Interaction-Driven Insulators and Superconductors in Holography, and Homes Law

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May 27, 2014

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Introduction: High T_c and AdS/CFT

- Difficulty in understanding High T_c via AdS/CFT: Don't know the IR field theory (DOFs, operator content, action, scaling properties) - very different from AdS/QCD
- Need universal signatures to guide AdS/CFT model building
- Most prominent: Linear temperature resistivity

  • Many tunable bottom-up models , free parameters

  [Hyperscaling Violating Lishitz 1005.4690, Probe Branes 1005.4690, Holographic Lattices 1302.6586, p-wave Herzog etal. 1405.3714]

  • Different more or less generic mechanisms , also with free parameters, assumptions

  [Hartnoll+Hofman 1201.3917, Davison+Schalm+Zaanen 1311.2451, Hartnoll 1405.3651]
- Need additional universal signatures to distinguish between models/mechanisms
- Homes' law provides such a universal signature

Introduction: Homes' Law

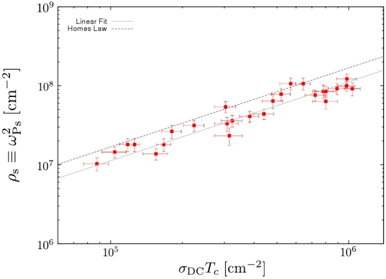
- **Homes' Law** [Homes etal Nature 2004]

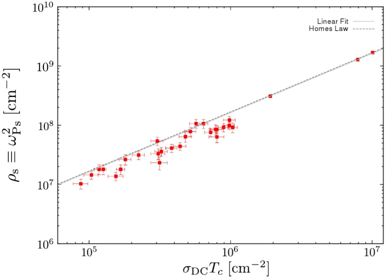
$$\rho_S|_{T=0} = C\sigma_{DC}(T_{c,+})T_c$$

C is universal for families of superconductors

In-Plane Cuprates, Elemental BCS: $C \approx 35$

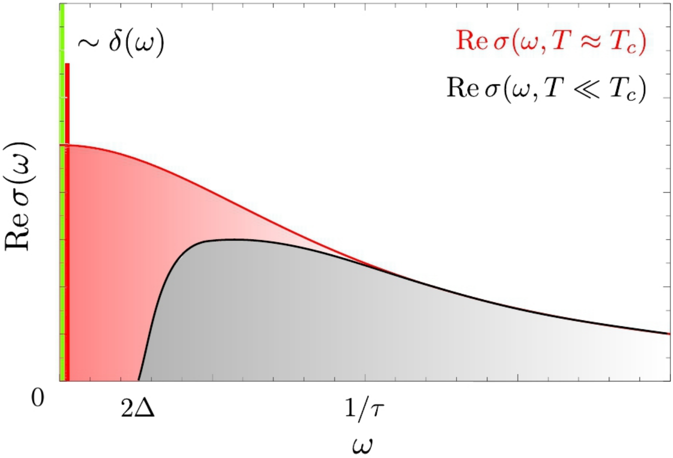
C-Axis Cuprates, Dirty Limit BCS (Nb,Pb): $C \approx 65$





- Experimentally verified to good accuracy
- In Holographic Superconductors?

Towards Homes' Law in AdS/CFT [1206.5305]



$$\boxed{Re\sigma_n = K\delta(\omega) + \ldots , \quad Re\sigma_s = (K' + \rho_s)\delta(\omega) + \ldots}$$

Idea: Rewrite Homes' Law by sum rules, τ from diffusion

[1206.5305]

Towards Homes' Law in AdS/CFT [1206.5305]

- Three Assumptions
  - Oscillator/FGT sum rules hold in Holography [1302.6586]
  - Tanner's law: $\boxed{n_s \sim n_n \quad \Rightarrow \quad \omega_{Pn}^2 \sim \omega_{Ps}^2}$
  - Drude model applicable (two scales ω_P^2 and τ)
- Using $\sigma_{DC} \sim \omega_{Pn}^2 \tau(T_c)$ one can rewrite

$$\boxed{\rho_s = \omega_{Ps}^2 \sim \sigma_{DC} T_c \sim \rho_s \tau(T_c) T_c \quad \Rightarrow \quad T_c \tau_c = const.}$$

- Relaxation time τ_c from Momentum or R-charge diffusion

$$\boxed{\tau_c \propto D_{M/R}(T_c) \quad \Rightarrow \quad D_{M/R}(T_c) T_c = \text{const.}}$$

- s-wave holographic Superconductor in probe limit:

[1,34-36] in 1206.5305

$$\boxed{D_M = \frac{1}{4\pi T} \qquad D_R = \frac{1}{4\pi T} \frac{d}{d-2}}$$

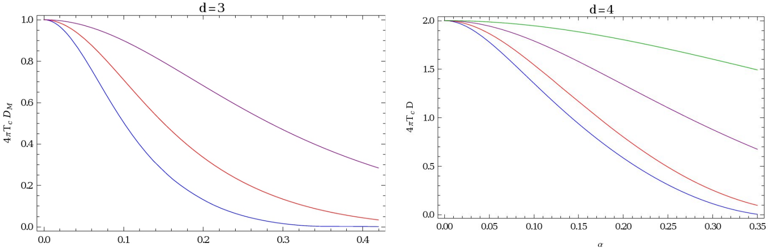
- Homes' law holds in probe limit

Towards Homes' Law in AdS/CFT [1206.5305]

Finite backreaction changes T_c and $D_{M/R}$

$\rightarrow$ Changing the charge of the order parameter

Homes' Law does not hold any more: (for D_M and D_R, p wave)



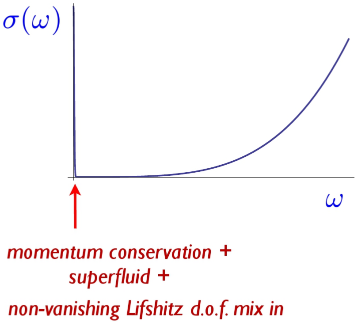
What could have gone wrong?

- Sum rules may not hold (at $T = 0$?) . [Horowitz+Santos]
- Diffusion and T_c depend on backreaction.
- Relaxation time not given by diffusion constant alone?
- Lifshitz IR = Extra IR DOFs!

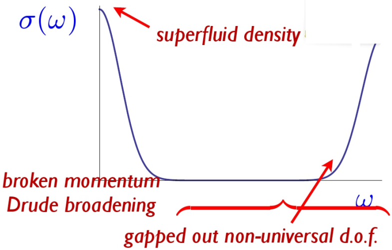
Homes' Law in Holographic Insulator/Superconductor

- Two puzzles:
  - separate delta (momentum breaking/superconductor)
  - lift extra IR d.o.f. ("insulating/gapped geometry")

Standard Hol. SC



Real SC

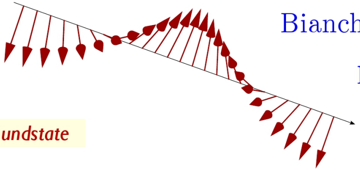


Homes' Law in Holographic Insulator/Superconductor

- Can we make the holographic superconductor more like a real superconductor by
  - Getting rid of the $\delta(\omega)$ in the normal phase (momentum breaking)?
  - Lift extra d.o.f. $\rightarrow$ insulating/gapped geometry [Donos,Hartnoll]

$$S=\int d^5x\sqrt{-g}\left(R+12-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}W_{\mu\nu}W^{\mu\nu}-\frac{m^2}{2}B_\mu B^\mu\right)-\frac{\kappa}{2}\int B\wedge F\wedge W$$

$$ds^2=-U(r)dt^2+\frac{dr^2}{U(r)}+e^{2v_1(r)}\omega_1^2+e^{2v_2(r)}\omega_2^2+e^{2v_3(r)}\omega_3^2$$



Bianchi VII_0 helix geometry

period p; strength λ

has an "insulating" groundstate

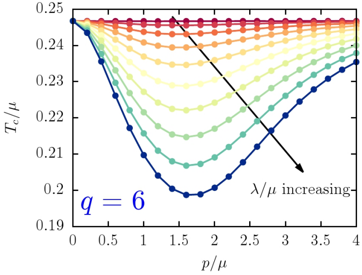
Homes' Law in Holographic Insulator/Superconductor

Erdmenger, Herwerth, Meyer, Mueller/Klug, KS

- Add a charged scalar

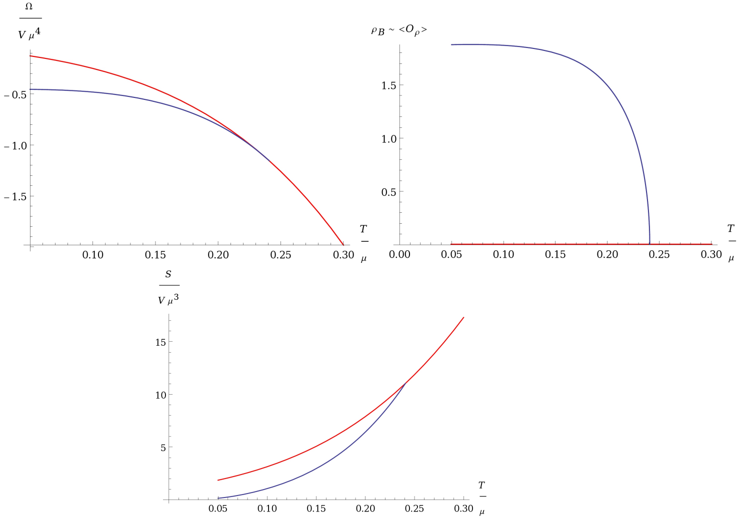
$$S = \int d^5x\sqrt{-g}\left(R + 12 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}W_{\mu\nu}W^{\mu\nu} - \frac{m^2}{2}B_\mu B^\mu\right) - \frac{\kappa}{2}\int B \wedge F \wedge W - |(\partial_\mu - 2iqA_\mu)\eta)|^2$$

  - Holographic superconductor in a translationally broken "insulating" background



Thermodynamics

- Second order phase transition of mean field type:



Homes' Law in Holographic Insulator/Superconductor

Erdmenger, Herwerth, Meyer, Mueller/Klug, KS

- Resistivity in Donos Hartnoll model

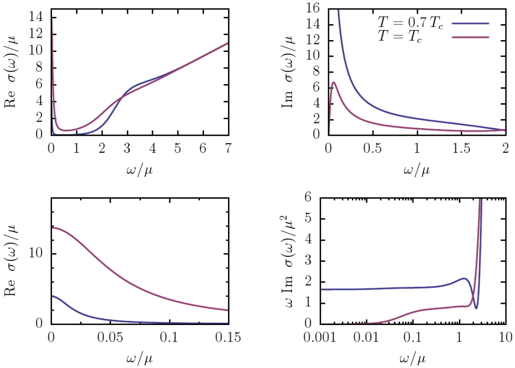
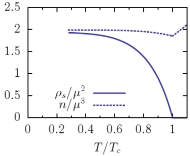


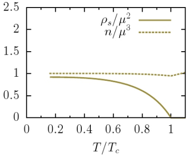
Figure 7: Optical conductivity for $T = 0.7\,T_c$ (blue line) and at the transition temperature (red line). The real part of the conductivity (top left and bottom left) exhibits a Drude peak for small frequencies. Additionally, there is a $1/\omega$ pole in the imaginary part in the broken phase (top right and bottom right, where $\text{Im}(\sigma)$ has been multiplied by ω). All graphs have $p/\mu = 2.4$, $\lambda/\mu = 1$, and $q = 6$.

Homes' Law in Holographic Insulator/Superconductor

Erdmenger, Herwerth, Meyer, Mueller/Klug, KS

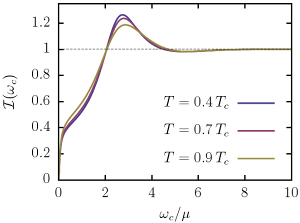
- Homes' Law in holography (DH model)





can now extract the density reliably

Figure 10: Superfluid density ρ_s (solid lines) and charge density n (dashed lines) as a function of T/μ. For $T > T_c$, the superfluid density vanishes, i.e. the DC conductivity is finite. As T is lowered, the superfluid density increases. The left graph has $q = 6$, $\lambda/\mu = 1$, and $p/\mu = 2.4$. In the right graph, $q = 4.1$, $\lambda/\mu = 1$, and $p/\mu = 1$.



Ferrell-Glover-Tinkham sum rule

$$I(\omega_c) = \frac{1}{\rho_s}\int_0^{\omega_c} d\omega\,[\mathrm{Re}\sigma_n(\omega) - \mathrm{Re}\sigma_s(\omega)]$$

$$p/\mu = 2.4,\ \ \lambda/\mu = 1,\ \ q = 6$$

Towards a T=0 solution

- Break translations by Bianchi VII_0 Helix [1212.2998]

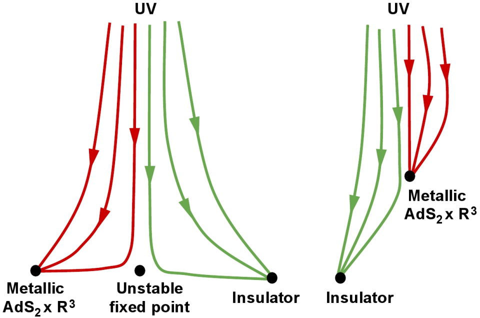
$$\omega_1 = dx\,, \quad \omega_2 + i\omega_3 = e^{ipx}(dy + idz)$$

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + \sum_{i=1}^{3} e^{2v_i(r)}\omega_i^2$$

- Charge density and Helix Field, Order Parameter

$$A = a(r)dt\,, \quad B = w(r)\omega_2\,, \quad \rho = \rho(r) \in \mathbb{R}$$

- QPT by μ or Helix source/pitch: $|\kappa_c| \approx 0.57$ [1212.2998]



Towards a T=0 solution

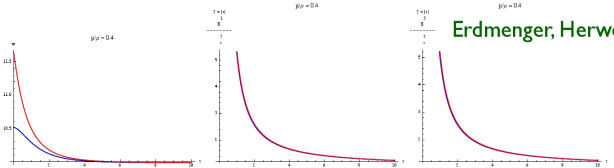
- Usual superconducting instability: $AdS_2 \times \mathbb{R}^3$ with the charged scalar mode becoming unstable (relevant) if
$$3 + m_\rho^2 - 2q^2 < 0$$
- Insulating geometry with charged hair: $(\kappa = 1/\sqrt{2})$
$$a = a_0 r^{5/3} + \dots\,, \quad w = w_0 + w_1 r^{4/3} + \dots\,, \quad U = U_0 r^2 + \dots$$
$$e^{v_1} = e^{v_{10}} r^{-1/3} + \dots\,, \quad e^{v_2} = e^{v_{20}} r^{2/3} + \dots\,, \quad e^{v_3} = e^{v_{30}} r^{1/3} + \dots\,,$$
$$\rho = \rho_0 + \rho_1 r^{4/3} + \dots$$
- Cohesive Phase (no flux): $\int_{\mathbb{R}^3} *F \Big|_{horizon} = 0$
- Has only marginal & irrelevant perturbations within this ansatz. [Iizuka's talk]
- Expected to be the superconducting ground state.

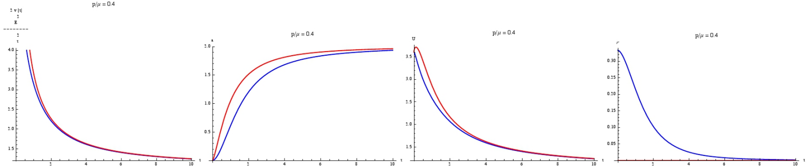
Towards a T=0 solution

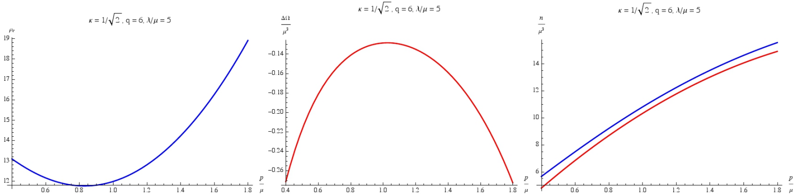
Erdmenger, Herwerth, Meyer, Mueller/Klug, KS

normal phase

broken phase







Conclusions & Outlook

- Homes' law as another candidate for a universal relation in high temperature superconductivity
- For s- and p-wave holographic superconductors in the probe limit, but not with backreaction included
- Problematic: Translation invariance & IR D.O.Fs
- Break translation invariance via Bianchi VII_0
  Calculate both sides of Homes' law independently
- Normal and superconducting ground states
- Thermodynamics - Second order mean field
- $\rho_s(T=0)$ extractable, sum rules hold
- To Do List:
  - DC conductivity in normal phase, Homes law?
  - Complete finite T & $T=0$ phase diagram
  - Other instabilities? [Iizuka's talk]
- Expect Homes' Law to hold