# Vortices in Holographic Superconductors (SCs) & Superfluids (SFs)

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**Based on:**

OD, Gary Horowitz, Nabil Iqbal & Jorge Santos, arXiv:1311.3673

**Holographic Vistas of Kyoto & Gravity** and **Strings, Yukawa Institute**

May 2014

- **Previous works considered the probe limit: dynamical $\Phi$, $A_\mu$ but NO backreaction on $g_{\mu\nu}$**
  - Albash, Johnson;
  - Montull, Pomorol, Silva;
  - Maeda, Natsuume, Okamura;
  - Kachru, Sachdev;
  - Bao, Harrison;

- **Here, I will consider the full nonlinear problem with backreaction on $g_{\mu\nu}$:**

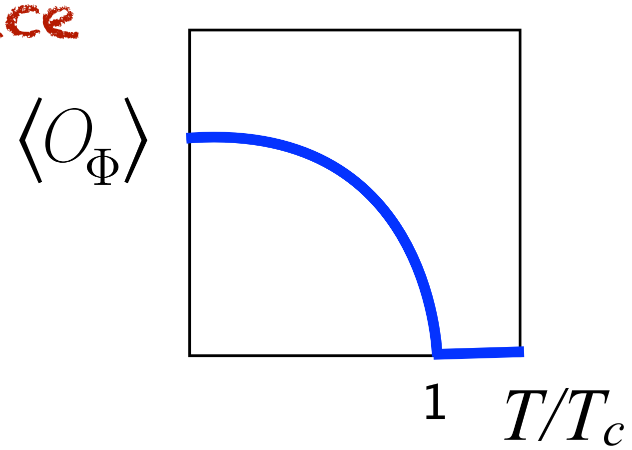
  **- fully characterise** the system: find **properties not seen** in **probe limit**

  - follow physics all way down to **low temperatures**,

  - learn about the **IR field theory** that **describes** the system **at zero Temperature**

## → Accommodating SCs in the gauge/gravity correspondence

- **Ginzburg–Landau (GL) theory:**




SC wavefunction $\Phi$ has **order parameter** properties:

its equilibrium value is zero above $T_C$ and increases gradually below $T_C$

$\Phi = |\Psi| e^{\tilde{\varphi}}$, $\quad |\Psi| \sim \rho_C^{1/2}$ (Cooper pair charge $2e$, mass $m$), $\tilde{\varphi}$ is macroscopic SC phase

GL free energy density for a SC expanded around $T = T_C$ for small expansion parameter $|\Psi|$:

$$F_s(\mathbf{r},T) = F_n(\mathbf{r},T) + \alpha|\Phi|^2 + \frac{\beta}{2}|\Phi|^4 + \frac{1}{2m}\left|(-ih\nabla - 2e\mathbf{A})\,\Phi\right|^2 + \frac{1}{2\mu_0}B^2$$

Minimize $F_s$, $\quad \delta F_s/\delta\Phi = 0$ :

$$\frac{1}{2m}\left|(-ih\nabla - 2e\mathbf{A})\,\Phi\right|^2 + \alpha\Phi + \beta|\Phi|^2\Phi = 0$$

**Ginzburg–Landau eq. I**

$\nabla\times\mathbf{B} = \mu_0\mathbf{J} \Rightarrow \mu_0\mathbf{J} = -\nabla^2\mathbf{A}$ (London gauge: $\nabla\times\mathbf{A} = 0$) & Minimize $F_s$, $\delta F_s/\delta\mathbf{A} = 0$ :

$$\mathbf{J} = \frac{e}{m}\left[\Phi^*\left(-ih\nabla - 2e\mathbf{A}\right)\Phi + c.c.\right]$$

**Ginzburg–Landau eq. II**

- Note that GL eqs follow from **Abelian Higgs model** (**Klein-Gordon** eq for **charged scalar**):

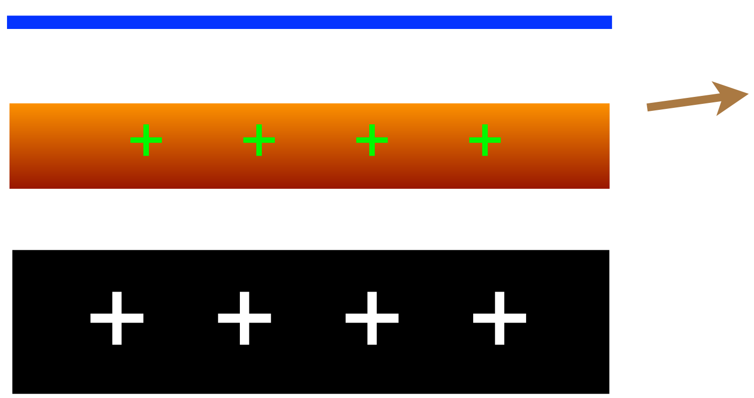
$$S = \int \mathrm{d}^4x\,\sqrt{-g}\left[-\frac{1}{2}F_{ab}F^{ab} - 2(D_a\Phi)(D^a\Phi)^\dagger - 2V(|\Phi|^2)\right]$$

- To discuss **SCs** in the **holographic** context **add bulk gravitational AdS background**

$$S = \int \mathrm{d}^4 x \sqrt{-g}\left[R + \frac{6}{L^2} - \frac{1}{2} F_{ab}F^{ab} - 2(D_a\Phi)(D^a\Phi)^\dagger - 2V(|\Phi|^2)\right]$$

***$T < T_c$* :**

***Hairy BH* ( HHH ) ↔ *SC phase***

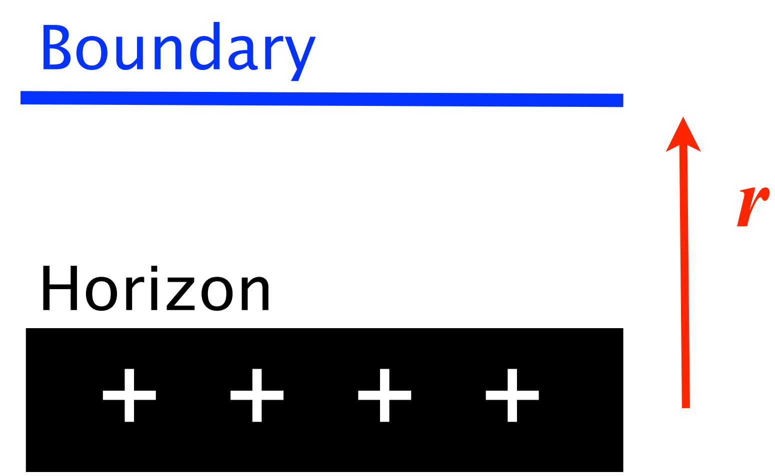


Floating condensate of scalar field

$\phi = \langle \mathcal{O} \rangle$

***$T > T_c$* :**

**Planar RN-AdS BH ↔ *normal phase***



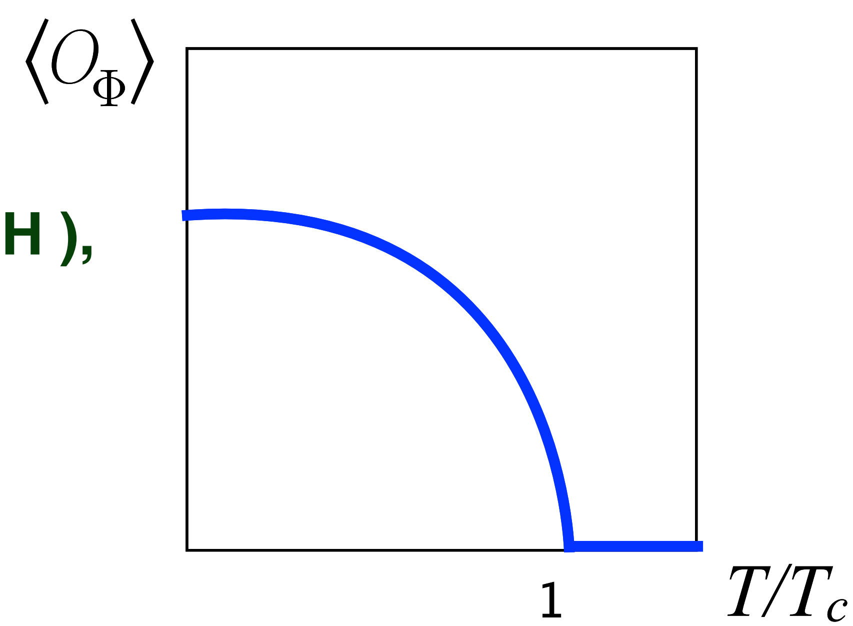

**Gubser,**

**Hartnoll, Herzog, Horowitz ( HHH ),**

**Horowitz, Roberts,**

**Murata, Kinoshita, Tanahashi**

**OD, Monteiro, Reall, Santos**




**RN-AdS BH unstable to scalar condensation if**

$$\mu_{2d\,NH}(\mu, q) < \mu_{2d\,BF}$$

**although**

$\mu > \mu_{4d\,BF}$ **(asymp. stable)**

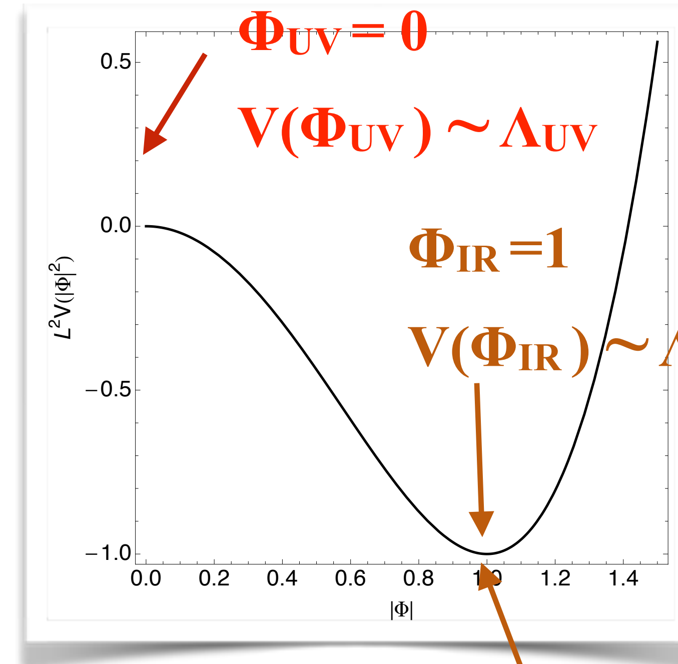
→ Normal / SC phase transition even in the absence of a chemical potential

$(A_t = 0)$

$$S = \int \mathrm{d}^4x \sqrt{-g}\left[R + \frac{6}{L^2} - \cancel{\frac{1}{2}F_{ab}F^{ab}} - 2(D_a\Phi)(D^a\Phi)^\dagger - 2V(|\Phi|^2)\right]$$

- **Mexican hat potential:**

$$V(\eta) = \eta\,\mu^2\left(1 - \frac{\eta\,\mu^2}{4\,V_0}\right), \qquad \eta = \Phi\Phi^\dagger$$




$$L_{\mathrm{IR}}^2 = \frac{3}{4}L^2$$

- **Asymptotic decay of scalar field**

$$\Phi|_{z=0} = \frac{\alpha}{r^{\Delta_-}} + \frac{\beta}{r^{\Delta_+}} + \cdots$$

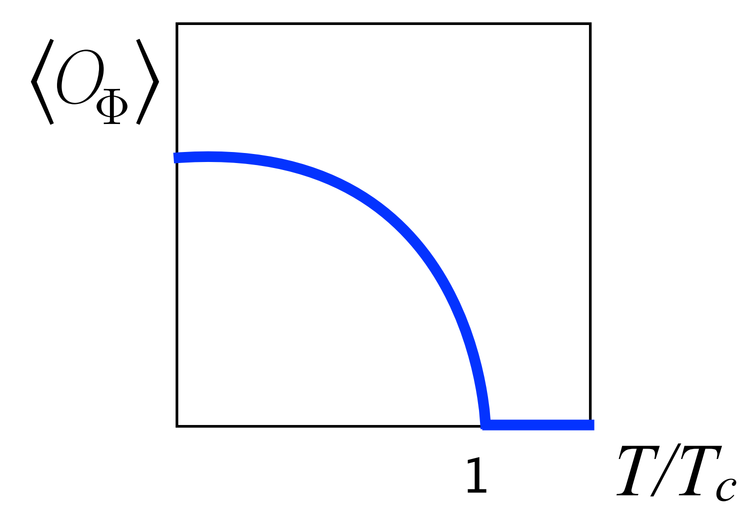
- For $\mu^2_{BF} < \mu^2 < \mu^2_{Unit}$ , impose **Robin BCs:** $\beta = \kappa\,\alpha$ $\quad (\alpha = \langle \mathcal{O} \rangle)$

- Corresponds to a **relevant double trace deformation** of the **boundary theory** of the form :

$$S_{bdry} \to S_{bdry} - \kappa \int d^3x\, \mathcal{O}^\dagger \mathcal{O}$$

- **Instability breaks a $U(1)$ symmetry at low $T$:**

$T_c \sim \kappa$ 2xTr HHH condensate forms

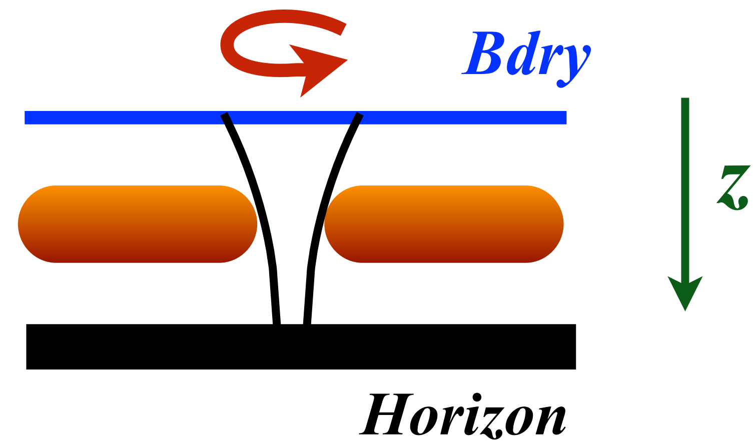



[BF], [Ishisbashi,Wald], [ Faulkner, Horowitz, Roberts ( 2xTr HHH ) ] , [Witten], [Sever, Shomer]

## → Adding SC & SF vortices: how to do ?

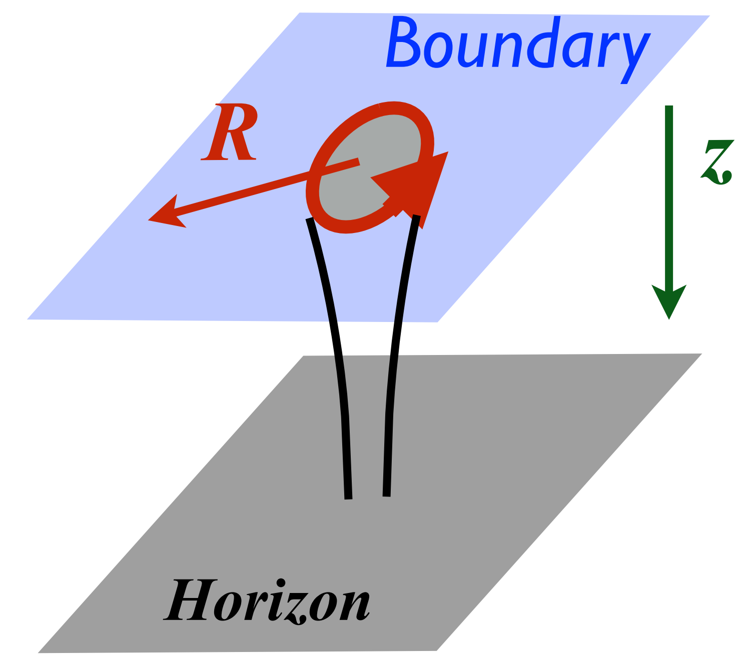
- Bdry: vortex is pointlike excitation around which phase of condensate winds

$$\Phi = |\Psi| e^{\widetilde{\varphi}}, \qquad \widetilde{\varphi} = in\varphi$$




- Static axisymmetric vortices: $\partial_t$ , $\partial_\varphi$ are KV.

- Problem depends on holog distance $z \sim 1/r$ , and bdry radius $R$.




- SF or SC vortex means that $A_\varphi \neq 0$

- Solve Einstein-deTurck PDEs for $\{ |\psi|, A_\varphi , g_{\mu\nu} \}$ [Wiseman] using Newton-Raphson relaxation algorithm on a pseudospectral collocation grid

- **BCs for $g_{\mu\nu}$ :** Fefferman-Graham form and its bdry expansion

$$g(x)\big|_{bdry} = g^{(0)} + \ldots + z^3 g^{(3)} + \cdots , \quad \text{with} \quad \langle T_{\alpha\beta}(x) \rangle = g^{(3)}_{\alpha\beta}(x)$$

**[ Haro, Solodukhin, Skenderis ]**

- **Dirichlet BC: fix $g^{(0)}$** to be **planar AdS.** **Find $< T > \sim g^{(3)}$**

**Previously in the Probe limit:**
- Albash, Johnson;
- Montull, Pomorol, Silva;
- Maeda, Natsuume, Okamura;
- Kachru, Sachdev;
- Bao, Harrison;

# → Distinguishing SC & SF vortices: BCs for Maxwell field

- **CFT has a gauge coupling:** $\nabla_a F^{ab} = g_c J^b$

**SF case: $g_c = 0$**

- **Global $U(1)$:** not gauged (just rotational sym)
- Do **not** want **external applied field**

$\Rightarrow$ **SF BC: $A_\varphi|_{z=0} = 0$**

- **$J_\varphi$ creates the vortex**

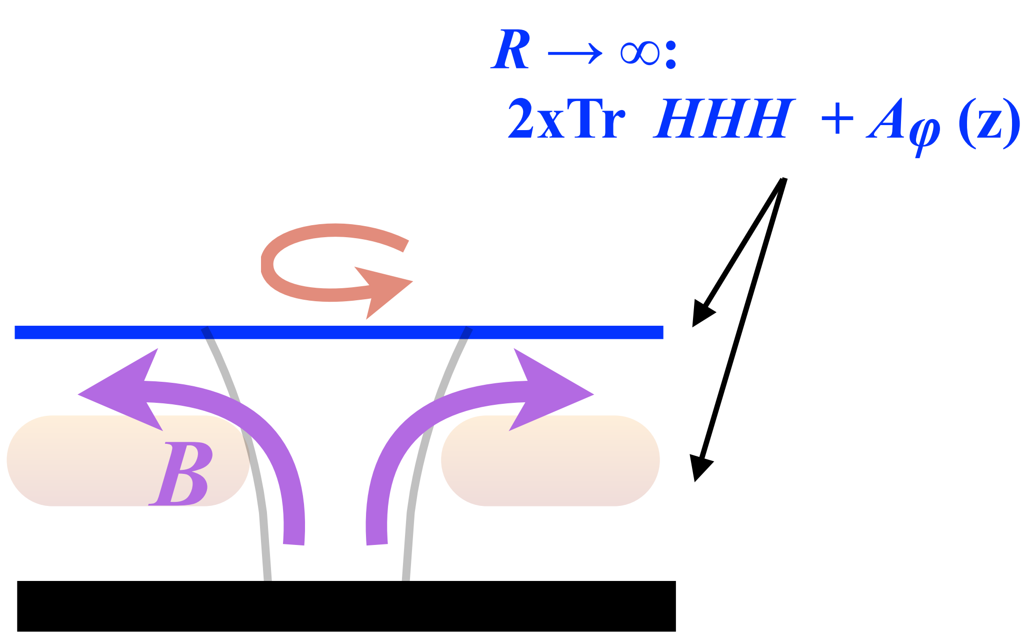
**SC case w/ $J$=0: $g_c \to \infty$**

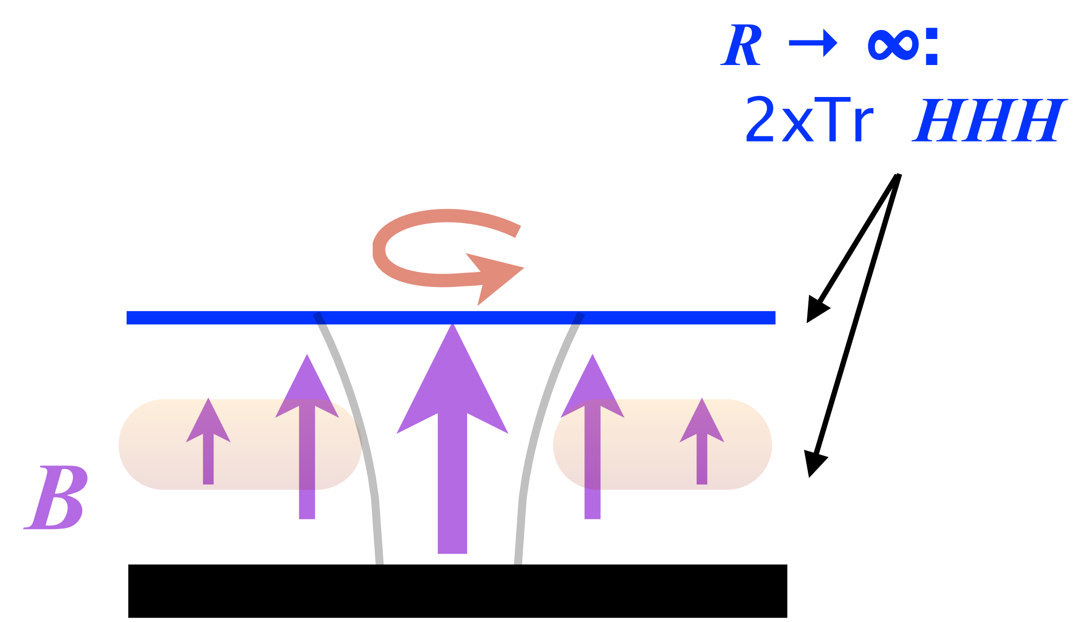
- **Gauged $U(1)$**
- Dynamical field $\nabla_a F^{ab}=0$

$\Rightarrow$ **SC BC: $J_\varphi|_{z=0} = 0$**

- **$A_\varphi$ creates the vortex**

$$A_\varphi = A_\varphi^{(0)} + J_\varphi\, z + \ldots$$



→ T=0, near-horizon & field theory considerations:

Standard Higgs mechanism

- **Conventional SFs** has few low-energy excitations:

  at $T$=0, there is ***single* gapless Goldstone mode** associated with spontaneous symmetry breaking.

  **Conventional SCs** does not even have this mode: it is eaten by the dynamical photon.

- However, typical **holographic SFs** or **SCs** have **many gapless dof**: their gravity dual has **BH horizon at low *T***.

- **Vortex** is **localised point** in the UV CFT3 directions.

  It becomes a **bulk cosmic string**, **carrying $\Phi_{\text{mag}}$** down to bulk horizon where it **interacts with IR dof.**

  This interaction is described by an IR CFT3 ( ≠ UV CFT3 ):

- We propose:

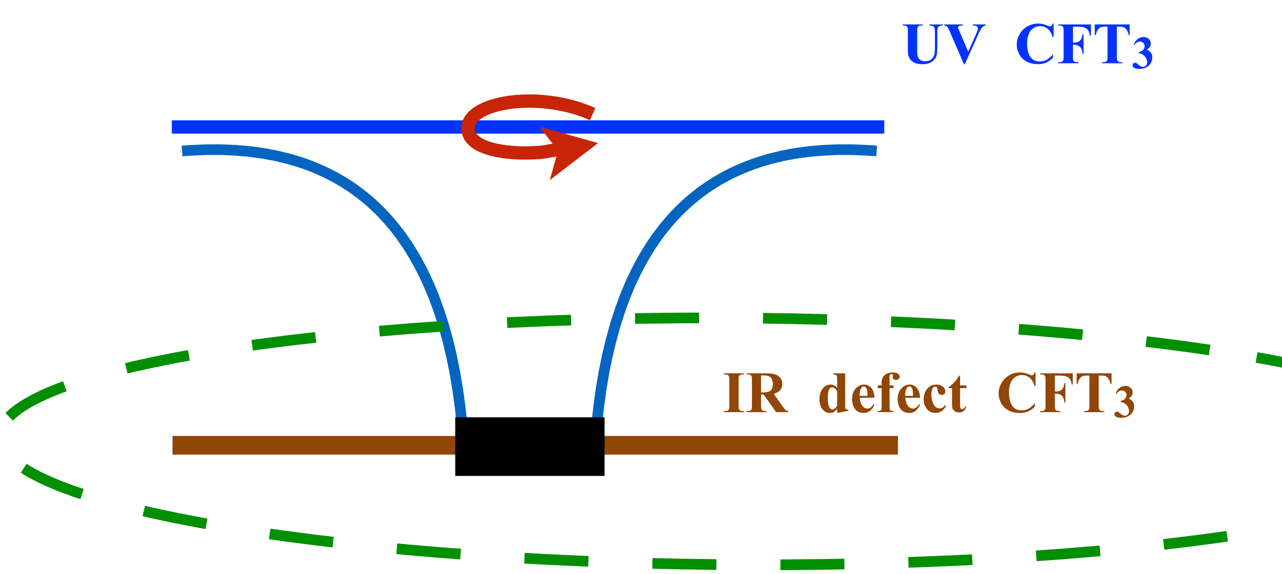
  Use ***defect* CFT** formalism to describe **interaction** of a **heavy, point like** object (e.g. a vortex) **with IR CFT3**

  Defect **breaks translational invariance** of the **conformal group**

- Construct the **Near-Horizon solution**

  of the ***T*=0 configuration**

  on the boundary of which the **defect IR CFT3 lives**



# → Near-Horizon solution (T=0) & IR defect $CFT_3$

- **Vortex (defect) breaks translational invariance** of the **conformal group** $SO(3,2)$**: broken** $SO(2,1) \times SO(2)$

- Most general element with these symmetries: [ **Vortex core:** $\theta=0$. $\partial AdS_4$: $\theta=\pi/2$ is conf to $AdS_2 \times S^1$ ]

$$ds^2 = \frac{L^2}{\cos^2\theta}\left[F(\theta)\left(\frac{-dt^2+d\rho^2}{\rho^2}\right) + H(\theta)d\theta^2 + G(\theta)\sin^2\theta d\varphi^2\right]$$

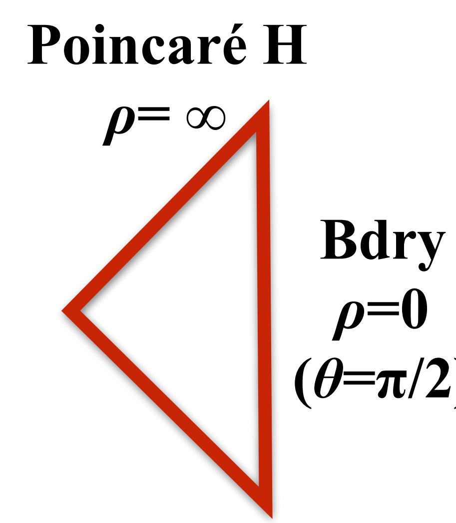
$\{F(\theta), G(\theta), H(\theta)\}$ ?
$A_\varphi(\theta), \Phi(\theta)$

- $SO(2,1)$ is sym. group of a $CFT_1$ **extending along vortex worldline: nontrivial** $CFT_1$ **lives on** the **defect.**

  **Defect is mapped to** $AdS_2$ **boundary and BCs therein. Phase of vortex should wind around** $S^1$**.**

- ∃ $AdS_2$ endows bulk solution with a **Poincaré horizon at** $\rho \to \infty$.

  There is an **entropy** associated with this horizon, which extends from $\theta = 0$ to $\theta = \pi/2$:




$$\Delta S_{imp} = \lim_{\theta_\Lambda \to \frac{\pi}{2}} \frac{\pi L_{IR}^2}{2G_N}\left(\int_0^{\theta_\Lambda} d\theta \frac{\sin\theta}{\cos^2\theta}\sqrt{H(\theta)G(\theta)} - \int_0^{\theta_\Lambda} d\theta \frac{\sin\theta}{\cos^2\theta}\right)$$

- Horizon ∩ conformal bdry $AdS_2 \times S^1$ at $\theta = \pi/2$: it's a **bulk minimal surface** that hangs down from bdry.
  $\Delta S_{imp}$ computes **entanglement entropy** via the **Ryu-Takayanagi prescription**

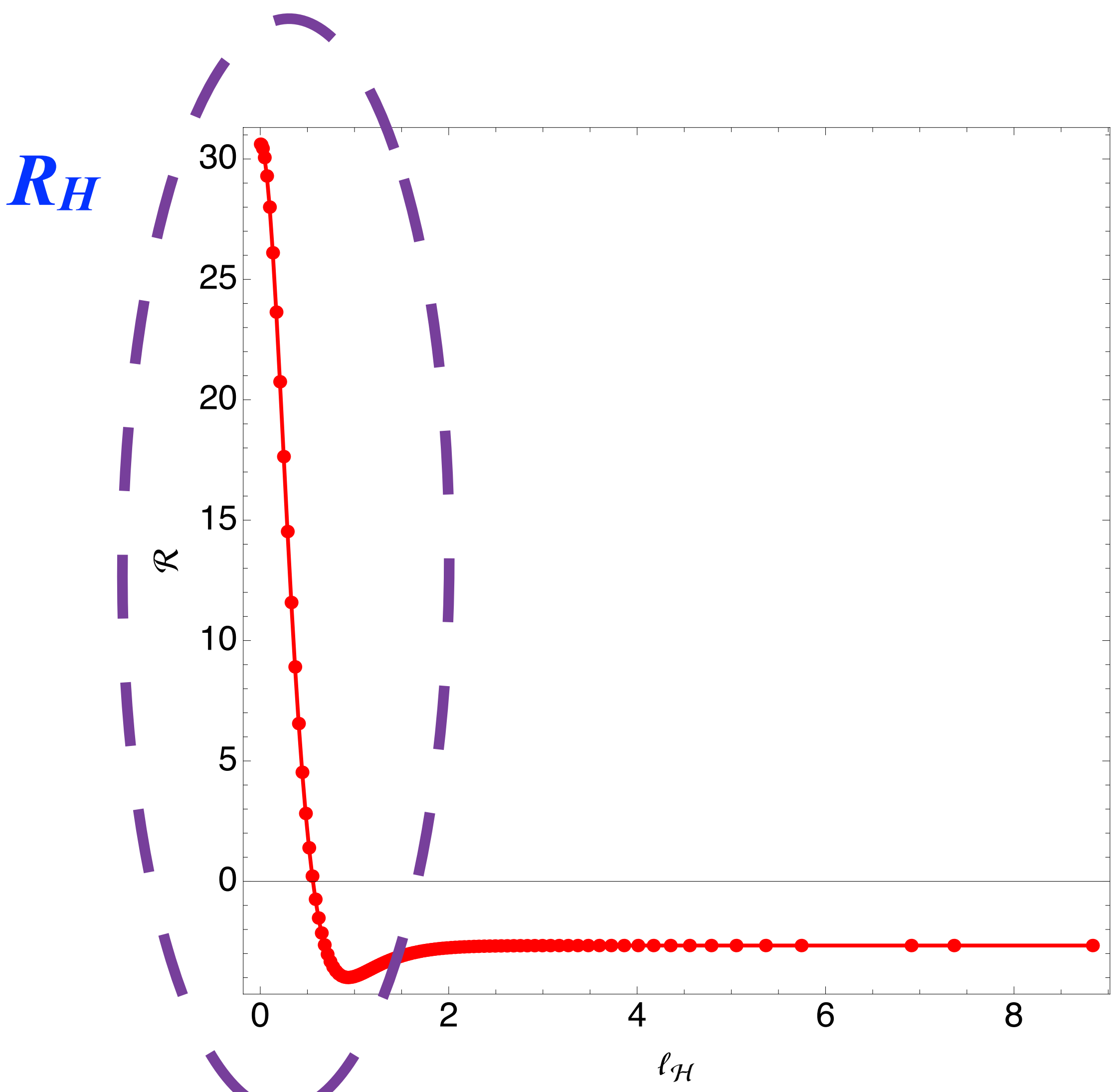
- **Minimal surface wraps the** $S^1$ that **surrounds the defect** on the bdry.
  So $\Delta S_{imp}$ is ***boundary*** or ***impurity*** **entropy** of the **defect with its surrounding**

## → Near-Horizon solution (T=0) & IR defect CFT₃

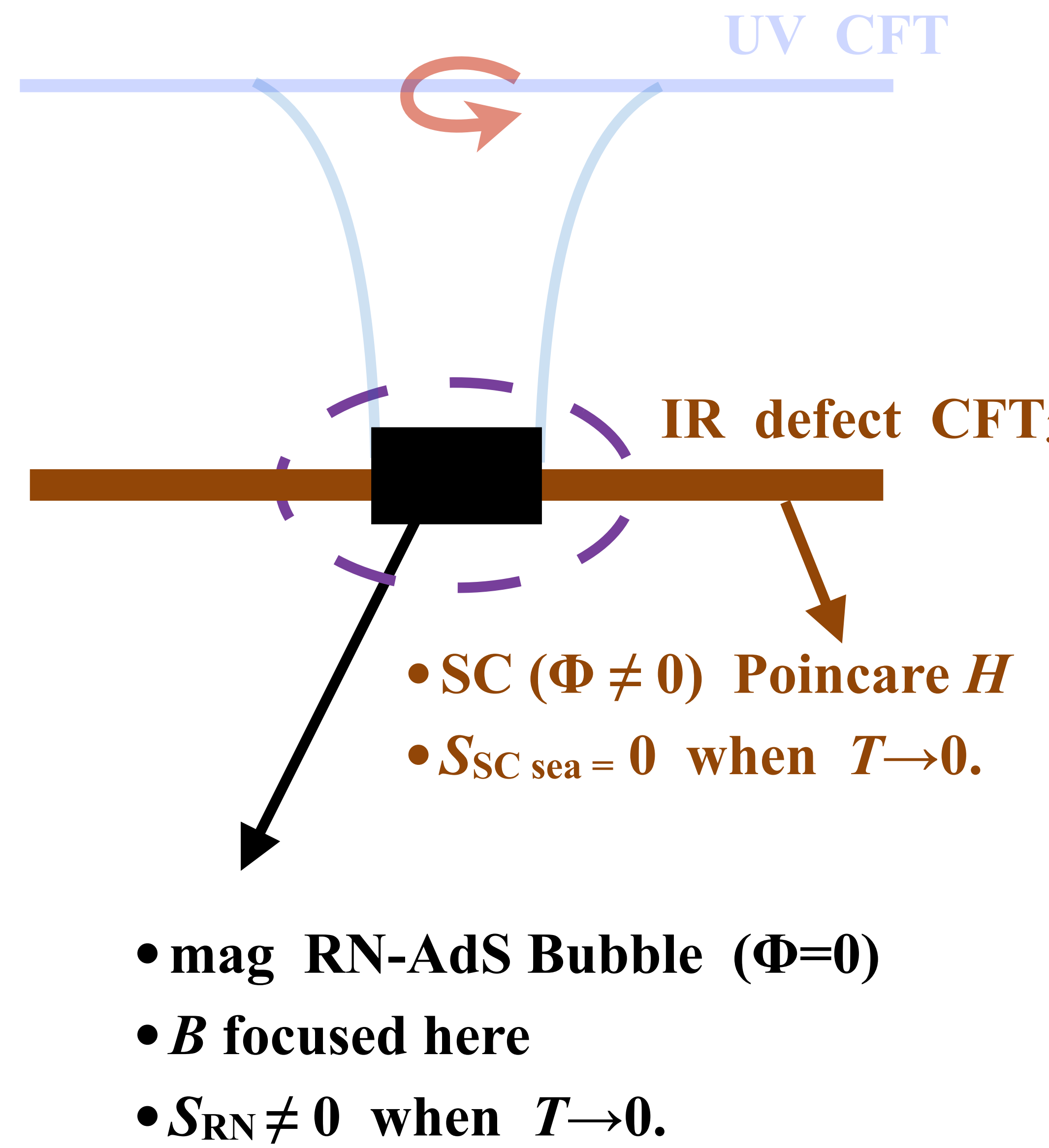
- **Scalar curvature** of the $T = 0$ horizon is **large** near the **core**:

  Defect is breaking the translational invariance of the 2xTr HHH near-horizon solution.

  Signals a "**bubble of RN-AdS horizon**" (carrying $S_{imp}$) **sticking out** of the **usual Poincaré horizon**.




**Proper distance** from **vortex core**



→ SC vortex (full geometry @ any T) vs $S_{imp}$ (NH solution T=0)

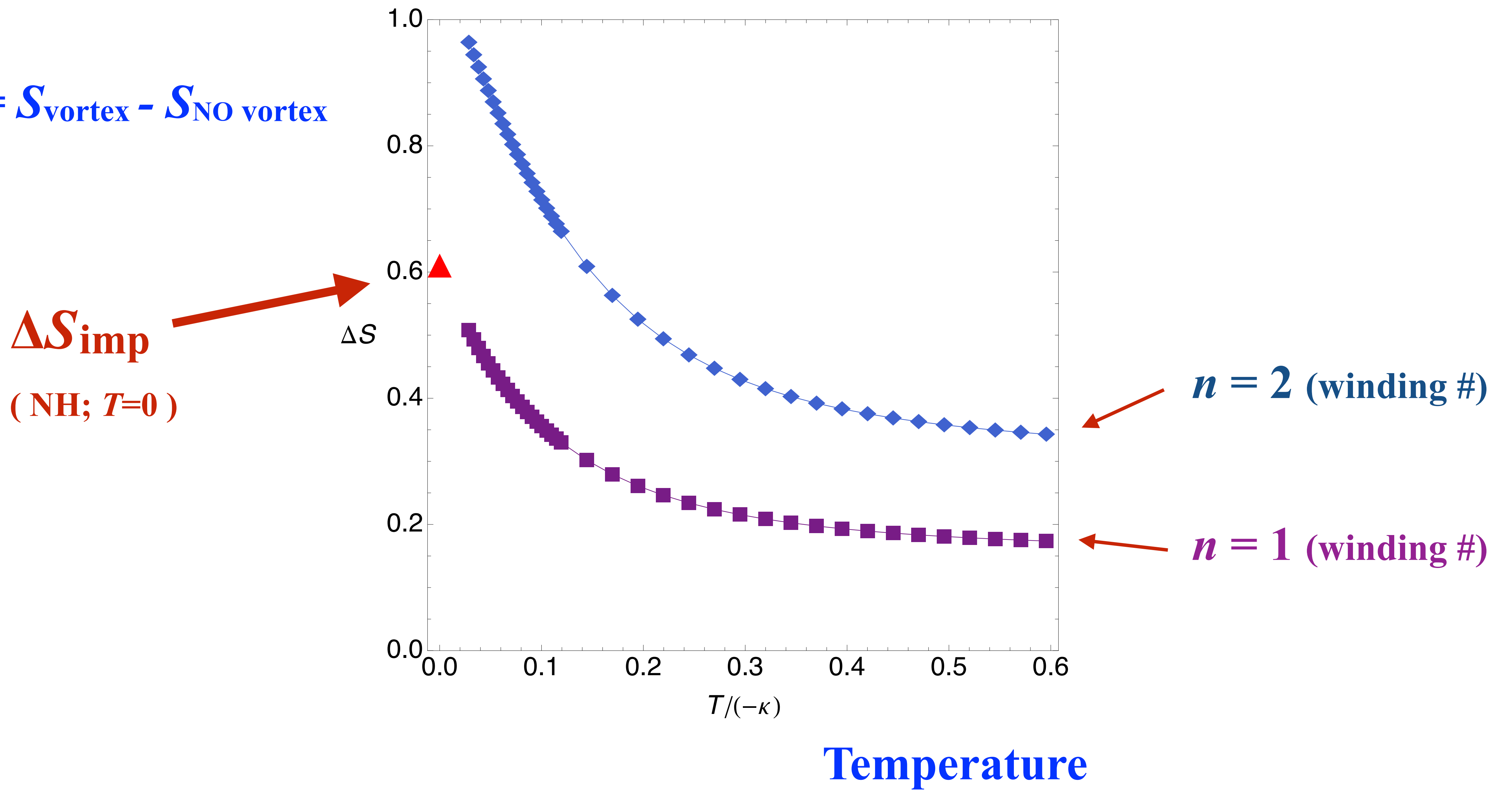
- Confirming we have the correct NH geometry:

Entropy of the **full** **vortex solution** (that extends to UV) as a function of Temperature

$\Delta S \nearrow$ as $T \searrow$: vortex causes RN horizon to bubble out as $T \to 0$

approaches $S_{\text{imp}}$ **as** $T \to 0$.

$\Delta S = S_{\text{vortex}} - S_{\text{NO vortex}}$



→ ( FULL geometry ) SC results: type I vs type II

- **Ginzburg–Landau parameter:**

$$\kappa_{LG} = \frac{\lambda}{\xi}$$

→ **London penetration depth:** how quickly $\boldsymbol{B}$ falls off

→ **Coherence length:** how quickly disturbances of the $\langle \mathcal{O} \rangle$ fall off

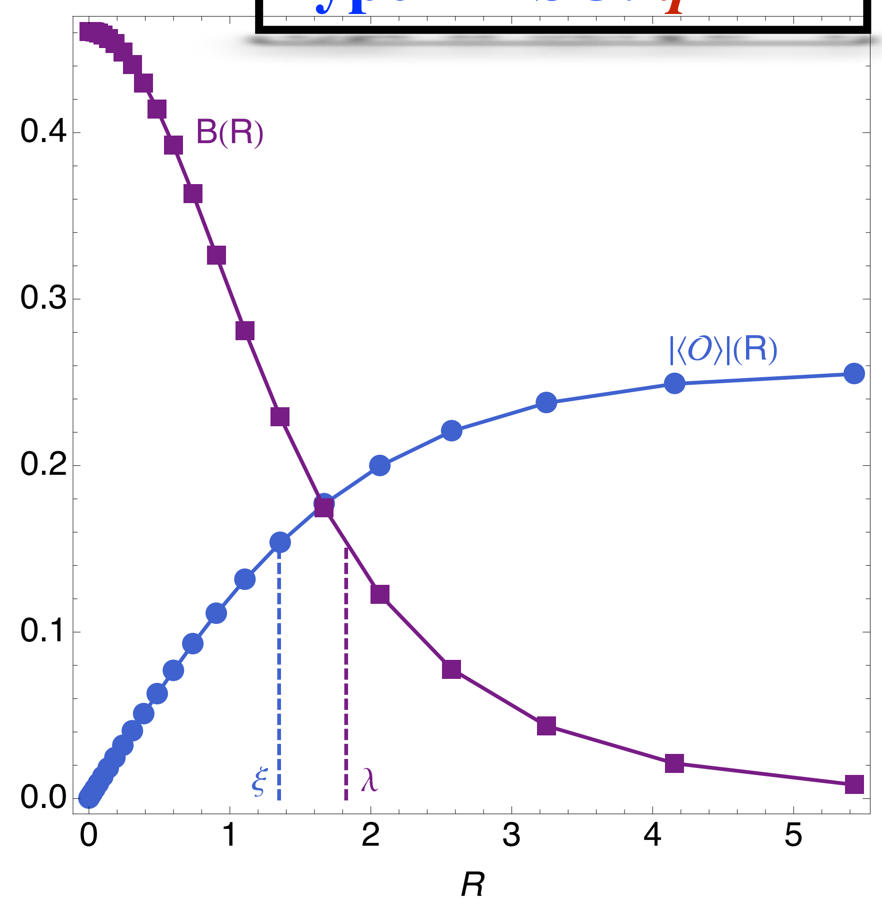
$$\kappa_{LG} > \frac{1}{\sqrt{2}} \;\to\; \text{Type II SC} \qquad\qquad \kappa_{LG} < \frac{1}{\sqrt{2}} \;\to\; \text{Type I SC}$$

- **Gravitational results:**

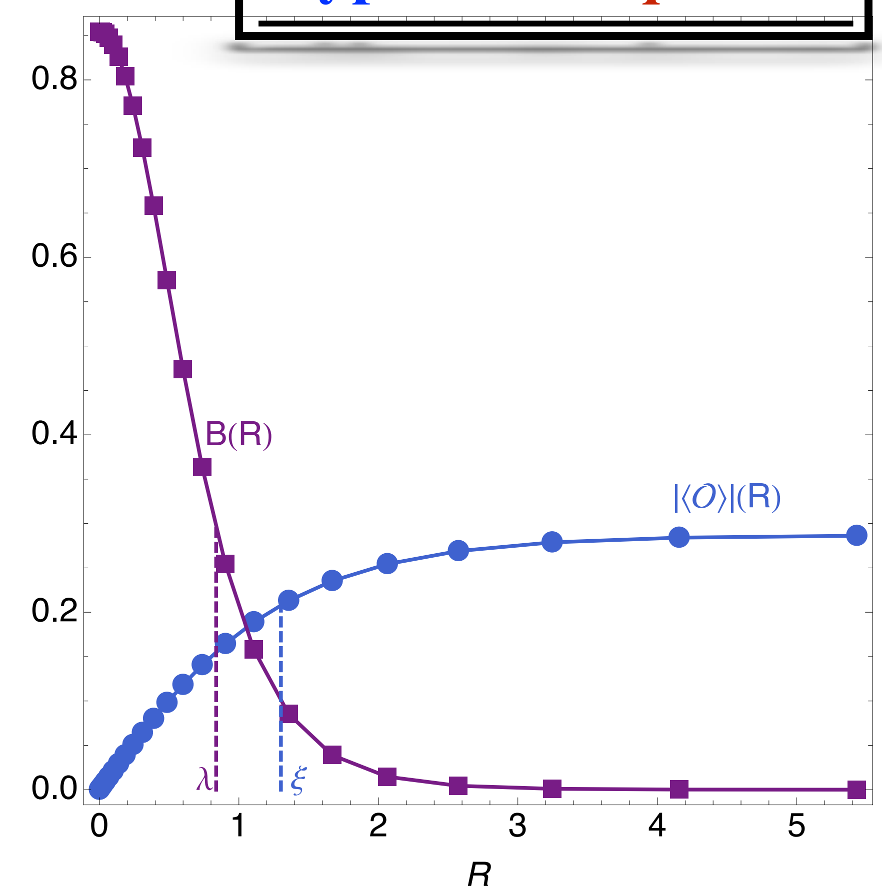
***R*: distance (@ bdry)**

to **vortex core**

**Type II SC: $qL = 1$**




**Type I SC: $qL = 3$**




**Holographic SCs can be either type I or type II, depending on the scalar charge**

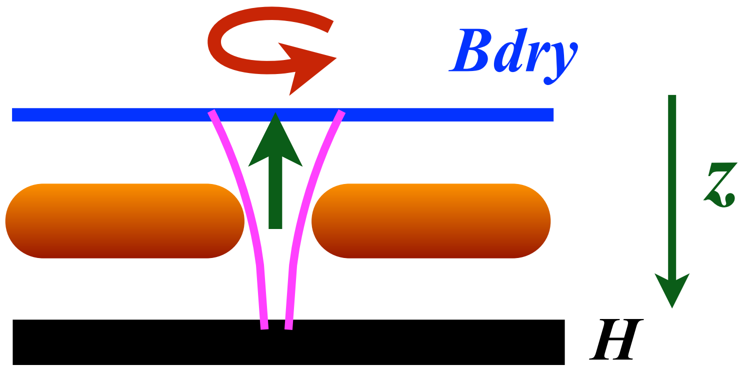
Probe limit is $q \to \infty$ **so misses it**

Conventional wisdom assumed all holographic SCs are type II

[ Umeh 2009 for earlier suggestion I / II ]

## → SC results: vortex thermodynamics & stability

- **Applied magnetic field** penetrates the condensate sample and creates region of normal phase with flux.




- **Domain wall** (DW) separating normal / SC phase **costs energy.**

- For **type I SCs**, DW costs **positive** $E$: to **minimize cost** system creates a **single large lump of Normal phase**.

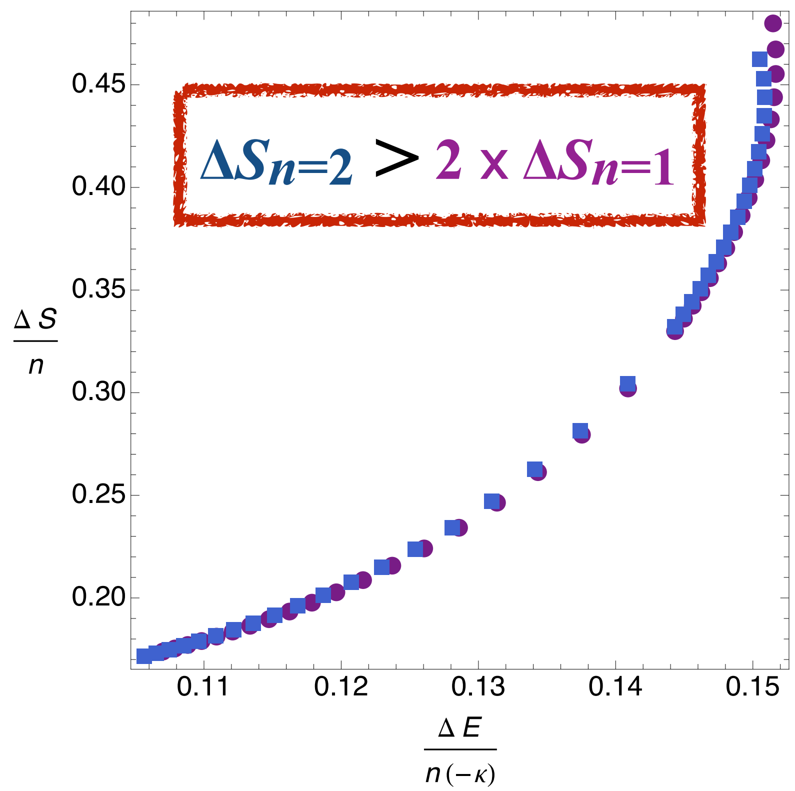
  ***n*=2 vortex** should be **energetically favoured** over two $n$=1 vortices

- For **type II SCs**, DW costs **negative** $E$: to **maximize DW length**, system tries to creates as **many vortices** with Normal phase as possible (eventually a Abrikosov lattice of vortices is favoured).

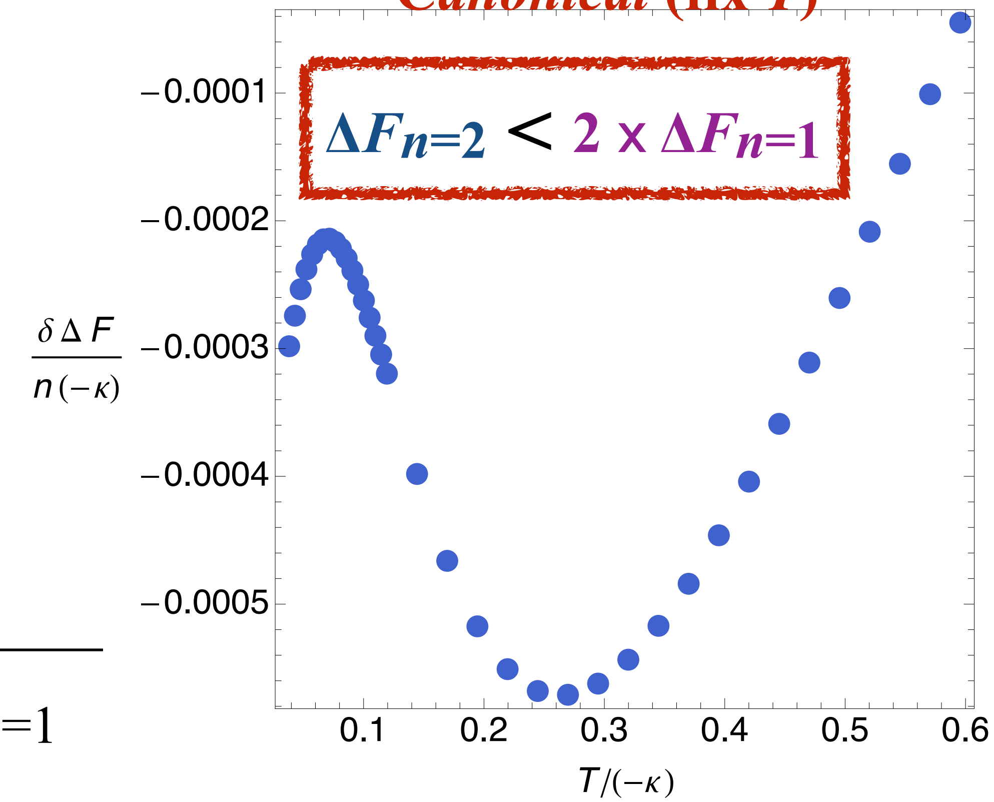
  ***n*=2 vortex** should be **unstable** to **fragmentation** into **two *n*=1 vortices**

- For **type I SCs** ( $qL \gtrsim 1.9$ ), ***n*=2 vortex** is **energetically favoured** over **two *n*=1 vortices**

*Microcanonical* (fix *E*)



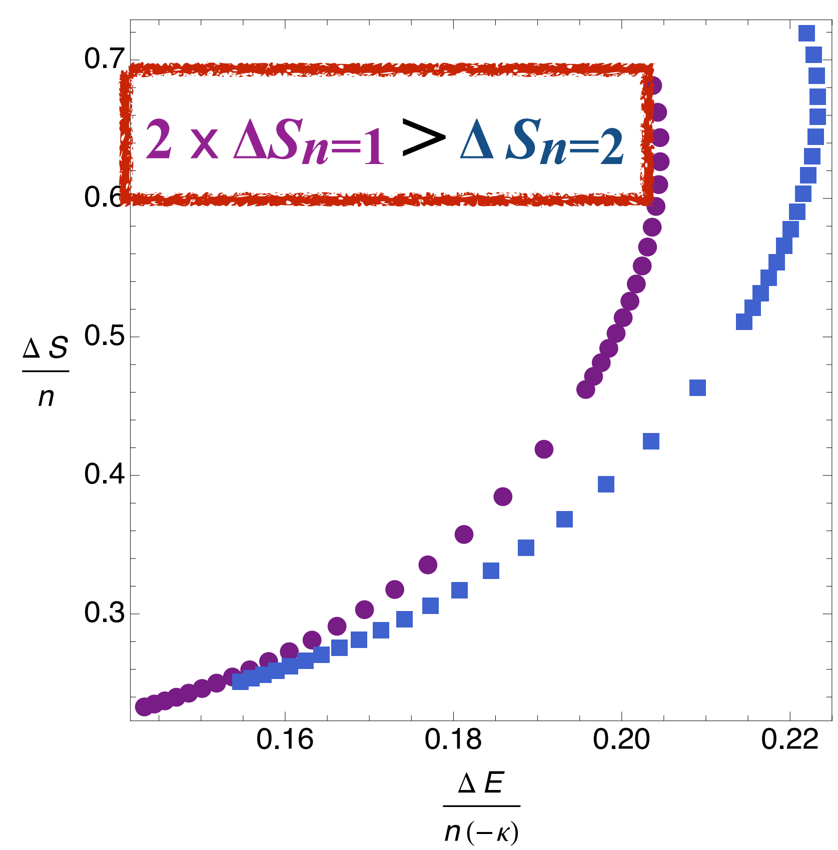

*Canonical* (fix *T*)

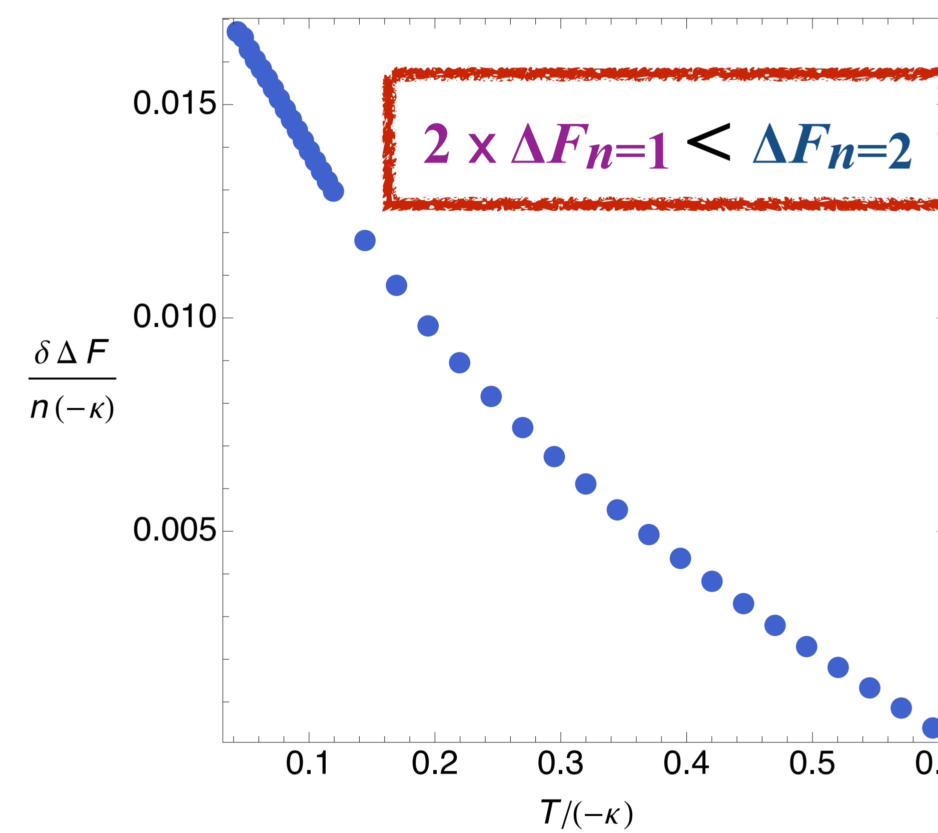



$F = E - TS$

$\delta F = F_{n=2} - 2F_{n=1}$

- For **type II SCs** ( $qL \lesssim 1.9$ ), ***n*=2 vortex** is **unstable** to **fragmentation** into **two *n*=1 vortices**

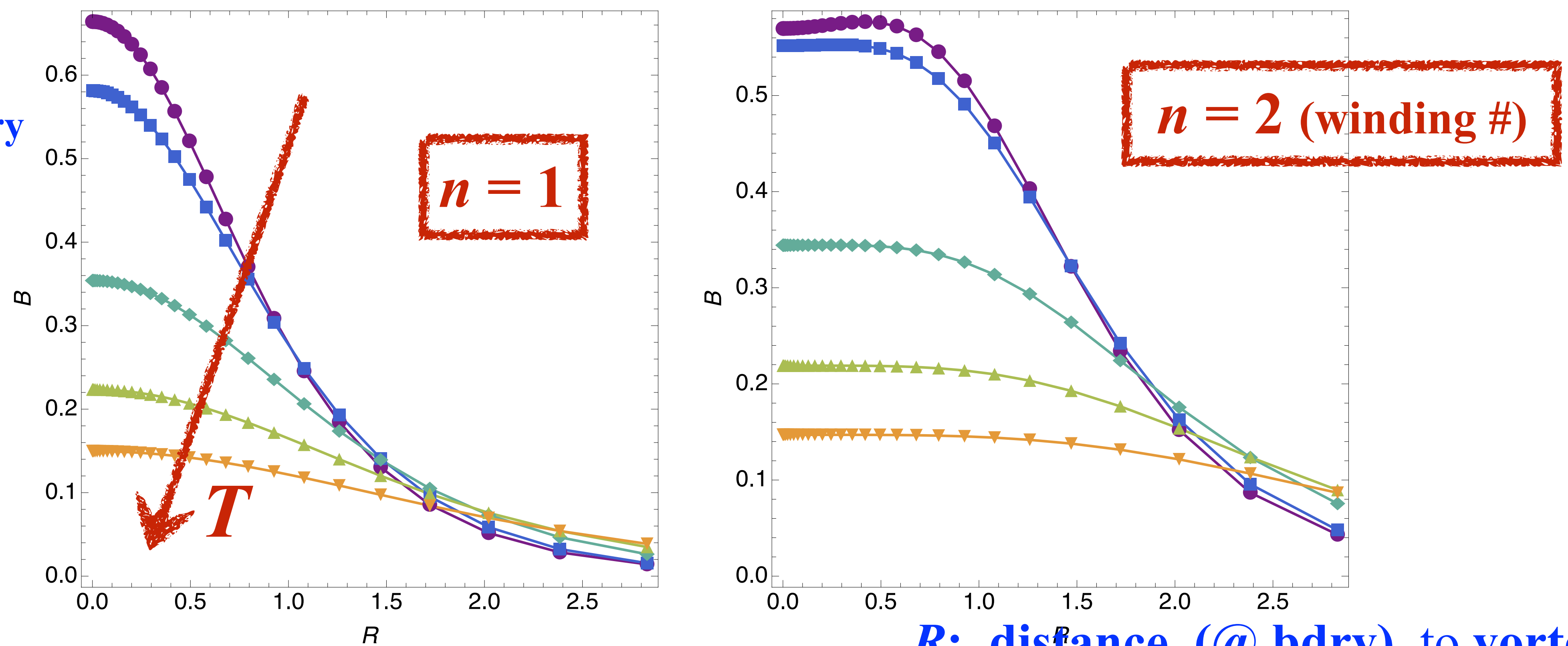


# → SC results: holographic SCs ≠ conventional SCs

- Boundary magnetic field $B_{\text{bdry}}$ as a function of distance $R$ to SC Vortex core for several Temperatures

$B_{\text{bdry}} = F_{xy}|_{\text{bdry}}$




**$R$: distance (@ bdry) to vortex core**

- $B$ falls-off exp outside a $\boldsymbol{R_{\text{core}}}$ ($\sim \kappa$ ). $\boldsymbol{R_{\text{core}}}$ remains finite even as $T \to \mathbf{0}$.
- This **contrasts** with the fall-off of **energy density**:

$\boldsymbol{\mathcal{E}(R) \sim e^{-\alpha(T)\, R}}$, inverse "energy screening length" $\boldsymbol{\alpha(T) \to 0}$ **as** $\boldsymbol{T \to 0}$**.**

- Thus @ $T$=0 the **vortex sources a long-range disturbance in the stress tensor** (requires back-reaction), due to its **interaction** with the **IR CFT**.
- Long-range tail demonstrates a **difference** between **Holographic & Conventional** SC vortices (latter source no long range fields).

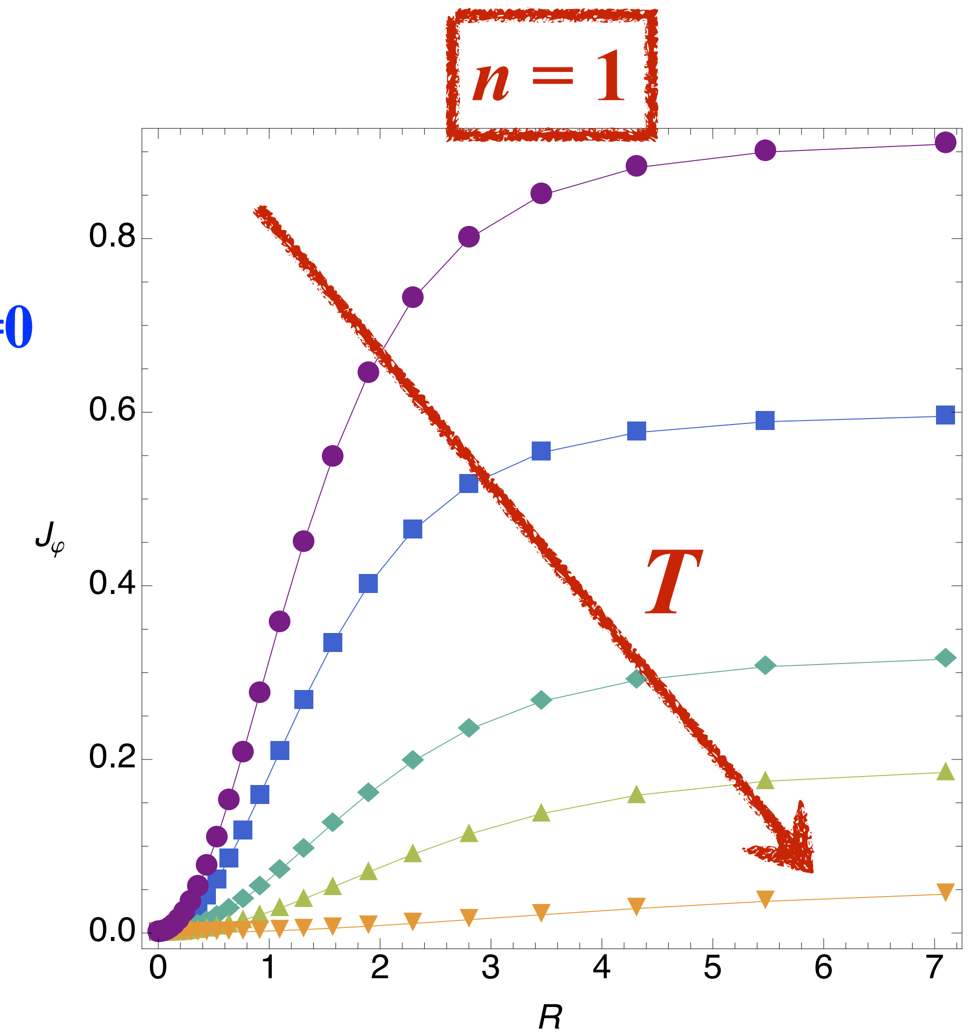
# → SUPERFLUID results:

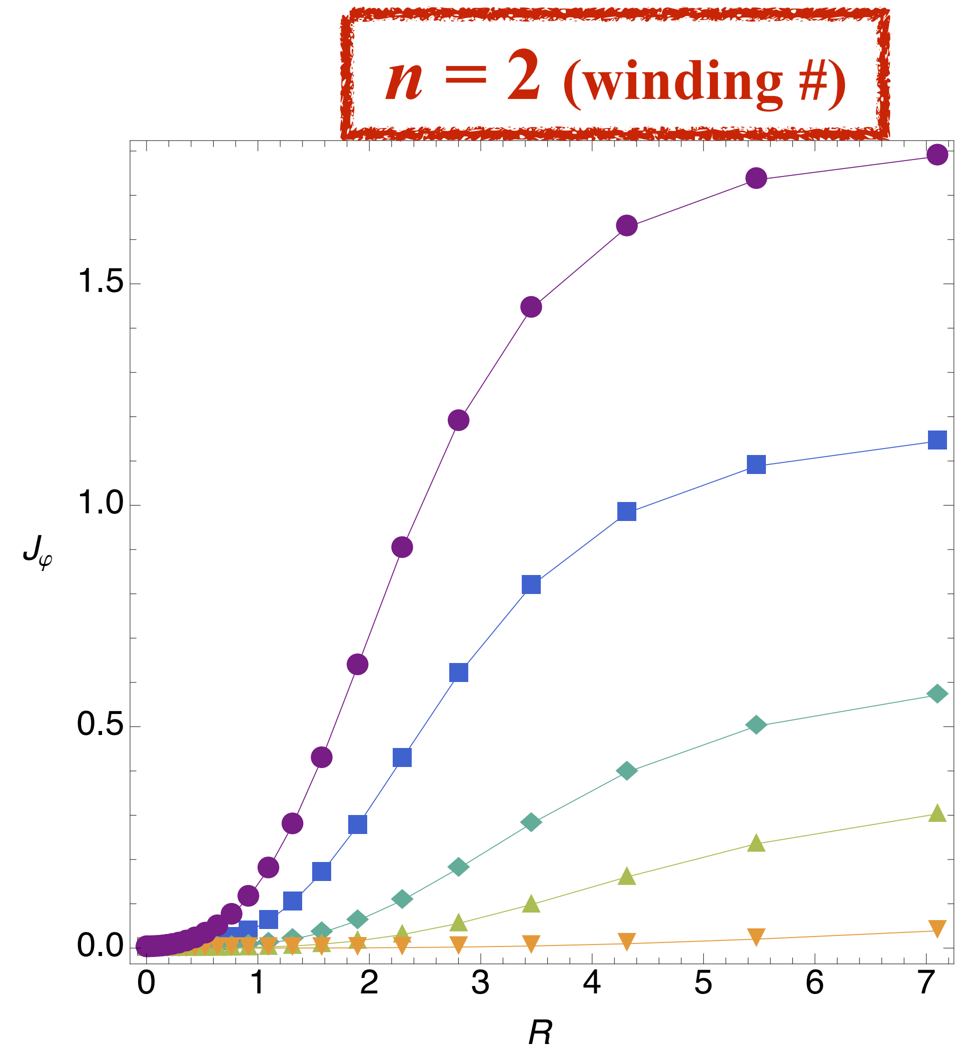
- Recall: **SC vortex** sourced by a boundary magnetic field $\boldsymbol{B} \sim \partial_R \mathbf{A}_\varphi$

  **SF vortex** sourced by boundary current $\boldsymbol{J_\varphi}$ (no applied $B$ field)

- Boundary Current $\boldsymbol{J_\varphi|_{z=0}}$ as a function of distance $R$ to the SC Vortex core for several Temperatures:

$\boldsymbol{J\varphi|_{z=0}}$




***R*: distance (@ bdry) to vortex core**

## → SUPERFLUID results:

- **SF** low-energy dynamics is given by the **action for a Goldstone mode $\theta$**: $S = \rho_s \int d^3x \, (\nabla\theta)^2$
- A vortex with winding charge $n$ has $\boldsymbol{\theta\,(R \to \infty) \sim n\,\varphi}$. It has **energy:**

$$E \sim \rho_s \int dR \frac{1}{R} n^2 \sim \rho_s \, n^2 \, \log\left(\frac{R_{IR\,cutoff}}{R_{core}}\right)$$

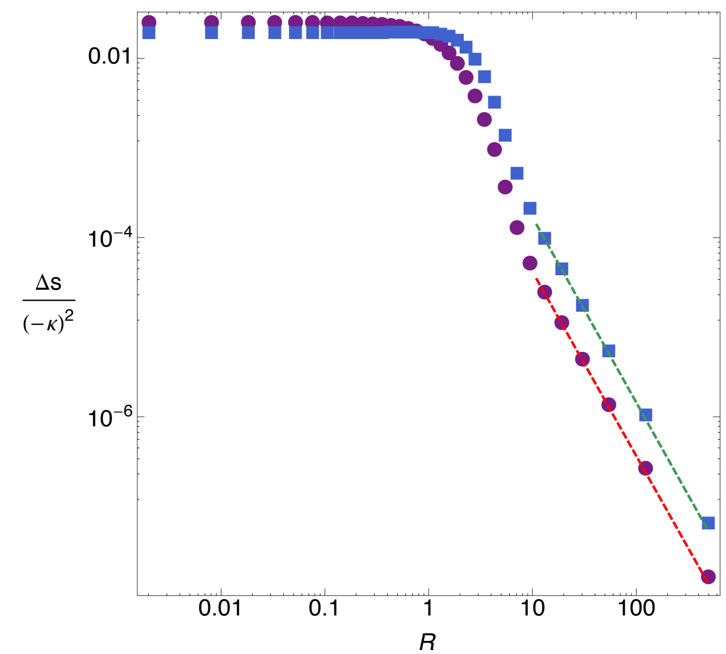
- **d$E$ = $T$ d$S$: divergent $E$ $\Rightarrow$ divergent $S$**

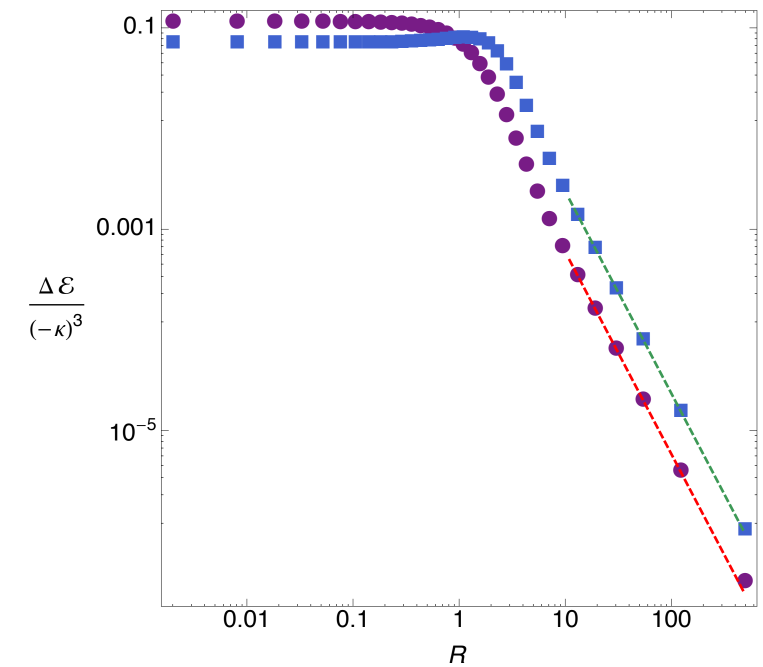
Bdry FT

## Gravity:

- Entropy & energy **densities** decay polynomially as $1/R^2$ when $R \to \infty$ (in SC case they decay exponentially)

$\Rightarrow$ $S$ & $E$ diverge ($\int_{\mathrm{vol}} \sim \log R$)




- But $\Delta s \sim f(T)\, n^2 / R^2$ with $f(T) \to 0$ as $T \to 0$ $\Rightarrow$ $\boldsymbol{\Delta S_{\mathrm{SF}} \to \Delta S_{\mathrm{imp}}}$ **as** $T \to 0$ (like SC; same NH)
- $S \sim n^2 \Rightarrow$ any **high winding charge SF vortex** is **unstable** to **fragmentation into $n$ = 1 vortices**

[ agrees with time evolution of **Adams, Chesler, Liu, 1212.0281** ]

## ➙ Take-home messages:

- Constructed **nonlinear (backreaction) holographic vortices** at **any** temperature $\boldsymbol{T}$ and condensate charge $\boldsymbol{q}$.
- **Superfluid** [ Global $U(1)$; $A_\varphi|_{z=0}=0$ ] & **Superconductor** [ Gauged $U(1)$; $J_\varphi|_{z=0}=0$ ] **vortices**.
- SC vortices **can be type I** or **type II depending** on the **scalar charge** [so far it was thought they were type II]
- **Type I / II SC** classification is **correlated** with thermodynamic **stability**
- Vortex carries magnetic flux down to horizon where it interacts with IR dof: IR $CFT_3$ ( $\neq$ UV $CFT_3$ )
- Use formalism of ***defect* CFT** to describe interaction of vortex (breaks translational invariance) with IR $CFT_3$
- Constructed the associated **near-horizon solution @ $\boldsymbol{T}$=0:**
  - "**bubble of RN-AdS horizon**" **sticking out** of the **usual Poincaré horizon**.
  - There is an **entropy** ($S_{imp}$) associated with this bubble horizon
  - $S_{imp}$ computes **entanglement entropy** via the **Ryu-Takayanagi prescription**
  - $S_{imp}$ is ***boundary*** or ***impurity*** **entropy of the defect with its surrounding**
- @ $T$=0 the **vortex sources a long-range disturbance in the stress tensor** (requires back-reaction),
  due to its **interaction with the many gapless dof of the IR CFT**.
- Long-range tail demonstrates a **difference** between **Holographic** & **Conventional** SC vortices (latter: no tail).

# Canary Wharf, London, 2014

**Ooki ni !**

("**Arigato**" in Kyoto's dialect)