Decoupling and non-decoupling dynamics of large D black holes

Roberto Emparan
ICREA & U. Barcelona & YITP Kyoto

w/ Kentaro Tanabe, Ryotaku Suzuki

Why black hole dynamics is difficult,

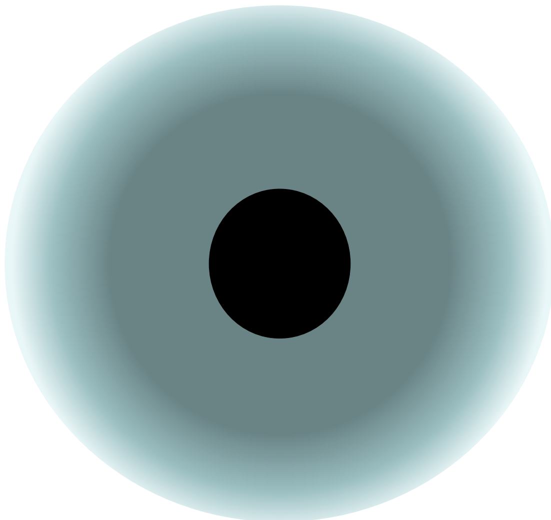
and

how $D \to \infty$ can help

BH is an extended object whose dynamics mixes strongly with background

BH's own dynamics not well-localized

Quasinormal modes spread to distance $\sim r_H$ from the horizon



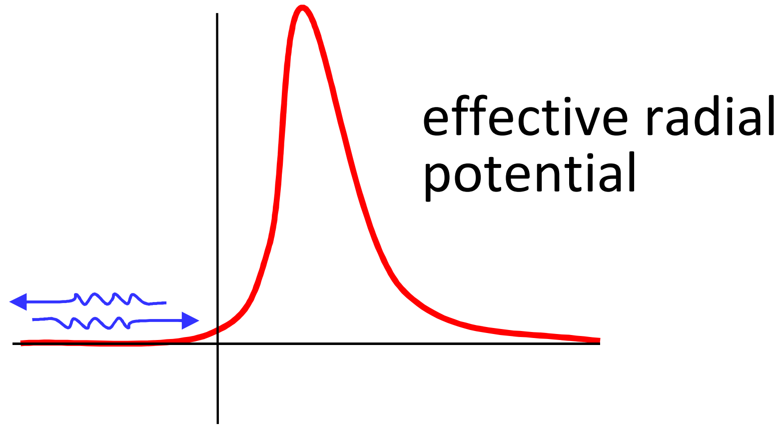
Near-extremality (w/ charges or rotation)

Localizes dynamics near-horizon

AdS/CFT-type decoupling limit

Develop a throat with

decoupled dynamics



Near-extremality → Small parameter

But this is not *generically* possible

eg for Schwarzschild: only scale is r_0

BH dynamics lacks a *generically small* parameter

Large D limit

Kol+Miyamoto et al
RE+Suzuki+Tanabe

1/D as small parameter

Separates bh's own dynamics from background spacetime

- *sharp* localization of bh dynamics

Well-defined near-horizon geometry

- a very special $2D$ bh
- *not* a decoupling geometry
- but distinct decoupled/non-decoupled dynamics

Large *D* black holes

Basic solution

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right) dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

length scale r_0

Large *D* black holes

r_0 not the only scale

Small *parameter* $1/D \Longrightarrow$ scale hierarchy

$$r_0/D \ll r_0$$

Localization of interactions

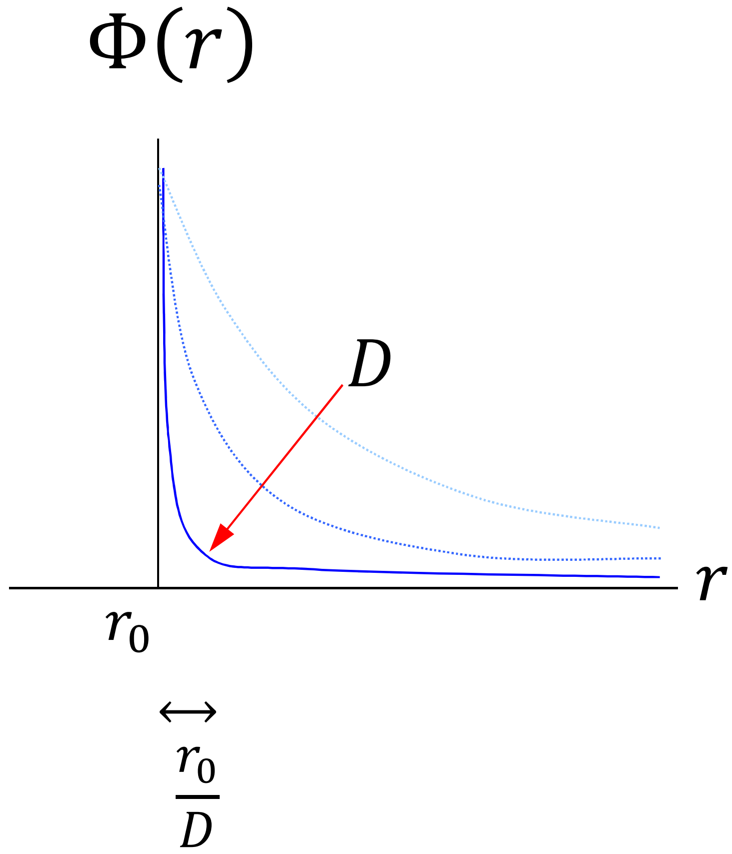
Large potential gradient:

$$\Phi(r) \sim \left(\frac{r_0}{r}\right)^{D-3}$$

$$\nabla\Phi\Big|_{r_0} \sim D/r_0$$

$\Longrightarrow$ Hierarchy of scales

$$\frac{r_0}{D} \ll r_0$$



Far zone

Fixed $r > r_0$ $\quad D \to \infty$

$$f(r) = 1 - \left(\frac{r_0}{r}\right)^{D-3} \to 1$$

$$ds^2 \to -dt^2 + dr^2 + r^2 d\Omega_{D-2}$$

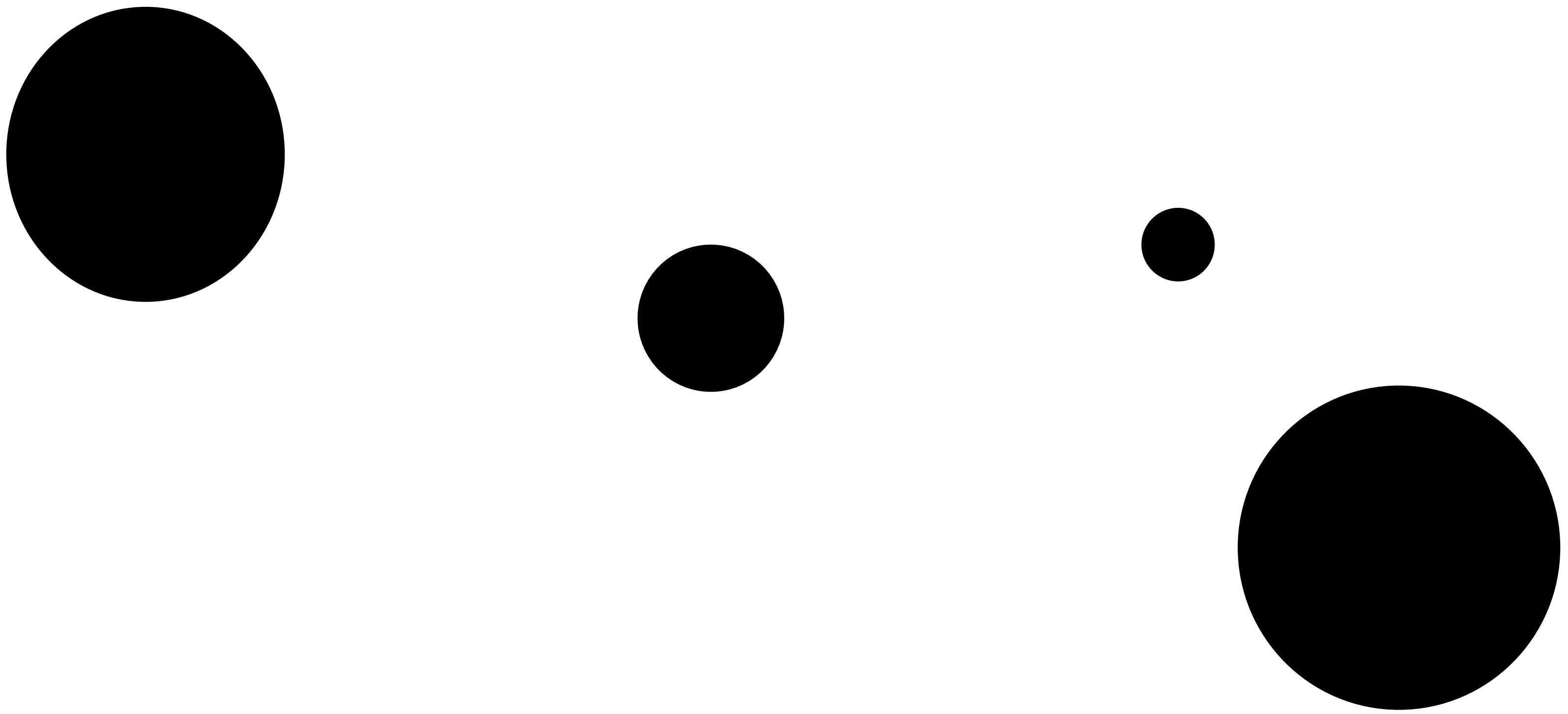
Flat, empty space at $r > r_0$

no gravitational field

Far zone geometry

scale $\mathcal{O}(r_0 D^0)$

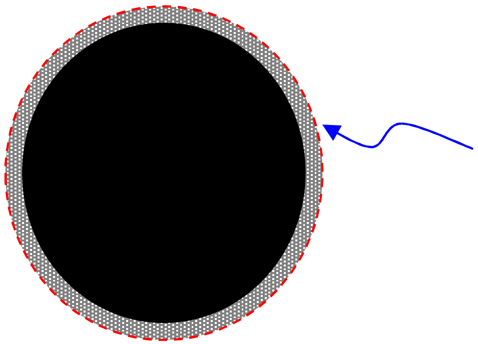
Holes cut out in Minkowski space



Near zone

Gravitational field appreciable only in *thin* near-horizon region

$$\left(\frac{r_0}{r}\right)^{D-3} = \mathcal{O}(1) \;\Leftrightarrow\; r - r_0 < \frac{r_0}{D}$$



$$r - r_0 \sim \frac{r_0}{D}$$

Near zone

$$ds^2 = -\left(1-\left(\frac{r_0}{r}\right)^{D-3}\right)dt^2 + \frac{dr^2}{1-\left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

$$\left.\begin{aligned} \left(\frac{r}{r_0}\right)^{D-3} &= \cosh^2\rho \\ t_{near} &= \frac{D}{2r_0}t \end{aligned}\right\} \begin{array}{l}\text{finite} \\ \text{as } D\to\infty\end{array}$$

Near zone

$$ds_{nh}^2 \to \frac{4r_0^2}{D^2}\Big(\underbrace{-\tanh^2\rho \; dt_{near}^2 + d\rho^2}_{\textbf{2d string black hole}}\Big) + r_0^2 d\Omega_{D-2}^2$$

Elitzur et al
Mandal et al
Witten

Soda
Grumiller et al

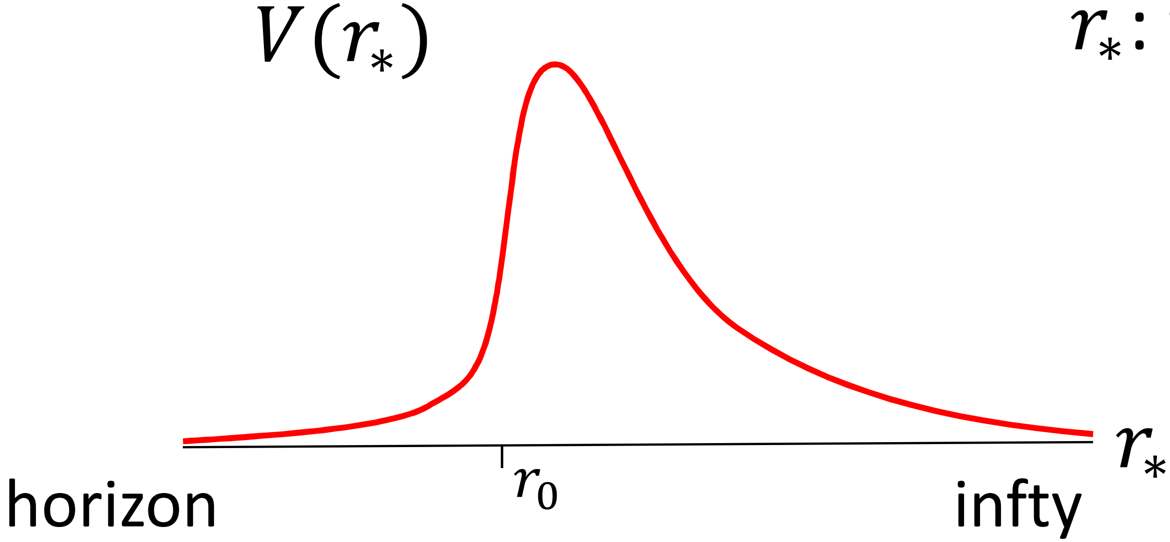
$$\ell_{string} \sim \frac{r_0}{D}, \qquad \alpha' \sim \left(\frac{r_0}{D}\right)^2$$

Black hole perturbative dynamics @ large D

Massless scalar field

$\Box\Phi = 0$ $\qquad$ $\Phi = r^{-\frac{D-2}{2}}\,\phi(r)\,e^{-i\omega t}\,Y_\ell(\Omega)$

$$\frac{d^2\phi}{dr_*^2} + (\omega^2 - V(r_*))\phi = 0$$



r_*: tortoise coord

Schwarzschild bh grav perturbations

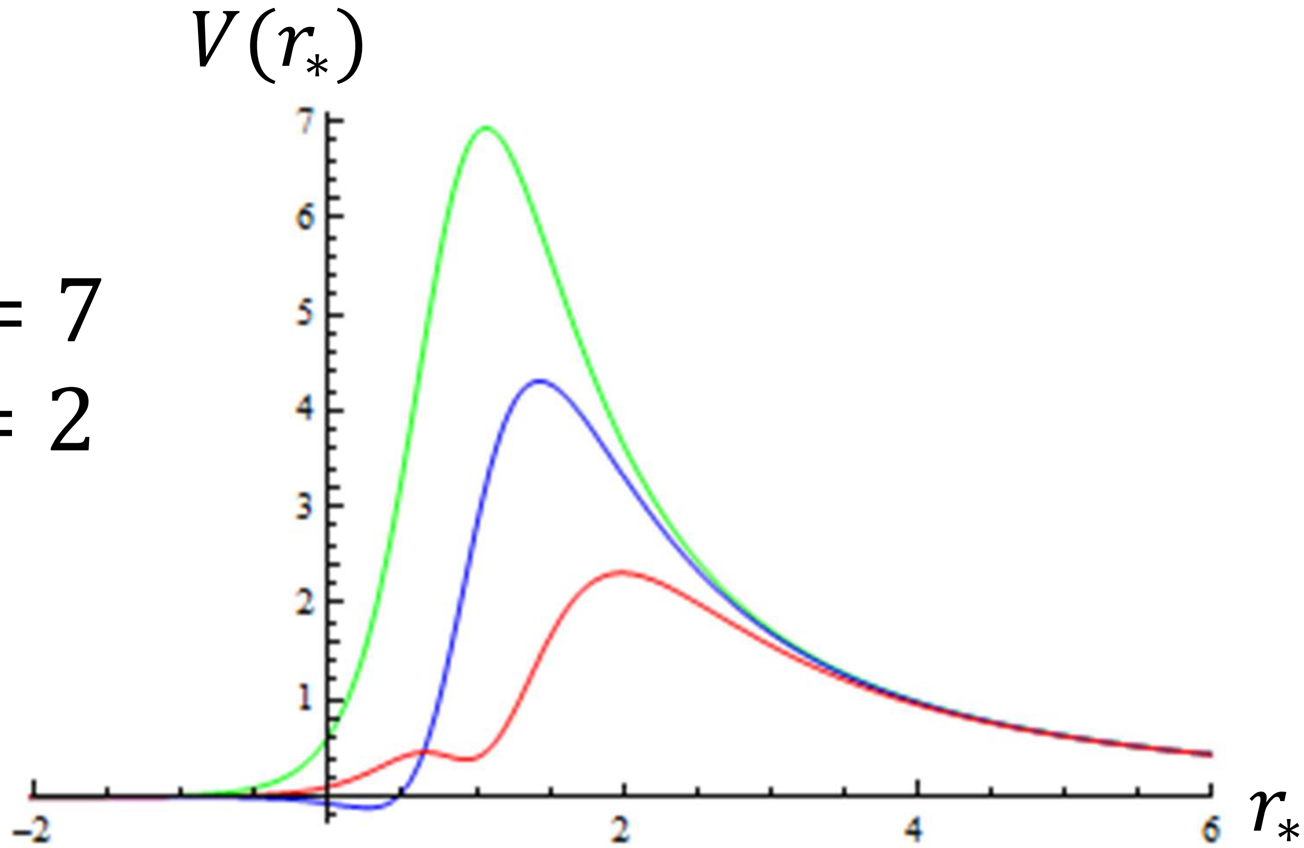
Kodama+Ishibashi

Gravitational scalar, vector, tensor modes

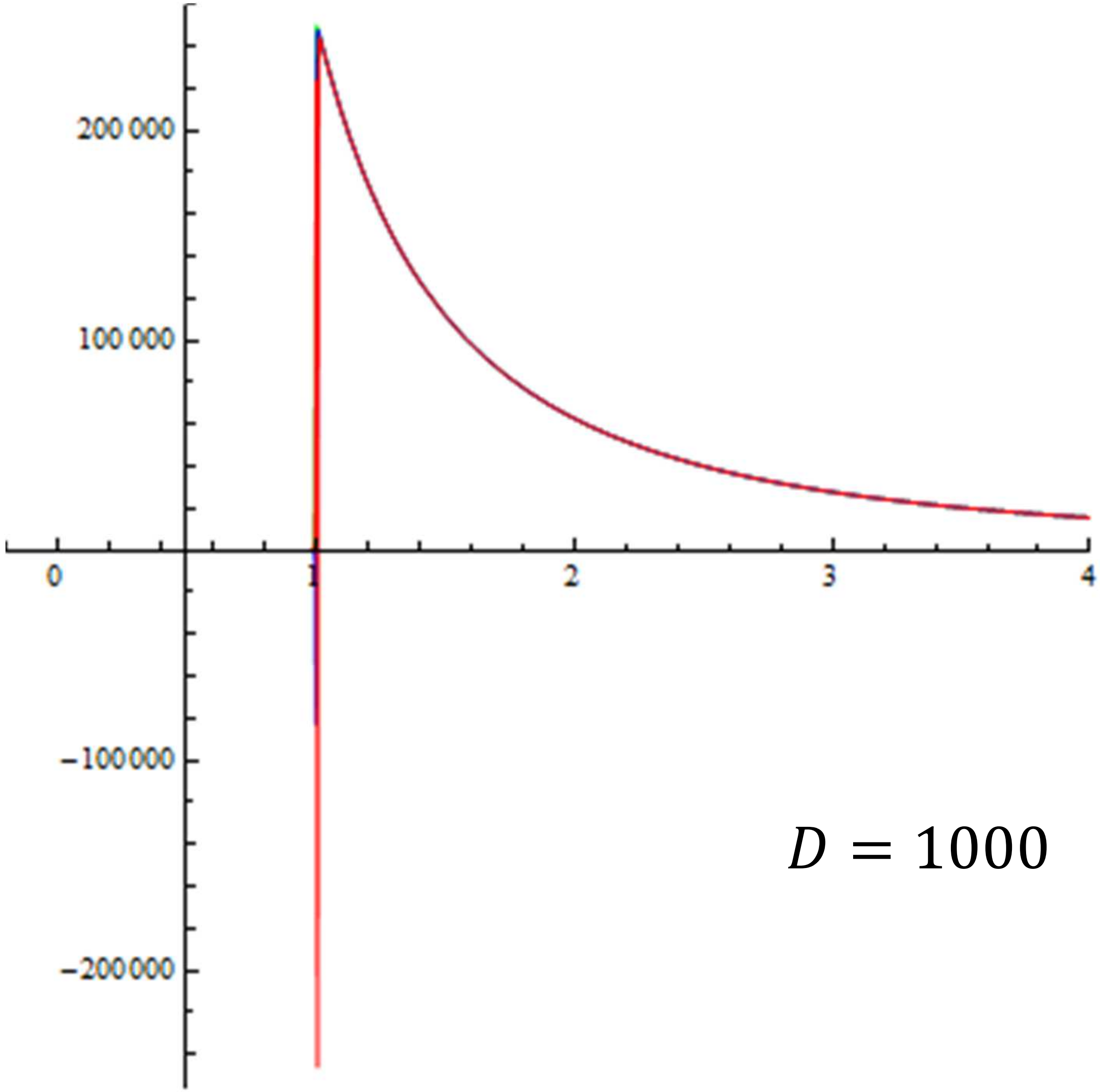
$SO(D-1)$ reps

$D = 7$

$\ell = 2$

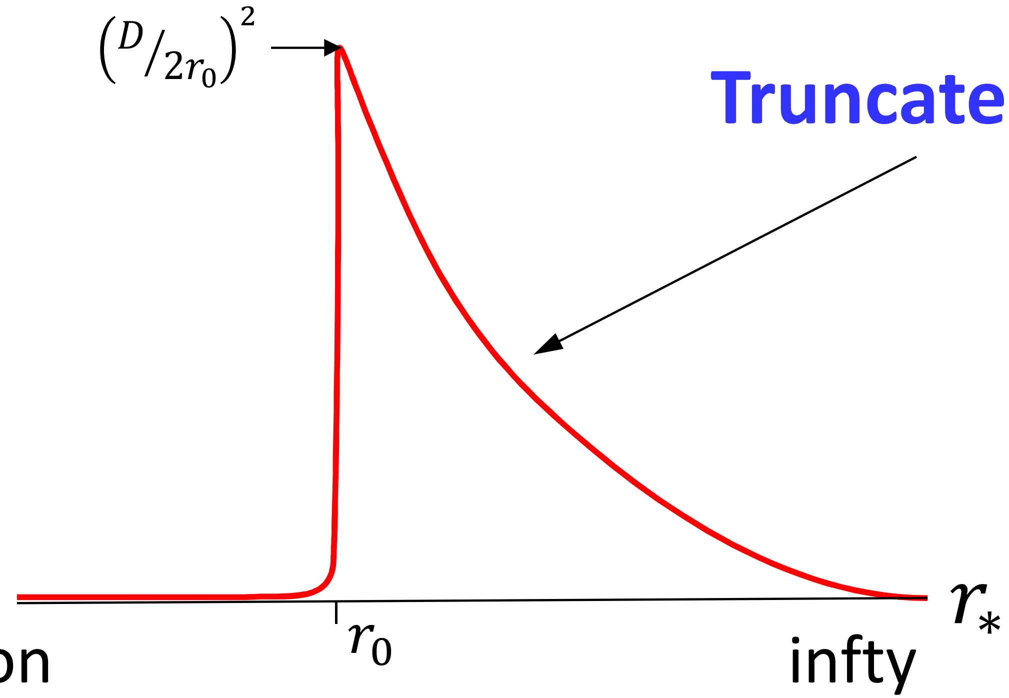


scalar vector tensor @ large D



$D \to \infty$

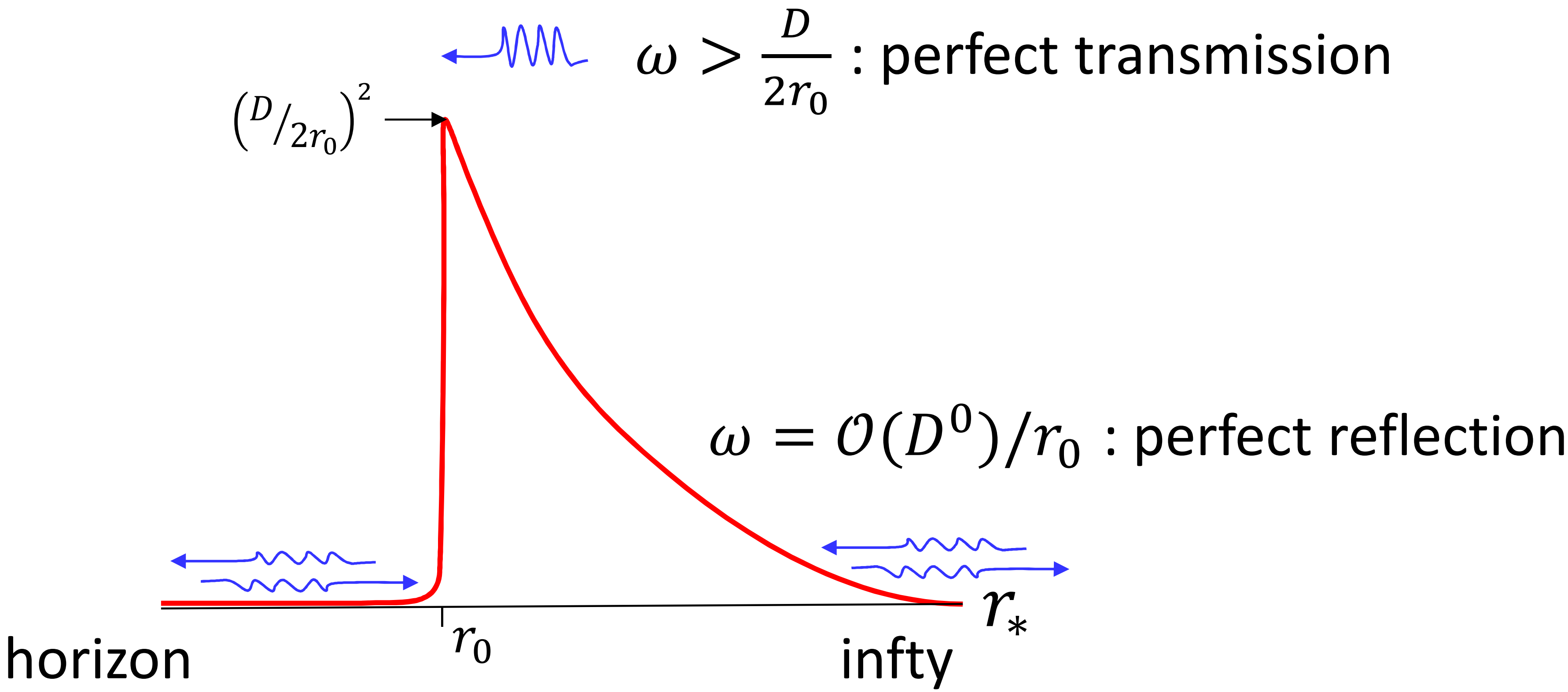
$$V(r_*) \to \frac{D^2}{4r_*^2}\Theta(r_* - r_0)$$



$\ell = \mathcal{O}(1)$

(for simplicity)

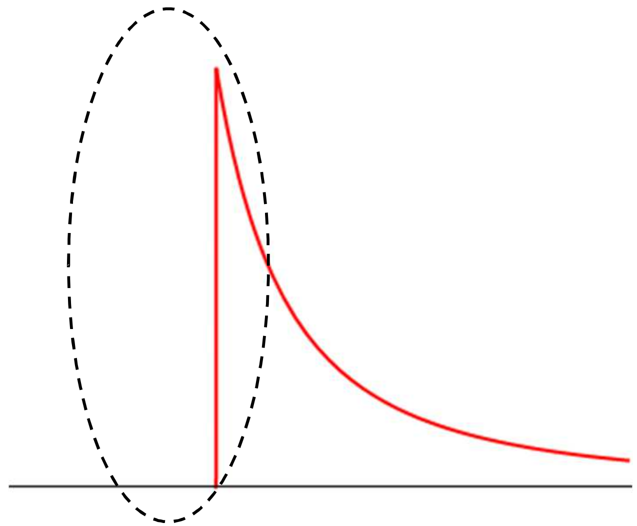
$$V(r_*) \to \frac{D^2}{4r_*^2}\Theta(r_* - r_0)$$



Near-horizon view

$t_{near} \sim D\, t$: fast n-h time

$\omega_{near} \sim \hat{\omega} \equiv \frac{\omega}{D}$



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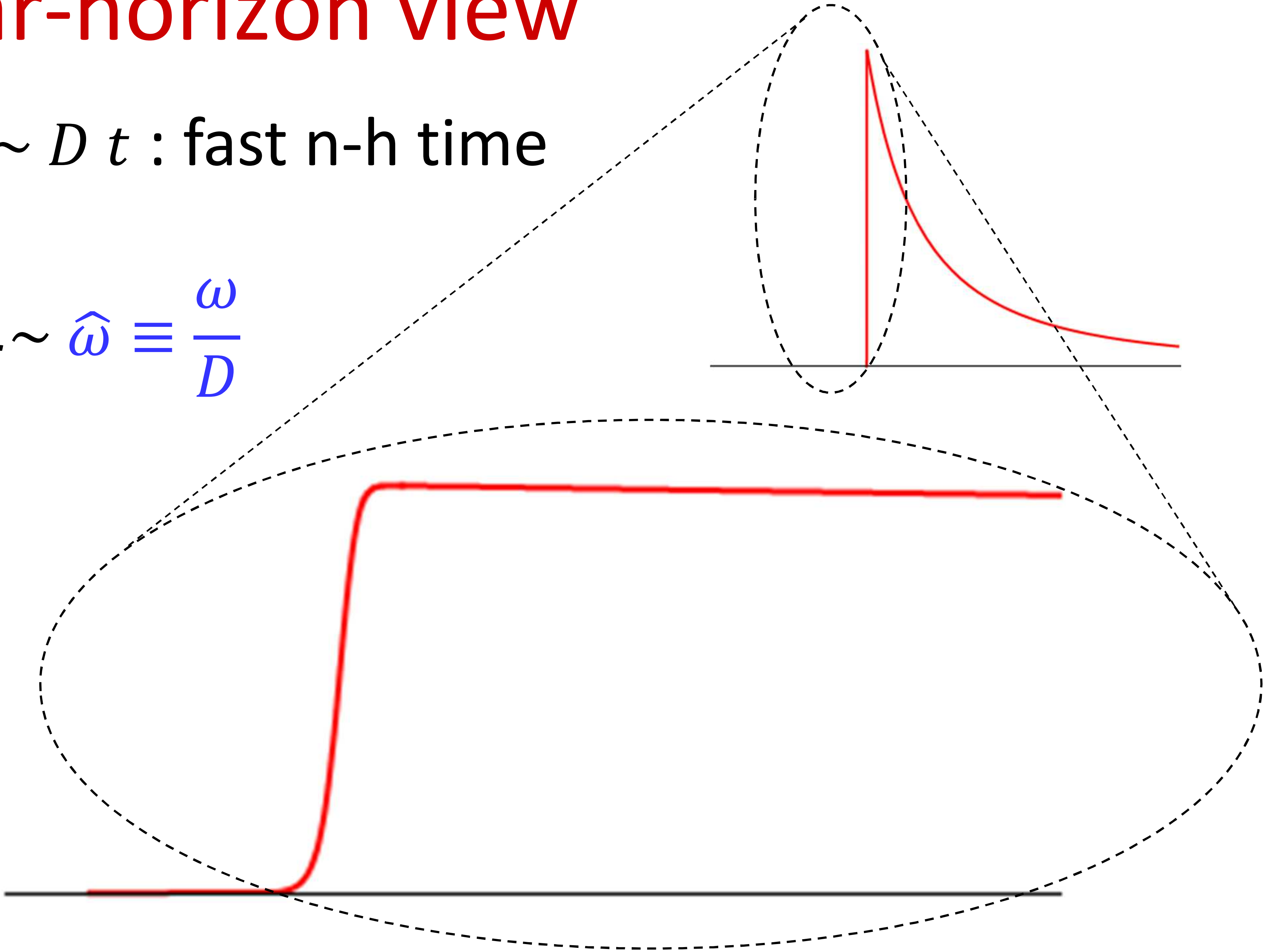
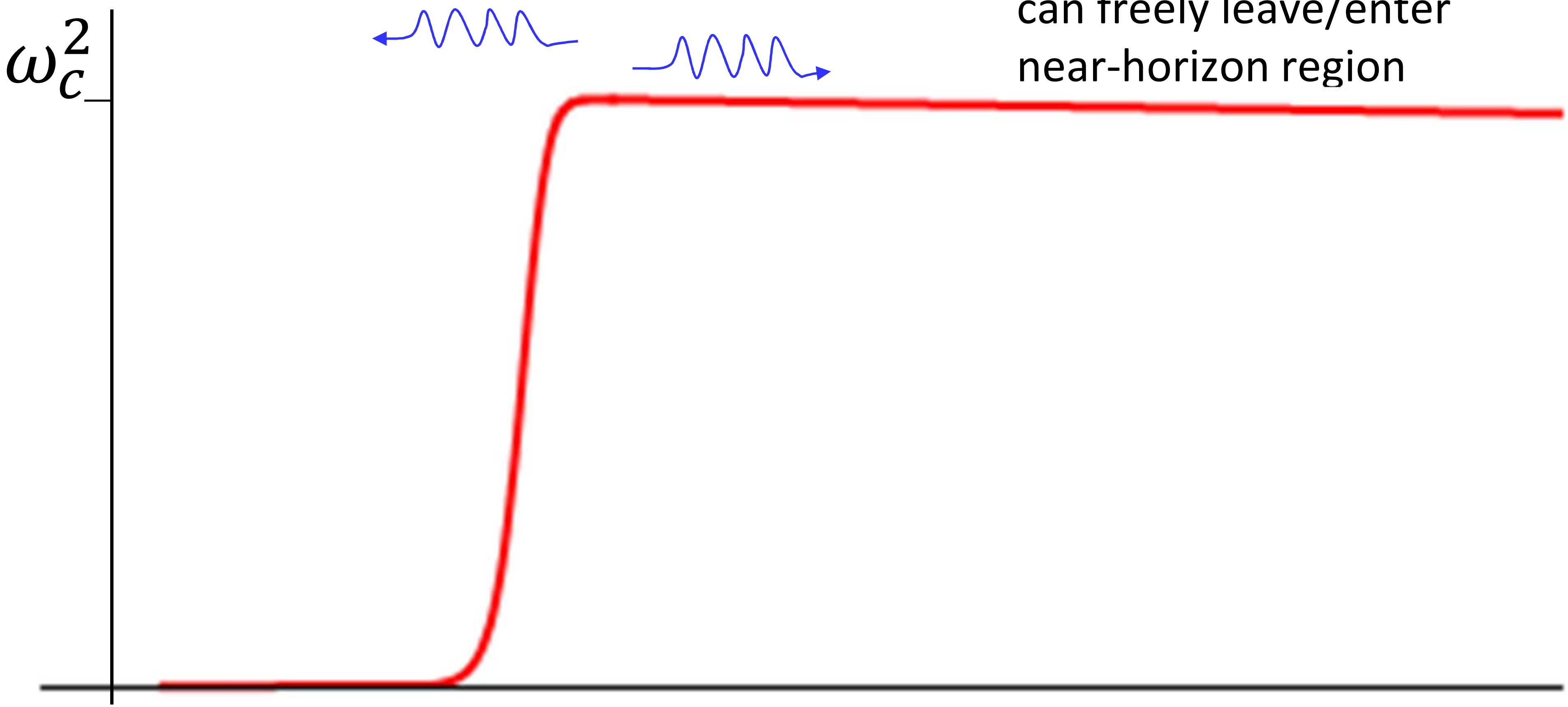


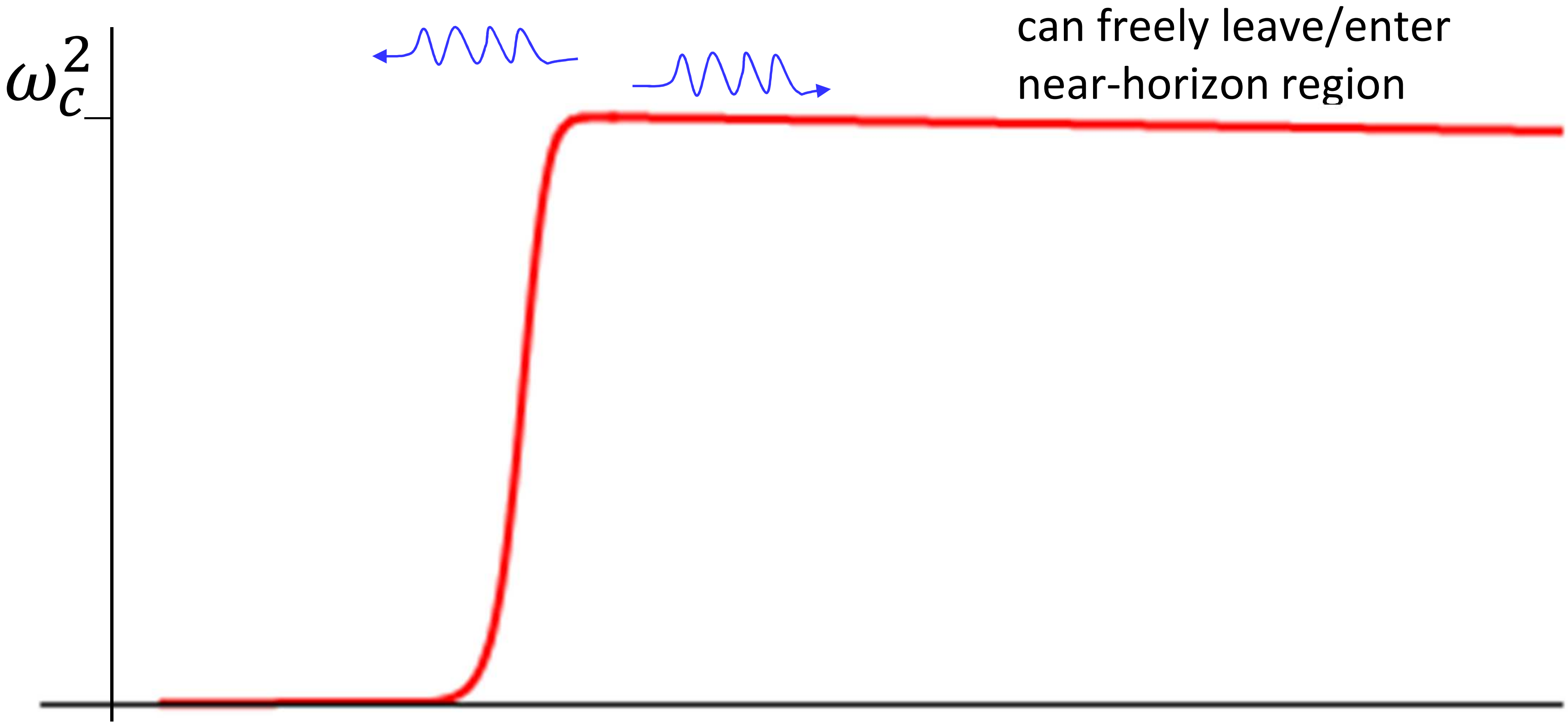
Near-horizon excitations

$\hat{\omega} > \omega_c$



Near-horizon excitations

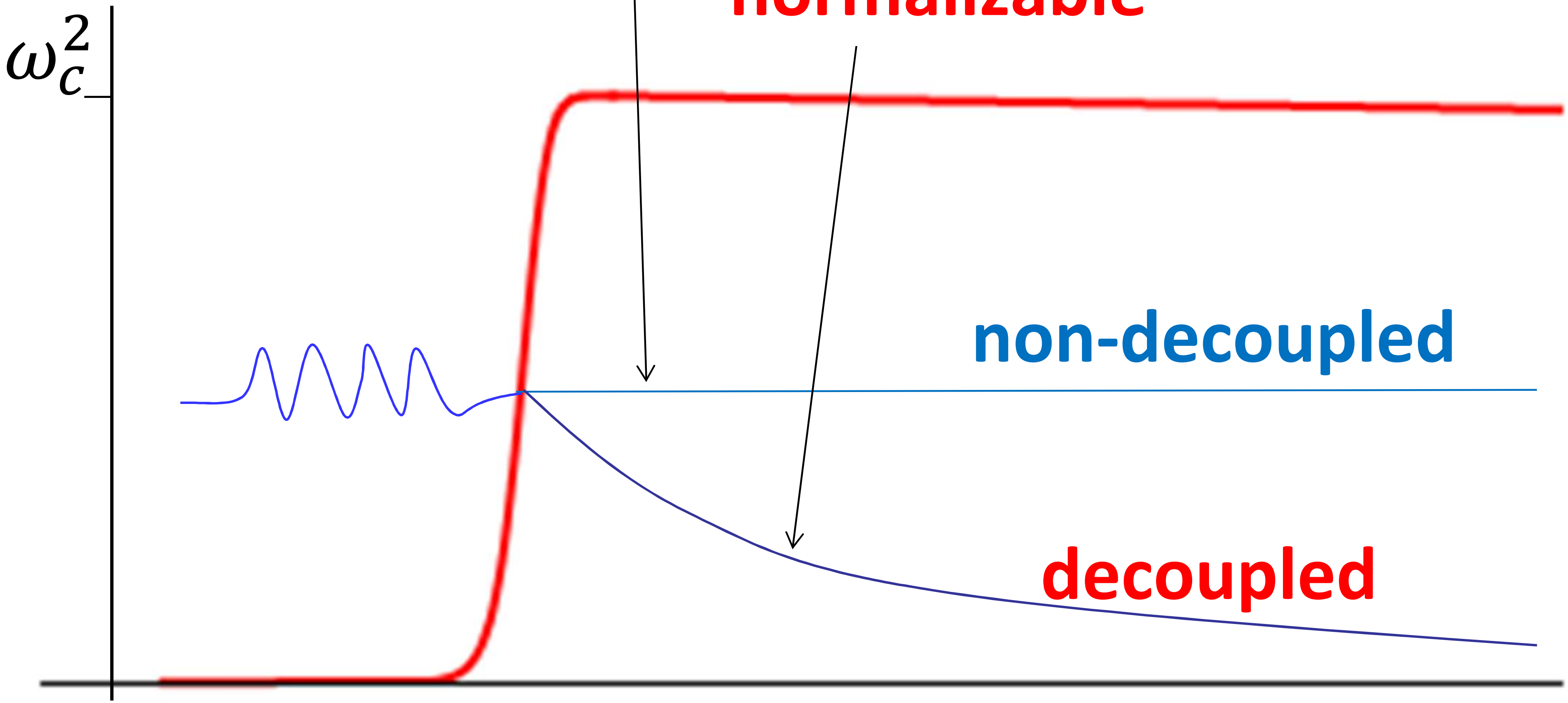
$\widehat{\boldsymbol{\omega}} > \boldsymbol{\omega_c}$: violate near-horizon unitarity (BF) bound



Near-horizon excitations

$\mathbf{0} < \widehat{\boldsymbol{\omega}} < \boldsymbol{\omega_c}$: **non-normalizable**

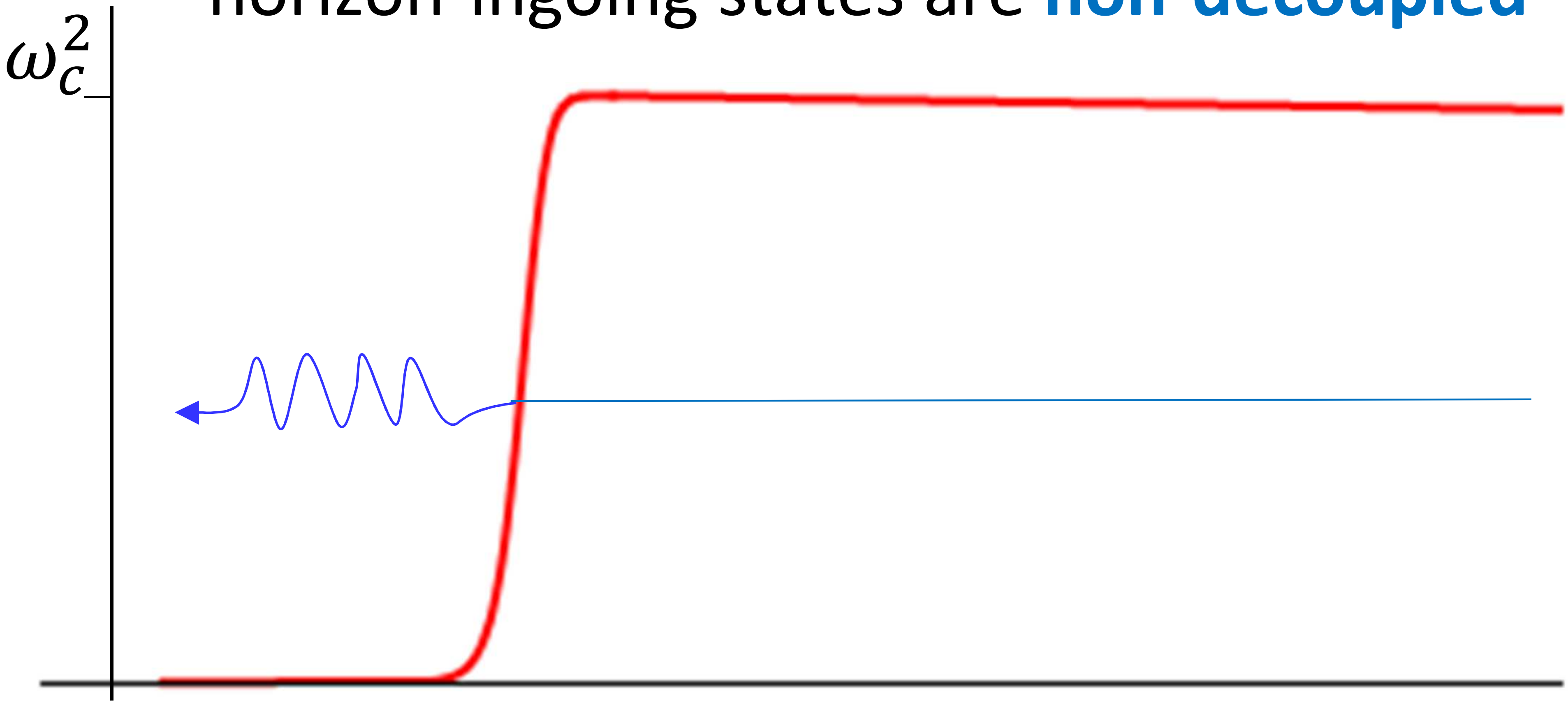
normalizable



Near-horizon excitations

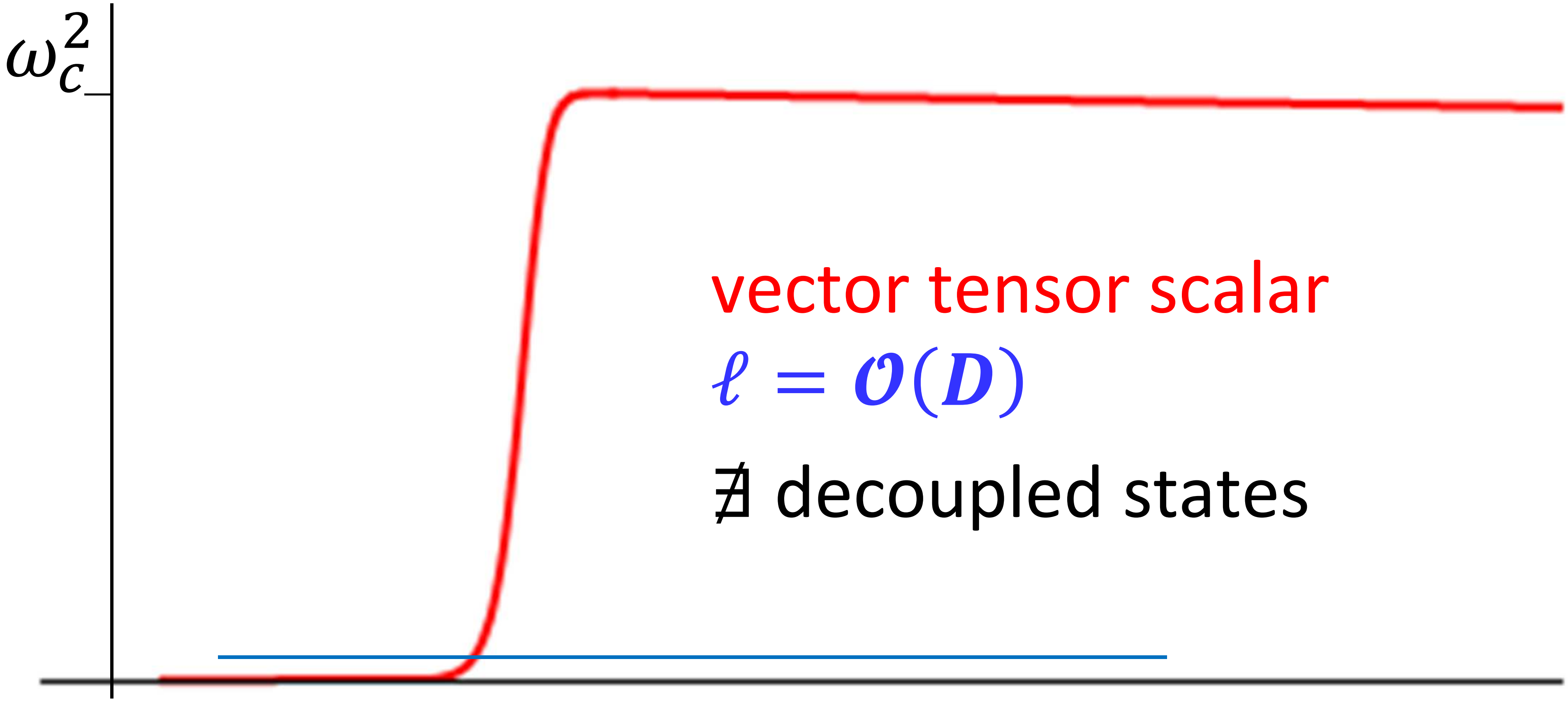
$$\mathbf{0} < \hat{\boldsymbol{\omega}} < \boldsymbol{\omega_c}$$

horizon-ingoing states are **non-decoupled**



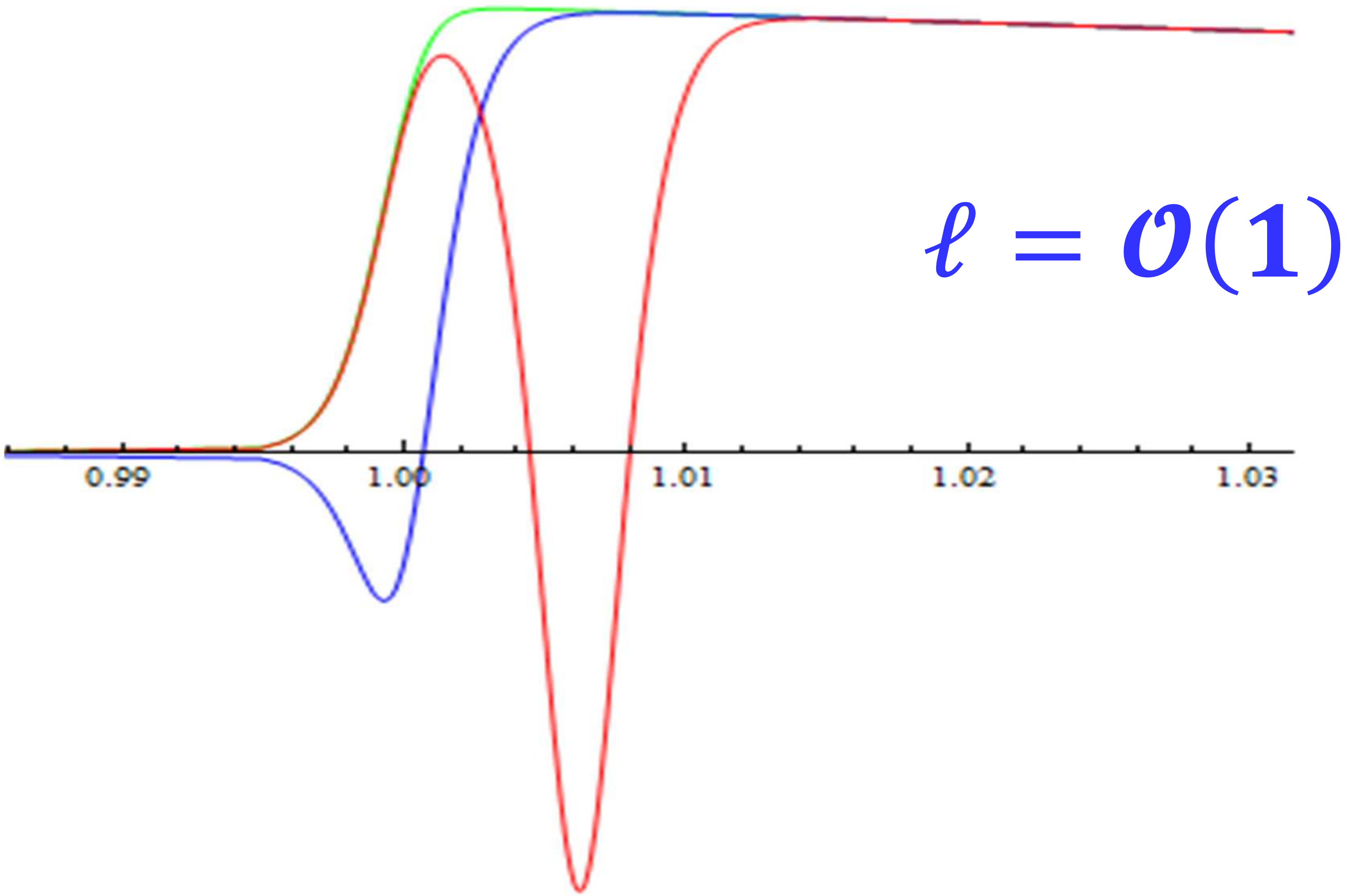
$$\boldsymbol{\omega} = \mathcal{O}\left(\boldsymbol{D}^{\mathbf{0}}\right) \;\rightarrow\; \widehat{\boldsymbol{\omega}} = \mathbf{0}:$$

normalizable zero-energy states = decoupled



$\boldsymbol{\omega} = \mathcal{O}\left(D^{0}\right) \;\rightarrow\; \widehat{\boldsymbol{\omega}} = \mathbf{0}$:   zero-energy states

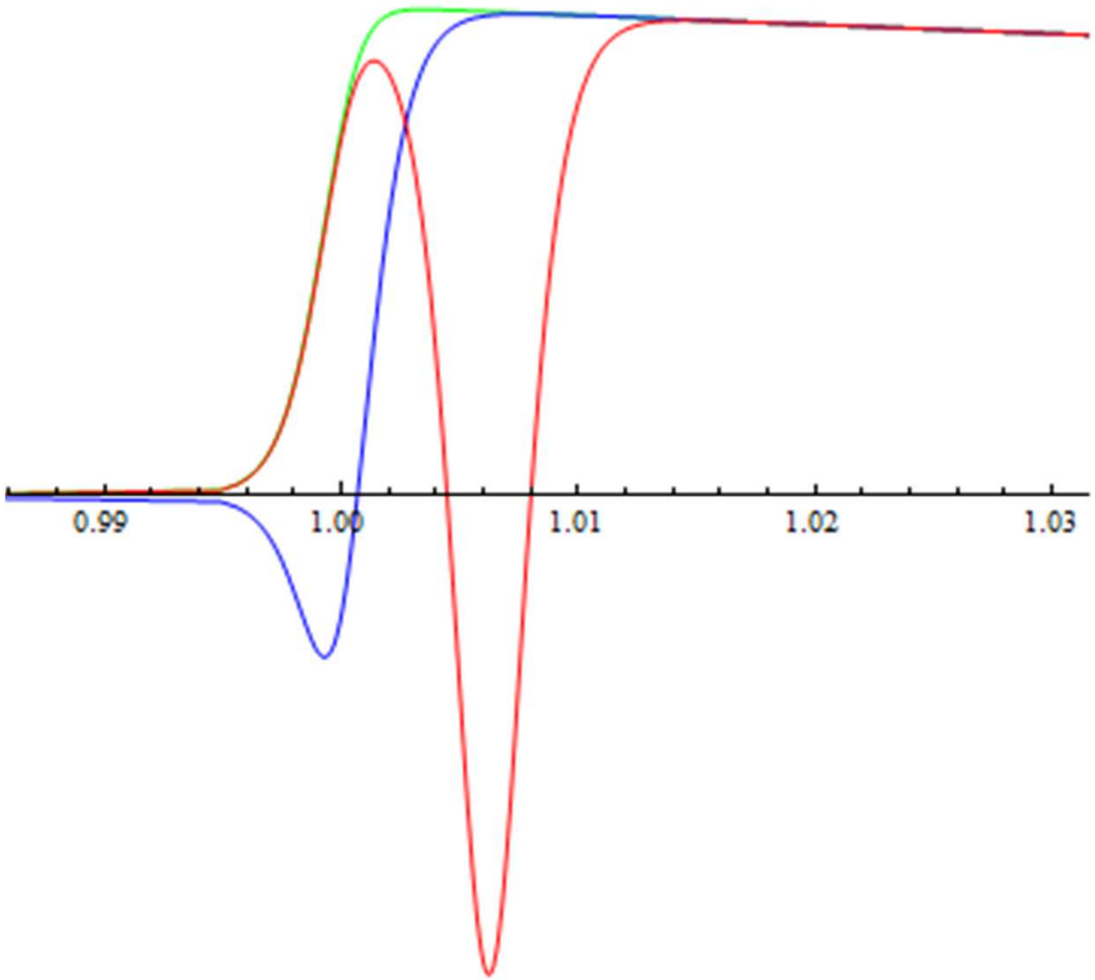
scalar vector tensor



$$\boldsymbol{\omega = \mathcal{O}\left(D^{0}\right) \;\rightarrow\; \widehat{\omega} = 0}:$$ zero-energy states

scalar vector tensor

$$\boldsymbol{\ell = \mathcal{O}(1)}$$



scalar & vector admit zero-energy states

∃ decoupled states

Summing up so far

BH dynamics can be classified according to near-horizon (n-h) geometry:

Unitarity (B-F)-violating states: freely leave n-h

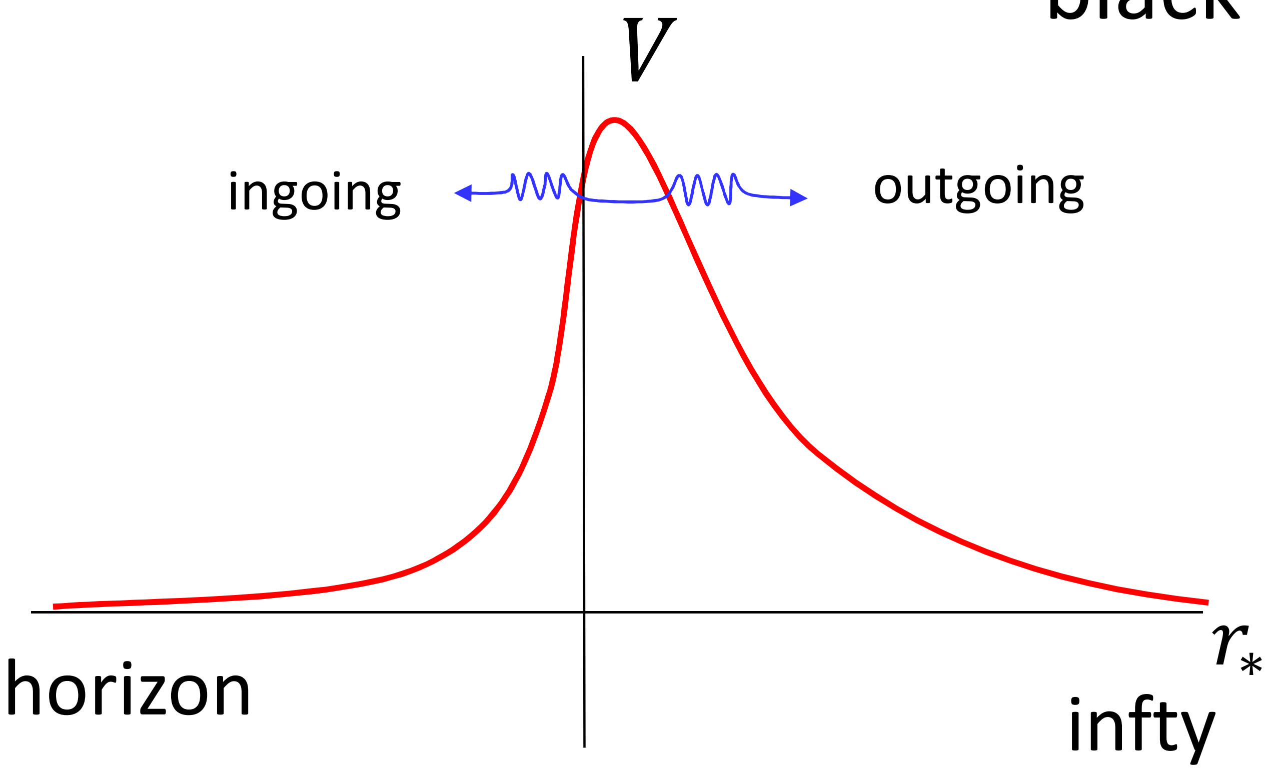
Non-normalizable states: **non-decoupled** dynamics

Normalizable states: **decoupled** non-dynamics
(dynamical @ NLO in 1/D)

BH quasinormal modes

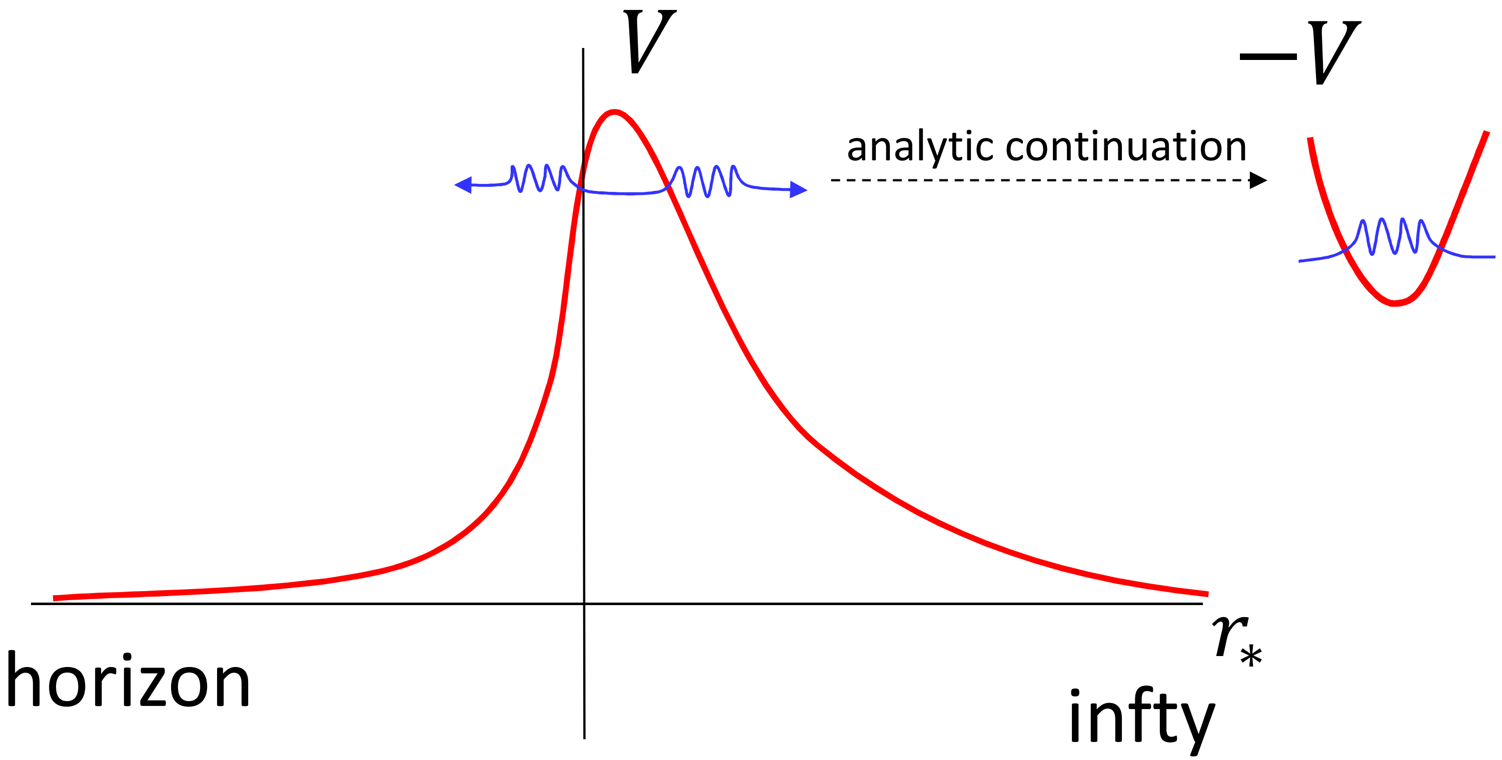
Quasinormal modes

Free, damped oscillations of black hole

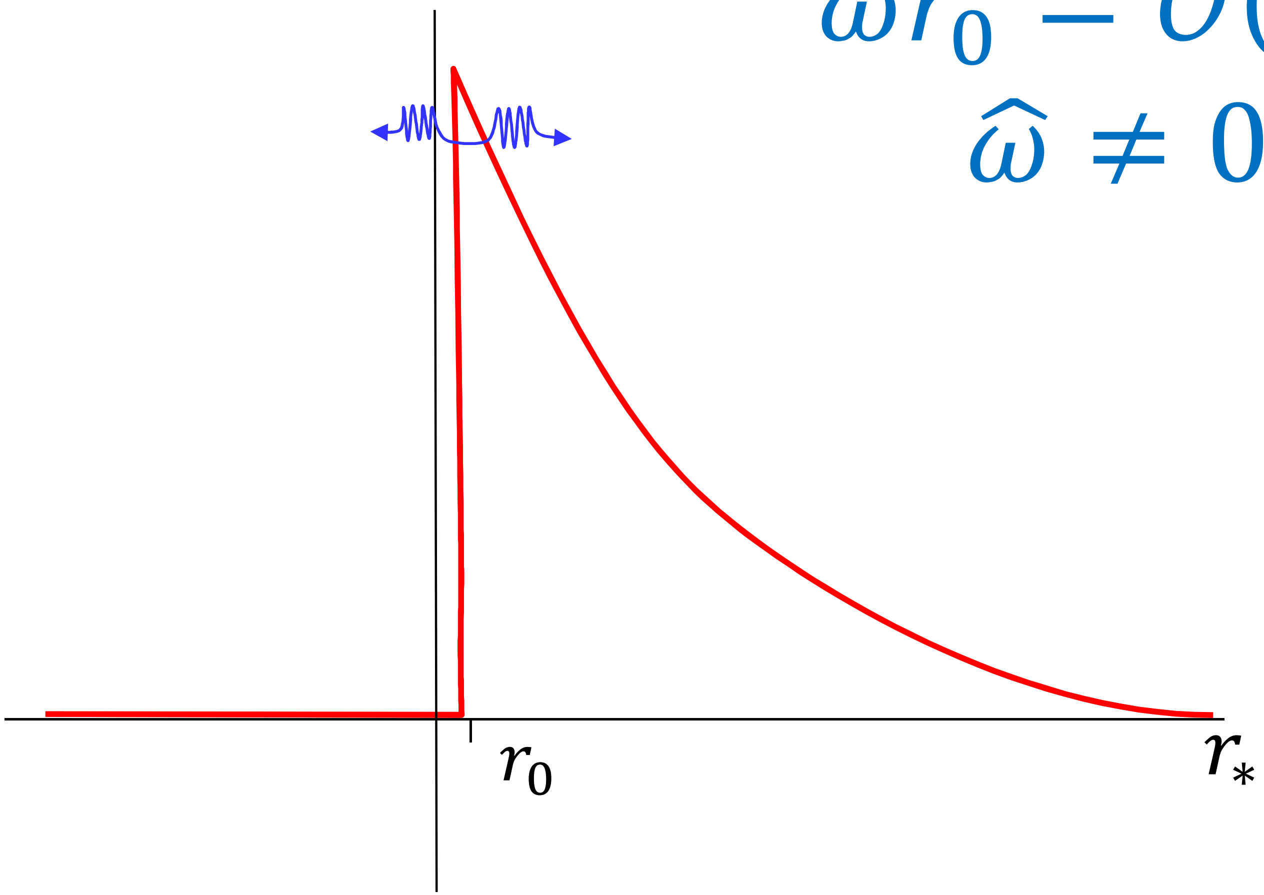


Quasinormal modes

QNMs as bound states in
inverted potential



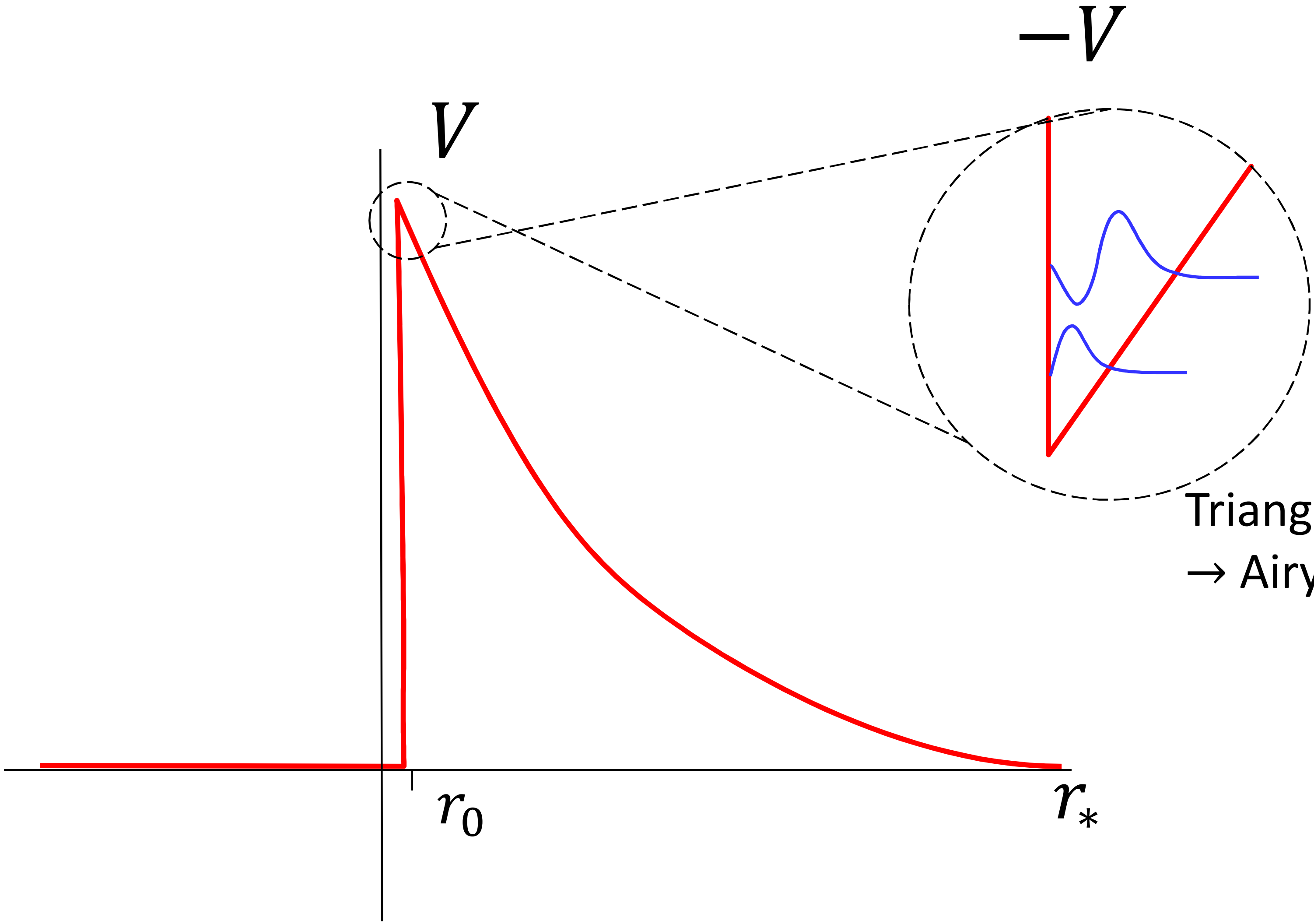
Non-decoupled QNMs



$$\omega r_0 = \mathcal{O}(D)$$

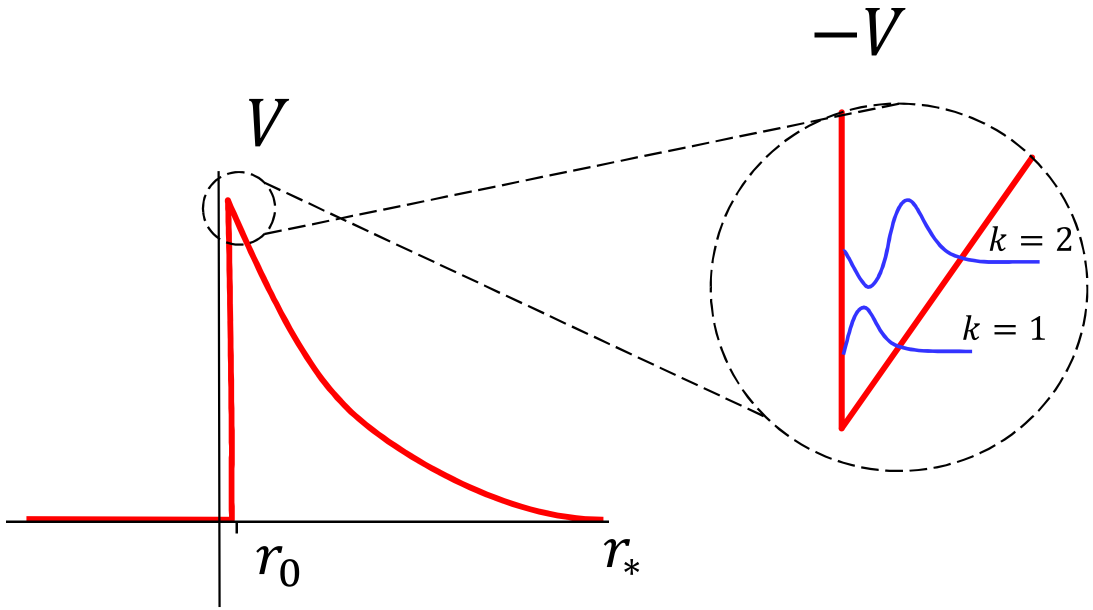
$$\hat{\omega} \neq 0$$

Non-decoupled QNMs $\omega r_0 = \mathcal{O}(D)$



Triangular well
→ Airy wavefns

Non-decoupled QNMs $\omega r_0 = \mathcal{O}(D)$



Airy zeroes

$$\Rightarrow \omega_{(\ell,k)}\, r_0 = \frac{D}{2} + \ell - \left(\frac{e^{i\pi}}{2}\left(\frac{D}{2} + \ell\right)\right)^{\frac{1}{3}} a_k$$

Universal spectrum @ large D

$$\omega_{(\ell,k)} r_0 = \frac{D}{2} + \ell - \left(\frac{e^{i\pi}}{2} \left(\frac{D}{2} + \ell \right) \right)^{\frac{1}{3}} a_k$$

Depends **only on** bh radius $\boldsymbol{r_0}$

Same spectrum for:

- any charges, dilaton coupling etc
- scalar, vector, tensor perturbations

Universal spectrum @ large D

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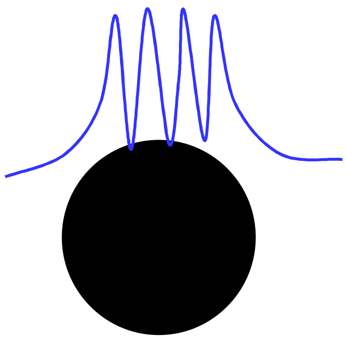


spectrum of scalar
oscillations of a hole
in space

$$\frac{\mathrm{Im}\omega}{\mathrm{Re}\omega} \sim D^{-2/3} \to 0:$$

sharp resonances

'normal modes' of bh

Decoupled QNMs

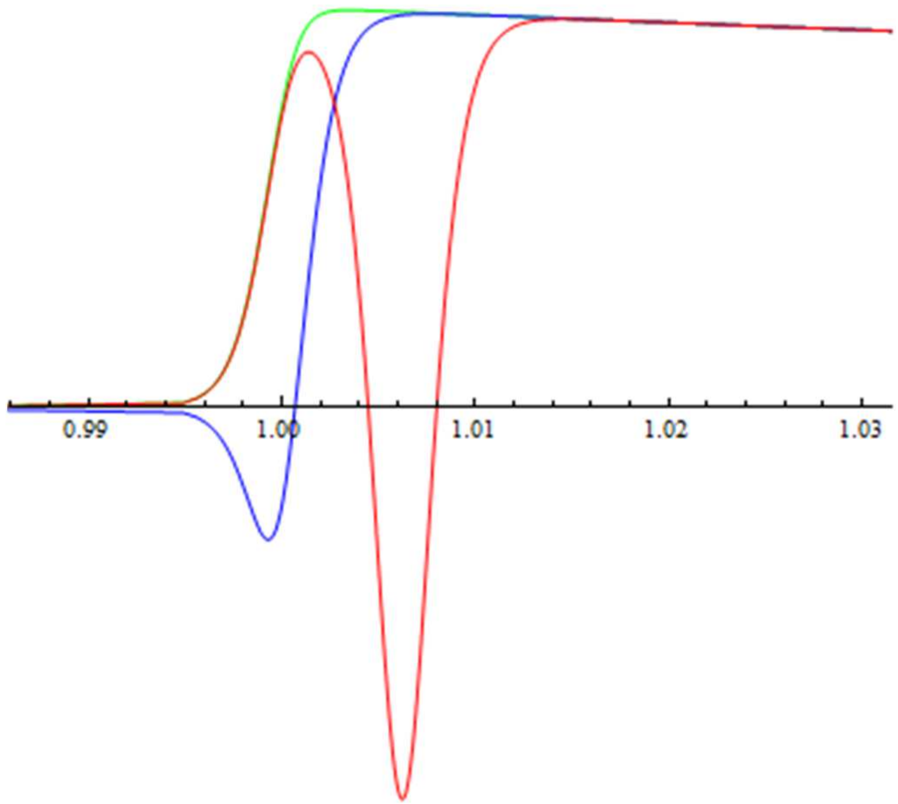
$\omega r_0 = \mathcal{O}(D^0)$

zero-energy states

at leading order

∃ for vectors and scalars

no tensors



Decoupled QNMs

$$\omega r_0 = \mathcal{O}(D^0)$$

zero-energy states

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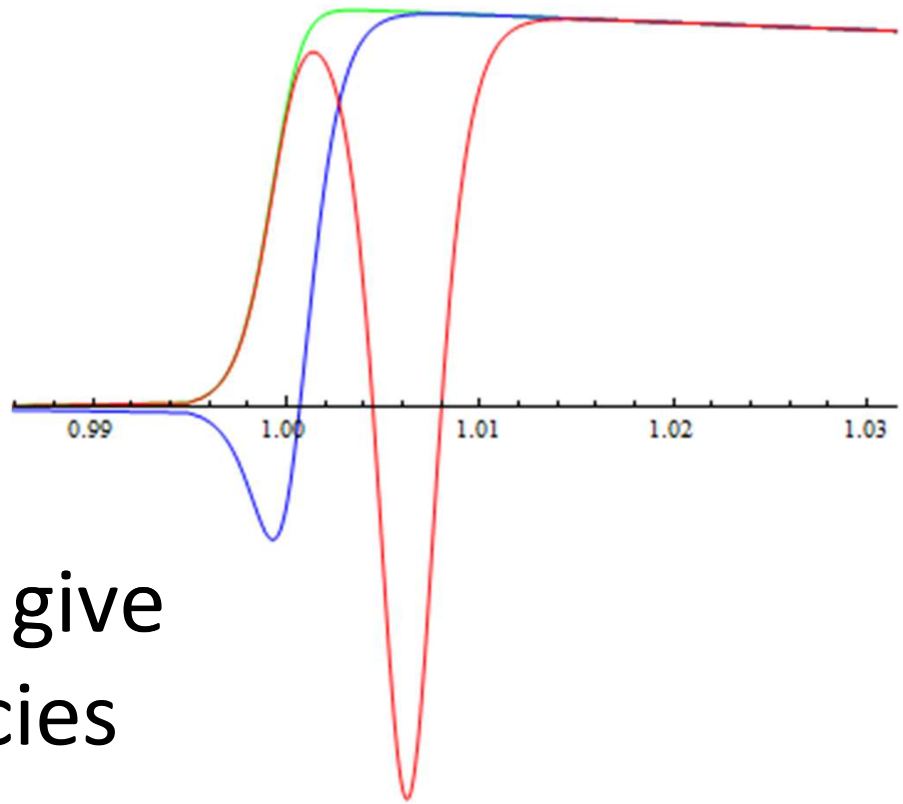
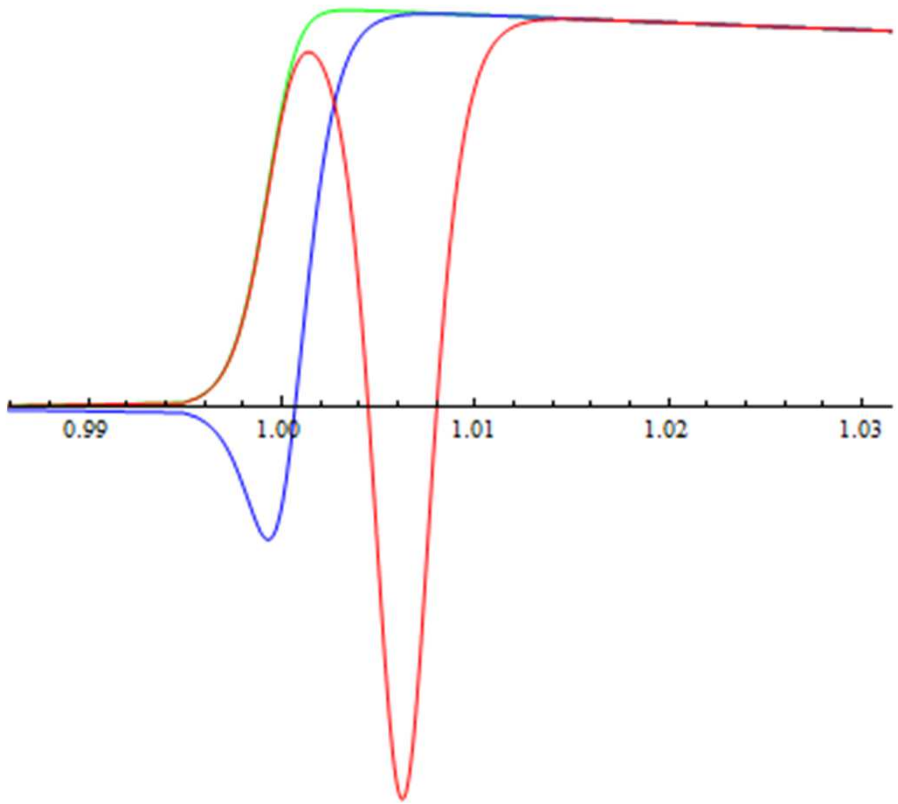


1/D corrections to state give non-zero QNM frequencies

Decoupled QNMs $\omega r_0 = \mathcal{O}(D^0)$

We've computed the qnm frequencies up to $1/D^3$



Quantitative accuracy

Decoupled spectrum $\boldsymbol{\omega r_0 = \mathcal{O}(1)}$

Vector mode (purely imaginary)

- At $D = 100$:

$\ell = 2$ mode $\mathrm{Im}\,\omega r_0 =$ -1.01044742 (analytical)

-1.01044741 (numerical *Dias et al*)

Quantitative accuracy

Decoupled spectrum $\omega r_0 = \mathcal{O}(1)$

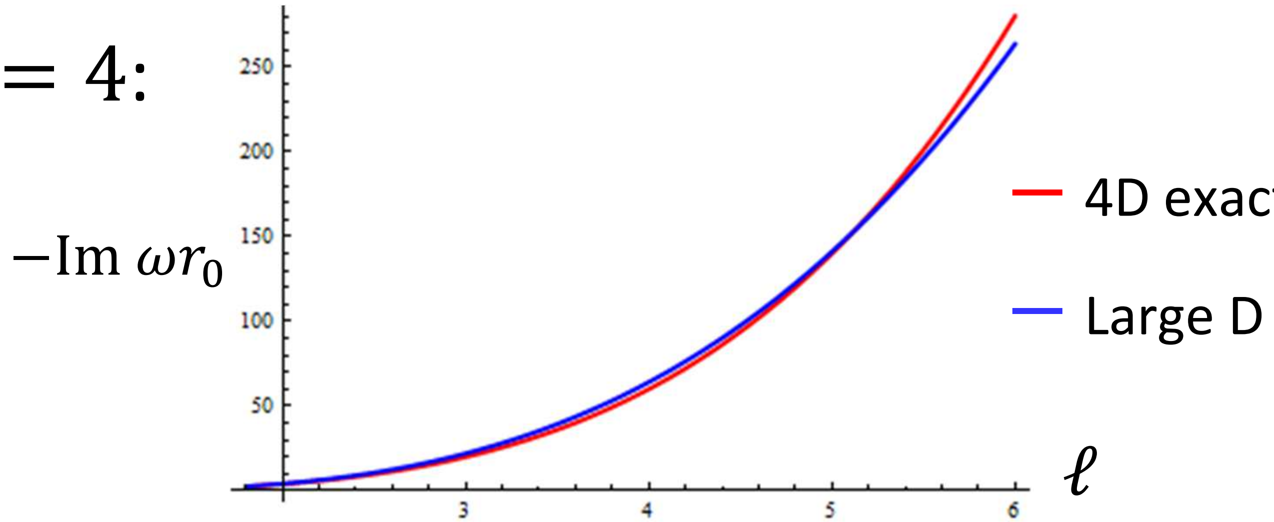
Vector mode (purely imaginary)

- At $D = 100$:

$\ell = 2$ mode $\mathrm{Im}\,\omega r_0 =$ -1.01044742 (analytical)

-1.01044741 (numerical *Dias et al*)

- At $D = 4$:



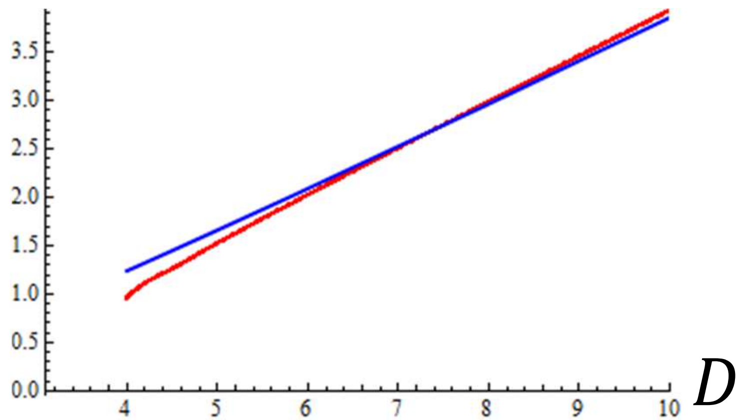
'algebraically special' mode

Quantitative accuracy

Non-decoupled spectrum $\boldsymbol{\omega r_0 = \mathcal{O}(D)}$

$\mathrm{Re}\,\omega r_0$: good at moderate D

$\mathrm{Re}\,\omega r_0$



$\ell = 2$

$\mathrm{Im}\,\omega r_0 \sim D^{1/3}$: only good at *very* high D

Outlook

Universal features @ large D

Far region

∀bhs: empty space

Near-horizon region

∀neutral bhs: 2D string bh

BH dynamics splits into:

$\omega r_0 = \mathcal{O}(D)$: non-decoupled dynamics

– scalar field **oscillations of a hole** in space

– *universal normal* modes

$\omega r_0 = \mathcal{O}(D^0)$: decoupled dynamics

– localized in near-horizon region

$\omega r_0 = \mathcal{O}(D^0)$: decoupled dynamics

– *specific* of each bh

– less numerous

– for rapidly rotating bhs, instabilities appear in this sector *Tanabe's talk*

$\omega r_0 = \mathcal{O}(D)$: non-decoupled dynamics

– **universal** normal modes of hole in space

– much more **numerous**

– describe **interaction** of bh **w/ environment**

– connection to BH entropy?