M5-branes and Wilson Surfaces in AdS_7/CFT_6 Correspondence

Hironori Mori (Osaka Univ.)

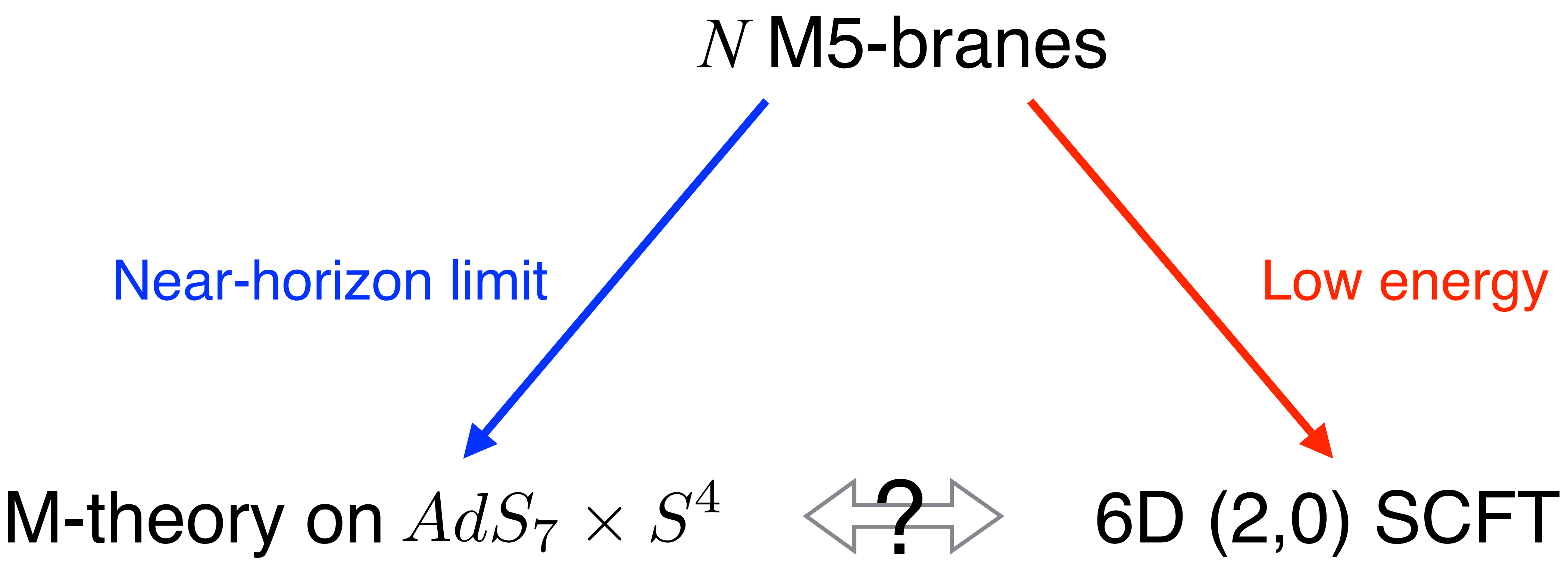
OSAKA UNIVERSITY

based on arXiv:1404.0930 with Satoshi Yamaguchi (Osaka Univ.)

2014/05/28, Holographic vistas on Gravity and Strings @ YITP

AdS_7/CFT_6

AdS$_7$/CFT$_6$


N M5-branes
Near-horizon limit
Low energy
M-theory on $AdS_7 \times S^4$
?
6D (2,0) SCFT

6D (2,0) theory

equivalent || S^1 compactification

[Douglas '10] [Lambert, Papageorgakis, Schmidt-Sommerfeld '10]

5D maximally SYM

Exact results on curved backgrounds

[Källén, Zabzine '12] [Hosomichi, Seong, Terashima '12]
[Källén, Qiu, Zabzine '12] [Kim, Kim '12] [Imamura '12]
[Kim, Lee '12] [Fukuda, Kawano, Matsumiya '12] [Kim, Kim, Kim '12]
[Qiu, Zabzine '13] [Kim, Kim, Kim, Lee '13]
[Schmude '14] …

AdS$_7$/CFT$_6$

M-theory on $AdS_7 \times S^4$

(Boundary = $S^5 \times S^1$)

M-branes

Bubbling geometry

⇔ ?

6D (2,0) SCFT

on $S^5 \times S^1$

Wilson surfaces

||

Wilson loops $\langle W_R \rangle$

5D MSYM on S^5

Summary

AdS7/CFT6

M-branes		Wilson surfaces
M2-brane wrapping $\begin{matrix} AdS_3 \\ \cap \\ AdS_7 \end{matrix}$	⟺ [Minahan, Nedelin, Zabzine, '13]	Fundamental rep $\langle W_{\square} \rangle \sim \exp\left[\frac{\beta N}{2}\right]$
M5-brane wrapping $\begin{matrix} AdS_3 \times S^3 \\ \cap \\ AdS_7 \end{matrix}$	New ⟺	Symmetric rep $\langle W_{\underbrace{\square\square\cdots\square}_{k}} \rangle \sim \exp\left[\frac{\beta N}{2} k \left(1 + \frac{k}{2N}\right)\right]$
M5-brane wrapping $\begin{matrix} AdS_3 \times \tilde{S}^3 \\ \cap \\ AdS_7 \times S^4 \end{matrix}$	⟺	Anti-symmetric rep $\langle W_{k\left\{\begin{smallmatrix}\square \\ \square \\ \vdots \\ \square\end{smallmatrix}\right.} \rangle \sim \exp\left[\frac{\beta N}{2} k \left(1 - \frac{k}{N}\right)\right]$

Plan

1. Review: Wilson surface

2. CFT side

3. Gravity side: From M5-branes

4. Summary

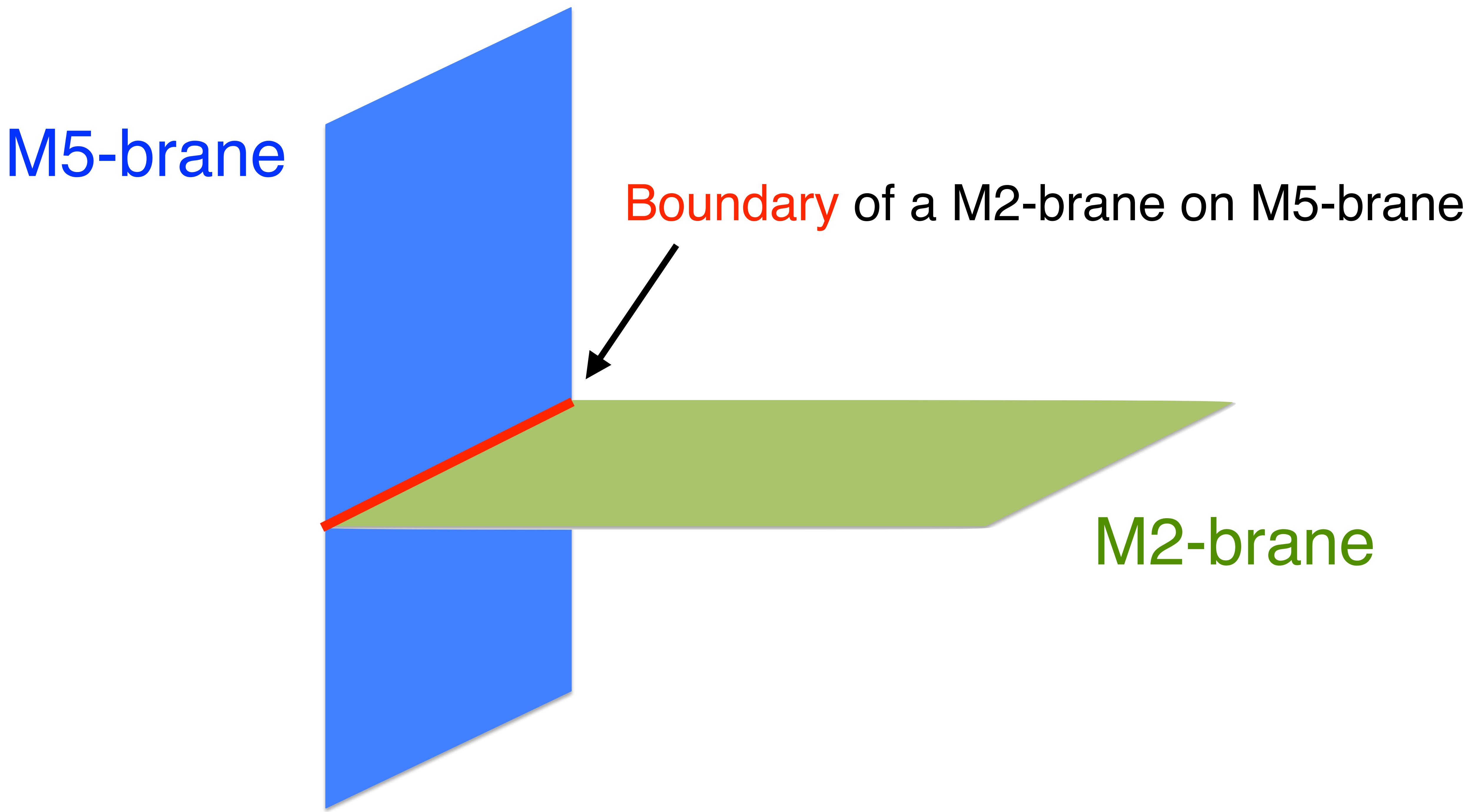
1. Review

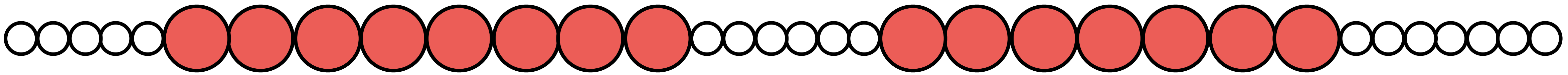
- Wilson surface
- Maya diagram

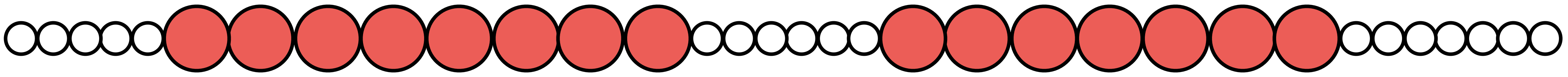
6d (2,0) SCFT : Effective theory on N M5-branes

(but no Lagrangian)

Wilson surface : Non-local operator
extending on 2-dimensional space







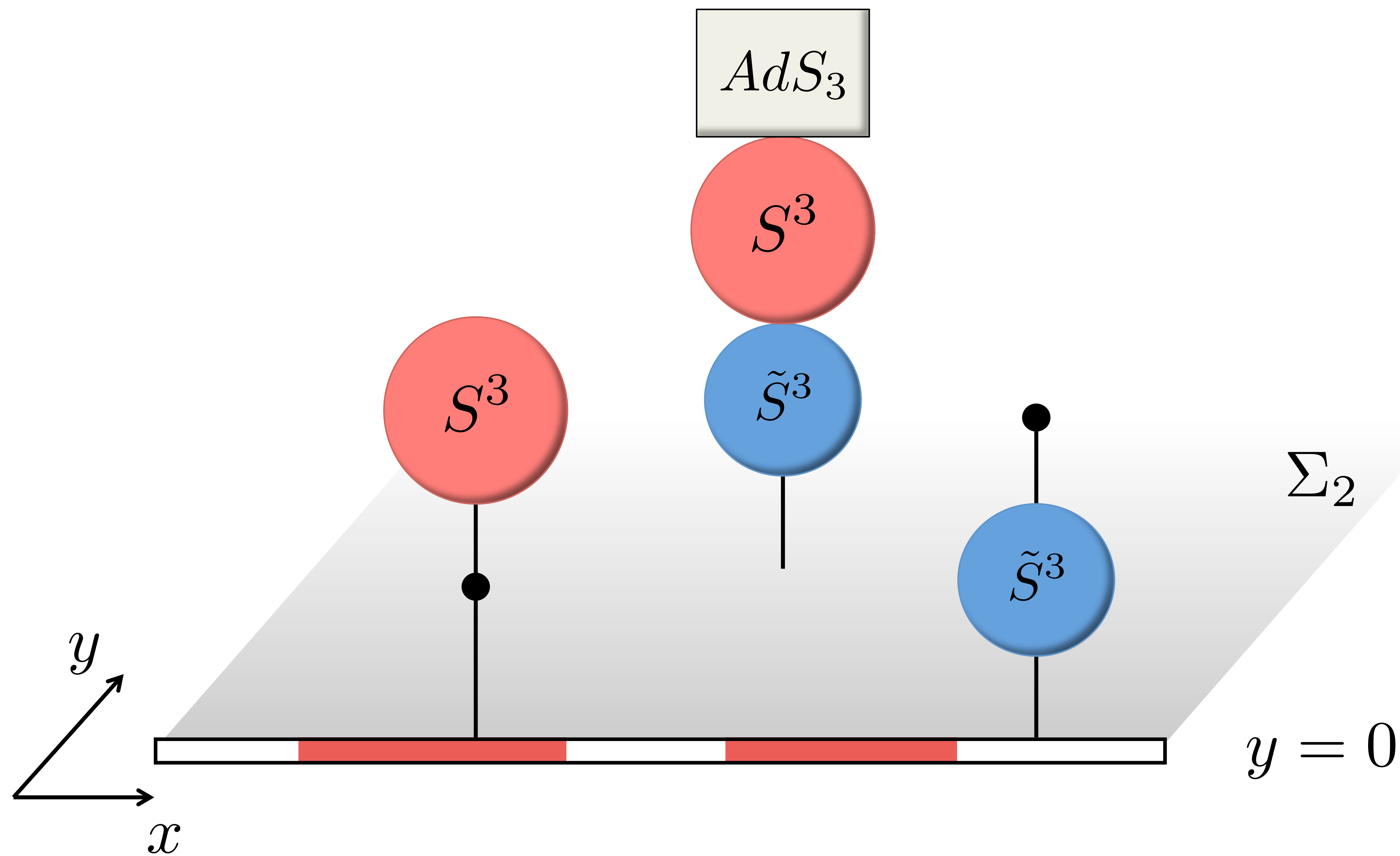
“Maya diagram”

"Bubbling geometry"

: A class of classical half-BPS solutions of 11D supergravity

[Lin, Lunin, Maldacena, '04]
[Yamaguchi, '06] [Lunin, '06]
[D'Hoker, Estes, Gutperle, Krym, '08]

$$\mathcal{M}_{11} = AdS_3 \times S^3 \times \tilde{S}^3 \times \Sigma_2$$



Bubbling geometry is labeled by

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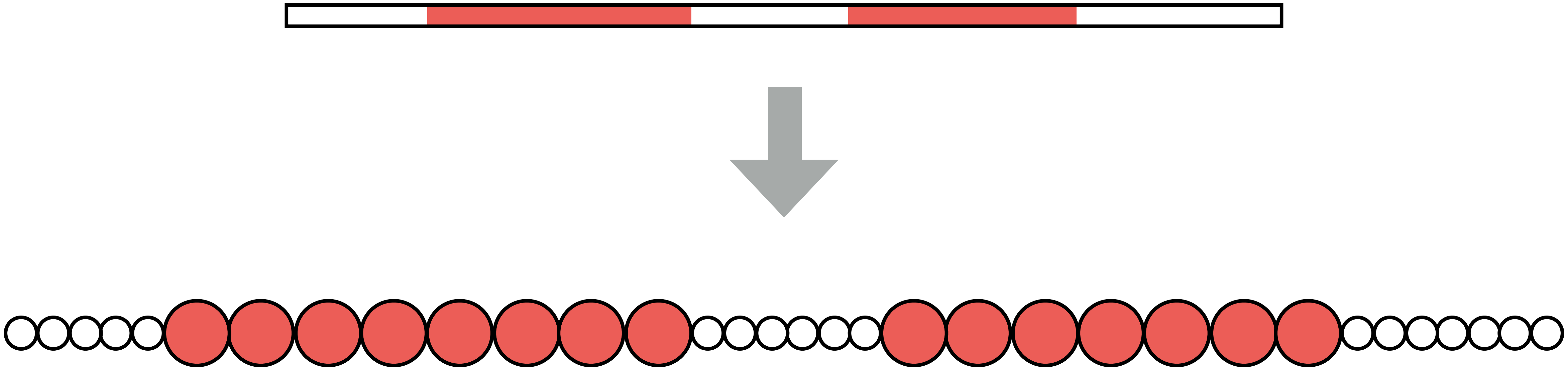

“Maya diagram”

The dual field theory may have
this eigenvalue distribution of the “matrix model.”

Plan

✓ 1. Review: Wilson surface

2. CFT side

3. Gravity side: From M5-branes

4. Summary

2. CFT side

- Chern-Simons matrix model
- Calculations of Wilson surfaces

6D (2,0) theory on $S^5 \times S^1$

equivalent ||

$$R_6 = \frac{g_{YM}^2}{8\pi^2}$$

$(R_6$: radius of $S^1)$

5D maximally SYM on S^5

⇩

Chern-Simons matrix model [Källén, Qiu, Zabzine '12] [Kim, Kim '12]

$$\mathcal{Z} \sim \int \prod_{i=1}^{N} d\nu_i \exp\left[-\frac{N^2}{\beta}\sum_{i=1}^{N}\nu_i^2 + \sum_{i\neq j}\ln\left|2\sinh\frac{N(\nu_i-\nu_j)}{2}\right|\right]$$

raduius of S^5 : r, $\quad \beta = \frac{g_{YM}^2}{2\pi r}$

6D (2,0) theory on $S^5 \times S^1$

Wilson surface $(S^1 \times S^1)$

equivalent ||

5D maximally SYM on S^5

Wilson loop (S^1)

$$\langle W_R \rangle = \langle \mathrm{Tr}_R e^{N\nu} \rangle$$

⇩

Chern-Simons matrix model [Källén, Qiu, Zabzine '12] [Kim, Kim '12]

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Chern-Simons matrix model

$O(N^3)$

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Large N limit (β fixed)

$$\Rightarrow \mathcal{Z} \sim \int \prod_{i=1}^{N} d\nu_i \exp\left[-\frac{N^2}{\beta}\sum_{i=1}^{N}\nu_i^2 + \frac{N}{2}\sum_{i\neq j}|\nu_i-\nu_j|\right]$$

- Saddle point equations

$$0 = -\frac{2N^2}{\beta}\nu_i + N\sum_{j,i\neq j}\mathrm{sign}\left(\nu_i-\nu_j\right)$$

Chern-Simons matrix model

$O(N^3)$

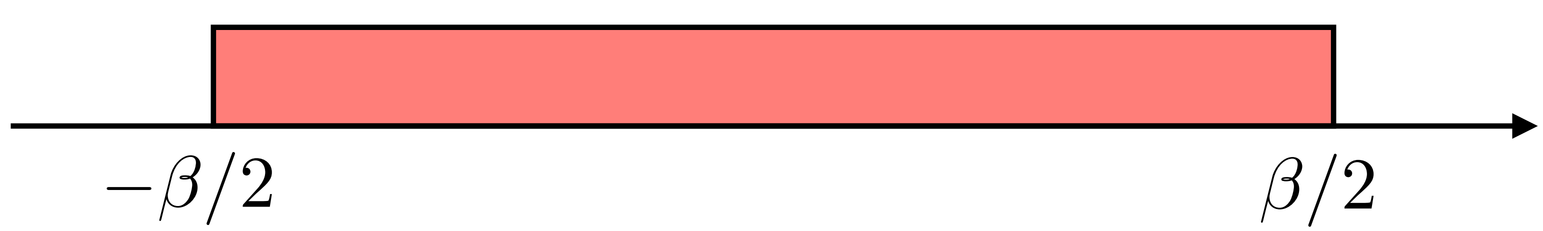
$$\mathcal{Z} \sim \int \prod_{i=1}^{N} d\nu_i \exp\left[-\frac{N^2}{\beta} \sum_{i=1}^{N} \nu_i^2 + \sum_{i\neq j} \ln \left| 2\sinh \frac{N(\nu_i - \nu_j)}{2} \right| \right]$$

Large N limit (β fixed)

$$\Rightarrow \mathcal{Z} \sim \int \prod_{i=1}^{N} d\nu_i \exp\left[-\frac{N^2}{\beta} \sum_{i=1}^{N} \nu_i^2 + \frac{N}{2} \sum_{i\neq j} |\nu_i - \nu_j| \right]$$

- Eigenvalue distribution ➡ consistent with bubbling geometry!

$$\nu_i = \frac{\beta}{2}\left(1 - \frac{2i}{N}\right) \qquad (\nu_1 > \nu_2 > \cdots > \nu_N)$$



Symmetric rep

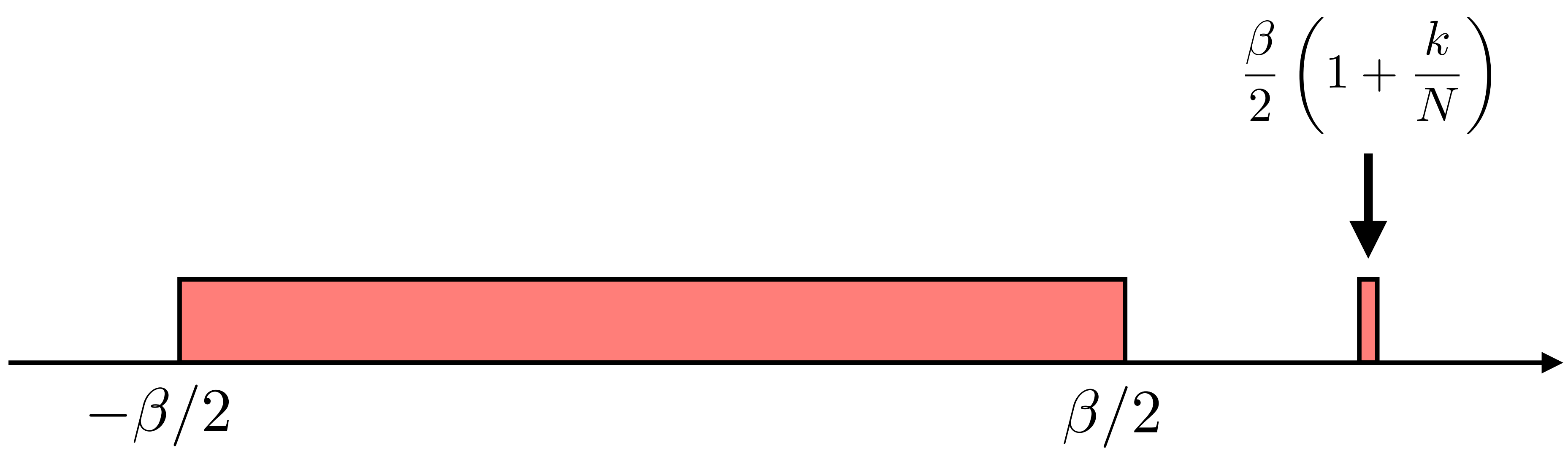
$(\nu_1 > \nu_2 > \cdots > \nu_N)$

$$\mathrm{Tr}_{\underbrace{\square\square\cdots\square}_{k}} e^{N\nu} = \sum_{1\le i_1 \le \cdots \le i_k \le N}^{N} \exp\left[N\sum_{l=1}^{k}\nu_{i_l}\right] \xrightarrow{\text{leading}} \exp\left[Nk\nu_1\right]$$

$$\Longrightarrow \nu_1 \text{ changes : } \nu_1 = \frac{\beta}{2}\left(1+\frac{k}{N}\right)$$

$$\Longrightarrow \langle W_{\underbrace{\square\square\cdots\square}_{k}} \rangle \sim \exp\left[-\frac{N^2}{\beta}\nu_1^2 + N\sum_{j=2}^{N}|\nu_1-\nu_j| + Nk\nu_1 + (\text{terms independent of } k)\right]\Bigg|_{\text{saddle point}}$$

$$\sim \exp\left[\frac{\beta N}{2}k\left(1+\frac{k}{2N}\right)\right]$$



Anti-symmetric rep

$(\nu_1 > \nu_2 > \cdots > \nu_N)$

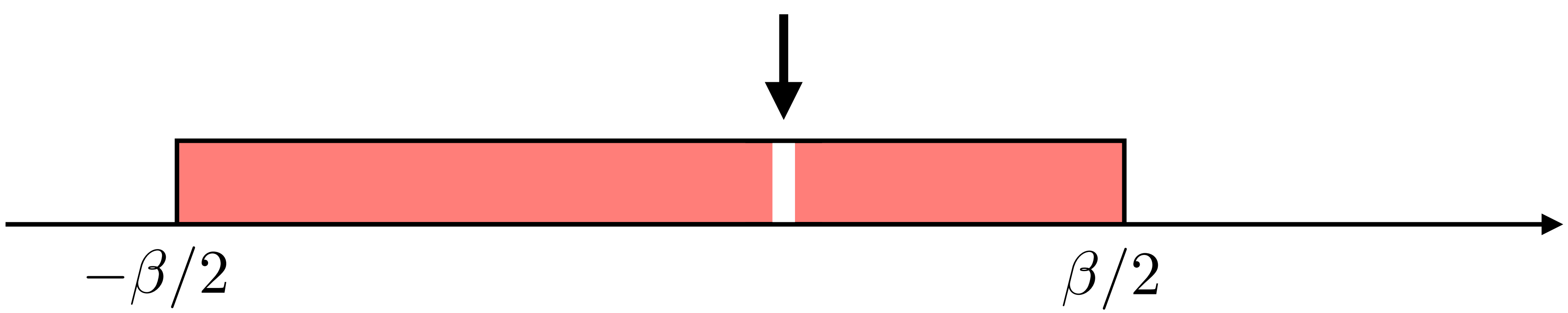
$$\mathrm{Tr}_{k\left\{\begin{smallmatrix}\square\\ \square\\ \vdots\\ \square\end{smallmatrix}\right.} e^{N\nu} = \sum_{1\le i_1<\cdots<i_k\le N}^{N} \exp\left[N\sum_{l=1}^{k}\nu_{i_l}\right] \xrightarrow{\text{leading}} \exp\left[N(\nu_1+\cdots+\nu_k)\right]$$

$\Longrightarrow$ ν_i do not change : $\nu_i = \dfrac{\beta}{2}\left(1-\dfrac{2i}{N}\right)$

$$\Longrightarrow \left\langle W_{k\left\{\begin{smallmatrix}\square\\ \square\\ \vdots\\ \square\end{smallmatrix}\right.}\right\rangle \sim \exp\left[N(\nu_1+\cdots+\nu_k)\right]\Big|_{\text{saddle point}}$$

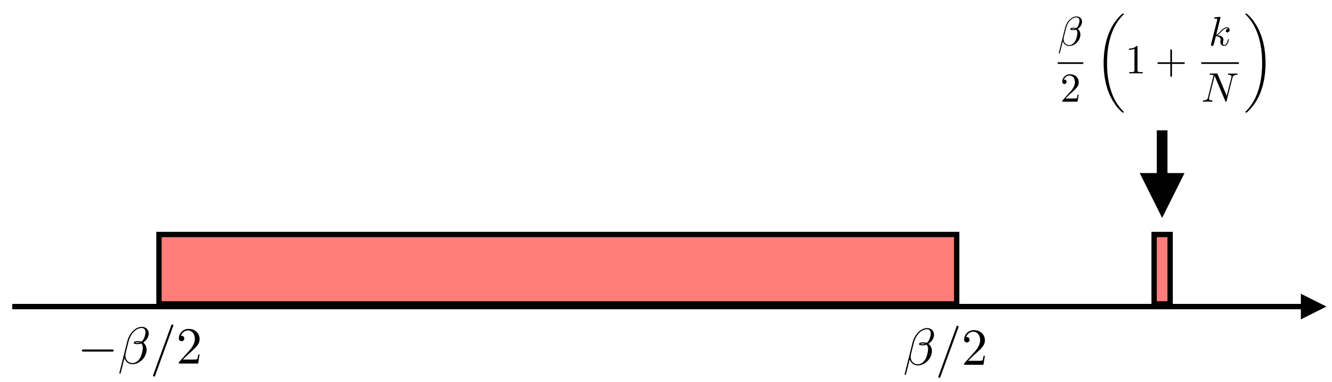
$$\sim \exp\left[\frac{\beta N}{2}k\left(1-\frac{k}{N}\right)\right]$$

$$\frac{\beta}{2}\left(1-\frac{2k}{N}\right)$$

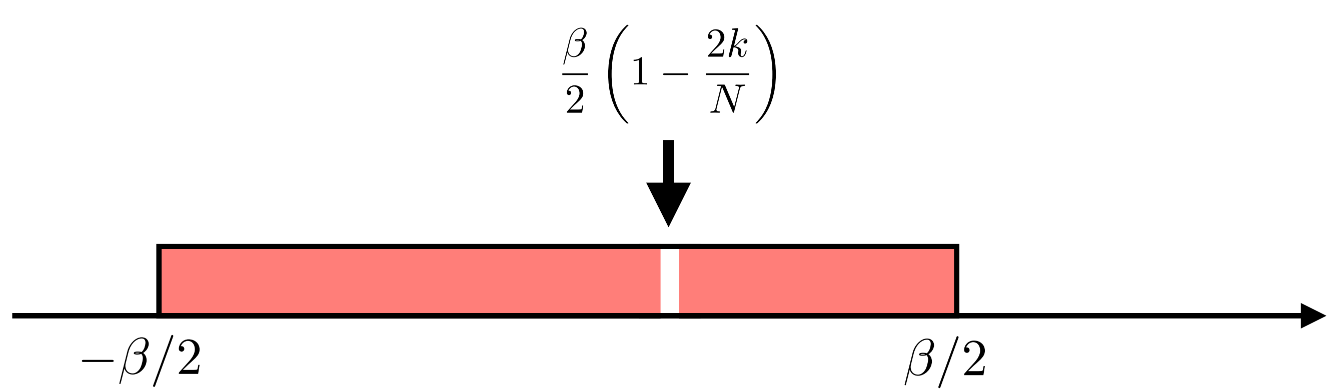


Summary for $\langle W_R \rangle$

Symmetric : $\langle W_{\underbrace{\square\square\cdots\square}_{k}} \rangle \sim \exp\left[\frac{\beta N}{2} k \left(1 + \frac{k}{2N}\right)\right]$



Anti-symmetric : $\langle W_{k\left\{\begin{smallmatrix}\square\\ \square\\ \vdots\\ \square\end{smallmatrix}\right.} \rangle \sim \exp\left[\frac{\beta N}{2} k \left(1 - \frac{k}{N}\right)\right]$



Plan

✓ 1. Review: Wilson surface

✓ 2. CFT side

3. Gravity side: From M5-branes

4. Summary

3. Gravity side

- Calculations of Wilson surfaces from probe M5-branes

Summary for $\langle W_R \rangle$

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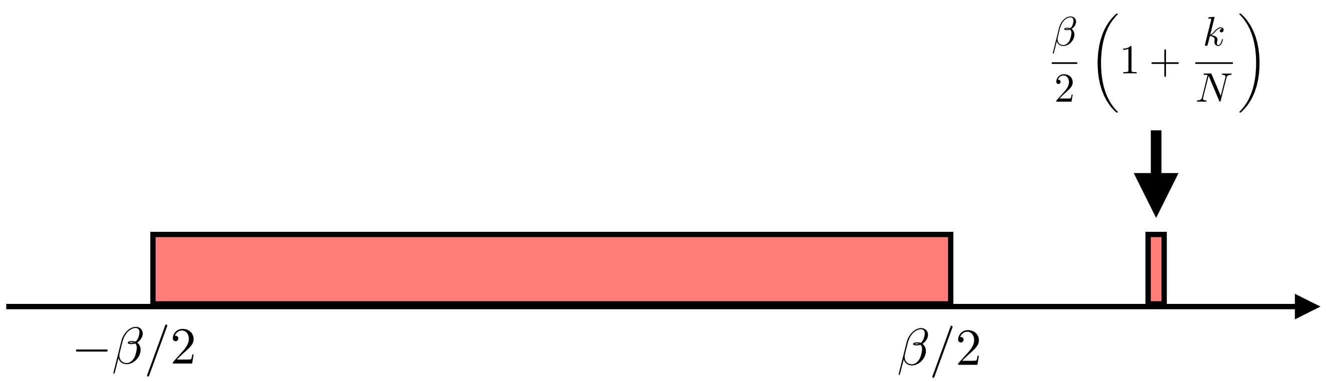

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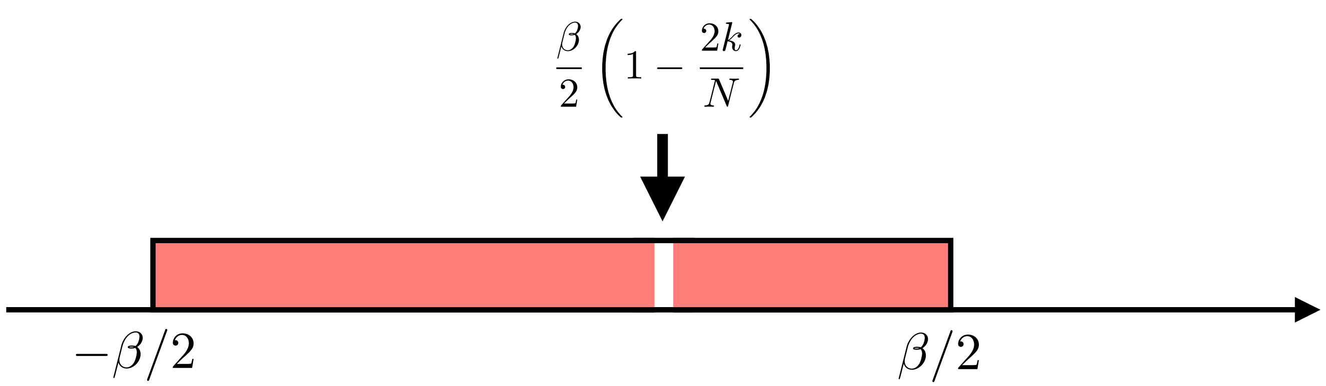

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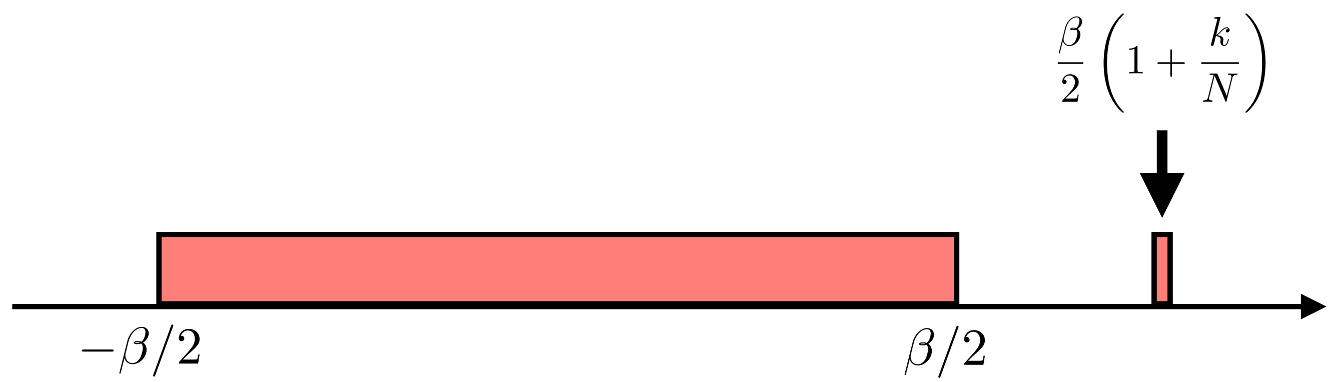

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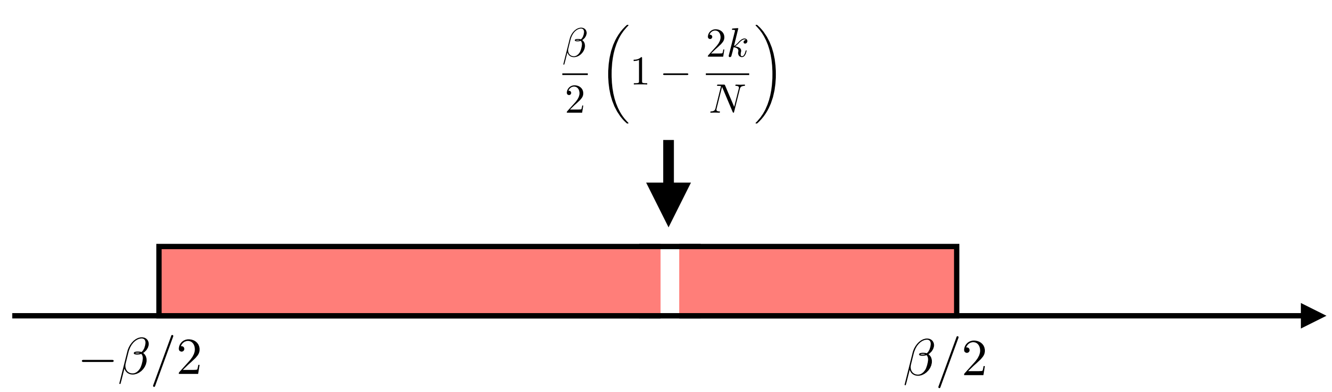


Bubbling geometry

M5-brane

$AdS_3 \times S^3$

M5-brane

$AdS_3 \times \tilde{S}^3$

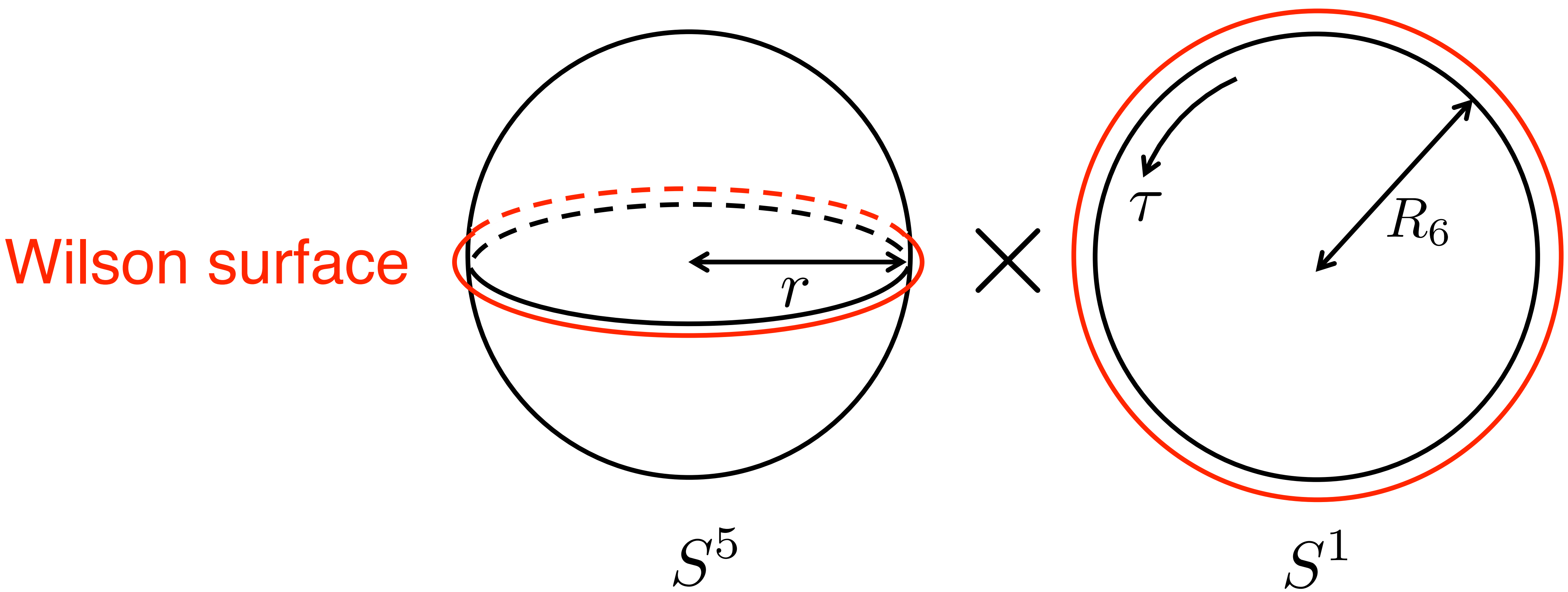
SUGRA on $AdS_7 \times S^4$

Metric

$$ds^2 = L^2 \left(\cosh^2 \rho d\tau^2 + d\rho^2 + \sinh^2 \rho d\Omega_5^2\right) + \frac{L^2}{4}\left(d\theta^2 + \sin^2 \theta d\tilde{\Omega}_3^2\right)$$

Boundary = $S^5 \times S^1$

Identification : $\tau \sim \tau + 2\pi \frac{R_6}{r}$



PST action [Pasti, Sorokin, Tonin '97] ← on a single M5-brane

$$S_{\rm M5} = T_5 \int d^6x \sqrt{-g} \left[\mathcal{L} + \frac{1}{4} \tilde{H}^{mn} H_{mn} \right] + T_5 \int \left(C_6 - \frac{1}{2} C_3 \wedge H_3 \right)$$

Notice:

We must add the boundary term $S_{\rm bdy}$ to regularize $S_{\rm M5}$.

We carefully determine the boundary term $S_{\rm bdy}$.

$$AdS_7 \supset AdS_3 \times S^3$$

Poincare coordinate : $ds^2 = \frac{L^2}{y^2}\left(dy^2 + dr_1^2 + r_1^2 d\phi^2 + dr_2^2 + r_2^2 d\Omega_3^2\right) + \frac{L^2}{4} d\Omega_4^2$

Ansatz : $r_2 = \kappa y, \quad y = y(\lambda)$

Flux quantization : $\kappa = \sqrt{\frac{k}{2N}}, \qquad k \in \mathbb{Z}$

$\Longrightarrow$ PST action

$S_{\mathrm{M5}} \propto$ (Volume of the M5-brane on the boundary)

||

Boundary term S_{bdy}

$\Longrightarrow$ regularized PST action $S_{\mathrm{M5}}^{\mathrm{reg}} = S_{\mathrm{M5}} + S_{\mathrm{bdy}} = 0$

$$\begin{array}{c} AdS_7 \\ \cup \\ AdS_3 \times S^3 \end{array}$$

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$$\begin{array}{c} AdS_7 \\ \cup \\ AdS_3 \times S^3 \end{array}$$

Global coordinate : $ds^2 = L^2\left[\cosh^2 u(\cosh^2 w d\tau^2 + dw^2 + \sinh^2 w d\phi^2) + du^2 + \sinh^2 u d\Omega_3\right] + \frac{L^2}{4} d\Omega_4^2$

Flux quantization : $\sinh u_k = \sqrt{\frac{k}{2N}}$

$$\Rightarrow \quad S_{\mathrm{M5}} = \frac{4\pi R_6}{r} N k \left(1 + \frac{k}{2N}\right) \sinh^2 w_0$$

Boundary term $\propto \sinh w_0 \cosh w_0$

$R_6 = \frac{g_{YM}^2}{8\pi^2}$

$\beta = \frac{g_{YM}^2}{2\pi r}$

$$\Rightarrow \quad S_{\mathrm{M5}}^{\mathrm{reg}} = -\frac{\beta N}{2} k \left(1 + \frac{k}{2N}\right)$$

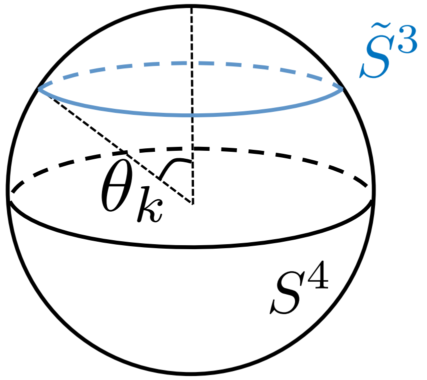
⟷ agree

$$\langle W_{\underbrace{\square\square\cdots\square}_{k}} \rangle$$

$$\begin{array}{ccc} AdS_7 & & S^4 \\ \cup & & \cup \\ AdS_3 & \times & \tilde{S}^3 \end{array}$$

Global coordinate : $ds^2 = L^2\left(\cosh^2\rho d\tau^2 + d\rho^2 + \sinh^2\rho d\Omega_5^2\right) + \frac{L^2}{4}\left(d\theta^2 + \sin^2\theta d\tilde{\Omega}_3^2\right)$

Flux quantization : $\cos\theta_k = 1 - \frac{2k}{N}$



$$\Rightarrow \quad S_{\mathrm{M5}} = \frac{4\pi R_6}{r} N k \left(1 - \frac{k}{N}\right) \sinh^2\rho_0$$

Boundary term $\propto \sinh\rho_0 \cosh\rho_0$

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$$\langle W_{k\left\{\begin{smallmatrix}\square\\\square\\\vdots\\\square\end{smallmatrix}\right.} \rangle$$

4. Summary

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M5-brane wrapping $\begin{matrix} AdS_3 \times \tilde{S}^3 \\ \cap \\ AdS_7 \times S^4 \end{matrix}$	⟺	Anti-symmetric rep $\langle W_{k\left\{\begin{smallmatrix}\square \\ \square \\ \vdots \\ \square\end{smallmatrix}\right.} \rangle \sim \exp\left[\frac{\beta N}{2} k \left(1 - \frac{k}{N}\right)\right]$