Band structure in the spectrum of the collision operator of one-dimensional protein molecule

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1.Comparison with the Redfield equation

$$i\frac{\partial}{\partial t}|\rho_{tot}(t)\rangle\rangle = (\underbrace{\mathcal{L}_S + \mathcal{L}_B}_{\mathcal{L}_0} + \mathcal{L}_{SB})|\rho_{tot}(t)\rangle\rangle \qquad (1.1)$$

$$\begin{aligned}|\rho_{tot}(t)\rangle\rangle &= e^{-i\mathcal{L}_0 t}|\rho(0)\rangle\rangle \\ &+(-i)\int_0^t dt_1 e^{-i\mathcal{L}_0(t-t_1)}\mathcal{L}_{SB}e^{-i\mathcal{L}_0 t_1}|\rho(0)\rangle\rangle \\ &+(-i)^2\int_0^t dt_1\int_0^{t_1} dt_2 e^{-i\mathcal{L}_0(t-t_1)}\mathcal{L}_{SB}e^{-i\mathcal{L}_0(t_1-t_2)}\mathcal{L}_{SB}e^{-i\mathcal{L}_0 t_2}|\rho(0)\rangle\rangle + O(\lambda^3)\end{aligned} \qquad (1.2)$$

Initial state $\quad |\rho_{tot}(t)\rangle\rangle = |\rho_S(0)\rangle\rangle \otimes |\rho_{ph}^{eq}\rangle\rangle \qquad (1.3)$

$$|\rho_{ph}^{eq}\rangle\rangle = \sum_N |0, N\rangle\rangle \rho_{ph}^{eq}(N)$$

Wigner basis

Reduced Density Matrix

$$|\rho_S(t)\rangle\rangle = \mathrm{Tr}_B[\rho_{tot}(t)] = \sum_N \langle\langle 0, N|\rho_{tot}(t)\rangle\rangle \qquad (1.4)$$

Taking trace of (1.2) for Bath

$$|\rho_S(t)\rangle\rangle = e^{-i\mathcal{L}_S t}|\rho_S(0)\rangle\rangle$$
$$+(-i)\int_0^t dt_1 e^{-i\mathcal{L}_S(t-t_1)} \sum_N \underbrace{\langle\langle 0, N|e^{-i\mathcal{L}_B(t-t_1)}\mathcal{L}_{SB}e^{-i\mathcal{L}_B t_1}|\rho_{ph}^{eq}\rangle\rangle}_{=0}|\rho_S(0)\rangle\rangle$$
$$+(-i)^2\int_0^t dt_1\int_0^{t_1} dt_2 e^{-i\mathcal{L}_S}$$
$$\times\sum_N \langle\langle 0, N|e^{-i\mathcal{L}_B(t-t_1)}\mathcal{L}_{SB}e^{-i\mathcal{L}_0(t_1-t_2)}\mathcal{L}_{SB}e^{-i\mathcal{L}_B t_2}|\rho_{ph}^{eq}\rangle\rangle|\rho_S(0)\rangle\rangle \qquad (1.5)$$

$$|\rho_S(t)\rangle\rangle = e^{-i\mathcal{L}_S t}|\rho_S(0)\rangle\rangle + (-i)^2\int_0^t dt_1\int_0^{t_1} d\tau e^{-i\mathcal{L}_S(t-t_1)}$$
$$\times\underbrace{\sum_N \langle\langle 0, N|\mathcal{L}_{SB}e^{-i\mathcal{L}_0\tau}\mathcal{L}_{SB}e^{-i\mathcal{L}_0\tau}|\rho_{ph}^{eq}\rangle\rangle}_{\equiv C(\tau)} e^{-i\mathcal{L}_S t_1}|\rho_S(0)\rangle\rangle \qquad (1.6)$$

Differentiating (1.6)

$$i\frac{\partial}{\partial t}|\rho_S(t)\rangle\rangle = \mathcal{L}_S e^{-i\mathcal{L}_S t}|\rho_S(0)\rangle\rangle$$
$$+\mathcal{L}_S e^{-i\mathcal{L}_S t}\int_0^t dt_1 \int_0^{t_1} d\tau e^{i\mathcal{L}_S t_1} C(\tau) e^{-i\mathcal{L}_S t_1}|\rho_S(0)\rangle\rangle$$
$$-\int_0^t d\tau C(\tau) \underline{e^{-i\mathcal{L}_S t}|\rho_S(0)\rangle\rangle} \tag{1.7}$$
$$\simeq |\rho_S(t)\rangle\rangle$$

Assumption

$$\therefore\ i\frac{\partial}{\partial t}|\rho_S(t)\rangle\rangle = \left[\mathcal{L}_S - \int_0^t d\tau C(\tau)\right]|\rho_S(t)\rangle\rangle \tag{1.8}$$

Non-Markov effect

c.f., Generalized master equation

$$i\frac{\partial}{\partial t}|\rho_S(t)\rangle\rangle = \mathcal{L}_S|\rho_S(t)\rangle\rangle - \int_0^t d\tau \hat{\Psi}(\tau)|\rho_S(t-\tau)\rangle\rangle + \mathcal{D} \tag{1.9}$$

Kinetic equation

$$i\frac{\partial}{\partial t}|\rho_S(t)\rangle\rangle = \underbrace{\left[\mathcal{L}_S - \int_0^\infty d\tau C(\tau)\right]}_{\equiv \mathcal{L}_S^{Red}}|\rho_S(t)\rangle\rangle \tag{1.10}$$

2. Application to 1D polaron model with non-spatial correlation assumption

$$H = H_S + H_B + H_{SB} \tag{2.1}$$

$$H_S = -J \sum_l (a_{l+1}^\dagger a_l + a_l^\dagger a_{l+1}) = \sum_p \varepsilon_p a_p^\dagger a_p \tag{2.2}$$

$$H_B = \sum_q \omega_q b_q^\dagger b_q \tag{2.3}$$

$$H_{SB} = \frac{1}{\sqrt{N}} \sum_{k,q} g_q a_{k+q}^\dagger a_k (b_q + b_{-q}^\dagger) \tag{2.4}$$

$$g_q = \frac{1}{\sqrt{2M\omega_q}} \Delta |q| \tag{2.5}$$

Deformation potential interaction

Site representation

$$a_k^\dagger = \frac{1}{\sqrt{N}} \sum_k e^{ikl} \tilde{a}_l^\dagger \ , \ a_k = \frac{1}{\sqrt{N}} \sum_k e^{-ikl} \tilde{a}_l \tag{2.6}$$

$$\begin{aligned} H_{SB} &= \frac{1}{N\sqrt{N}} \sum_{k,q} \sum_{l,l'} g_q e^{i(k+l)} \tilde{a}_l^\dagger e^{-ikl'} \tilde{a}_{l'} (b_q + b_{-q}^\dagger) \\ &= \sum_l \tilde{a}_l^\dagger \tilde{a}_l \underline{\frac{1}{\sqrt{N}} \sum_q g_q e^{iql} (b_q + b_{-q}^\dagger)} \\ & \qquad\qquad \equiv \hat{\Gamma}_l \end{aligned} \tag{2.7}$$

$$\therefore \quad H_{SB} = \sum_l \hat{S}_l \hat{\Gamma}_l \tag{2.8}$$

$$\hat{S}_l = \tilde{a}_l^\dagger \tilde{a}_l \tag{2.9}$$

$$\hat{\Gamma}_l \equiv \frac{1}{\sqrt{N}} \sum_q g_q e^{iql} (b_q + b_{-q}^\dagger) \tag{2.10}$$

Note $\hat{S}_l^\dagger = \hat{S}_l \ , \ \hat{\Gamma}_l^\dagger = \hat{\Gamma}_l$ (2.11)

$$
\begin{aligned}
& C(\tau)|\rho_S(t)\rangle\rangle \\
= & \sum_N \sum_{l,l'} \langle\langle 0,N|(\hat{S}_l\hat{\Gamma}_l)^\times e^{-i\mathcal{L}_0\tau}(\hat{S}^\dagger_{l'}\hat{\Gamma}^\dagger_{l'})^\times e^{i\mathcal{L}_0\tau}|\rho^{eq}_{ph}\rangle\rangle|\rho_S(t)\rangle\rangle \\
= & \sum_N \sum_{l,l'} \Big\{ \langle\langle 0,N|\hat{S}_l\hat{\Gamma}_l e^{-iH_0\tau}\hat{S}^\dagger_{l'}\hat{\Gamma}^\dagger_{l'}e^{iH_0\tau}\rho^{eq}_{ph}\rho_S\rangle\rangle - \langle\langle 0,N|e^{-iH_0\tau}\hat{S}^\dagger_{l'}\hat{\Gamma}^\dagger_{l'}e^{iH_0\tau}\rho^{eq}_{ph}\rho_S\hat{S}_l\hat{\Gamma}_l\rangle\rangle \\
& -\langle\langle 0,N|\hat{S}_l\hat{\Gamma}_l\rho^{eq}_{ph}\rho_S e^{-iH_0\tau}\hat{S}^\dagger_{l'}\hat{\Gamma}^\dagger_{l'}e^{iH_0\tau}\rangle\rangle + \langle\langle 0,N|\rho^{eq}_{ph}\rho_S e^{-iH_0\tau}\hat{S}^\dagger_{l'}\hat{\Gamma}^\dagger_{l'}e^{iH_0\tau}\hat{S}_l\hat{\Gamma}_l\rangle\rangle \Big\} \\
= & \sum_{l,l'} \Big\{ |\hat{S}_l e^{-iH_S\tau}\hat{S}^\dagger_{l'}e^{iH_S\tau}\rho_S\rangle\rangle \sum_N \underline{\langle\langle 0,N|\hat{\Gamma}_l e^{-iH_B\tau}\hat{\Gamma}^\dagger_{l'}e^{iH_B\tau}\rho^{eq}_{ph}\rangle\rangle} \\
& -|e^{-iH_S\tau}\hat{S}^\dagger_{l'}e^{iH_S\tau}\rho_S\hat{S}_l\rangle\rangle \sum_N \underline{\langle\langle 0,N|e^{-iH_B\tau}\hat{\Gamma}^\dagger_{l'}e^{iH_B\tau}\rho^{eq}_{ph}\hat{\Gamma}_l\rangle\rangle} \\
& -|\hat{S}_l\rho_S e^{-iH_S\tau}\hat{S}^\dagger_{l'}e^{iH_S\tau}\rangle\rangle \sum_N \underline{\langle\langle 0,N|\hat{\Gamma}_l\rho^{eq}_{ph}e^{-iH_B\tau}\hat{\Gamma}^\dagger_{l'}e^{iH_B\tau}\rangle\rangle} \\
& +|\rho_S e^{-iH_S\tau}\hat{S}^\dagger_{l'}e^{iH_S\tau}\hat{S}_l\rangle\rangle \sum_N \underline{\langle\langle 0,N|\rho^{eq}_{ph}e^{-iH_B\tau}\hat{\Gamma}^\dagger_{l'}e^{iH_B\tau}\hat{\Gamma}_l\rangle\rangle} \Big\}
\end{aligned}
\tag{2.12}
$$

Two point and two time correlation functions for bath modes

$$C_{l,l'}(\tau) \equiv \mathrm{Tr}\left[e^{iH_B\tau}\hat{\Gamma}_l e^{-iH_B\tau}\hat{\Gamma}^\dagger_{l'}\right] = \langle\hat{\Gamma}_l(\tau)\hat{\Gamma}^\dagger_{l'}\rangle_B \qquad (2.13)$$

$\therefore$

$$\int_0^\infty d\tau C(\tau)|\rho_S(t)\rangle\rangle$$

$$= \sum_{l,l'}\int_0^\infty d\tau\Big\{|\hat{S}_l e^{-iH_S\tau}\hat{S}^\dagger_{l'}e^{iH_S\tau}\rho_S\rangle\rangle C_{l,l'}(\tau) - |e^{-iH_S\tau}\hat{S}^\dagger_{l'}e^{iH_S\tau}\rho_S\hat{S}_l\rangle\rangle C_{l,l'}(\tau)$$

$$-|\hat{S}_l\rho_S e^{-iH_S\tau}\hat{S}^\dagger_{l'}e^{iH_S\tau}\rangle\rangle C^*_{l,l'}(\tau) + |\rho_S e^{-iH_S\tau}\hat{S}^\dagger_{l'}e^{iH_S\tau}\hat{S}_l\rangle\rangle C^*_{l,l'}(\tau)\Big\} \qquad (2.14)$$

$\therefore$ master equation (Redfield equation)

$$i\frac{\partial}{\partial t}|\rho_S(t)\rangle\rangle = \mathcal{L}_S|\rho_S(t)\rangle\rangle$$

$$-\sum_l\Big\{|\hat{S}_l\hat{T}_l\rho_S\rangle\rangle - |\hat{T}_l\rho_S\hat{S}_l\rangle\rangle - |\hat{S}^\dagger_l\rho_S\hat{T}^\dagger_l\rangle\rangle + |\rho_S\hat{T}^\dagger_l\hat{S}^\dagger_l\rangle\rangle\Big\} \qquad (2.15)$$

$$\hat{T}_l \equiv \sum_{l'}\int_0^\infty d\tau e^{-iH_S\tau}\hat{S}^\dagger_{l'}e^{iH_S\tau}C_{l,l'}(\tau) \qquad (2.16)$$

Explicit form of $C_{l,l'}(\tau)$

$$C_{l,l'}(\tau) = \langle \hat{\Gamma}_l(\tau)\hat{\Gamma}_{l'}\rangle$$
$$= \frac{1}{N}\sum_{q,q'} g_q e^{iql} g_{q'} e^{iq'l'} \langle (b_q(\tau) + b^{\dagger}_{-q}(\tau))(b_{q'} + b^{\dagger}_{-q'})\rangle$$
$$= \frac{1}{N}\sum_{q} g_q^2 \underline{\exp[iq(l-l')]} \Big\{ (n_q+1)e^{-i\omega_q \tau} + n_q e^{i\omega_q\tau} \Big\} \qquad (2.17)$$

- Loss term of the collision term: (1st)+(4th) in (2.14)

(Loss term)

$$= \sum_{l,l'} \int_0^{\infty} d\tau \Big\{ |\hat{S}_l e^{-iH_S\tau} \hat{S}^{\dagger}_{l'} e^{iH_S\tau}\rho_S\rangle\rangle \mathcal{C}_{l,l'}(\tau) + |\rho_S e^{-iH_S\tau}\hat{S}^{\dagger}_{l'} e^{iH_S\tau}\hat{S}_l\rangle\rangle \mathcal{C}^*_{l,l'}(\tau) \Big\}$$

Taking <<0,P| component

$$\frac{2\pi}{N}\langle\langle 0, P|\rho_S\rangle\rangle \sum_q |g_q|^2 \Big\{ (n_q+1)\delta(\varepsilon_P - \varepsilon_{P+q} + \omega_q) + n_q \delta(\varepsilon_P - \varepsilon_{P-q} - \omega_q) \Big\}$$

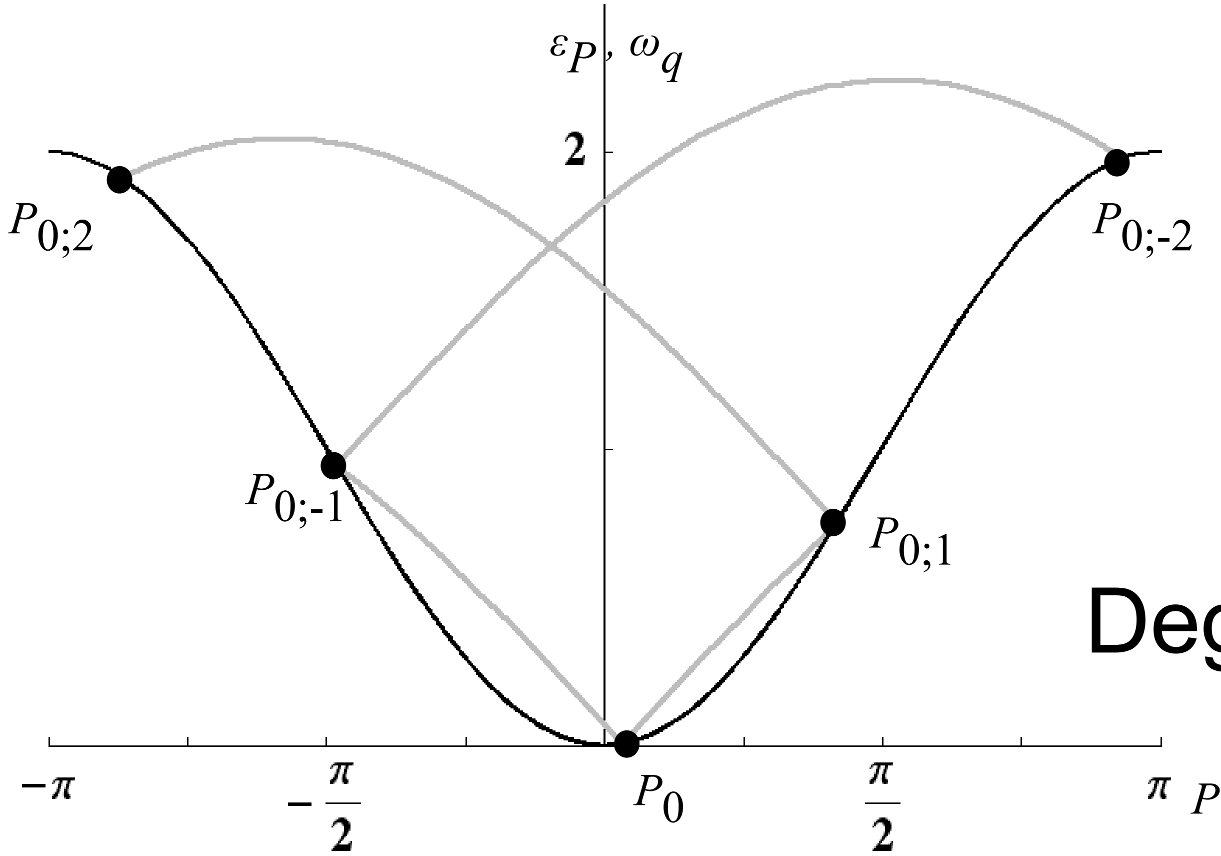
- Gain term of the collision term: (2nd)+(3rd) in (2.14)

$$(\text{Gain term}) = \sum_{l,l'} \int_0^\infty d\tau \Big\{ |e^{-iH_S\tau}\hat{S}_{l'}^\dagger e^{iH_S\tau}\rho_S \hat{S}_l\rangle\!\rangle \mathcal{C}_{l,l'}(\tau) + |\hat{S}_l \rho_S e^{-iH_S\tau}\hat{S}_{l'}^\dagger e^{iH_S\tau}\rangle\!\rangle \mathcal{C}_{l,l'}^*(\tau) \Big\}$$

Taking <<0,P| component

$$\frac{2\pi}{N}\sum_{P'}\sum_q |g_q|^2 \langle\!\langle 0,P'|\rho_S\rangle\!\rangle \delta_{P',P+q}\Big\{(n_q+1)\delta(\varepsilon_P-\varepsilon_{P'}+\omega_q)+n_q\delta(\varepsilon_P-\varepsilon_{P'}-\omega_q)\Big\}$$

momentum & energy conservation



disjoint sets of momentum states

Degeneracy of the collision invariant

Assumption: no spatial correlation

$$C_{l,l'}(\tau) = \delta_{l,l'}\mathcal{G}(\tau) \tag{2.20}$$

$$\mathcal{G}(\tau) = \frac{1}{N}\sum_q g_q^2\left\{(n_q+1)e^{-i\omega_q\tau} + n_q e^{i\omega_q\tau}\right\} \tag{2.21}$$

(2.22)

(Loss term)

$$= -\sum_l \int_0^\infty d\tau\left\{|\hat{S}_l e^{-iH_S\tau}\hat{S}_l^\dagger e^{iH_S\tau}\rho_S\rangle\!\rangle + |\rho_S e^{-iH_S\tau}\hat{S}_l^\dagger e^{iH_S\tau}\hat{S}_l\rangle\!\rangle\right\}\mathcal{G}(\tau)$$

$\cdot\ \langle\!\langle 0, P|$ component

$$= -\langle\!\langle 0, P|\rho_S\rangle\!\rangle\frac{2\pi}{N^2}\sum_{k,q}|g_q|^2\left\{(n_q+1)\delta(\varepsilon_k - \varepsilon_P + \omega_q) + n_q\delta(\varepsilon_k - \varepsilon_P - \omega_q)\right\}$$

(Gain term)

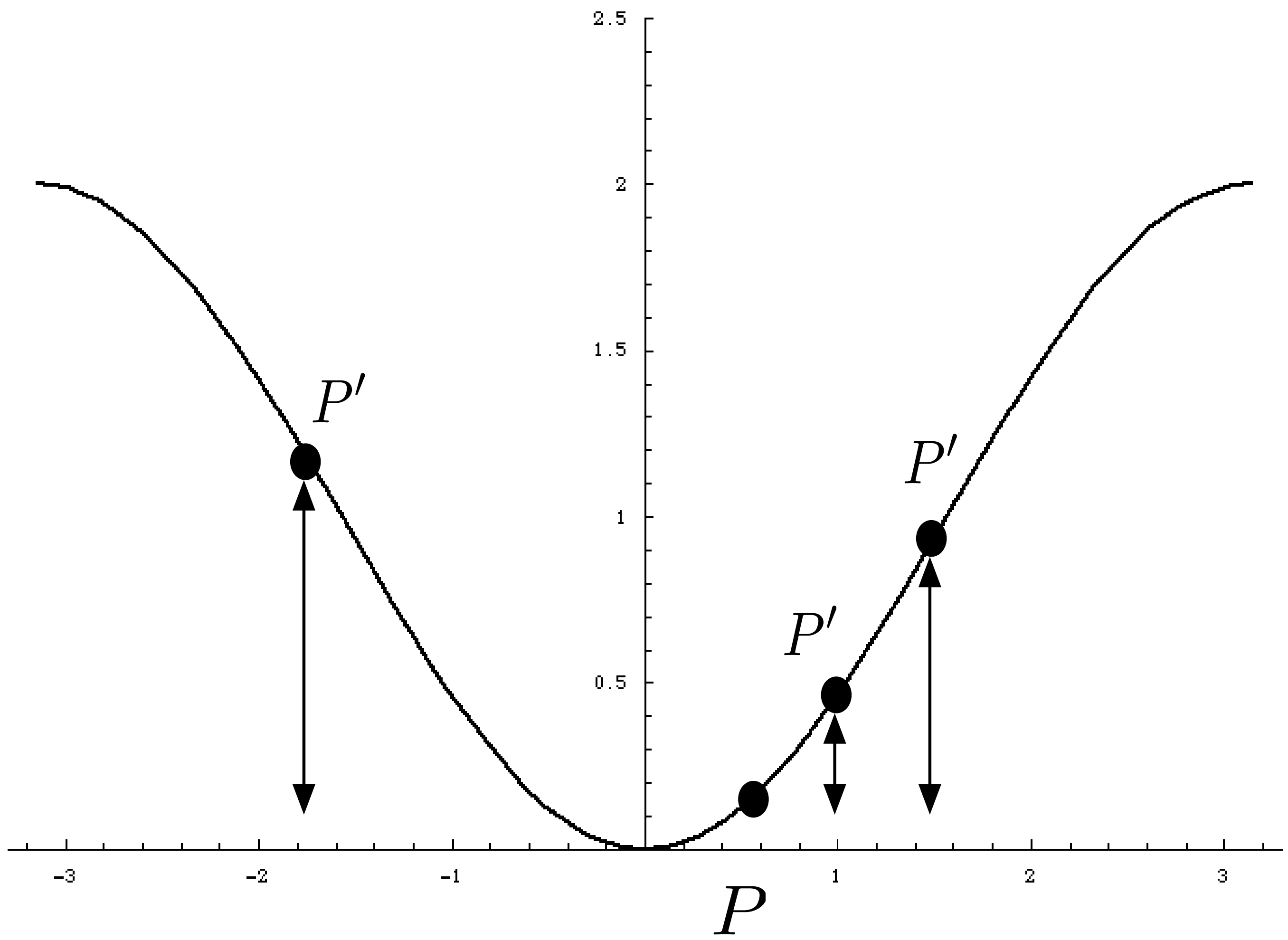
$$= \sum_l \int_0^\infty d\tau\left\{|e^{-iH_S\tau}\hat{S}_l^\dagger e^{iH_S\tau}\rho_S\hat{S}_l\rangle\!\rangle + |\hat{S}_l\rho_S e^{-iH_S\tau}\hat{S}_l^\dagger e^{iH_S\tau}\rangle\!\rangle\right\}\mathcal{G}(\tau)$$

$\cdot\ \langle\!\langle 0, P|$ component

energy conservation ONLY

$$= \frac{2\pi}{N^2}\sum_{P'}\sum_q |g_q|^2\langle\!\langle 0, P'|\rho_S\rangle\!\rangle\left\{(n_q+1)\underline{\delta(\varepsilon_P - \varepsilon_{P'} + \omega_q)} + n_q\underline{\delta(\varepsilon_P - \varepsilon_{P'} - \omega_q)}\right\}$$

All the states are coupled=> there is no degeneracy.



Evaluation of $C_{l,l'}(\tau)$

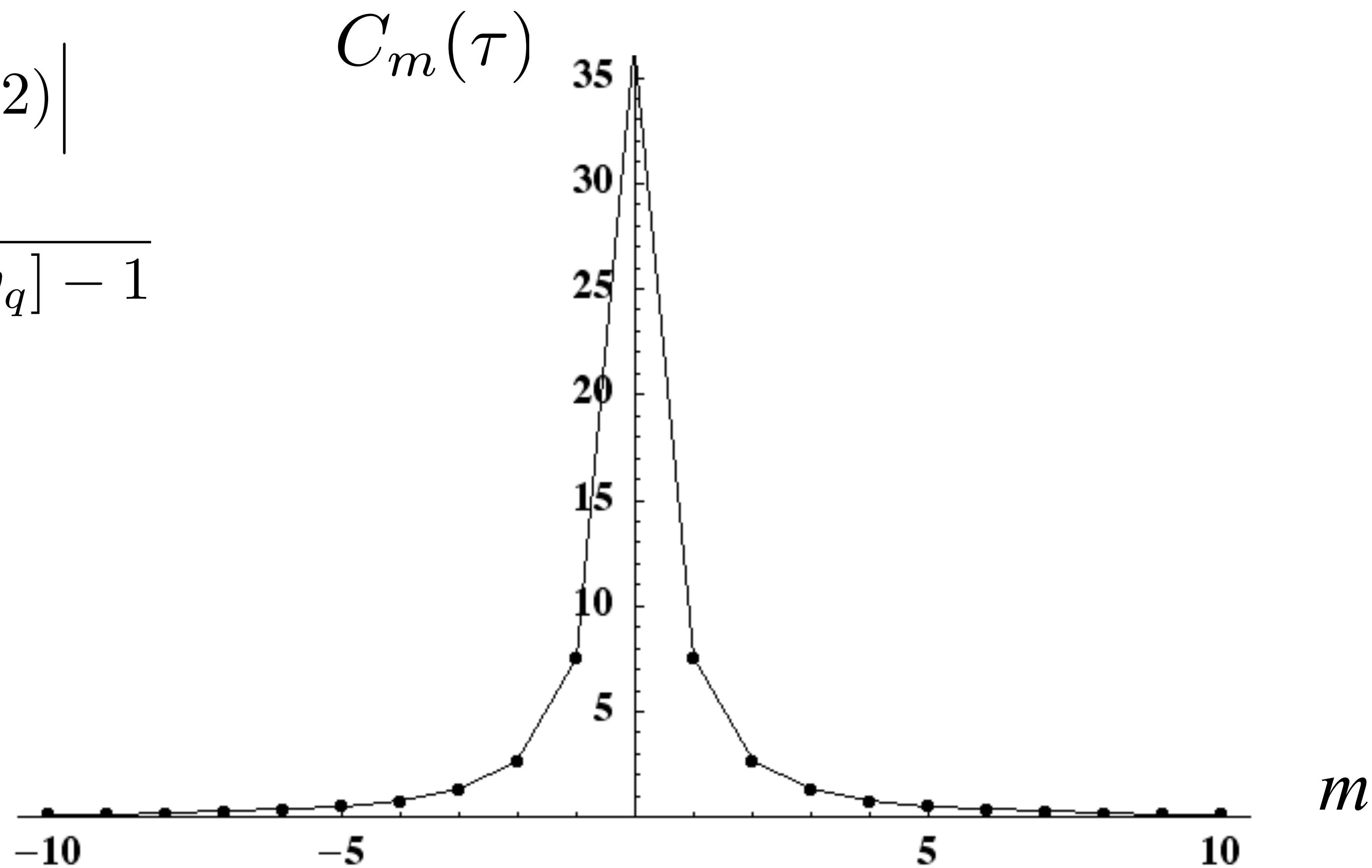
$$C_{l,l'}(\tau) = \mathcal{C}_{l-l'}(\tau)$$

$$= \mathcal{C}_m(\tau) = \int_{-\pi}^{\pi} g_q^2 \exp[iqm]\Big\{(n_q+1)e^{-i\omega_q\tau} + n_q e^{i\omega_q\tau}\Big\} \tag{2.23}$$

$$g_q^2 = \frac{\Delta^2 |q|^2}{2M\omega_q}$$

$$\omega_q = \Big|\sin(q/2)\Big|$$

$$n_q = \frac{1}{\exp[\beta\omega_q] - 1}$$



Spatial correlation between the bath mode is critical to cause the quantum kinetic sound wave.

Quantum sound wave propagates upon a propagation of a thermal phonon.

3.Band structure of the spectrum of the collision operator

$$H_{vib} = \sum_n \Omega_0 B_n^\dagger B_n - J \sum_n (B_{n+1}^\dagger B_n + B_n^\dagger B_{n+1}) = \sum_p \varepsilon_p B_p^\dagger B_p$$

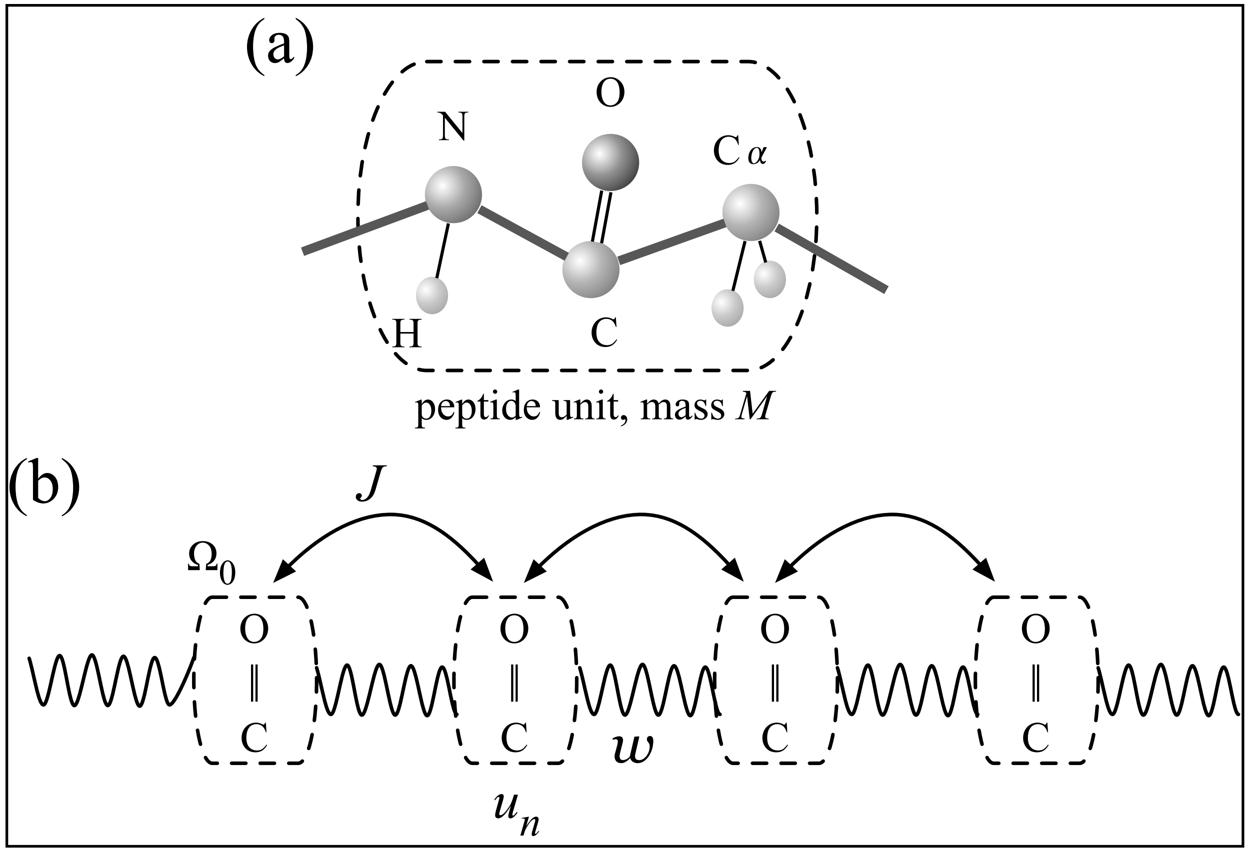
$$H_{ph} = \frac{1}{2} \sum_n \left[w(u_{n+1} - u_n)^2 + \frac{p_n^2}{M} \right] = \sum_q \hbar\omega_q a_q^\dagger a_q \ ,$$

$$H' = \chi \sum_n B_n^\dagger B_n (u_{n+1} - u_{n-1})$$

$$= \sqrt{\frac{2\pi}{L}} \sum_{p,q} g_q B_{p+\hbar q}^\dagger B_p (a_q + a_{-q}^\dagger) \ ,$$

$$\varepsilon_p = \hbar\Omega_0 - 2J\cos(pd/\hbar)$$

$$\omega_q = \frac{2c}{d} \left| \sin(qd/2) \right| \quad ; c \equiv d\sqrt{\frac{w}{M}}$$



Model Hamiltonian

$$\bar{H} \equiv H/2J = \sum_{\bar{p}} \bar{\varepsilon}_{\bar{p}} |\bar{p}\rangle\langle\bar{p}| + \sum_{\bar{q}} \bar{\omega}_{\bar{q}} a^{\dagger}_{\bar{q}} a_{\bar{q}} + \sqrt{\frac{2\pi}{N}} \sum_{\bar{p},\bar{q}} \bar{g}_{\bar{q}} |\bar{p}+\bar{q}\rangle\langle\bar{p}|(a_{\bar{q}} + a^{\dagger}_{-\bar{q}}) \ ,$$

$$\bar{\varepsilon}_{\bar{p}} \equiv \frac{\varepsilon_p}{2J} = \frac{\hbar\Omega_0}{2J} - \cos(\bar{p}) \ , \ \bar{\omega}_{\bar{q}} \equiv \frac{\omega_q}{1/t_u} = 2B\left|\sin\left(\frac{\bar{q}}{2}\right)\right|$$

Reduced density operator

$$f(t) \equiv \mathrm{Tr}_{\mathrm{ph}}[\rho(t)] \qquad f_k(P,t) \equiv \langle\langle k, P | f(t) \rangle\rangle = \langle P + k/2 | f(t) | P - k/2 \rangle$$

Kinetic equation for the momentum distribution

$$\frac{\partial}{\partial t} f_0(P,t) = \hat{\mathcal{K}} f_0(P,t)$$

$$\hat{\mathcal{K}}(P, \frac{\partial}{\partial_P})$$

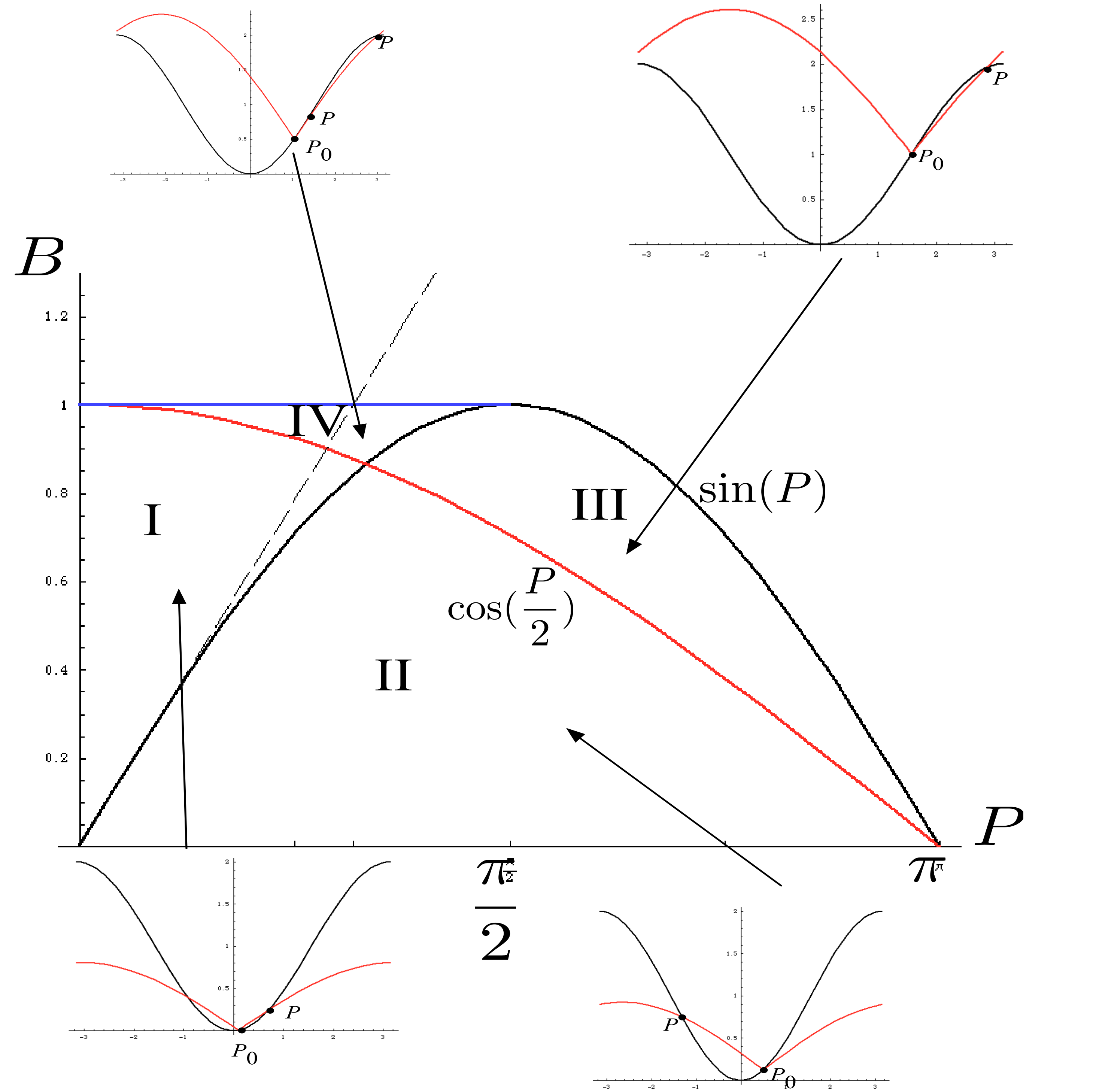
$$= -\frac{2\pi}{\hbar^2} \int dq |g_q|^2 \left\{ \delta\left(\frac{\varepsilon_P - \varepsilon_{P+\hbar q}}{\hbar} + \omega_q\right) n_q + \delta\left(\frac{\varepsilon_{P-\hbar q} - \varepsilon_P}{\hbar} + \omega_q\right)(n_q + 1) \right\}$$

$$-\frac{2\pi}{\hbar^2} \int dq |g_q|^2 \left\{ \delta\left(\frac{\varepsilon_{P-\hbar q} - \varepsilon_P}{\hbar} + \omega_q\right) n_q \exp\left[-\hbar q \frac{\partial}{\partial_P}\right] + \delta\left(\frac{\varepsilon_P - \varepsilon_{P+\hbar q}}{\hbar} + \omega_q\right)(n_q + 1) \exp\left[\hbar q \frac{\partial}{\partial_P}\right] \right\}$$

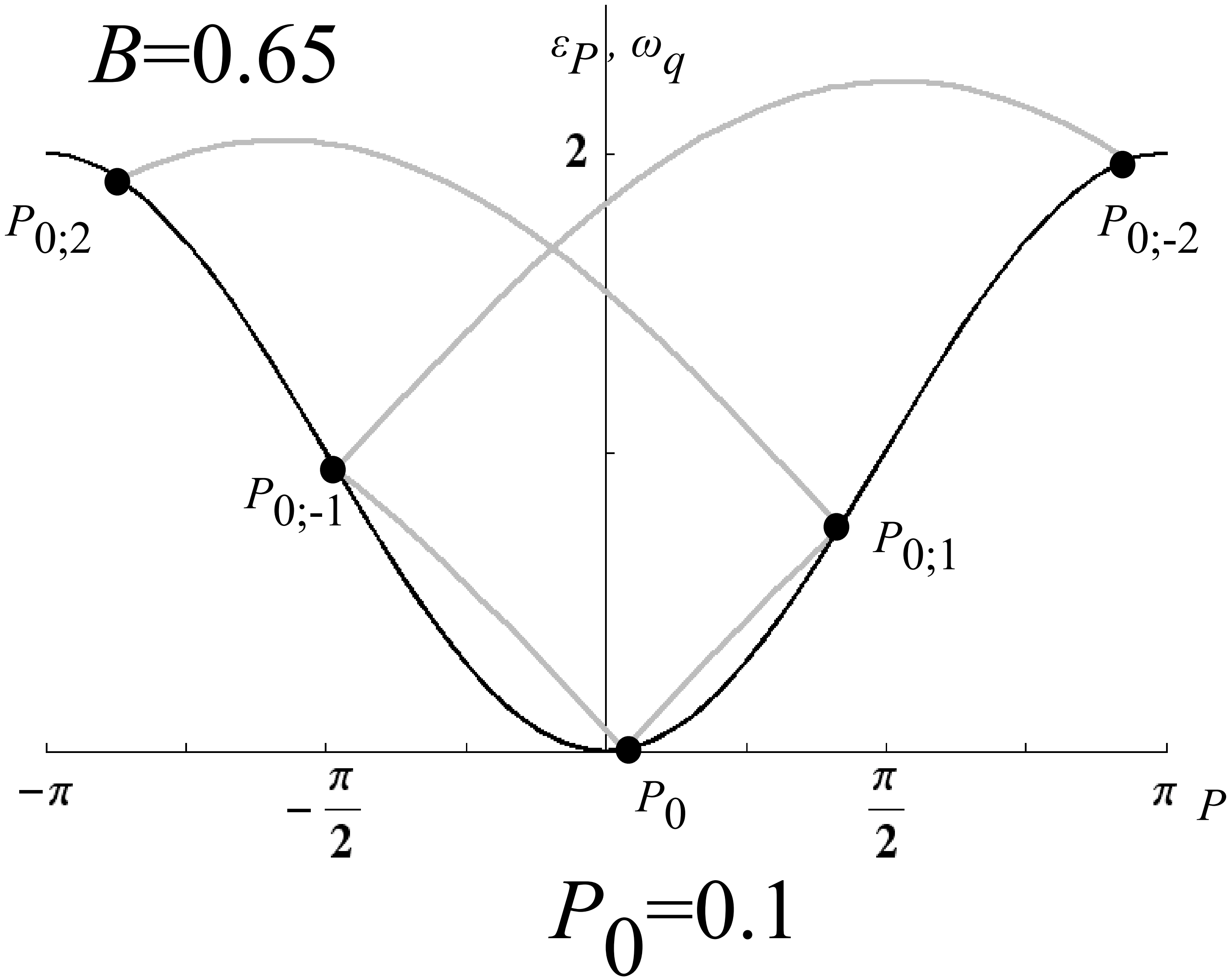
<u>Resonance condition</u> $\varepsilon_{P+q} - \varepsilon_P = \omega_q \Longleftrightarrow \varepsilon_P - \varepsilon_{P_0} = \omega_{P-P_0}$

$$\Longleftrightarrow (1-\cos(P)) - (1-\cos(P_0)) = 2B\left|\sin\left(\frac{P-P_0}{2}\right)\right|$$

$$\Longleftrightarrow \sin\left(\frac{P+P_0}{2}\right) = \pm B$$



Multiple resonance



$$P_{0,n} = (-1)^n (P_0 - 2n \arcsin(B))$$

- For each P_0 within $|P_0| < \arcsin(B)$, we have a disjoint momentum set.
- Collision operator is diagonalized for each set of momentum states.
- The number of the resonance points depend on P_0 and B.

A matrix of Collision operator

$\langle\langle P_{0;n}|\hat{\mathcal{K}}|P_{0;n'}\rangle\rangle$ Non-symmetric tridiagonal matrix

	$\vert P_{0;-2}\rangle\rangle$	$\vert P_{0;-1}\rangle\rangle$	$\vert P_{0;0}\rangle\rangle$	$\vert P_{0;1}\rangle\rangle$	$\vert P_{0;2}\rangle\rangle$
$\langle\langle P_{0;-2}\vert$	loss	gain		0	
$\langle\langle P_{0;-1}\vert$	gain	loss	gain		
$\langle\langle P_{0}\vert$		gain	loss	gain	
$\langle\langle P_{0;1}\vert$	0		gain	loss	gain
$\langle\langle P_{0;2}\vert$				gain	loss

(for a particular P_0 and B)

Symmetrization

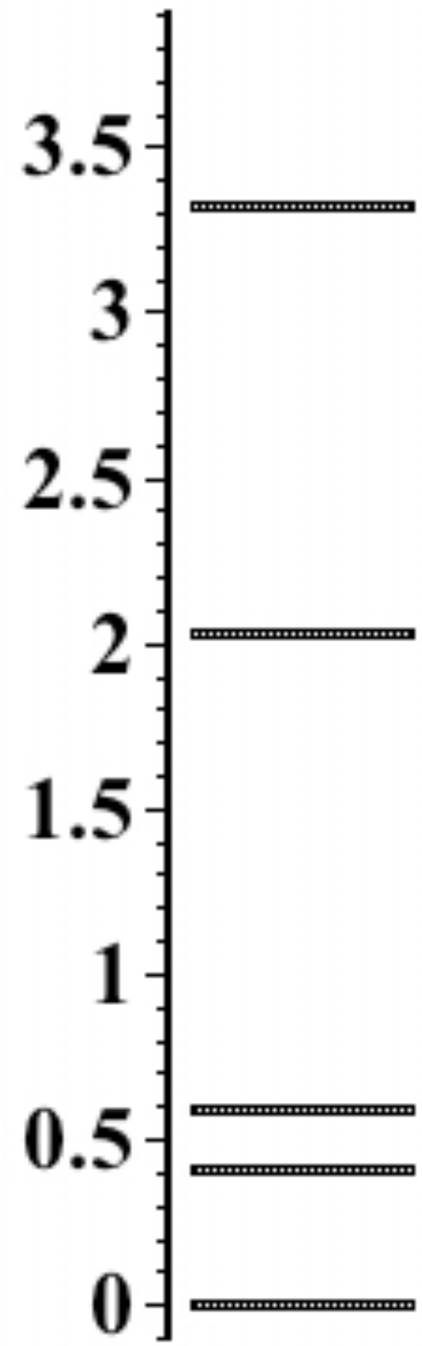
$$\langle\langle P_{0;n}|\bar{\mathcal{K}}|P_{0;n'}\rangle\rangle \equiv \exp[\beta\varepsilon_{P_0;n}/2]\langle\langle P_{0;n}|\hat{\mathcal{K}}|P_{0;n'}\rangle\rangle \exp[-\beta\varepsilon_{P_0;n'}/2]$$

Eigenvalue problem

$$\bar{\mathcal{K}}|\phi_j\rangle\rangle = \lambda_j|\phi_j\rangle\rangle$$

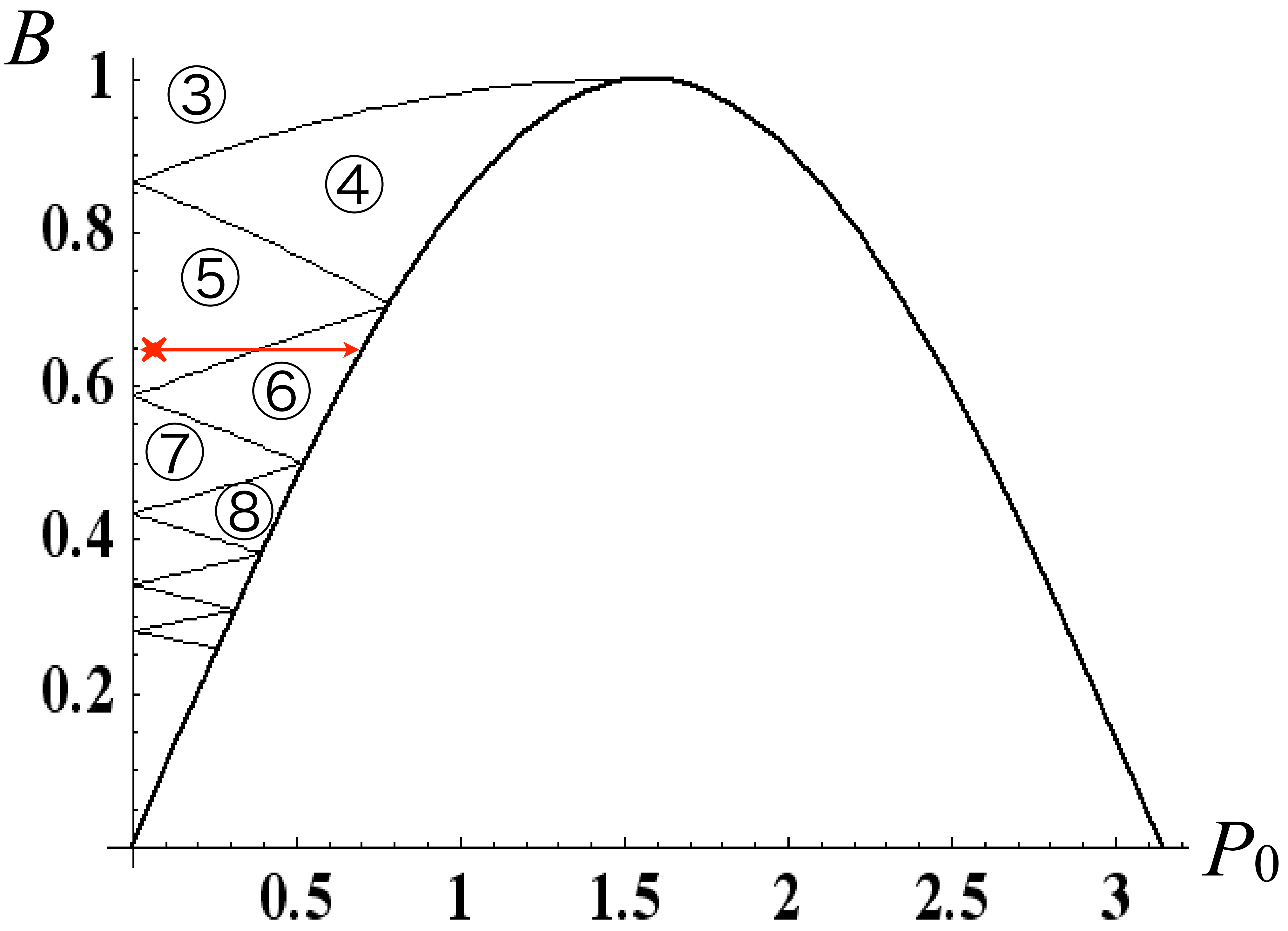
$\underline{B=0.65}$

$P_0=0.1$

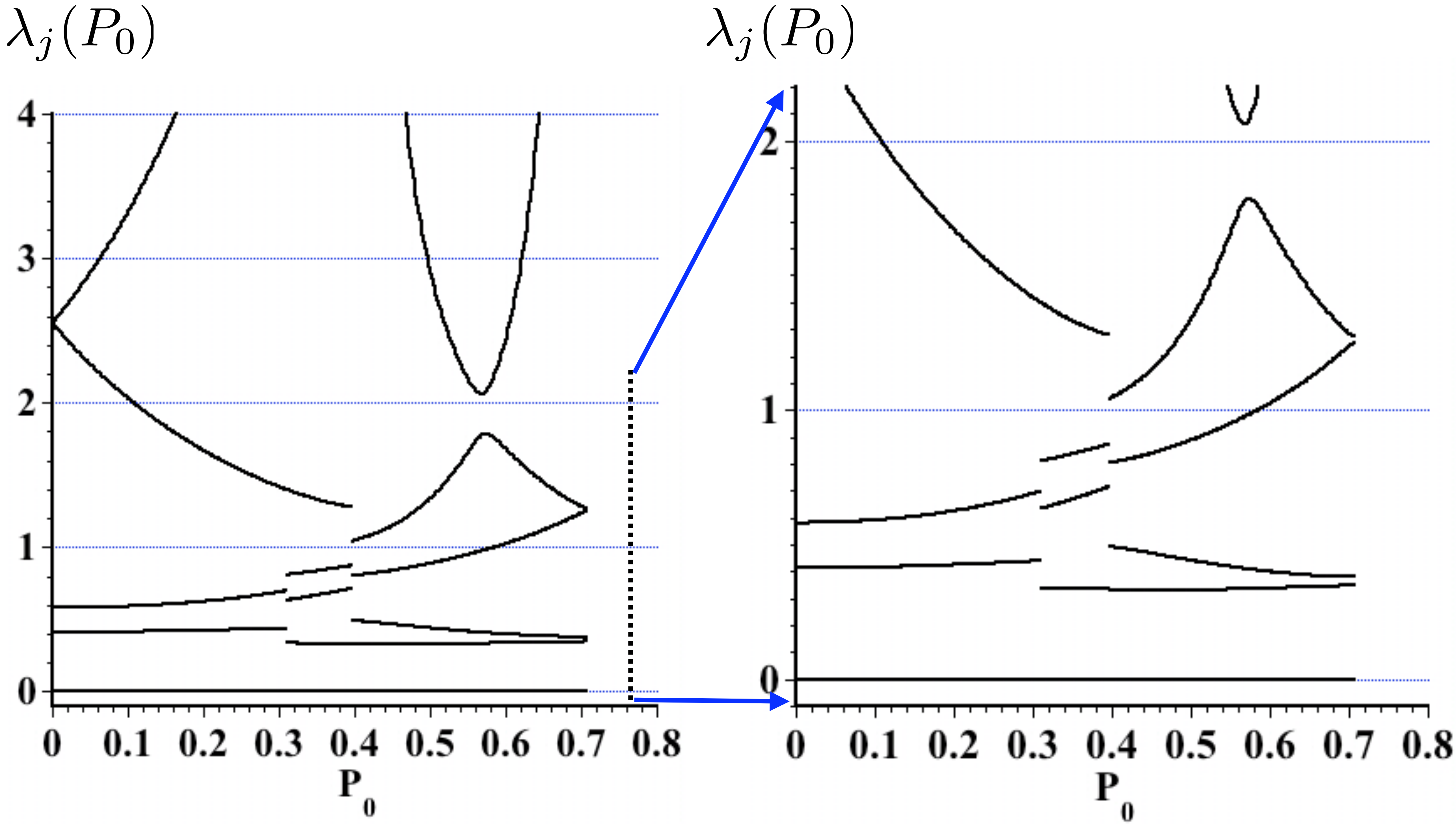


Spectrum of $\hat{\mathcal{K}}$

- For each P_0 within $|P_0| < \arcsin(B)$, we have a disjoint momentum set.
- Collision operator is diagonalized for each set of momentum states.
- The number of the resonance points depend on P_0 and B.



Band structure of the eigenvalues of the collision operator



What is the relaxation dynamics when the spectrum of the collision operator possesses a band structure?

Early stage of the relaxation reflects the band structure?

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