Entangled Operators in AdS/CFT

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based on arXiv:1405.5946 [hep-th]
with M.Nozaki and T.Takayanagi

Outline

- Motivation
- Renyi entropies for excited states
- Large N and large c
- Holographic analysis
- Summary/Future directions

AdS/CFT ("early")

[Maldacena],
[Witten,GKP…]

Physics of local operators in CFT $\longleftrightarrow$ Physics of Strings on AdS

$$\langle \mathcal{O}(x_1)\cdots\mathcal{O}(x_n)\rangle = \frac{\delta}{\delta\phi_0(x_1)}\cdots\frac{\delta}{\delta\phi_0(x_1)} Z_{\mathrm{Gravity}}[\phi_0(x)]$$

Lots of developments: 2 & 3-pt functions, Integrability, Operators with large R-charge and LLM geometries

Seems well understood and explored…

AdS/CFT ("modern")

Entanglement ⟷ Spacetime Geometry

$$S_A = \frac{\mathrm{Area}(\gamma_A^d)}{4G_N^{d+2}}$$

[Ryu,Takayanagi'06]
[Hubeny,Rangamani, Takayanagi'07]

A common platform for holography, quantum many-body physics, QFT, information theory….

Time dependence and thermalization, global and local quench, Universal Laws of entanglement

[Cardy&Calabrese;Solodukhin; Myers Sinha; Sinha et al.; Maldacena,Harman; Iizuka,Ogawa;Takayanagi,Bhattacharya;Ugajin; Das,Galante,Myers;Lopez et al.; Nozaki,Numasawa,Takayanagi;…….]

This Talk:

[intro: M.Nozaki's talk]

Universal physics of excitations created by a local operator acting on the ground state
(milder version of a local quench)

$$|\psi\rangle = O(z,\bar{z})\,|0\rangle$$

Connection with AdS/CFT: Large N or large c

Program: Modern look on early holography

(Can we learn anything new about local operators from the perspective of entanglement ?)

Our Results:

- von-Neuman (n=1) and Renyi entropies behave very differently
- Naive large N limit breaks down for the Entanglement Entropy
- Universal scaling of the Renyi entropy for excited states at large c
- Holographic analysis consistent with large c and large N analysis…

Main Tool:

[M.Nozaki's talk]

Relative Renyi entropies for states excited by local operators

$$
\begin{aligned}
\rho(t) &= \mathcal{N} \cdot e^{-iHt} e^{-\epsilon H} O(0, x^i)|0\rangle\langle 0|O^\dagger(0, x^i) e^{-\epsilon H} e^{iHt} \\
&= \mathcal{N} \cdot O(w_2, \bar{w}_2, \mathbf{x})|0\rangle\langle 0|O^\dagger(w_1, \bar{w}_1, \mathbf{x}),
\end{aligned}
$$

Reduced $\rho_A = \mathrm{Tr}_B(\rho)$

$$
\boxed{\Delta S_A^{(n)} = \frac{1}{1-n} \log \left(\frac{\mathrm{Tr}(\rho_A^n)}{\mathrm{Tr}(\rho_A^{(0)})^n} \right) = \frac{1}{1-n} \log \left[\frac{\langle O(w_1, \bar{w}_1) O^\dagger(w_2, \bar{w}_2) \ldots O(w_{2n}, \bar{w}_{2n}) \rangle_{\Sigma_n}}{\left(\langle O(w_1, \bar{w}_1) O^\dagger(w_2, \bar{w}_2) \rangle_{\Sigma_1} \right)^n} \right]}
$$

Connection with AdS/CFT: Large N or large c

CFTs and large N limit

Operator: $$Tr(\mathcal{Z}^J) = Tr(\phi_1 + i\phi_2)^J$$

e.g. J=2 (Free Field or L-R decomposition)

$$\Delta S_R^{(n)} = \frac{1}{1-n} \log\left(2^{1-2n} + \frac{1}{2^n N^{2(n-1)}}\right)$$

!!

n $\geq$ 2

$$\Delta S_A^{(n)} = \frac{Jn-1}{n-1} \log 2$$

(n=1) von-Neuman entropy

$$\Delta S_R^{(1)} = \log\left(2\sqrt{2}N\right)$$

N^2 d.o.f

<u>Ground State</u>

<u>Both scale as c~N^2</u>

2d CFTs

[He,Numasawa,Takayanagi,Watanabe'14]

(n=2)

$$\frac{\langle O(w_1,\bar{w}_1)O(w_2,\bar{w}_2)O(w_3,\bar{w}_3)O(w_4,\bar{w}_4)\rangle_{\Sigma_2}}{\left(\langle O(w_1,\bar{w}_1)O(w_2,\bar{w}_2)\rangle_{\Sigma_1}\right)^2} = |z|^{2\Delta_O}|1-z|^{2\Delta_0}G_O(z,\bar{z})$$

At late time $(z,\bar{z}) \to (1,0)$

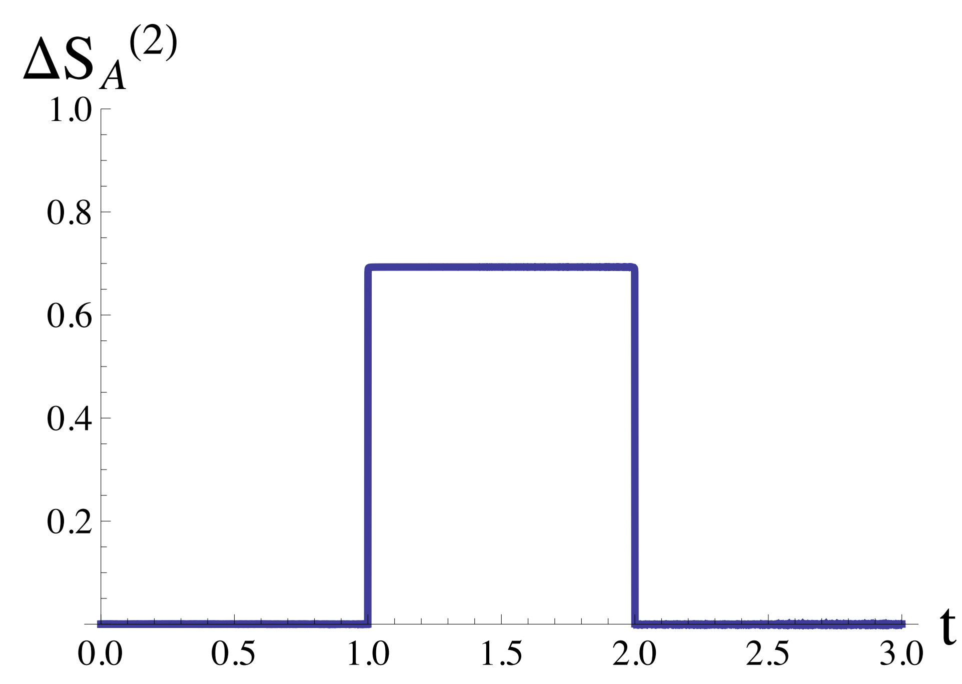
In rational CFTs

$$G(z,\bar{z}) \simeq F_{00}[O]\cdot(1-z)^{-2\Delta_O}\,\bar{z}^{-2\Delta_O}$$

$$\Delta S_A^{(2)} = -\log F_{00}[O] = \log d_O$$

$$d_\Delta = \frac{S_{0\Delta}}{S_{00}}$$

quantum dimension



2d CFTs at large c

[PC,M.Nozaki,T.Takayanagi14]

Conformal block expansion

$$G(z,\bar{z}) = \sum_b (C^b_{OO^\dagger})^2 F_O(b|z)\bar{F}_O(b|\bar{z})$$

at large central charge c

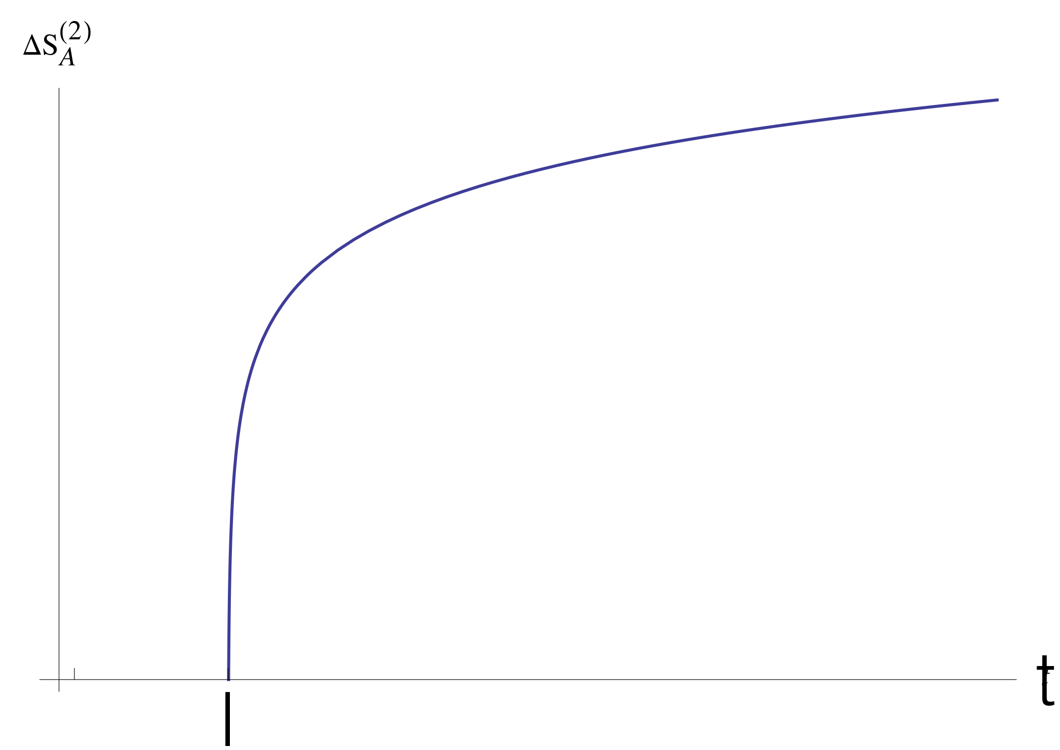
[Fateev,Ribault'11]

$$F_O(b|z) \simeq z^{\Delta_b - 2\Delta_O} \cdot {}_2F_1(\Delta_b, \Delta_b, 2\Delta_b, z)$$

at late time

$$\Delta S_A^{(2)} \simeq 4\Delta_O \cdot \log \frac{2t}{\epsilon}$$

similar to a local quench



Holographic Checks

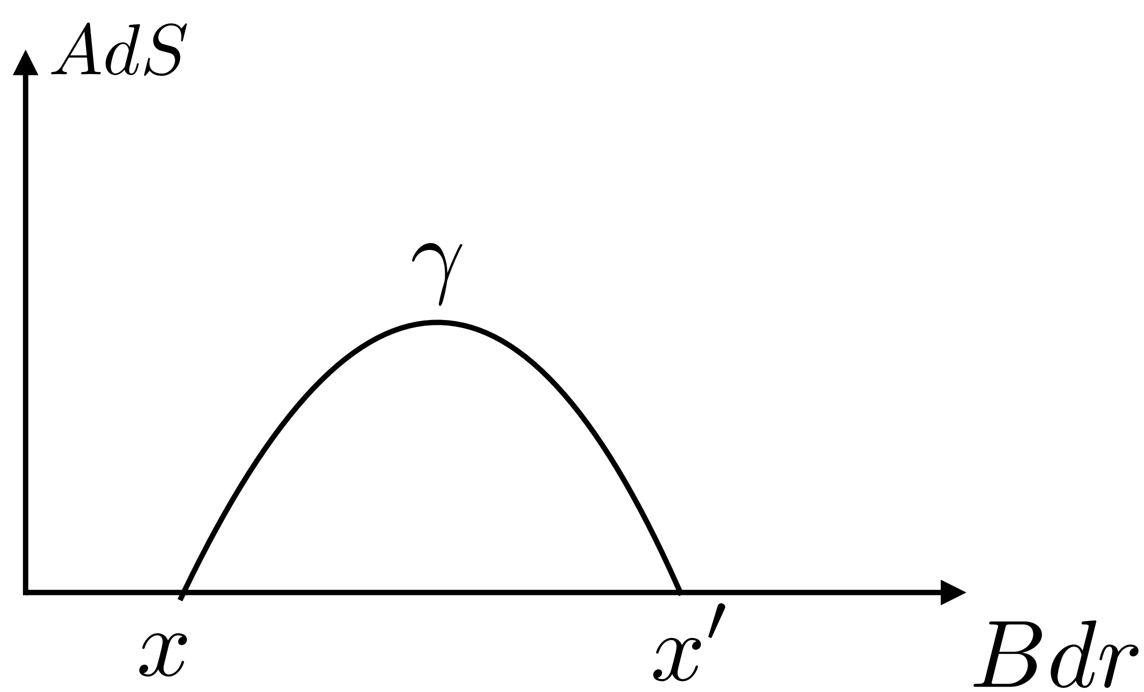
Geodesics and propagators

$$\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle = \frac{\delta}{\delta \phi_0(x_1)} \cdots \frac{\delta}{\delta \phi_0(x_1)} Z_{\text{Gravity}}[\phi_0(x)]$$

for operators with "large" conformal dimension (semiclassical)

$$\langle \Phi_\Delta(x) \Phi_\Delta(x') \rangle \sim e^{-\frac{\Delta}{R} L(\gamma)}$$

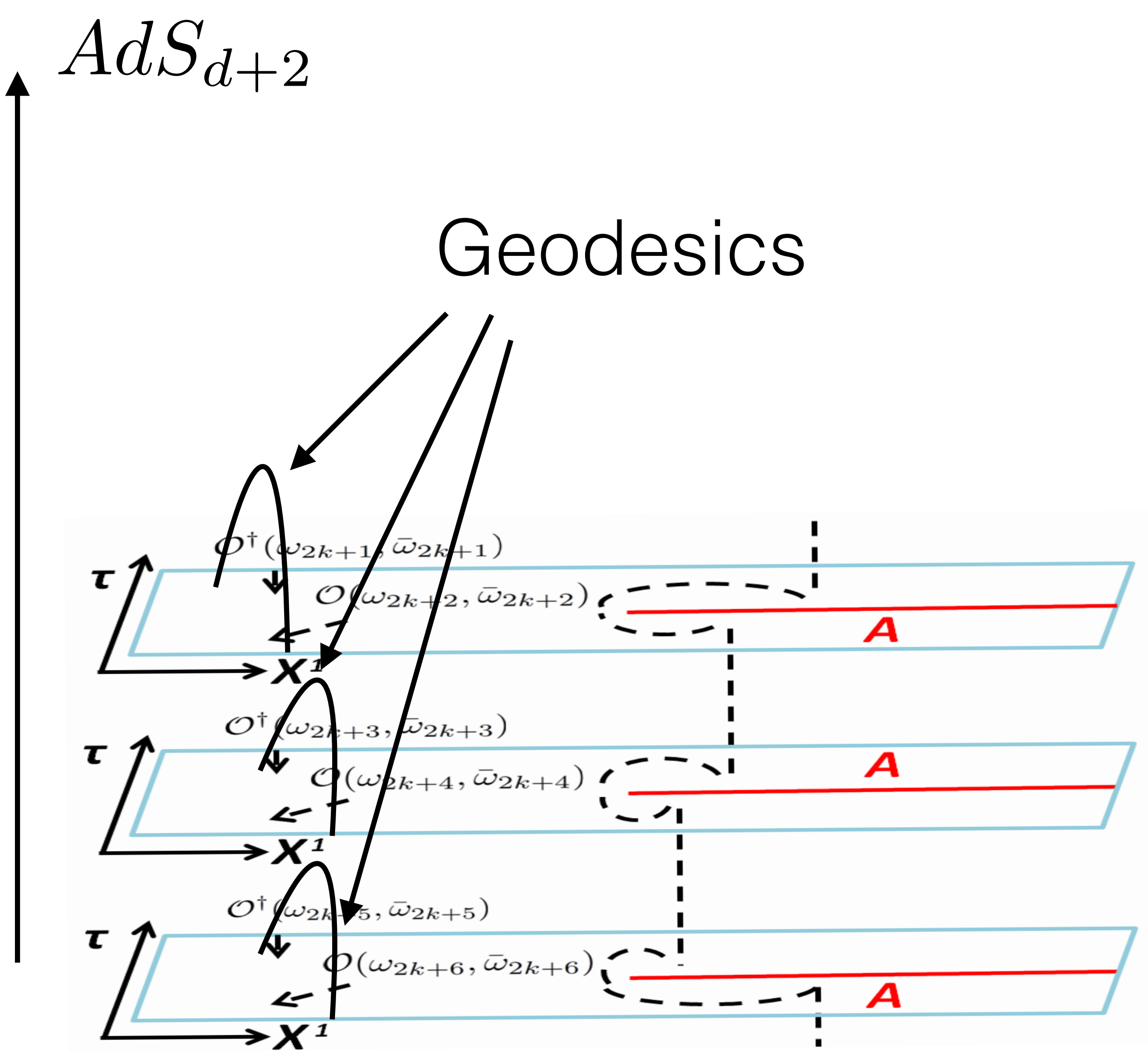
[Balasubramanian,Ross'99]



Geodesics in topological Black Holes

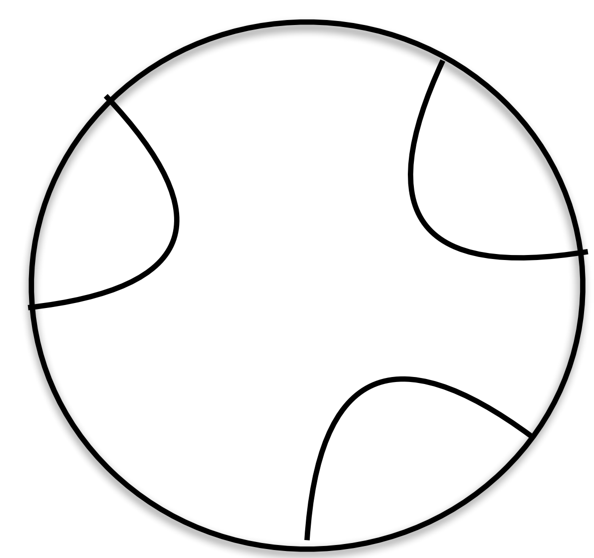
[Emparan'99]

horizon H_d^n



H_d^n

Witten Diagram



Geodesics in topological Black Holes

[Emparan'99]
[Cassini,Huerta,Myers' 11]

$$ds^2 = f(r)d\tau^2 + \frac{dr^2}{f(r)} + r^2 d\phi^2 + r^2 e^{-2\phi} dx_i^2$$

$$f(r) = -1 - \frac{\mu}{r^{d-2}} + \frac{r^2}{R^2}$$

Temperature and “n”

$$\beta = 2\pi n R$$

holographic ratio of the propagators (eg d=2)

$$e^{-\frac{2\Delta_O}{R}\left(L^{(n)}-L^{(1)}\right)} = \left(\frac{\cosh\left(\Delta\phi\right) - \cos\left(\frac{\Delta\tau}{R}\right)}{n^2\left(\cosh\left(\frac{\Delta\phi}{n}\right) - \cos\left(\frac{\Delta\tau}{n\,R}\right)\right)}\right)^{2\Delta_O}$$

perfectly matches the 2d CFT result

Results $n \geq 2$

AdS_3

$$\Delta S_A^{(n)} \simeq \frac{2n\Delta_O}{n-1} \log\left(\frac{n\sin\left(\frac{\pi}{n}\right)t}{\epsilon}\right) - \frac{1}{n-1}\log 2$$

higher d at late time

$$\Delta S_A^{(n)} \simeq \frac{4n\Delta}{d(n-1)} \log\left(\frac{F_{(d,n)}\,t}{2\,\epsilon}\right) + C_{(n,d)} - \frac{1}{n-1}\log 2$$

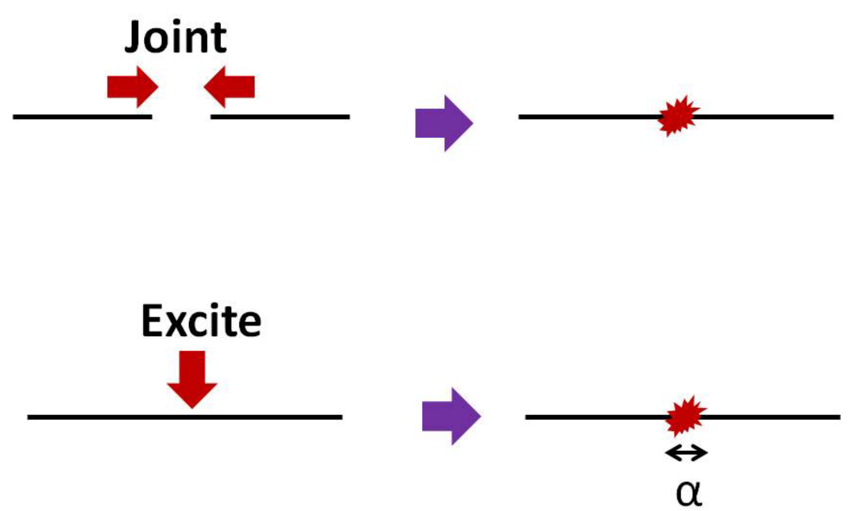
n->1

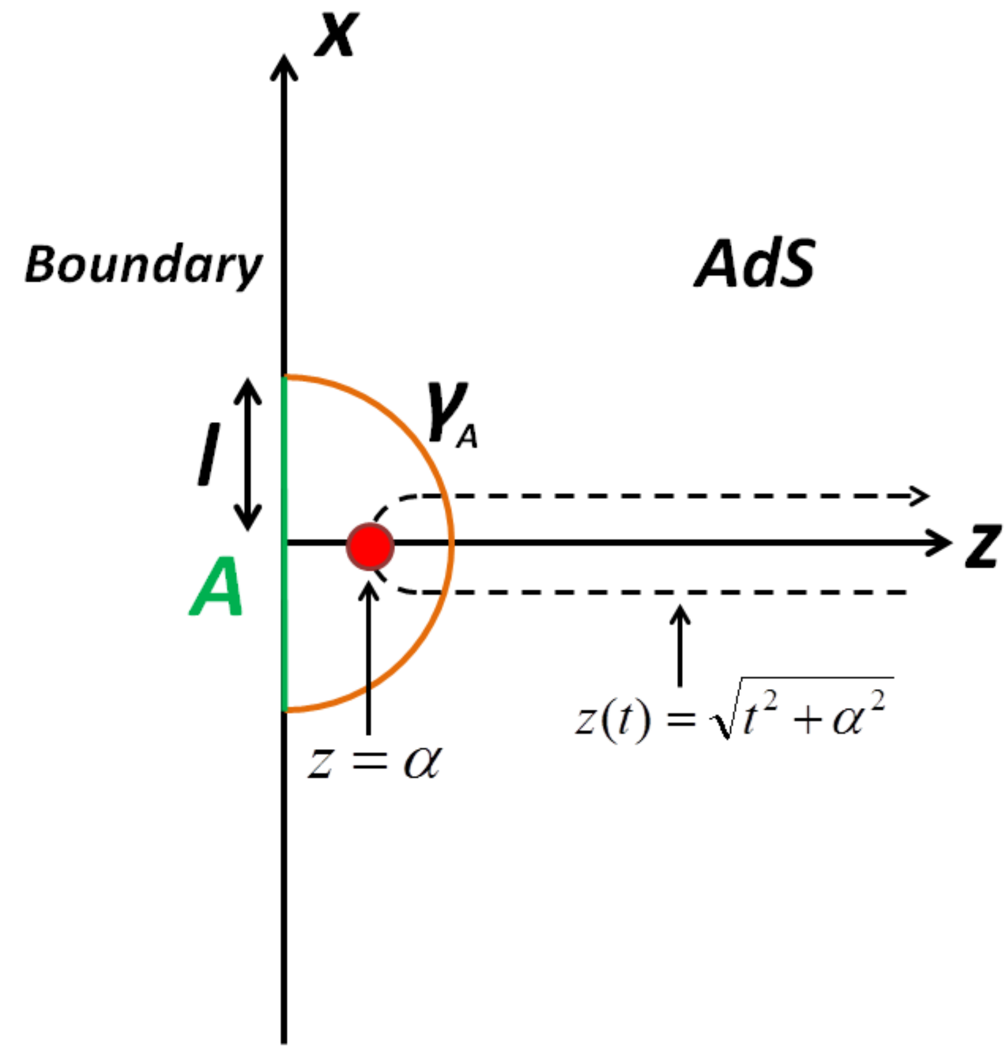
n->1 from holography?

Falling particle in AdS (n=1)

[Nozaki,Numasawa,Takayanagi'13]

CFT (chain)





In our setup: $\alpha \equiv \epsilon$

$$\Delta S_A^{(1)} = 0 \qquad (t < l)$$

$$\Delta S_A^{(1)} \simeq \frac{c}{6} \log \frac{t}{\epsilon} + \frac{c}{6} \log \frac{\Delta}{c} \qquad (t \gg l)$$

General picture (summary)

rational CFT

$$D_n = (d_O)^{n-1}$$

free field (large N)

$$D_n = 2^{Jn-1} + O(N^{-2})$$

$$\Delta S_A^{(n)} = -\frac{1}{n-1} \log \frac{\langle O^\dagger O \cdots O^\dagger O \rangle_{\Sigma_n}}{(\langle O^\dagger O \rangle_{\Sigma_1})^n}$$

$$\simeq -\frac{1}{n-1} \log \left[\frac{1}{D_n} + \mu_n \cdot \left(\frac{\epsilon}{t} \right)^{\nu_n} \right]$$

Large c
(strongly interacting)

$$\Delta S_A^{(n)} \simeq \frac{\nu_n}{n-1} \log \frac{t}{\epsilon}$$

$$\nu_n \simeq \frac{4n\Delta}{d} + O\left(\frac{\Delta^2}{c} \right)$$

$$D_n \sim e^{b_n \cdot N^{a_n}}$$

$$\lim_{n \to 1} \frac{\nu_n}{n-1} \simeq c \quad ?$$

Thank You