $\Rightarrow$ 10次元type IIB sugra

開弦＋閉弦理論(16 susy以下）

- IIA型理論＋ブレーン $\Rightarrow$ 10次元type II sugra + brane上の (chiral)SYM
- IIB型理論＋ブレーン $\Rightarrow$ 10次元type II sugra + brane上の (chiral)SYM
- I型 SO(32) $\Rightarrow$ IIB型理論＋D_9 ブレーン / Orientifold projection

M理論

??? $\Rightarrow$ 11次元sugra

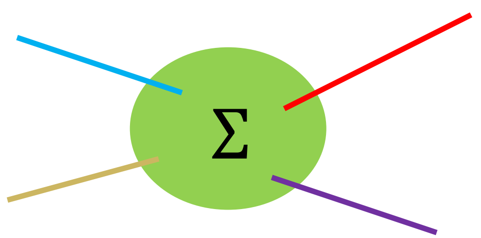
Classical Limit of String Theory

- String action (bosonic part)

$$S_E = \frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma(-g)^{1/2}\left[\left(g^{ab}G_{MN}(X)+i\epsilon^{ab}B_{MN}(X)\right)\partial_a X^M\partial_b X^N\right] + \frac{1}{4\pi}\int_\Sigma d^2\sigma(-g)^{1/2}R\phi(X)+i\int_{\partial\Sigma}d\sigma^a\partial_a X^M A_M(X).$$

- QFT on the cylinder WS

⇒ Asymptotic free particle spectrum



- S-matrix

$$S_{c_1\cdots c_n o_1\cdots o_m} = \sum_{\text{WStopologies}}\frac{1}{V_{\text{diff}\times\text{Weyl}}}\int[dXdg]e^{-S_m[X,g:\text{BG}]-\chi(\Sigma)\phi(X)} \times\prod_{i=1}^{n}\int_\Sigma d^2\sigma_i g(\sigma_i)^{1/2}\mathcal{V}_{c_i}(\sigma_i)\prod_{j=1}^{m}\int_{\partial\Sigma}ds_j\mathcal{V}_{o_j}(s_j)$$

- Low Energy Effective Action (bosonic string)

$$Z = \int [dG][dB][d\phi] \sum_{\chi=-\infty}^{2} \exp\left[i S_{\text{eff}}^{(\chi)} \right]$$

$\Leftarrow$ $\boldsymbol{Z_{\text{eff}}(\text{BGF}) = Z_{\text{string}}(\text{BGF})}$

$$S_{\text{eff}}^{(2)} = \frac{1}{2\kappa_0^2} \int d^D x \, (-G)^{1/2} e^{-2\phi} \left[-\frac{2(D-26)}{3\alpha'} + R - \frac{1}{2} H_{[3]}^2 + 4(\nabla\phi)^2 + \mathrm{O}(\alpha'^2) \right].$$

+ RR field contributions.

- Corrections to the classical supergravity limit
  - α’ corrections (higher-derivative/-dimension terms)
  - fermion contribution (condensates)
  - WS instanton effects
  - Brane contributions (D-brane, O-plane)
  - Euclidean D-brane (Brane instanton)
  - loop corrections ($\chi<2$)

Type IIA Sugra in 10D

- Fundamental fields

Bose fields

- NS-NS fields: $g_{MN}, \quad \phi, \quad B_2$
- RR fields: $C_1, \quad C_3$

Spinor fields

- Two Majorana 1/2-fields: $\lambda \in \mathbf{16}, \quad \lambda' \in \mathbf{16}'$
- Two Majorana 3/2-fields: $\psi_M \in \mathbf{16} \otimes \mathbf{10}, \ \psi'_M \in \mathbf{16}' \times \mathbf{10}$

- Action (string frame)

$$
\begin{aligned}
S_{\mathrm{IIA,bosonic}} &= S_{\mathrm{NS}} + S_{\mathrm{R}} + S_{\mathrm{CS}}; \\
S_{\mathrm{NS}} &= \frac{1}{2\kappa_{10}^2} \int d^{10}x (-g)^{1/2} e^{-2\phi} \left(R + 4(\nabla\phi)^2 - \frac{1}{2} H_3 \cdot H_3 \right), \\
S_{\mathrm{R}} &= -\frac{1}{4\kappa_{10}^2} \int d^{10}x (-g)^{1/2} \left(\tilde{F}_2 \cdot \tilde{F}_2 + \tilde{F}_4 \cdot \tilde{F}_4 \right), \\
S_{\mathrm{CS}} &= -\frac{1}{4\kappa_{10}^2} \int B_2 \wedge F_4 \wedge F_4 .
\end{aligned}
$$

where

$$
H_3 = dB_2, \quad F_2 = dC_1 + m_0 B_2, \quad F_4 = dC_3,
$$
$$
\tilde{F}_4 = F_4 - C_1 \wedge H_3 . - \frac{m_0}{2} B_2 \wedge B_2
$$

Type IIB Sugra in 10D

- Fundamental fields

Bose fields

- NS-NS fields: $g_{MN}, \quad \phi, \quad B_2$
- RR fields: C_0, C_2, C_4

Spinor fields

- Two Majorana 1/2-fields: $\lambda = \lambda^{(1)} + i\lambda^{(2)},\ \Gamma_{11}\lambda = \pm\lambda$
- Two Majorana 3/2-fields: $\psi_M = \psi_M^{(1)} + i\psi_M^{(2)},\ \Gamma_{11}\psi_M = \mp\psi_M$

- Action (Bosonic part: impose $*F_5=F_5$ after variation) string frame:

$$
\begin{aligned}
S_{\mathrm{IIB}} &= S_{\mathrm{NS}} + S_{\mathrm{R}} + S_{\mathrm{CS}}; \\
S_{\mathrm{NS}} &= \frac{1}{2\kappa^2}\int dx^{10}(-g)^{1/2}e^{-2\phi}\left(R+4(\nabla\phi)^2-\frac{1}{2}H_3\cdot H_3\right), \\
S_{\mathrm{R}} &= -\frac{1}{4\kappa^2}\int dx^{10}(-g)^{1/2}\left(F_1\cdot F_1+\tilde{F}_3\cdot\tilde{F}_3+\frac{1}{2}\tilde{F}_5\cdot\tilde{F}_5\right), \\
S_{\mathrm{CS}} &= \pm\frac{1}{4\kappa^2}\int C_4\wedge H_3\wedge F_3.
\end{aligned}
$$

where

$$
F_1 := dC_0, \quad F_3 = dC_2, \quad H_3 = dB_2,
$$
$$
\tilde{F}_3 := F_3 - C_0\wedge H_3,
$$
$$
\tilde{F}_5 = dC_4 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3
$$

⇦ Self-dual 5-form

- ## Action (Einstein frame)

String frame $g \Rightarrow e^{\phi/2} g$

$$
\begin{aligned}
2\kappa^2 S_{\rm IIB} \quad &= \quad \int *1 \left[R - \frac{\nabla\tau \cdot \nabla\bar{\tau}}{2({\rm Im}\,\tau)^2} \right] - \frac{1}{2{\rm Im}\,\tau} *G_3 \wedge \bar{G}_3 - \frac{1}{4} *\tilde{F}_5 \wedge \tilde{F}_5 \\
&\pm \frac{i}{4{\rm Im}\,\tau} C_4 \wedge G_3 \wedge \bar{G}_3 .
\end{aligned}
$$

where

$$
\tau = C_0 + i e^{-\phi}, \quad G_3 := \tau H_3 - F_3
$$

Type I Sugra in 10D

- Fundamental fields

  - Gravity multiplet
    - Bose fields: $g_{MN}, \quad \phi, \quad B_2$
    - Majorana 3/2-field $\psi_M \in \mathbf{16} \otimes \mathbf{10}$; $\Gamma_{11}\psi_M = \pm\psi_M$
    - 1/2-field $\lambda \in \mathbf{16}$; $\Gamma_{11}\lambda = \mp\lambda$
  - Gauge multiplet
    - Gauge field $A_1 \in \mathrm{ad}(G)$ (Gauge group G can be arbitrary)
    - Majorana 1/2-field $\chi \in \mathrm{ad}(G) \otimes \mathbf{16}$

Anomaly free $\Rightarrow$ $G = SO(32),\ E_8 \times E_8$

- Action

$$S=\int_M e^{-2\phi}\Big[R_s(\omega_+)+4|\nabla\phi|^2-\frac{1}{2}|T|^2-\frac{\alpha'}{4}\left(\mathrm{tr}|F|^2-\mathrm{tr}|\mathscr{R}(\omega_+)|^2+2\mathrm{tr}(\bar\chi \not{D}\chi)\right)+\cdots\Big]$$

$$\omega_\pm^{AB}=\omega^{AB}\pm\tfrac{1}{2}H^{AB}{}_M dx^M+\mathrm{O}(\alpha'^2)\,,$$

$$T=H_3+\frac{\alpha'}{8}\mathrm{tr}(\bar\chi\Gamma_{[3]}\chi),$$

$$H_3=dB_2+\frac{\alpha'}{4}[\mathrm{CS}(\omega_+)-\mathrm{CS}(A)],$$

$$\mathrm{CS}(A)=\mathrm{tr}\left(A\wedge dA+\tfrac{2}{3}A\wedge A\wedge A\right).$$

アノーマリー相殺条件

$$dH_3=\frac{\alpha'}{4}\left[\mathrm{Tr}(\mathscr{R}\wedge\mathscr{R})-\mathrm{Tr}(F\wedge F)\right]$$

- Gauge transformation

$$\begin{aligned}\delta A_1 &= d\lambda-i[A_1,\lambda],\\ \delta\omega_1 &= d\Theta+[\omega_1,\Theta],\\ \delta B_2 &= \frac{\alpha'}{4}\left[\mathrm{tr}(\lambda dA_1)-\mathrm{tr}(\Theta d\omega_1)\right]\end{aligned}$$

D=11 Supergravity

- Fundamental fields

$$\text{Metric/frame field}: \quad e_A = (e_A^M); \quad g^{MN} = \eta^{AB} e_A^M e_B^N, \quad \theta^A(e_B) = \delta_B^A,$$

$$\text{Form gauge fields}: \quad A_3 = \frac{1}{3!} A_{MPQ} dx^M \wedge dx^P \wedge dx^Q,$$

$$\text{Majorana 3/2 field}: \quad \Psi_M; \quad \Gamma_{(11)} \Psi_M = +\Psi_M.$$

- Action

$$\begin{aligned} 2\kappa^2 S = & \int d^{11}x |\theta| \Big[R_s(g) - \frac{1}{2}|F_4|^2 + \frac{1}{6} * (F_4 \wedge F_4 \wedge A_3) \\ & -i\bar{\Psi}_M \Gamma^{MNP} D_N \Psi_P + \Psi^4 \text{ terms} \\ & + \frac{i}{96} \left(\bar{\Psi}_M \Gamma^{MN****} \Psi_N + 12 \bar{\Psi}^* \Gamma^{**} \Psi^* \right) F_{****} \Big] . \end{aligned}$$

- Field equations

$$R_{MN}(\tilde{\omega}) - \frac{1}{2} g_{MN} R(\tilde{\omega}) = \frac{1}{12} \tilde{F}_{MPQR} \tilde{F}_N{}^{PQR} - \frac{1}{4} g_{MN} |F_4|^2,$$

$$\Gamma^{MNP} \tilde{D}_N(\tilde{\omega}) \Psi_P = 0,$$

$$d * \tilde{F} + \frac{1}{2} \tilde{F} \wedge \tilde{F} = 0.$$

where

$$\tilde{D}_M(\tilde{\omega}) = D_M(\tilde{\omega}) + \frac{1}{288} \left(\Gamma_M{}^{****} - 8\delta^*_M \Gamma^{***} \right) \tilde{F}_{****}.$$

$$\tilde{\omega}_{ABM} = \omega_{ABM} - \frac{i}{8} \bar{\Psi}_N \Gamma_{MAB}{}^{NP} \Psi_P$$

$$\tilde{F}_{MNPQ} = F_{MNPQ} + \frac{3i}{2} \bar{\Psi}_{[M} \Gamma_{NP} \Psi_{Q]}$$

2. 3 D-Brane

D-braneとは？

- 開弦に対する境界条件: Dp = Σ_{p+1}
  - Neumann: ∂_σ X$_{//}$=0
  - Dirichlet: X$_\perp$=0

- D-ブレーン上に誘導される場

  N 枚のDp-ブレーン $\Rightarrow$
  - U(N)-ゲージ場: A_μ
  - adj(U(N)) -ヒグス場: Φ_i (i=1,..,D-p-1)

分類

- D次元時空におけるDp-braneは，C_{p+1}ポテンシャルと電気的に，$C_{D\text{-}p\text{-}3}$ポテンシャルと磁気的に結合する．すなわち，$dC_{D\text{-}p\text{-}3}={}^* d\, C^*_{p+1}$として，

$$\text{Electric} \quad : \quad \mu_p \int_{Dp} C_{p+1},$$

$$\text{Magnetic} \quad : \quad \mu'_p \int_{Dp} C^*_{p+1}.$$

- フォーム場$F_{D\text{-}p\text{-}2}$の磁荷ないしF_{p+2}の電荷を持ちうる：

$$\text{Electric charge} \quad : \quad \int_{S_{D-p-2}} *F_{p+2} = 2\kappa_{10}^2 \mu_p,$$

$$\text{Magnetic charge} \quad : \quad \int_{S_{D-p-2}} F_{D-p-2} = 2\kappa_{10}^2 \mu'_p.$$

IIA

Potential	Flux	electric	magnetic
Φ	$d\Phi$	—	NS7(?)
B_2	H_3	F1	NS5
C_1	F_2	D0	D6
C_3	F_4	D2	D4
(C_5)	(F_6)	D4	D2
(C_7)	(F_8)	D6	D0
C_9	F_{10}	D8	—

IIB

Potential	Flux	electric	magnetic
Φ	$d\Phi$	—	NS7(?)
B_2	H_3	F1	NS5
C_0	F_1	D-1	D7
C_2	F_3	D1	D5
$(C_4)_+$	$(F_5)_+$	D3	D3
(C_6)	(F_7)	D5	D1
(C_8)	(F_9)	D7	D-1
C_{10}	0	D9	—

量子化

D(*D*-*p*-4)ブレーンを囲む多様体 S_{p+2} 上をD*p*が，$\Sigma_{p+1}=\partial N_{p+2}$に沿ってD(*D*-*p*-4)の周りを一周すると，D*p*の波動関数はこの軌道に沿った運動により，

$$\exp\left(i\mu_p\int_{\Sigma_{p+1}}C_{p+1}\right)=\exp\left(i\mu_p\int_{N_{p+2}}F_{p+2}\right)$$

だけ位相が変化する．いま，軌道をだんだん小さくして，$N_{p+2}\to S_{p+2}$となるよう1点に縮めると，位相の変化はなくならないといけないので，

$$\exp\left(i\mu_p\int_{\Sigma_{p+2}}F_{p+2}\right)=\exp\left(i\mu_p\times 2\kappa_{10}^2\mu'_{D-p-4}\right)=1.$$

これより，次のDirac型量子化条件を得る：

$$2\kappa_{10}^2\mu_p\mu'_{D-4-p}=2n\pi,\ n\in\mathbb{Z}.$$

作用積分

- 電磁場中の荷電粒子

作用積分は，Cを時空軌道として

$$S = -mc \int_C ds - q \int_C A$$

- Dpブレーンに対する作用積分

荷電粒子をD0ブレーンとみなすと，一般のDpブレーンの作用積分も同じ構造をもち，大まかには，第1項を弧長からDブレーンの面積に(DBI作用積分），第2項の1形式AをC_{p+1}に比例した微分形式のブレーン上での積分(Chern-Simons作用積分)に置き換えたものになる.

$$S_{Dp} = S_{\mathrm{DBI}} + S_{\mathrm{CS}}$$

Abelian Case

- DBI作用積分

$$S_{\mathrm{DBI,Dp}} = -\mu_p \int_{\Sigma^{p+1}} d^{p+1}\xi\, e^{-\Phi(X)} \sqrt{-\det(g_{ab}(X) + 2\pi\alpha' \mathscr{F}_{ab}(X))}.$$

$$2\pi\alpha'\mathscr{F} = 2\pi\alpha' F + B,$$

$$\mu_p = \mu_p \ell_s^{-p-1} \times \begin{cases} 1 & \text{for type II} \\ \frac{1}{\sqrt{2}} & \text{for type I} \end{cases}$$

- CS作用積分

$$S_{\mathrm{CS}}(Dp) = 2\pi \int_{B^{p+1}} C \wedge \mathrm{Tr} e^{\frac{B}{\ell_s^2} + \frac{F}{2\pi}} \frac{\sqrt{\hat{A}(TB)}}{\sqrt{\hat{A}(NB)}}$$

$$C \equiv \sum_q C_q / \ell_s^q$$

$$S_{\mathrm{CS}}(Op) = -2^{p-4} 2\pi \int_{B^{p+1}} C \wedge \frac{\sqrt{L(\mathscr{R}_T/4)}}{\sqrt{L(\mathscr{R}_N/4)}}$$

Non-Abelian Case

*N*枚重なったDpブレーン

$\Rightarrow$ SU(*N*)ゲージ場A_M, 非可換スカラ場Φ^i(*i*=1,.., 9-*p*)

$$\lambda = 2\pi\ell_s^2 = 2\pi\alpha', \quad \mu_p = \frac{2\pi}{g_s(2\pi\ell_s)^{p+1}}.$$

DBI作用積分

$$S_{\rm DBI} = -\mu_p \int d^{p+1}\sigma {\rm Tr}\left(e^{-\phi}\sqrt{-(\det P + \lambda F)\det Q}\right).$$

$$P_{ab} = E_{ab} + E_{ai}(Q^{-1} - \delta)^{ij}E_{jb}, \quad E_{MN} = g_{MN} + B_{MN},$$
$$Q^i{}_j = \delta^i_j + i\lambda[\Phi^i, \Phi^j]E_{k,j}$$

CS作用積分

$$S_{\rm CS} = \mu_p \int {\rm Tr}\left(\mathscr{P}\left[e^{i\lambda I_\Phi I_\Phi} C e^{B/\ell_s^2}\right] e^{F/(2\pi)}\right) \frac{\sqrt{\hat{A}(TB)}}{\sqrt{\hat{A}(NB)}}.$$

Backup Slides