Thermal Helicity in Fluid/Gravity Correspondence

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based on works arXiv:1311.2940 [hep-th] + to appear
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@ YITP workshop Holographic vistas on Gravity and Strings, YITP, Kyoto Japan, May 26-28, 2014

Introduction

Anomalies in QFT

(Quantum) Anomalies

- Break down of symmetries at quantum level

Zero Temperature

- Anomalies are one-loop exact (Adler-Bardeen)
- Important information is encoded

  (ex) Weyl anomaly and central charges

Finite Temperature

- Not well-understood at the level of QFT

  → Getting understood through fluid description

Setup in QFT

Setup

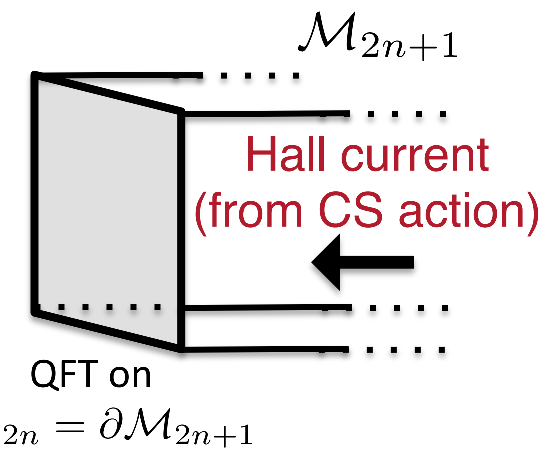
(2n)-d QFT on $\mathbf{R}^{2n-1,1}$ at zero temperature

with (1) translation and rotation invariance
(2) U(1)/gravitational/mixed anomaly

Effective action $W[g,A] = -i \log Z[g,A]$

background fields $g_{\mu\nu}, A_\mu$

Anomaly and CS term



<u>How much non-conserved ?</u> [Callan, Harvey]

→ Chern-Simons term in (2n+1)-d bulk

$$W_{cov}[g, A] = W[g, A] + \int_{\mathcal{M}_{2n+1}} I_{CS}$$

Total current is conserved

→ Conservation equations

$$\nabla_\rho \Sigma^{\rho[\mu\nu]} = 2T^{[\mu\nu]}_{orbital} + \underline{\Sigma^{\perp[\mu\nu]}_{H}} \qquad D_\mu J^\mu = \underline{J^\perp_H}$$

(note : Bardeen-Zumino contribution)

$$\nabla_\nu T^{\mu\nu}_{orbital} = \frac{1}{2}\Sigma_{\nu[\alpha\beta]} R^{\mu\nu\beta\alpha} + J_\nu F^{\mu\nu}$$

charge/spin Hall current: $\star J_H = \dfrac{\partial P}{\partial F}$ $\qquad \star\Sigma_H = 2\dfrac{\partial P}{\partial R}$

with <u>**Anomaly polynomial**</u> $P[F, R] = dI_{CS}$

Anomaly polynomial encodes/classifies anomaly

Anomaly-induced Transports

Higher-dim interacting QFT at finite temp. is hard to deal with

Hydrodynamic limit

variables $u^{\mu}, T, \mu, g_{\mu\nu}, A_{\mu}$

EoM = conservation equations

Anomaly-induced transports

stress-energy tensor $$(T_{\alpha\beta})_{anom} = -\underline{n\mathfrak{F}_{anom}[\mu,T]}(u_\alpha V_\beta + u_\beta V_\alpha) + \cdots$$

U(1) current $$(J_\alpha)_{anom} = -\underline{\frac{\partial \mathfrak{F}_{anom}}{\partial \mu}} V_\alpha + \cdots$$

with $$V^{\mu} = \underline{\epsilon^{\mu\nu\rho_1\cdots\rho_{2n-2}}} u_\nu (\nabla_{\rho_1} u_{\rho_2}) \cdots (\nabla_{\rho_{2n-3}} u_{\rho_{2n-2}})$$

Thermal Helicity

Thermal helicity

[Loganayagam]

$$\frac{1}{\mathrm{Vol}_{2n-1}}\langle\hat{\mathfrak{L}}_{12}\hat{\mathfrak{L}}_{34}\ldots\hat{\mathfrak{L}}_{2n-3,2n-2}\hat{\mathcal{P}}_{2n-1}\rangle = -(n-1)!(2T)^{n-1}\mathfrak{F}_{anom}[T,\mu]$$

$\hat{\mathfrak{L}}_{2k-1,2k}$: angular momentum op. in (x^{2k-1},x^{2k})

$\hat{\mathcal{P}}_{2n-1}$: translation op. in x^{2n-1}-direction

‘Replacement rule’

[Loganayagam] [Loganayagam-Surowka] [Jensen-Loganayagam-Yarom]

$$\mathfrak{F}_{anom}[T,\mu] = P[F\to\mu,\; trR^{2k}\to 2(2\pi T)^{2k}]$$

Determined completely by anomaly polynomial!

Relation

<u>Gibbs current</u>

↓

<u>Partition function for fluid</u>

↓ (generating function)

<u>Thermal helicity</u>

Question

Replacement Rule from Gravity
(Dual to CFT with Anomalies)?

Steps

(0) Start with a gravity theory with CS-term

(1) Prepare a BH solution dual to
a charged rotating fluid configuration

(2) Compute anomaly-induced transports

((3) Gibbs current & thermal helicity)

Potential Difficulties

(1) Kerr-Newman-AdS solution in higher-dim is not known …

→ Fluid/gravity expansion

(2) Anomaly-induced transports are (very) higher-derivative contributions…

→ 2nd order solution (+ symmetry) is enough for leading order computation

Gravity Side

Setup on Gravity Side

Setup

(2n+1)-d Einstein-Maxwell + negative c.c. + CS terms

$$R_{ab} - \frac{1}{2}(R - 2\Lambda)g_{ab} = 8\pi G_N[(T_M)_{ab} + (T_H)_{ab}]$$
$$\nabla_b F^{ab} = g_{YM}^2 (J_H)^a$$

- Maxwell contribution T_M^{ab}
- Hall contributions (from CS terms)

$$\star J_H = J_H^c \star dx_c = \frac{\partial P}{\partial F} \qquad T_H^{ab} = \nabla_c(\Sigma)^{(ab)c}$$

with $\quad \star(\Sigma_H)^b{}_a = (\Sigma_H)^{cb}{}_a \star dx_c = 2\frac{\partial P}{\partial R^a{}_b}$

$P[F,R]$: Anomaly polynomial

BH Solution

Gravity dual of charged rotating fluid

→ charged-rotating-AdS BH (boosted AdS-RN + correction)

$$ds^2 = -2u_\mu dx^\mu dr + r^2[-f(r,m,q)u_\mu u_\nu + P_{\mu\nu}]dx^\mu dx^\nu + \cdots$$

computed up to 2nd order

$$A = \Phi(r,q)u_\mu dx^\mu + \cdots$$

computed up to 2nd order

with

$$\Phi(r,q) = \frac{q}{r^{2n-2}} \qquad f(r,m,q) = 1 - \frac{m}{r^{2n}} + \frac{1}{2}\kappa_q \frac{q^2}{r^{2(2n-1)}}$$

‘Bulk Replacement Rule’

Hall contributions to bulk current

→ evaluated directly from the fluid/gravity solution

$$(T_H)_{ab}dx^a dx^b = T_H^{(V)}(dr + r^2 f u_\mu dx^\mu)V_\nu dx^\nu + \cdots$$

$$(J_H)_a dx^a = J_H^{(V)} V_\mu dx^\mu + \cdots$$

$$J_H^{(V)} = \frac{1}{r^{2n-3}}\frac{d}{dr}\frac{\partial \mathbb{G}^{(V)}}{\partial \Phi} \qquad T_H^{(V)} = -\frac{1}{2r^{2n-1}}\frac{d}{dr}\left(r^2 \frac{d}{dr}\frac{\partial \mathbb{G}^{(V)}}{\partial \Phi_T}\right)$$

with $\mathbb{G}^{(V)} = P[F \to \Phi, tr[R^{2k}] \to 2\Phi_T^{2k}]$

$$\Phi_T(r,m,q) = \frac{1}{2}r^2\frac{df}{dr} = \frac{1}{2r^{2n-1}}\left[(2n)m - \kappa_q(2n-1)\frac{q^2}{r^{2n-2}}\right]$$

Replacement Rule for Bulk!

BH Solution

Gravity dual of charged rotating fluid with anomaly

→ charged-rotating-AdS BH dressed by ‘Hall cloud’

$$ds^2 = -2u_\mu dx^\mu dr + r^2[-f(r,m,q)u_\mu u_\nu + P_{\mu\nu}]dx^\mu dx^\nu + \cdots$$
$$+\underline{g_V(r,m,q)(u_\mu V_\nu + u_\nu V_\mu)dx^\mu dx^\nu} + \cdots$$

$$A = \Phi(r,q)u_\mu dx^\mu + \cdots$$
$$\underline{+a_V(r,m,q)V_\mu dx^\mu} + \cdots$$

Hall contribution at the leading order

with $V^\mu = \epsilon^{\mu\nu\rho_1\cdots\rho_{2n-2}}u_\nu(\nabla_{\rho_1}u_{\rho_2})\cdots(\nabla_{\rho_{2n-3}}u_{\rho_{2n-2}})$

$$\Phi(r,q) = \frac{q}{r^{2n-2}} \qquad f(r,m,q) = 1 - \frac{m}{r^{2n}} + \frac{1}{2}\kappa_q\frac{q^2}{r^{2(2n-1)}}$$

Boundary Replacement Rule

Boundary Replacement Rule

$$(T_{\alpha\beta})_{anomaly} = -\lim_{r\to\infty}\frac{r^{2n-2}}{8\pi G_N}(K^{Brown-York}_{\alpha\beta})_{anomaly}$$

(Note: CS contribution vanishes at infinity)

$$= \underline{\left(\mathbb{G}^{(V)} - \Phi\frac{\partial\mathbb{G}^{(V)}}{\partial\Phi} - \Phi_T\frac{\partial\mathbb{G}^{(V)}}{\partial\Phi_T}\right)_{r=r_H}}(V_\alpha u_\beta + V_\beta u_\alpha)$$

$$= -n\mathbb{G}^{(V)}(r=r_H)$$

$$(J_\alpha)_{anomaly} = -\lim_{r\to\infty}\frac{r^{2n-1}}{g^2_{YM}}g_{\mu\alpha}(F^{r\mu})_{anomaly} = -\underline{\left(\frac{\partial\mathbb{G}^{(V)}}{\partial\Phi}\right)_{r=r_H}}V_\alpha$$

At the horizon $\Phi_T(r_H) = 2\pi T \qquad \Phi(r_H) = \mu$

$$\mathbb{G}^{(V)}(r=r_H) = P[F\to\mu, tr[R^{2k}]\to 2(2\pi T)^{2k}]$$

$\longrightarrow$ Gibbs current $\mathbb{G}^{(V)}_\alpha(r=r_H) = \mathbb{G}^{(V)}(r=r_H)V_\alpha$

[(cf) thermodynamic relation

$$\mathcal{M} = G + \mu\mathcal{Q} + T\mathcal{S} = G - \mu\left(\frac{\partial G}{\partial\mu}\right)_T - T\left(\frac{\partial G}{\partial T}\right)_\mu \qquad \mathcal{Q} = -\left(\frac{\partial G}{\partial\mu}\right)_T$$]

Replacement Rule for Boundary !

On Higher-Order Terms

Why is 2nd order solution enough to determine the leading order anomaly-induced transports (ω^{n-1}–order)?

<u>Some key observation</u>

- *dr* and *u* are allowed only one time in a given wedge product *F's* & *R's*
  → many 0th order terms in *F* & *R* → vanish

  (ex) $F^{(0)} \wedge F^{(0)} \wedge (F^{(1)})^{n-2} \sim (dr \wedge u) \wedge (dr \wedge u) \wedge (F^{(1)})^{n-2} = 0$

- Evaluation of a given wedge product at the leading (fixed) order
  → To add higher order terms in *F* & *R* = To add 0th order terms in *F* & *R*

  (ex) $F^{(0)} \wedge (F^{(1)})^{n-1} \neq 0 \qquad F^{(0)} \wedge F^{(0)} \wedge (F^{(1)})^{n-3} \wedge F^{(2)} = 0$

- Few exceptions treated directly by using 2nd order solution + symmetry

Summary

<u>Replacement rule from gravity side</u>

→ Fluid/gravity expansion to compute
(1) AdS-charged-rotating solution up to 2nd order
(2) thermal helicity and anomaly-induced transports at the leading order

- Any physical reason for this simplicity ?