Two Hydrodynamic Models of Granular Convection

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ABSTRACT: We present two continuum models A and B to study the convective instability of granular materials subjected to vibrations. We carry out the linear stability analysis for model A and uncover the instability mechanism as a supercritical bifurcation of a bouncing solution. We also explicitly determine the onset of convection as a function of control parameters. The results of simulations are in excellent agreement with the stability analysis. Additional feature of the model B is the inclusion of the relaxation term in the momentum equation, which appears to be crucial in capturing what is missing in model A, in particular, in reproducing experimental convection patterns for large aspect ratio, both horizontally, in which case convective rolls move toward the surface, and vertically, in which case convective rolls survive near the wall but are suppressed in the bulk region.

1. INTRODUCTION

It was Faraday who first discovered the convective instability in a vibrated granular bed in 1831. Initially the stationary granular pile develops a heap and a pair of convection rolls inside the pile when the strength of oscillation exceeds the critical value. Unlike Rayleigh-Bernard convection in fluids the origin of this instability has remained relatively unexplored since its discovery, but recently two simultaneous push from experimental side (Clement et al, 1992; Pak and Behringer, 1994) with the use of MRI or X-ray method (Knight et al, 1993) from computer simulations based on the distinct element method (Taguchi, 1992; Gallas et al, 1992) have aided our understanding through visualization. However, the theoretical efforts(Haff,1983; Bourzutschky and Miller, 1995) to uncover the basic mechanism of this convective instability have not been remarkable, still largely focused on producing convective patterns through computer models and simulations.

We have recently undertaken steps, based on two continuum models, to remedy this situation, which appear to have captured the essence of granular convection. This is a brief summary of our effort along this direction. For details, see Hayakawa et al(1995) and Yue(1995).

2. MODEL A

We have studied two models. Both are based on Navier-Stokes type continuum eqs, but ignore temperature equation assuming the existence of a global temperature throughout the bed. We first present the model A.

The staring point of model A is the recognition that the most fundamental aspect of the vibrating bed, apart from the obvious fixed bed solution with no external driving, is the existence of a uniform bouncing of a collection of particles, a solid or a fluidized block with no internal degrees of freedom. In such a case, the bouncing solution, represented by the motion of a ball on a vibrating platform, satisfies $\ddot{z} = (-1+\Gamma\sin t)\theta(-1+\Gamma\sin t)$ where z is the vertical coordinate of the granular block and the $\theta(x)=1$ for $x>0$ and $\theta(x)=0$ for otherwise. Next, in the presence of internal degrees of freedom such as rotation and/or translation, we define two coarse-grained dynamical variables: the density $\rho(\mathbf{r},t)$ and the velocity $\mathbf{v}(\mathbf{r},t)$ of the granular system. In the box fixed frame, the continuity equations then modifies into:

$$\partial_t\rho + \nabla\cdot(\rho\mathbf{v}) = 0 \qquad (1)$$

$$\partial_t \mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = \hat{z}(\Gamma \sin t - 1 - \lambda) - \frac{1}{\rho}\nabla P + \frac{1}{R}[\nabla^2 \mathbf{v} + \chi \nabla(\nabla\cdot\mathbf{v})] \quad (2)$$

where $\hat{z}$ is the unit vector in the vertical direction and λ is the Lagrange multiplier. $\lambda = 0$ for free motion and $\lambda = \Gamma \sin t - 1$ for stationary state. Note that the first term in the right hand side of (2) is due to the uniform bouncing and the third term is the energy dissipation effectively represented by the Reynolds number R and the bulk viscosity χ. The pressure term P requires some discussion(Hayakawa et al, 1995) but the Van der Waals model $P = T\rho/(1 - b\rho)$ is a reasonable choice, where T represents the effective temperature which might be a global variable and b is a constant of order unity. The result of the linear stability analysis is shown in Fig.1.

Figure 1:The effective growth rate $\sigma_{eff}(q)$ of the fluctuations as a function of the wave number q for $\Gamma = 1.05$(diamond),for which $\sigma_{eff}(q) < 0$ for all values of q, while for $\Gamma = 1.2 > \Gamma_c = 1.12$, $\sigma_{eff}(q)$ becomes positive for a band of q(square). Γ_c is determined by the condition that the maximum of $\sigma_{eff}(q)$ becomes zero at Γ_c.(cross) The parameters used are: $T_e = R = 10$ and $L = 10$.

To check the validity of our picture, we have solved (1) and (2) numerically in two dimension with no slip boundary conditions at the side walls as well as at the top and the bottom plates. Note that the top plate suppresses complicated surface motion of vibrating beds and allows us to use the simplified picture. Since the granular fluid is confined in a box, we do not introduce λ explicitly in the simulations. The absence of λ and the presence of the top wall is expected to cause the appearance of the bouncing solution for $\Gamma \leq 1$ in contrast with the real situation but its omission would not change the essence of the dynamics. In the same spirit, we have ignored χ and b in our simulations.

For $\Gamma < \Gamma_c$, the bouncing solution is expected to appear inside the bed and the density and the velocity at a given point oscillates with the same frequency of the vibration.(See Fig.2a of Hayakawa et al.) Upon increasing Γ further to $\Gamma = 1.2$, which is beyond the predicted $\Gamma_c = 1.12$ determined by $\sigma_M(\Gamma_c) = 0$, we find that the bouncing solution has disappeared and the permanent convective rolls have developed inside the bulk (Fig.2). The wavelength of the most unstable mode by the linear stability analysis is about $q_m \approx 0.4$, which is not far from the actual wavelength of the convective rolls: $q = 2\pi/\lambda = 2\pi/L \approx 0.6$.

Figure 2: For $\Gamma = 1.2 > \Gamma_c = 1.12$, the bouncing solution becomes unstable and the permanent convective rolls appear inside the box. The arrows are the velocity vectors pointing upward.

3. MODEL B

We now introduce another model as Model B. Although the mass conservation remains same as in (1), the momentum conservation (2) now changes into

$$\begin{aligned}\partial_t v_x + (\mathbf{v}\cdot\nabla)v_x &= -(c_0^2/\rho)\partial_x\rho + \mu\nabla^2 v_x \quad (3)\\ \partial_t v_z + (\mathbf{v}\cdot\nabla)v_z &= (V(\rho,t) - v_z))/\tau \\ &\quad -(c_0^2/\rho)\partial_z\rho + \mu\nabla^2 v_z \quad (4)\end{aligned}$$

where $c_0^2 \simeq T$ is the sound speed. The difference between model A and B is the presence of a relaxation term in the z direction (4), which is represented by an average function $V(\rho)$ with the relaxation time τ. The origin of such a term has been discussed in Hong et al(1994) in an attempt to introduce correlations among grains or voids in the diffusing void model(DVM). In the DVM, the void speed is only a function of the local density, namely $v_z = V(\rho)$ + diffusion term. However, a void is a compressible hydrodynamic object that changes and adjusts its shape to conform to the surrounding, not instantaneously, but in a given finite time. So, it may be more appropriate to write down the time dependent equation for the velocity in a manner given by (4) than simply assuming a fixed value at a given local density. The presence of such relaxation process may be effectively equivalent to assuming a drag force acting on a void.

Figure 3:For $\Gamma = 1.2 > \Gamma_c$ and for large aspect ratio along the vertical, the convective rolls move toward the surface.

These coupled equations (3) and (4) are also known as the traffic model or the two-phase model for fluidized beds that have been widely used for mixtures of gas and granular particles. Functions in the model may be inferred from the Enskoq equation; namely $-v_z/\tau$ is the drag term imposed externally on the particle. In the case of no interstitial fluid, its origin lies in the frictions of the front and the rear glass of the container and from the wall. Further, the Enskoq pressure, $T\rho(1 + f(\rho)\rho/2)$ with $f(\rho)$ the correlation function, produces an extra term $V(\rho)$ in addition to the hard sphere pressure $T\rho$. In this case, the coefficient of $V(\rho)$ is proportional to the gravity g. The net effect is for the void (or particle) to adjust its speed, v_z, around the average value $V(\rho)$ in a given time τ. While deriving the exact form for the function $V(\rho)$ is nontrivial, we know it must be a decreasing function of density and have a cut off at the closed packed density ρ_c. Hence, we have chosen a simple form: $V(\rho) = V_0(\rho)(-1 + \Gamma\sin(\omega t))$ $V_0(\rho) = (\rho_c - \rho)^\beta\theta(\rho_c - \rho)$ with the step function and $\beta \leq 1$.

Figure 4:For $\Gamma = 1.2 > \Gamma_c$ and for large aspect ratio along the horizontal, the convective rolls in the bulk are suppressed and survive only near the wall.

We have investigated the eqs.(3) and (4) numerically. The mechanism for the convection seems to be similar to model A, namely the superciritcal bifurcation of a bouncing solution. However, two notable differences emerge. First, when the aspect ratio increases vertically from, say one to two, the convection rolls that initially occupied the whole box move toward the surface as shown in Fig.3 and the motion of the particles are fairly confined near the surface. This is consistent with the experiments and MD simulations.

Second, when the aspect ratio increases horizontally, then convective rolls inside the bulk are suppressed and they appear only near the wall as shown in Fig.4. This again is consistent with the MD simulation results of Taguch(1992). Hence, the role of drag appears to be important in granular convection.

4. DISCUSSION

We review the present status of the convective instability of vibrating beds and simulations of two models presented here. From our analysis, we confirm that the bouncing motion of particle collections plays an important role in both models A and B. We have, however, two problems which should be resolved in the near future.

First, the role of boundary conditions. We have employed no slip boundary conditions at the walls and the plate. Further, we have put the rigid wall at the top and thus suppressed the surface motion. Granular materials have been shown to exhibit very different motion near the wall in a zet flow experiment under gravity(Caram and Hong, 1993) and there has been some attempt to use the negative or positive slip to control the convective patterns (Bourzutschky and Miller, 1995). More detailed studies to derive reasonable boundary conditions at the wall are required.

Second, the role of interstitial fluid. Our model A assumes no interstitial fluid such as air, and it predicts a series of rolls for the vertically large aspect ratio. Model B on the other hand predicts that the convection is suppressed in the bulk region but is confined near the surface, which is in accordance with experiments and with the results of the two-fluid model with an interstitial fluid. Perhaps, the origin of drag, whether it is coming from friction at the walls, or from the viscous effect of the interstitial fluid, may not be relevant once it is present. The suppression of the convection in the bulk is due to the locking mechanism of grains for near closed packed density, which was taken into account in model B by a cut off in $V(\rho)$, namely $V(\rho) = 0$ for $\rho \geq \rho_c$. We need more extensive studies of both models A and B to make quantitative comparison with experiments.

HH is, in part, supported by Grant-in-Aid for Scientific Research from Japanese Ministry of Education, Science and Culture (No. 08226206 and 08740308).

REFERENCES

Bourzutschky, M. & Miller,J. 1995. Granular convection in a vibrating fluid.*Phys. Rev. Lett.* 74: 2216-2219.

Caram,H., & Hong, D. 1992. Diffusing void model for granular flow. *Mod. Phys. Lett. B.* 6: 761-771.

Clement,E., Duran J., & Rajchenbach,J. 1992. Experimental study of heaping in a two dimen- sional sand pile. *Phys. Rev. Lett.* 69:1189-1192.

Gallas,J.A.C., Hermann,H.,& Sokolowski,S. 1992. Convection cells in vibrating granular media. *Phys. Rev. Lett.* 69: 1371-1374.

Haff,P. 1983. Granular flow as a fluid mechanical phenomena. *J. Fluid. Mech.* 134: 401- 430.

Hayakawa,H,S. Yue,& Hong,D. 1995. Hydrodynamic description of granular convection. *Phys. Rev. Lett.* 75:2328-2331.

Hong, D.C., Yue S., Rudra, J., Choi, M.Y.,& Kim, Y. 1994. Granular relaxation under tapping and the traffic equation. *Phys. Rev. E.* 50: 4123-4135.

Knight, J., Jaeger.H, & Nagel,S. 1993. Vibration induced size separation in granular media: convection connection *Phys. Rev. Lett.* 70: 3728-3731.

Pak, H., & R. P. Behringer 1994. *Bubbling in vertically vibrated granular materials.* Nature Vol 371: 231-233.

Taguch,Y-h. 1992. New origin of a convective motion. *Phys. Rev. Lett.* 69: 1367-1369.

Yue,S. 1995. Nonlinear phenomena in materials failure and granular dynamics. Ph.D thesis, Lehigh University.