Lessons from holographic relative entropy

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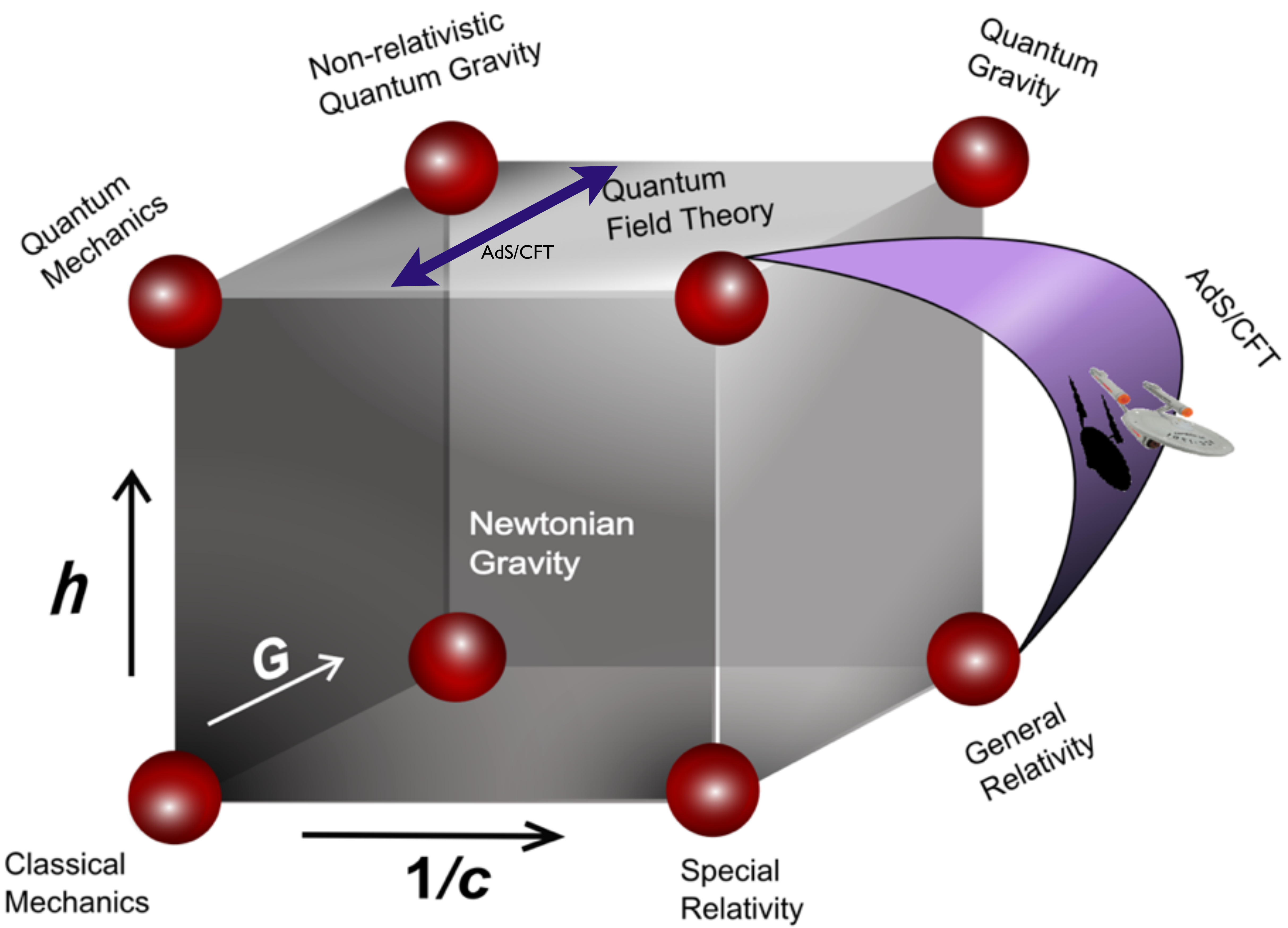
Thank you for inviting me!



Gamow-Ivanenko-Landau physics cube--holographic version

- Based on 1405.xxxx with Kallol Sen--This deals with holographic stress tensor correlation functions in general higher derivative gravity.
- and 1401.5089 with Shamik Banerjee, Arpan Bhattacharyya, Apratim Kaviraj, Kallol Sen and 1405.3743 with Shamik Banerjee, Apratim Kaviraj. This deals with constraining gravity using relative entropy.

Motivation--I

- AdS/CFT is frequently used to gain intuition about physics at strong coupling.
- For instance the famous viscosity bound conjecture due to Kovtun, Son, Starinets spurred experiments in cold atoms to set records for the substance with the lowest shear viscosity to entropy density ratio.
- The evidence for this conjecture was calculations done in AdS/CFT in IIB α'^3 sugra. (Buchel, Liu, Starinets; Myers, Paulos, AS)

- It is now known that this conjecture is incorrect (although the ratio is small). Calculations showing bound violations in toy models in AdS/CFT can be shown to be on firm footings and there are controlled 1/N violations of this bound. (Buchel, Myers, AS; for a 2 derivative example where this ratio vanishes see Sandip Trivedi's talk on Friday)

- This suggests the importance to understand 1/N as well as finite 't Hooft coupling effects in such calculations.

- It is important to understand what physical quantities can be considered universal even at finite N and finite 't Hooft coupling.

- Another example is the holographic c-theorems. Using intuition gained from certain toy models involving higher curvature terms Myers and I found evidence for the Cardy conjecture in even dim and proposed the finite part of the entanglement entropy across a sphere to play the role of the number of d.o.f. in odd dimensions. #UV>#IR. [Myers, AS 2010, 2011; EE proof by Casini, Huerta 2012; earlier related work by Emparan]

- However it was not clear to us how general these lessons were from the gravity point of view. Example: What about matter couplings at higher derivative order? ??NEC??

- For example it is still an unsolved problem what property in gravity guarantees c-theorems at finite coupling.

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- There are several more reasons why this problem is interesting. Another problem which has garnered a lot of recent attention is holographic entanglement entropy at finite coupling--namely corrections to the Ryu-Takayanagi formula due to higher curvature bulk terms. [Hung, Myers, Smolkin; de Boer, Parnachev, Kulaxizi; Bhattacharyya, Kaviraj, AS; Fursaev, Solodukhin, Patrushev; Bhattacharyya, Sharma, AS]

- Dong and Camps have proposed a general result but it appears to be at odds with an extension of Lewkowycz and Maldacena's derivations. Will not talk about this! [Bhattacharyya, Sharma]

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- How does one systematically proceed?

- Step 1, we need a general and useful procedure to write down the holographic stress tensor for an arbitrary higher derivative gravity.

- Step 2, we need a general and useful procedure to compute stress tensor correlation functions in arbitrary higher derivative gravity.

- Step 3,.....

- To accomplish step 1, we appeal to entanglement entropy.

- Recently Faulkner et al showed that for spherical entangling surfaces, using positivity of relative entropy, one recovers the linearized (general) gravity equations. As a by product they obtain a simple way to compute the renormalized holographic stress tensor for a general theory of gravity. [Faulkner, Guica, Hartman, Myers, Raamsdonk, 2013]

- The final result obtained is remarkably simple--it is given in terms of a few parameters in the linearized Wald functional.

- We will show that the parameters combine to a B-anomaly coefficient in even dimensions or equivalently to the two point function coefficient c_T in arbitrary dimensions.

- We will explain the simplicity using a background field approach (for anomalies, this was recently used by Rong-Xin Miao).

- We will further compute 2 and 3 point functions.

Motivation--II

- We wish to ask the following: Given a particular entanglement entropy functional what can we say about the gravity dual? For example, can we derive nonlinear Einstein equations from the Ryu-Takayanagi entropy functional?

- Partial progress has been achieved. Linearized Einstein equations can be shown to emerge. (Nozaki, Numasawa, Prudenziati, Takayanagi; Bhattacharya, Takayanagi; Lashkari, McDermott, Raamsdonk; Faulkner, Guica, Hartman, Myers, Raamsdonk)

- The tool at our disposal to study this problem is holographic relative entropy. (Blanco, Casini, Hung, Myers)

Relative entropy

Change in EE

Relative Entropy follows $S(\rho|\sigma) = \Delta\langle H\rangle - \Delta S$

Change in modular hamiltonian H

$\sigma = e^{-H}/\mathrm{tr}(e^{-H})$

From Klein's Inequality $S(\rho|\sigma) \geq 0$

We get $\Delta\langle H\rangle \geq \Delta S$

"1st law of entanglement" when saturated

From Holography we can compute ΔS

$$\Delta S = \underbrace{S(\rho)}_{\text{Area functonal}} - \underbrace{S(\sigma)}_{\text{Area functional}}$$

We can compute $S(\rho|\sigma)$ holographically provided we know H holographically [Blanco, Casini, Hung, Myers, 2013]

We know H in some special cases.

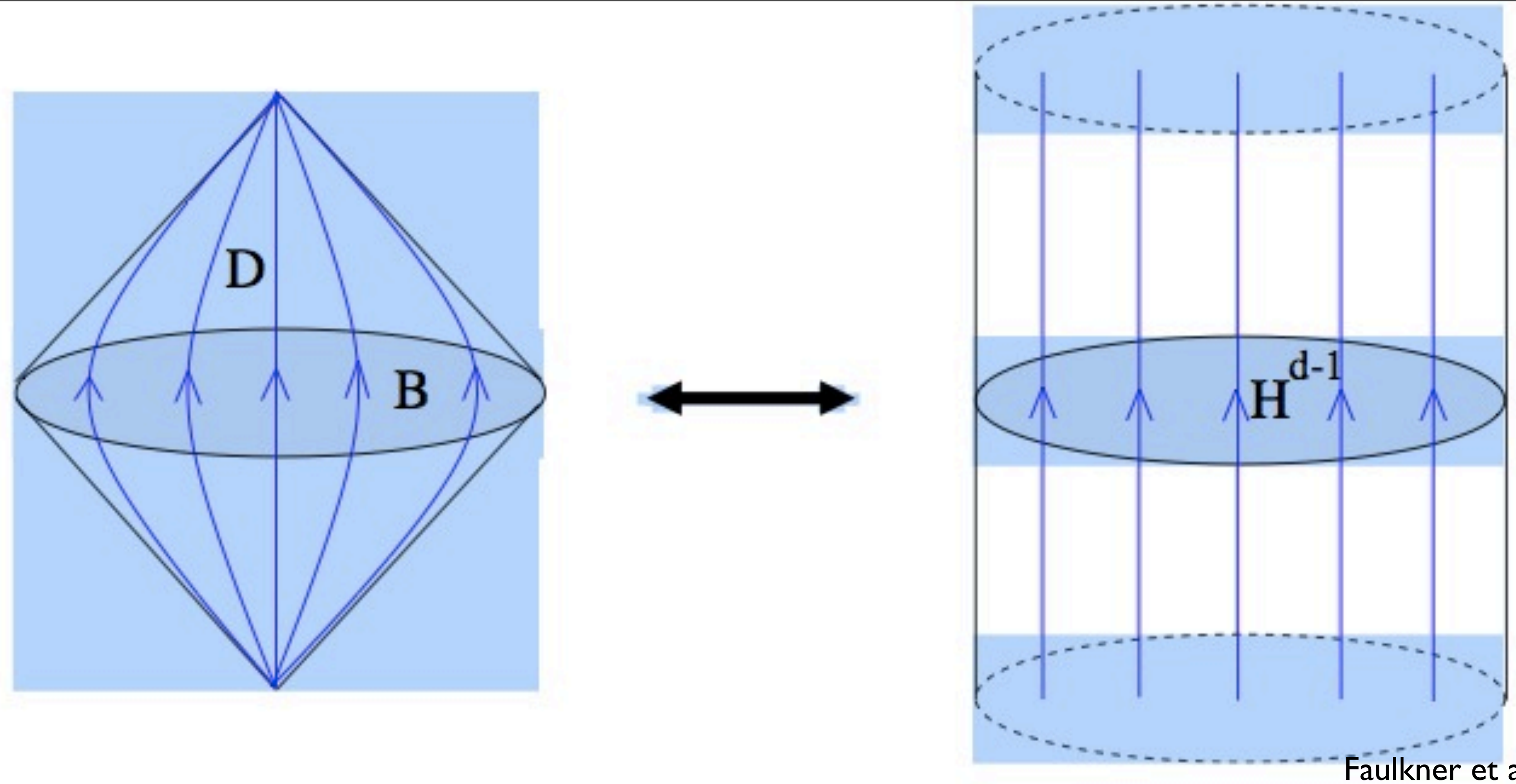
For a spherical entangling surface

If we knew how to calculate ΔS we would be able to infer the stress tensor.

$$H = 2\pi \int_{|x|<R} d^{d-1}x \, \frac{R^2 - r^2}{2R} \underbrace{T_{00}}(\vec{x})$$

Sphere radius

Can compute using holography can compute H



Faulkner et al, 2013.

- The causal development of a ball is mapped to the evolution generated by ordinary H in the hyperboloid. [Myers, AS; Casini, Huerta, Myers]

- EE gets mapped to thermal entropy.

- Can use Wald entropy to calculate the latter.[Myers, AS]

Holographic stress tensor from ΔS=ΔH

- More precisely we have

$$\delta T^{grav}_{tt}(\mathbf{x}_0) = \frac{d^2-1}{2\pi\Omega_{d-2}} \lim_{R\to 0}\left(\frac{1}{R^d}\delta S^{wald}_B\right)$$

renormalized stress tensor: want to know

Can calculate using linearization of Wald entropy functional. Small sphere limit makes integral local.

- Since linearization of the Wald entropy functional is involved in the underlying simplicity, this suggests that the calculation will be simpler if we do a background field expansion of the bulk lagrangian around a Riemann tensor of a maximally symmetric spacetime such that on the AdS background ΔRiemann is zero. [Kallol Sen, AS, to appear]

- So schematically we have the bulk lagrangian to have terms like $(\Delta \text{Riemann})^n$
- This suggests that the bulk action can be rearranged in terms of stress tensor correlation functions. For instance if we wanted n-point functions we only consider upto power n.
- The coefficients of these terms themselves would involve ALL higher derivative terms in the bulk lagrangian we started with.

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Full metric

$$\bar{R}_{abcd} = -\frac{1}{\tilde{L}^2}(g_{ac}g_{bd} - g_{ad}g_{bc})$$

$$\Delta R_{abcd} = R_{abcd} - \bar{R}_{abcd}$$

$$(\Delta R_{abcd})^{AdS} = 0$$

- There are some immediate advantages of doing this background field expansion. In TT gauge:

$$(\Delta R_{ab})^{linearized} = 0$$

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- Consider $(\Delta R)^2$ or $(\Delta R_{ab})^2$ in the action.

- To compute the effect of these terms on the n-point function, we have to expand this around AdS upto n'th order in the perturbation.

- This immediately tells us that these will start contributing only to 4-point functions. This is the reason for the simple expressions we will find for one, two and three point functions.

Background field Lagrangian from $\mathcal{L}(g^{ab}, R_{cdef})$

Kallol Sen, AS 2014

$$\mathcal{L} = (c_0 + c_1 \Delta R + \frac{c_4}{2}\Delta R^2 + \frac{c_5}{2}\Delta R^{ab}\Delta R_{ab} + \frac{c_6}{2}\Delta R^{abcd}\Delta R_{abcd} + \sum_{i=1}^{8} \tilde{c}_i \Delta \mathcal{K}_i + \cdots)$$

Two point function

$$\langle T_{ab}(x) T_{cd}(x') \rangle = \frac{\mathcal{C}_T}{|x-x'|^{2d}} \mathcal{I}_{ab,cd}(x-x')$$

$$\mathcal{C}_T = 2\frac{d+1}{d-1}\frac{\Gamma[d+1]}{\pi^{d/2}\Gamma[d/2]}\tilde{L}^{d-1}[c_1 + 2(d-2)c_6]$$

So simple!!

Stress tensor and geometry

$$\boxed{\langle T_{\mu\nu} \rangle = \frac{\pi^{d/2}}{2\tilde{L}^2}\frac{d-1}{d+1}\frac{\Gamma[d/2]}{\Gamma[d]}\mathcal{C}_T h^{(d)}_{\mu\nu}}$$

A complete piece in the AdS/CFT dictionary!!

$$ds^2 = \tilde{L}^2 \frac{dz^2}{z^2} + \frac{1}{z^2}(g^{(0)}_{\mu\nu} + z^2 g^{(2)}_{\mu\nu} + \cdots z^d h^{(d)}_{\mu\nu} + \cdots) dx^\mu dx^\nu$$

[Fefferman, Graham; de Haro, Skenderis, Solodukhin]

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A-type anomaly

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A-type anomaly

Two point function

$$\langle T_{ab}(x) T_{cd}(x') \rangle = \frac{\mathcal{C}_T}{|x - x'|^{2d}} \mathcal{I}_{ab,cd}(x - x')$$

B-type anomaly

$$\mathcal{C}_T = 2\frac{d+1}{d-1}\frac{\Gamma[d+1]}{\pi^{d/2}\Gamma[d/2]}\tilde{L}^{d-1}[c_1 + 2(d-2)c_6]$$

So simple!!

Stress tensor and geometry

$$\boxed{\langle T_{\mu\nu} \rangle = \frac{\pi^{d/2}}{2\tilde{L}^2}\frac{d-1}{d+1}\frac{\Gamma[d/2]}{\Gamma[d]}\mathcal{C}_T h^{(d)}_{\mu\nu}}$$

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[Fefferman, Graham; de Haro, Skenderis, Solodukhin]

- In even dimensions c_T is related to a B-anomaly coefficient. Let me sketch the argument. RG equation gives [Erdmenger, Osborn; Osborn, Petkou]

$$(\mu\partial_\mu + 2\int d^dx g^{\mu\nu}\frac{\delta}{\delta g^{\mu\nu}})W = 0.$$

Quantum effective action

- Hit twice more with $\dfrac{\delta}{\delta g_{\mu\nu}}$

$$\mu\partial_\mu\langle T_{ab}(x)T_{cd}(0)\rangle = -2\int d^dx\frac{\delta^2 A_{anomaly}}{\delta g^{ab}\delta g^{cd}}.$$

- Since RHS is to be computed around flat space, it is easy to see that we need terms in the anomaly that have at most 2 Riemann curvatures. In 4 dimensions an explicit calculation picks out the c-anomaly coefficient.

- In 6 dimensions the only anomaly term with 2 Riemann's is B_3. Hence this coefficient is picked out. All this we have explicitly check directly in holography.

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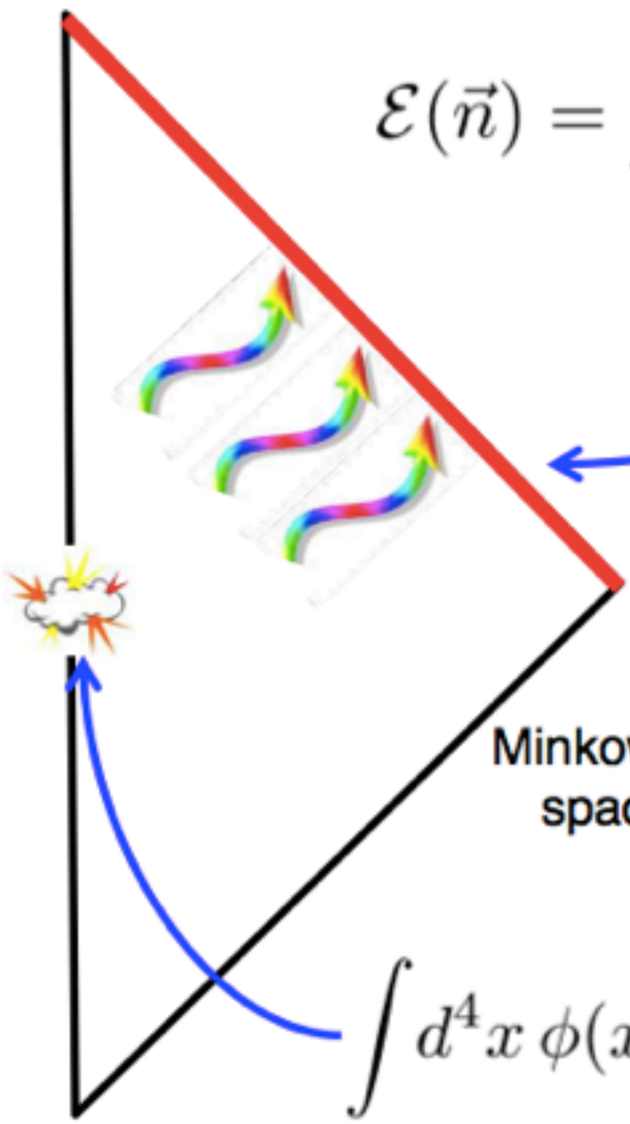
- In 6 dimensions the only anomaly term with 2 Riemann's is B_3. Hence this coefficient is picked out. All this we have explicitly check directly in holography.

We can also compute 3 point functions!

(Hofman & Maldacena, 0803.1467)

"Conformal Collider Physics"

- consider scattering "experiments" in d=4 CFT's
- insert disturbance with stress tensor
- measure energy flux at infinity



$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_{-\infty}^{\infty} dt\, n^i T_i{}^0(t, r n^i)$$

$$\int d^4x\, \phi(x)\, e^{-iEt}\, \epsilon_{ij} T_{ij}$$

$$\langle \mathcal{E}(\vec{n}) \rangle = \frac{\langle 0 | \epsilon^*_{ij} T_{ij}\, \mathcal{E}(\vec{n})\, \epsilon_{kl} T_{kl} | 0 \rangle}{\langle 0 | \epsilon^*_{ij} T_{ij}\, \epsilon_{kl} T_{kl} | 0 \rangle}$$

controlled by 3-pt function!!

"Conformal Collider Physics"

- result fixed by symmetry of "experiment":

$$\langle \mathcal{E}(\vec{n}) \rangle = \frac{\langle 0 | \epsilon^*_{ij} T_{ij}\, \mathcal{E}(\vec{n})\, \epsilon_{kl} T_{kl} | 0 \rangle}{\langle 0 | \epsilon^*_{ij} T_{ij} \epsilon_{kl} T_{kl} | 0 \rangle}$$

$$= \frac{E}{4\pi}\left[1 + t_2 \left(\frac{\epsilon^*_{ij}\epsilon_{il} n_j n_l}{\epsilon^*_{ij}\epsilon_{ij}} - \frac{1}{3} \right) + t_4 \left(\frac{|\epsilon_{ij} n_i n_j|^2}{\epsilon^*_{ij}\epsilon_{ij}} - \frac{2}{15} \right) \right]$$

Comments:

- only three independent parm's amongst $a,\ c,\ t_2,\ t_4$:

$$\frac{c-a}{c} = \frac{1}{6}\, t_2 + \frac{4}{45}\, t_4$$

- $t_4 = 0$ for supersymmetric CFT's

- consider scattering "experiments" in d=4 CFT's

$$\langle \mathcal{E}(\vec{n}) \rangle = \frac{\langle 0 | \epsilon^*_{ij} T_{ij} \, \mathcal{E}(\vec{n}) \, \epsilon_{kl} T_{kl} | 0 \rangle}{\langle 0 | \epsilon^*_{ij} T_{ij} \epsilon_{kl} T_{kl} | 0 \rangle}$$

$$= \frac{E}{4\pi} \left[1 + t_2 \left(\frac{\epsilon^*_{ij} \epsilon_{il} n_j n_l}{\epsilon^*_{ij} \epsilon_{ij}} - \frac{1}{3} \right) + t_4 \left(\frac{|\epsilon_{ij} n_i n_j|^2}{\epsilon^*_{ij} \epsilon_{ij}} - \frac{2}{15} \right) \right]$$

<u>Comments</u>:

- demanding $\langle \mathcal{E}(\vec{n}) \rangle \geq 0$ imposes nontrivial constraints

spin 2: $$1 - \frac{1}{3} t_2 - \frac{2}{15} t_4 \geq 0$$

spin 1: $$1 + \frac{1}{6} t_2 - \frac{2}{15} t_4 \geq 0$$

spin 0: $$1 + \frac{1}{3} t_2 + \frac{8}{15} t_4 \geq 0$$

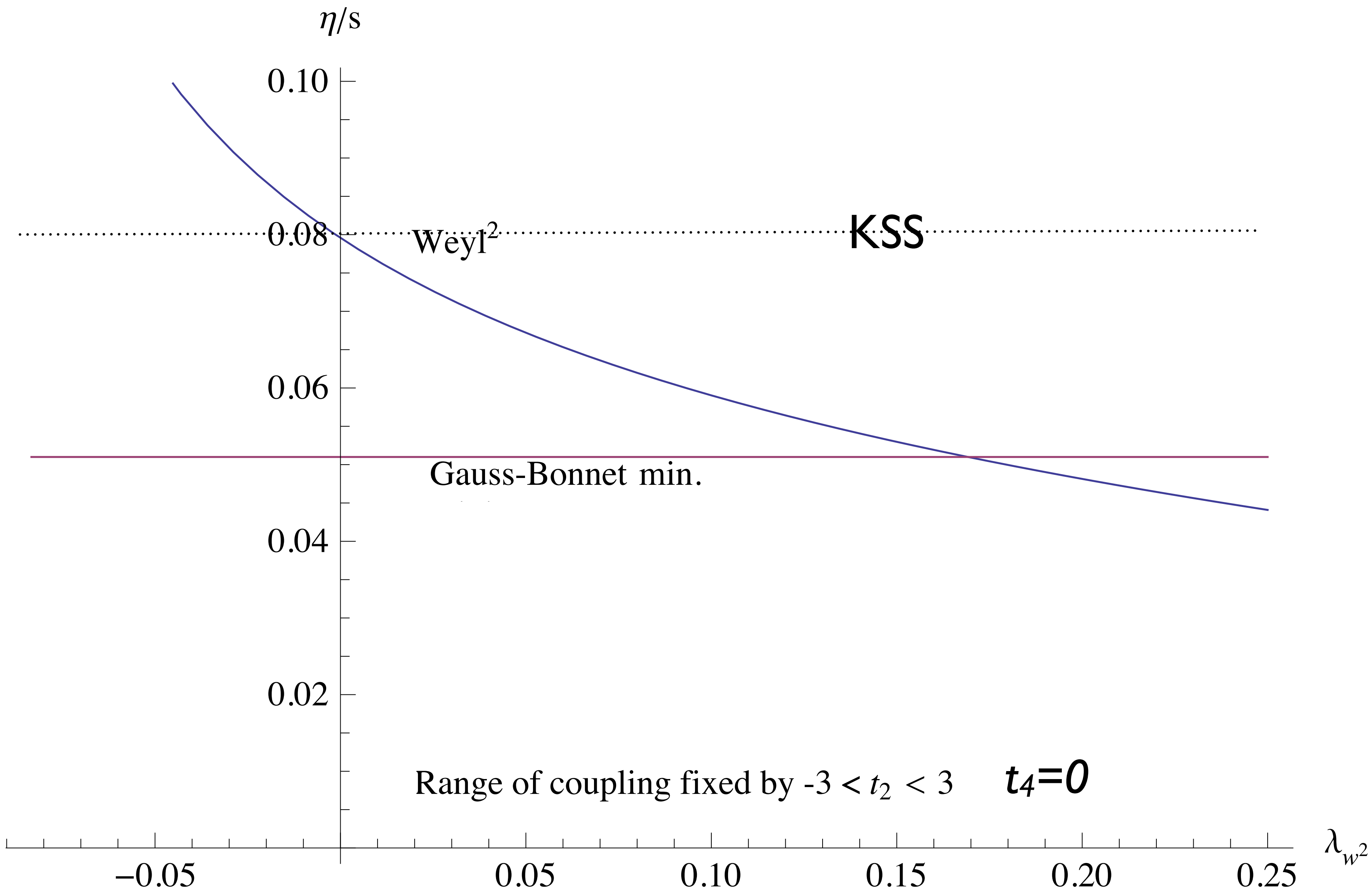
(Latorre & Osborn, hep-th/9703196)

$$t_2 = \frac{d(d-1)}{c_1 + 2(d-2)c_6}[2c_6 - 12(3d+4)\tilde{c}_7 + 3(7d+4)\tilde{c}_8],$$
$$t_4 = \frac{6d(d^2-1)(d+2)}{c_1 + 2(d-2)c_6}(2\tilde{c}_7 - \tilde{c}_8).$$

- When $t_4=0$ (& $c_7=c_8=0$)

$$(\frac{\eta}{s} - \frac{1}{4\pi})s \propto -c_6$$

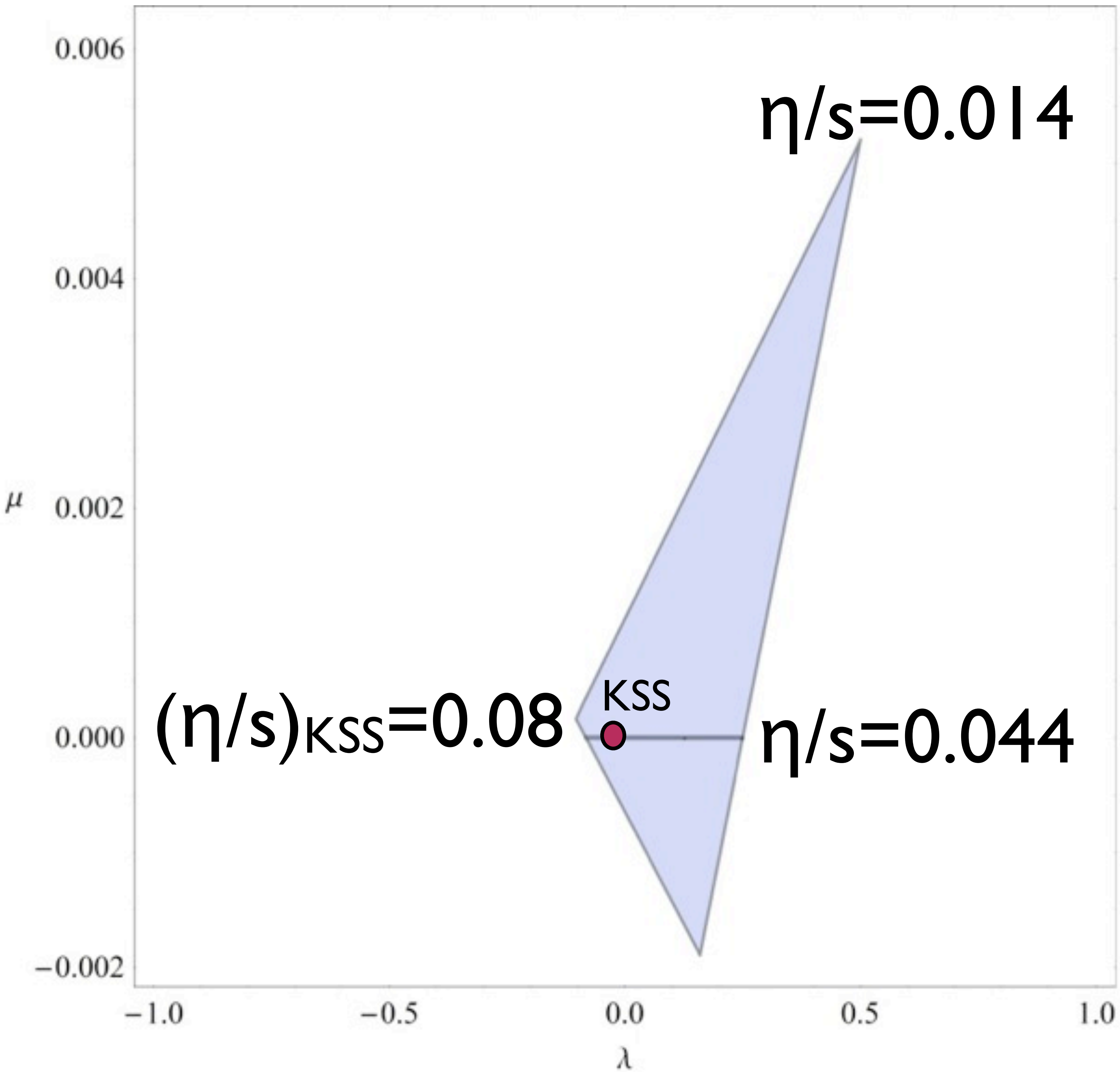
- Thus KSS bound is violated whenever we have $t_2>0$ in a perturbative expansion.



- Even without unitarity, positive energy correlations lead to a bound on the shear viscosity. Easily calculable using pole method. (Paulos)

- For general four derivative, it is possible to tune couplings so that positive energy correlations are satisfied but the ratio is zero with no other obvious pathologies except ghosts (boundary conditions to eliminate them??).

$$S = \frac{1}{2\ell_P^3}\int d^5x\sqrt{g}\left(R + \frac{12}{L^2} + \frac{\lambda}{2}L^2W^2 + \mu L^4 W^3\right)$$



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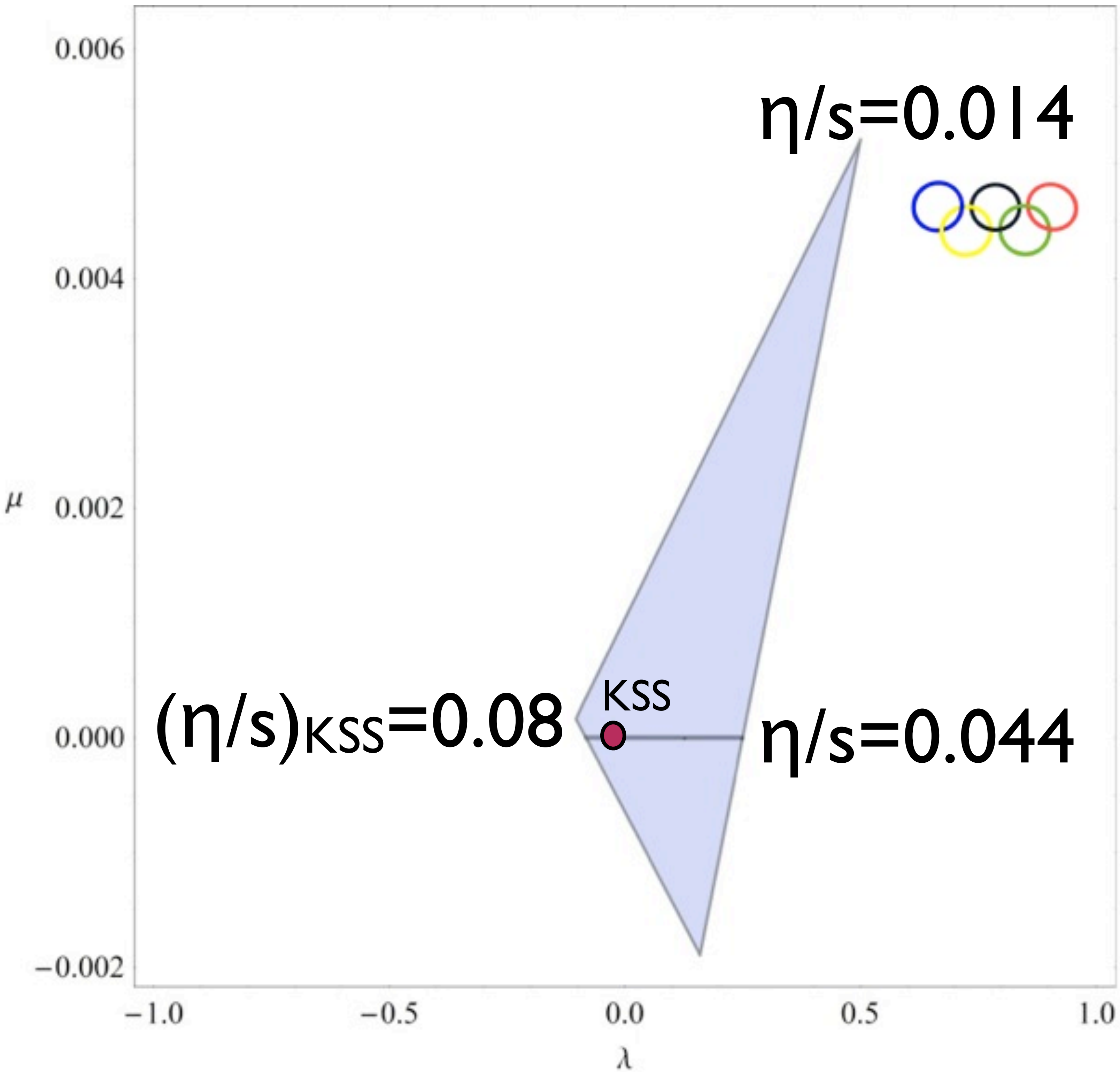


- Till now the lessons we have learnt used the saturation of the inequality, i.e., the 1st law of entanglement.

- What about the inequality?

$\Delta H > \Delta S$

Consider a scalar field in bulk

$$I = \frac{1}{2\, l_p^{d-1}} \int d^{d+1}x \sqrt{G} \left(R + \frac{d(d-1)}{L^2} - \frac{1}{2}(\partial \phi)^2 - \frac{1}{2} m^2 \phi^2 \right)$$

Choose σ : Empty AdS_{d+1}

ρ : Scalar field turned on

Asymptotically $\phi \sim \gamma\, O\, z^{\Delta}$

O: scalar operator of dim Δ

$$m^2 = \Delta(d - \Delta)$$

$$\frac{d}{2} - 1 < \Delta < \frac{d}{2}$$

Perturbed boundary metric:

$$ds^2 = \frac{L^2}{z^2}\left[dz^2 + \eta_{\mu\nu}\left(1 + z^{2\Delta}\frac{\gamma^2 O^2}{4d(d-1)}\right)dx^\mu dx^\nu\right]$$

In this case holographic stress tensor $T_{\mu\nu} = 0$

So $\Delta\langle H\rangle = 0$

And EE: $\Delta S = \dfrac{2\pi}{l_p^{d-1}}\int d^{d-1}x\,\Delta h$

$$= -\frac{\gamma^2 L^{d-1}}{l_p^{d-1}}\frac{\pi^{3/2}\left(\Delta - \frac{(d-2)^2}{2(d-1)}\right)\Gamma\left[\Delta - \frac{d}{2}+1\right]}{8\Gamma\left[\Delta - \frac{d}{2}+\frac{5}{2}\right]}\Omega^{d-2}R^{2\Delta}O^2$$

$\longrightarrow$ $\boxed{\Delta\langle H\rangle \geq \Delta S}$ if unitarity bound holds

Nonlinear constraints from ΔH>ΔS

- Consider AAdS in FG expansion
- n_1, n_2 below most general for constant stress tensor at quadratic order in stress tensor

$$ds^2 = \frac{L^2}{z^2} dz^2 + g_{\mu\nu} dx^\mu dx^\nu \,.$$

$$g_{\mu\nu} = \frac{L^2}{z^2}\left[\eta_{\mu\nu} + a z^d T_{\mu\nu} + a^2 z^{2d}(n_1 T_{\mu\alpha} T^\alpha_\nu + n_2\, \eta_{\mu\nu} T_{\alpha\beta} T^{\alpha\beta}) + \cdots\right]$$

- At linear order in stress tensor, inequality saturated.

- Next order ΔH will not contribute so ΔS<0. Correction to entangling surface is

$$z_1 = -\frac{R^2 z_0^{d-1}}{2(d+1)}(T+T_x)$$

- Leads to

$$\Delta^{(2)} S = 2\pi (L/\ell_P)^{d-1}\Omega_{d-2}\left(C_1 T^2 + C_2 T_{ij}^2 + C_3 T_{i0}^2\right)$$

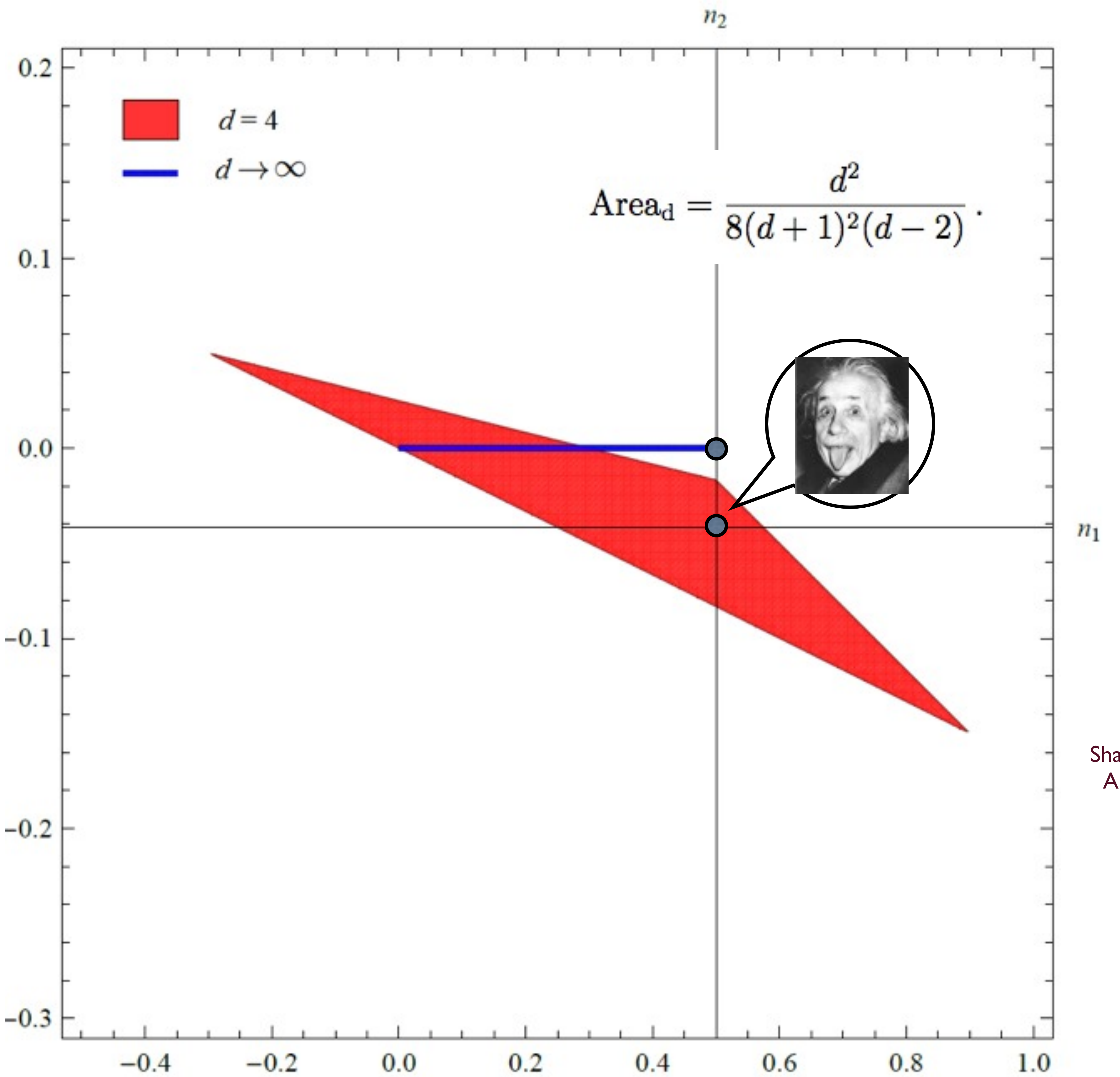
$$C_1 = \frac{2^{-3-d} d \left(1 + 4\left(d^2-1\right) n_2\right) \sqrt{\pi} R^{2d} \Gamma[d+1]}{\left(d^2-1\right) \Gamma\left[\frac{3}{2}+d\right]},$$

$$C_2 = \frac{2^{-3-d} d \sqrt{\pi} R^{2d} \Gamma[1+d]}{\left(d^2-1\right) \Gamma\left[\frac{3}{2}+d\right]} \left(-1 - 2d + 4(d+1) n_1 + 4\left(d^2-1\right) n_2\right),$$

$$C_3 = -\frac{2^{-1-d} d (n_1 + 2(d-1) n_2) \sqrt{\pi} R^{2d} \Gamma[1+d]}{(d-1) \Gamma\left[\frac{3}{2}+d\right]}.$$

- We get the following inequalities

$$\begin{aligned} n_1 + 2(d-1) n_2 &\geq 0 \\ 2d + 1 - 4(d+1) n_1 - 4(d^2-1) n_2 &\geq 0 \\ d + 2 - 4(d+1) n_1 - 4d(d^2-1) n_2 &\geq 0 \end{aligned}$$



Shamik Banerjee, Arpan Bhattacharyya, Apratim Kaviraj, Kallol Sen, AS 2014

For non-constant stress tensor we find

$$\frac{z^2}{L^2}\,g_{\mu\nu} = \eta_{\mu\nu} + z^4\left(T_{\mu\nu} - \frac{1}{12}z^2\Box T_{\mu\nu}\right) + z^8\left(n_1 T_{\mu\alpha}T_\nu{}^\alpha + n_2\eta_{\mu\nu}T_{\alpha\beta}T^{\alpha\beta} + z^2\mathcal{T}_{\mu\nu}\right)$$

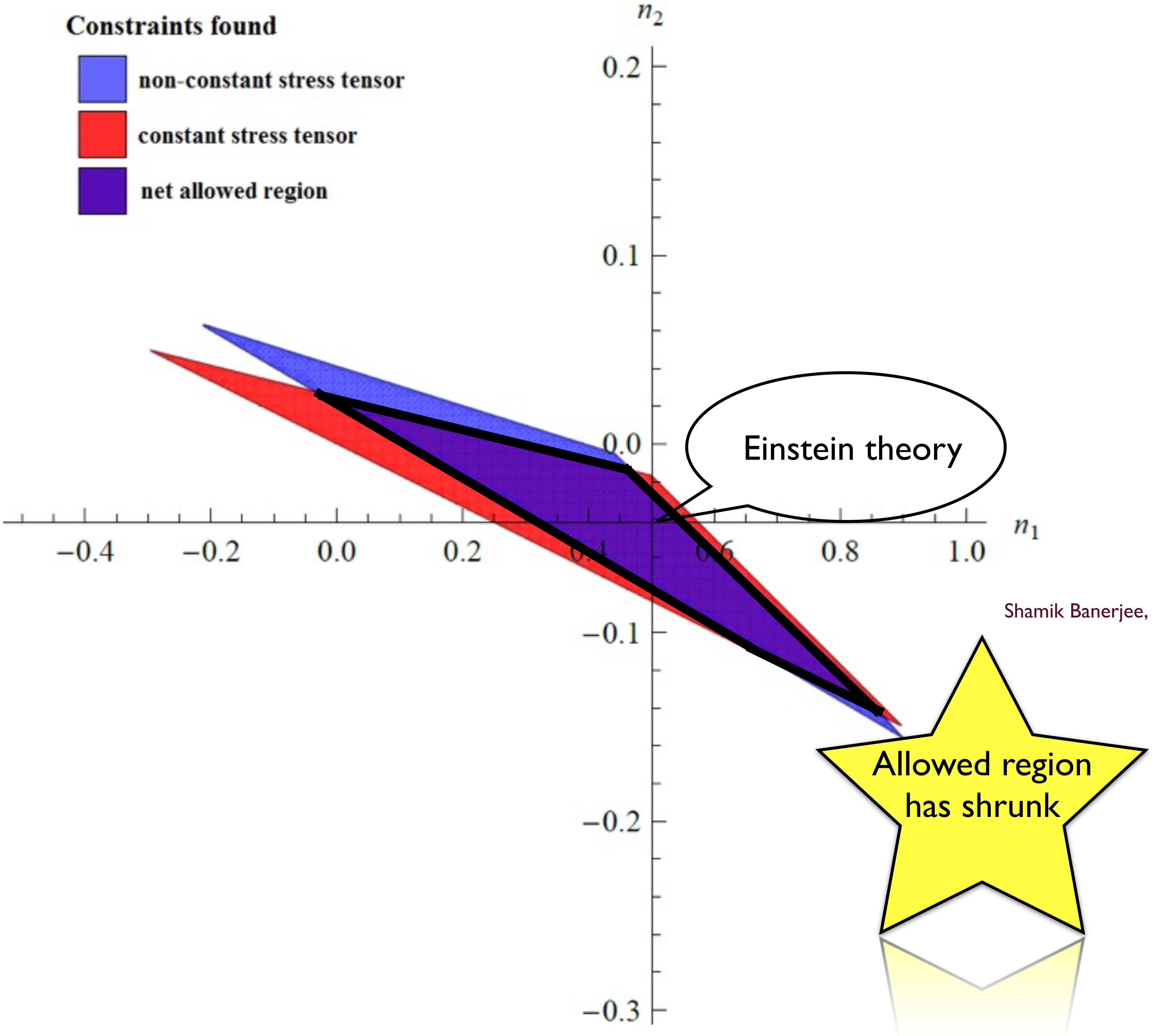
$$\begin{aligned}\mathcal{T}_{\mu\nu} =& n_3\left(T_{\mu\alpha}\Box T_\nu{}^\alpha + T_{\nu\alpha}\Box T_\mu{}^\alpha\right) + n_4\eta_{\mu\nu}T_{\alpha\beta}\Box T^{\alpha\beta} + n_5\partial_\mu T_{\alpha\beta}\partial_\nu T^{\alpha\beta} + n_6\partial_\alpha T_{\mu\beta}\partial^\beta T_\nu{}^\alpha + n_7\partial_\mu\partial_\nu T_{\alpha\beta}T^{\alpha\beta}\\ &+ n_8\partial_\alpha T_{\mu\beta}\partial^\alpha T_\nu{}^\beta + n_9\left(\partial_\mu T_{\alpha\beta}\partial^\beta T_\nu{}^\alpha + \partial_\nu T_{\alpha\beta}\partial^\beta T_\mu{}^\alpha\right) + n_{10}\eta_{\mu\nu}\partial_\alpha T_{\beta\gamma}\partial^\alpha T^{\beta\gamma} + n_{11}\partial_\alpha T_{\gamma\beta}\partial^\beta T^{\gamma\alpha}\\ &+ n_{12}\left(T^{\beta\alpha}\partial_\alpha\partial_\mu T_{\nu\beta} + T^{\beta\alpha}\partial_\alpha\partial_\nu T_{\mu\beta}\right) + n_{13}T^{\alpha\beta}\partial_\alpha\partial_\beta T_{\mu\nu}\,.\end{aligned} \tag{44}$$

Derivative expansion: $R\partial_i \ll 1$

$$\begin{aligned}z_1 = -{z_0}^3R^2\Bigg(&\frac{T+T_x}{10} + \frac{1}{12}\left(x^i\partial_i T + x^ix^jx^k\frac{\partial_k T_{ij}}{R^2}\right)\\ &+\frac{1}{28}\left(x^ix^j\partial_i\partial_j T + x^ix^jx^kx^l\frac{\partial_i\partial_j T_{kl}}{R^2}\right) - \frac{k^2\left(R^2-r^2\right)}{168}\left(\partial^2 T + x^ix^j\frac{\partial^2 T_{ij}}{R^2}\right)\Bigg)\end{aligned}$$

To get rid of constant T contribution consider $T_{\mu\nu}(\mathbf{x}=0)=0$

$$\begin{aligned}\Delta^{(2)}S = \frac{16\pi^2L^3R^{10}}{31185\,\ell_P^3}\Big(&-\left(\partial_i T_{0j}\right)^2\left(-1+60n_1+336n_2\right) + 6\partial_i T_{0j}\partial^j T_0{}^i\left(-1+n_1\right)\\ &+\left(\partial_i T_{jk}\right)^2\left(-28+60n_1+168n_2\right) + \partial_i T_{kj}\partial^k T^{ij}\left(5-6n_1\right) + \left(\partial_i T\right)^2\left(2+168n_2\right)\Big)\end{aligned}$$



Shamik Banerjee, Apratim Kaviraj, AS 2014

Lessons learnt

- Relative entropy taught us a clever way to do holographic renormalization.

- It led to using the background field method to compute one, two and three point functions for stress tensors in a large class of bulk theories quite easily arising from $\mathcal{L}(g^{ab}, R_{cdef}, \nabla_p R_{cdef})$.

- It led to interesting nonlinear constraints on the metric hinting at a possible derivation of Einstein equations from entanglement.