INSTABILITY OF ROTATING BLACK HOLES : LARGE D ANALYSIS

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based on: arXiv:1402.6215 (R.Emparan and R.Suzuki, KT)

WHAT WE DO AND FIND

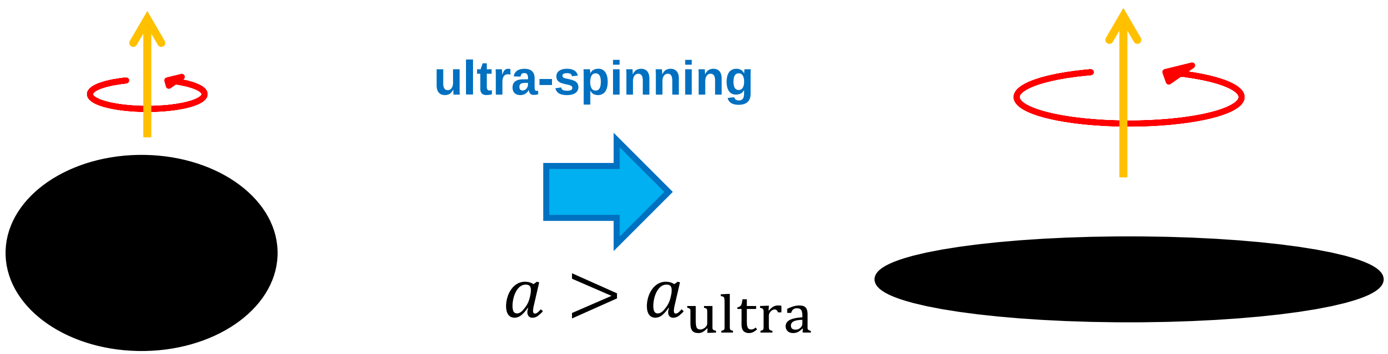
- study the **(in)stability of rotating black holes** by large D expansion
  - compute QNM frequency explicitly
    - MP-BH becomes unstable at $a > a_c$
    - relation with ultra-spinning condition a_{ultra}
    - satisfy superradiance condition
- consider odd-D MP-BH with equal angular momenta
  - cohomogenity-1 solution = perturbation eq. becomes ODE, but coupled

[Murata-Soda (2008), Dias, et.al. (2010)]

ULTRA-SPINNING

- MP-BH can be unstable (at D>5)

[Emparan-Myers (2003),
Dias, et.al. (2010)]



"black brane is unstable" = "MP-BH is also unstable"

e.g.

$$a_{\mathrm{ultra}} = \sqrt{\frac{D-3}{D-5}}$$

(MP-BH with single rotation)

$$a_{\mathrm{ultra}} = \sqrt{\frac{D-3}{D-1}} \leq a_{\mathrm{extreme}}$$

(MP-BH with equal spin)

- numerically confirmed for some modes

[Dias, et.al. (2010,2011)]

SUPERRADIANCE

- we are also interested in non-axisymmetric perturbation

$$\delta g_{\mu\nu} \propto e^{-i\omega t}\, e^{im\psi}$$

- unstable mode (if exists) with $\mathrm{Im}[\omega] > 0$ satisfies superradiance condition?

$$\mathrm{Re}[\omega] < m\Omega_H$$

- numerically checked this condition

single rotation $a_c > a_{\mathrm{superradiance}}$ [Shibata-Yoshino (2010), Dias-Hartnett-Santos (2014)]

equal rotation $a_c = a_{\mathrm{superradiance}}$ [Hartnett-Santos (2013)]

CHECK LIST

- [ ] we find threshold angular momentum of instability from QNM frequency

$$a_c^{m=0} \qquad a_c^{m>0}$$

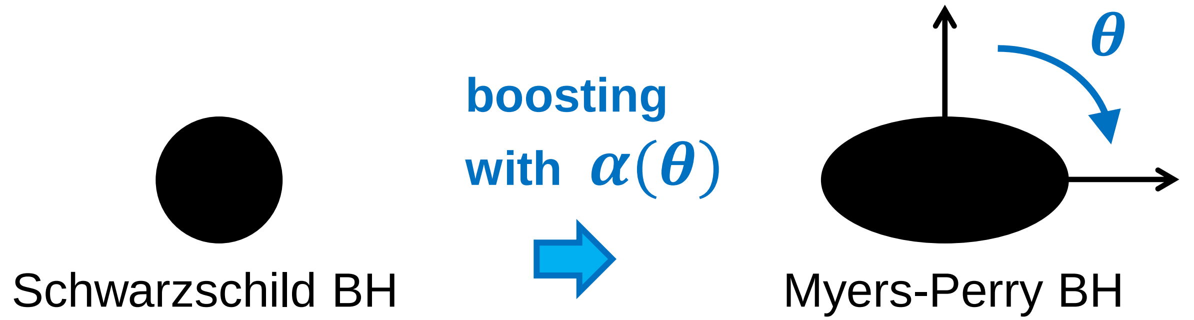
- [ ] relation with <u>ultra-spinning condition</u> and check <u>superradiance condition</u>

$$a_c > a_{\mathrm{ultra}} \quad \overset{?}{\text{or}} \quad a_c < a_{\mathrm{ultra}}$$

- [ ] what is the dominant instability? bar mode?

ROTATION=BOOST

- “rotation”, in general, makes a problem complicate (but so interesting)
- at large D, “**rotation**” can be treated as “**boost**”



[Emparan-Grumiller-KT (2013)]

- moving to “rest frame”, problem of MP-BH can be reduced to one of Schwarzschild BH

Notion!! **“boost symmetry” exists only at $D = \infty$**

MP-BH WITH EQUAL ANGULAR MOMENTA

- equal angular momenta case becomes much more tractable

"**cohomogenity-1**" ⇨ boost α is **constant**

※ dynamics of $\alpha(\theta)$ is trivial

D=2N+3 dim MP-BH with equal spin

$$ds^2 = -\frac{G(r)}{H(r)}dt^2 + \frac{dr^2}{G(r)} + r^2 H(r)\left(d\psi - \Omega(r)dt + A_a dx^a\right)^2 + r^2 \underbrace{\hat{g}_{ab}}_{\text{metric on CP^N}} dx^a dx^b$$

metric on CP^N

$$G(r) = 1 - \left(\frac{r_0}{r}\right)^{2N}\left(1 - \frac{a^2}{r^2}\right) \qquad H(r) = 1 + \frac{a^2}{r^2}\left(\frac{r_0}{r}\right)^{2N} \qquad \Omega(r) = \frac{a}{r^2 H(r)}\left(\frac{r_0}{r}\right)^{2N}$$

PERTURBATION EQ

- consider **scalar type** perturbations on CP^N

$$(D^2 + \lambda)Y_{\ell m} = 0$$ (scalar harmonics on CP^N)

$$\lambda = \ell(\ell + D - 3) - m^2 \qquad \ell = 0,2,4,\ldots \qquad -\ell \le m \le \ell$$

- tensor type perturbation is stable [Kunduri-Lucietti-Reall (2006)]

- perturbation can be expanded by $Y_{\ell m}$

$$\delta g_{\mu\nu} = F_{\mu\nu}(r)e^{-i\omega t}\, e^{im\psi}Y_{\ell m}$$

➢ perturbation equations become **coupled ODEs**

[Dias-Figueras- Monteiro-Reall-Santos (2010)]

$$\begin{aligned}
&-\left[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2N}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}\right.\\
&\qquad\left.-\frac{2}{g^2}\left(\left(\frac{f'}{f}\right)^2-\frac{1}{2}\left(\frac{h\Omega'}{f}\right)^2\right)\right]f_{00}\\
&\quad+2\,\mathrm{i}\left[\frac{2f'(\omega-m\Omega)}{f^2g}+\frac{m\Omega'}{fg}\right]f_{01}+2\left[\frac{h\Omega'}{fg^2}\frac{\partial}{\partial r}+\frac{1}{2g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)+\frac{Nh\Omega'}{fg^2r}\right]f_{02}\\
&\quad+\left[\left(\frac{h\Omega'}{fg}\right)^2-\frac{2}{g}\frac{\partial}{\partial r}\left(\frac{f'}{fg}\right)\right]f_{11}-\frac{2}{g^2}\left[\frac{f'h'}{fh}+\frac{1}{2}\left(\frac{h\Omega'}{f}\right)^2\right]f_{22}-\frac{4Nf'}{fg^2r}H_L=-k^2f_{00}\,,
\end{aligned}$$

$$\begin{aligned}
&-\left[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2N}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}\right.\\
&\qquad\left.-\frac{1}{g^2}\left(2\left(\frac{f'}{f}\right)^2+\left(\frac{h'}{h}\right)^2+\frac{2N}{r^2}\right)+\frac{2}{g}\frac{\partial}{\partial r}\left(\frac{f'}{fg}\right)\right]f_{01}\\
&\quad+\left[\frac{h\Omega'}{fg^2}\frac{\partial}{\partial r}+\frac{3}{2g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)+\frac{h\Omega'}{fg^2}\left(-\frac{f'}{f}+\frac{h'}{h}+\frac{N}{r}\right)\right]f_{12}\\
&\quad+\mathrm{i}\left[\frac{2f'(\omega-m\Omega)}{f^2g}+\frac{m\Omega'}{fg}\right](f_{00}+f_{11})-\mathrm{i}\left[\frac{h\Omega'(\omega-m\Omega)}{f^2g}-\frac{2mh'}{h^2g}\right]f_{02}\\
&\quad+\frac{1}{r^2g\sqrt{\lambda}}\left[W^+(\lambda-2mN)+W^-(\lambda+2mN)\right]=-k^2f_{01}\,,
\end{aligned}$$

$$\begin{aligned}
&-\left[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2N}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}\right.\\
&\qquad\left.-\frac{1}{g^2}\left(\left(\frac{f'}{f}-\frac{h'}{h}\right)^2-2\left(\frac{h\Omega'}{f}\right)^2\right)-\frac{2Nh^2}{r^4}\right]f_{02}\\
&\quad+\left[\frac{h\Omega'}{fg^2}\frac{\partial}{\partial r}+\frac{1}{2g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)+\frac{h\Omega'}{fg^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{N}{r}\right)\right]f_{00}+\mathrm{i}\left[\frac{h\Omega'(\omega-m\Omega)}{f^2g}-\frac{2mh'}{h^2g}\right]f_{01}\\
&\quad-\left[\frac{1}{g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)-\frac{h\Omega'}{fg^2}\left(\frac{f'}{f}-\frac{h'}{h}\right)\right]f_{11}+\mathrm{i}\left[\frac{2f'(\omega-m\Omega)}{f^2g}+\frac{m\Omega'}{fg}\right]f_{12}\\
&\quad+\left[\frac{h\Omega'}{fg^2}\frac{\partial}{\partial r}+\frac{1}{2g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)+\frac{Nh\Omega'}{fg^2r}\right]f_{22}\\
&\quad-\frac{\mathrm{i}h}{r^3\sqrt{\lambda}}\left[W^+(\lambda-2mN)-W^-(\lambda+2mN)\right]-\frac{2Nh\Omega'}{fg^2r}H_L=-k^2f_{02}\,,
\end{aligned}$$

$$\begin{aligned}
&-\left[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2N}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda-2(N+1)-2m}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}\right.\\
&\qquad\left.-\frac{1}{g^2}\left(\left(\frac{f'}{f}-\frac{1}{r}\right)^2-\frac{1}{2}\left(\frac{h\Omega'}{f}\right)^2\right)-\frac{2h^2}{r^4}\right]W^+\\
&\quad+\left[\frac{h\Omega'}{fg^2}\frac{\partial}{\partial r}+\frac{1}{2g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)+\frac{(N+1)h\Omega'}{fg^2r}\right]Z^+\\
&\quad+\mathrm{i}\left[\frac{2f'(\omega-m\Omega)}{f^2g}+\frac{m\Omega'}{fg}-\frac{2h^2\Omega'}{fgr^2}\right]X^++\frac{2\sqrt{\lambda}}{r^2g}f_{01}+\frac{\mathrm{i}2h\sqrt{\lambda}}{r^3}f_{02}=-k^2W^+\,,
\end{aligned}$$

$$\begin{aligned}
&-\left[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2N}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda-2(N+1)+2m}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}\right.\\
&\qquad\left.-\frac{1}{g^2}\left(\left(\frac{f'}{f}-\frac{1}{r}\right)^2-\frac{1}{2}\left(\frac{h\Omega'}{f}\right)^2\right)-\frac{2h^2}{r^4}\right]W^-\\
&\quad+\left[\frac{h\Omega'}{fg^2}\frac{\partial}{\partial r}+\frac{1}{2g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)+\frac{(N+1)h\Omega'}{fg^2r}\right]Z^-\\
&\quad+\mathrm{i}\left[\frac{2f'(\omega-m\Omega)}{f^2g}+\frac{m\Omega'}{fg}+\frac{2h^2\Omega'}{fgr^2}\right]X^-+\frac{2\sqrt{\lambda}}{r^2g}f_{01}-\frac{\mathrm{i}2h\sqrt{\lambda}}{r^3}f_{02}=-k^2W^-\,,
\end{aligned}$$

$$\begin{aligned}
&-\left[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2N}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}\right.\\
&\qquad\left.-\frac{2}{g^2}\left(\left(\frac{f'}{f}\right)^2+\left(\frac{h'}{h}\right)^2-\frac{1}{2}\left(\frac{h\Omega'}{f}\right)^2+\frac{2N}{r^2}\right)\right]f_{11}\\
&\quad-2\left[\frac{1}{g}\frac{\partial}{\partial r}\left(\frac{f'}{fg}\right)-\frac{1}{2}\left(\frac{h\Omega'}{fg}\right)^2\right]f_{00}+2\,\mathrm{i}\left[\frac{2f'(\omega-m\Omega)}{f^2g}+\frac{m\Omega'}{fg}\right]f_{01}\\
&\quad+2\left[\frac{1}{g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)-\frac{h\Omega'}{fg^2}\left(\frac{f'}{f}-\frac{h'}{h}\right)\right]f_{02}-2\,\mathrm{i}\left[\frac{h\Omega'(\omega-m\Omega)}{f^2g}-\frac{2mh'}{h^2g}\right]f_{12}\\
&\quad+2\left[\frac{1}{g}\frac{\partial}{\partial r}\left(\frac{h'}{hg}\right)+\frac{1}{2}\left(\frac{h\Omega'}{fg}\right)^2\right]f_{22}+\frac{2}{r^2g\sqrt{\lambda}}\left[X^+(\lambda-2mN)+X^-(\lambda+2mN)\right]\\
&\quad-\frac{4N}{g^2r}\left(\frac{g'}{g}+\frac{1}{r}\right)H_L=-k^2f_{11}\,,
\end{aligned}$$

$$\begin{aligned}
&-\left[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2N}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}\right.\\
&\qquad\left.-\frac{1}{g^2}\left(\left(\frac{f'}{f}\right)^2+2\left(\frac{h'}{h}\right)^2-2\left(\frac{h\Omega'}{f}\right)^2+\frac{2N}{r^2}\right)-\frac{2Nh^2}{r^4}+\frac{2}{g}\frac{\partial}{\partial r}\left(\frac{h'}{hg}\right)\right]f_{12}\\
&\quad+\left[\frac{h\Omega'}{fg^2}\frac{\partial}{\partial r}-\frac{1}{2g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)+\frac{h\Omega'}{fg^2}\left(\frac{2f'}{f}+\frac{N}{r}\right)\right]f_{01}\\
&\quad+\mathrm{i}\left[\frac{2f'(\omega-m\Omega)}{f^2g}+\frac{m\Omega'}{fg}\right]f_{02}+\mathrm{i}\left[\frac{h\Omega'(\omega-m\Omega)}{f^2g}-\frac{2mh'}{h^2g}\right](f_{11}-f_{22})\\
&\quad-\frac{\mathrm{i}h}{r^3\sqrt{\lambda}}\left[X^+(\lambda-2mN)-X^-(\lambda+2mN)\right]+\frac{1}{r^2g\sqrt{\lambda}}\left[Z^+(\lambda-2mN)+Z^-(\lambda+2mN)\right]\\
&\quad=-k^2f_{12}\,,
\end{aligned}$$

$$\begin{aligned}
&-\left[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2N}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda-2(N+1)-2m}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}\right.\\
&\qquad\left.-\frac{1}{g^2}\left(\left(\frac{f'}{f}\right)^2+\left(\frac{h'}{h}\right)^2-\frac{1}{2}\left(\frac{h\Omega'}{f}\right)^2+\frac{2N+3}{r^2}\right)-\frac{2h^2}{r^4}-\frac{2g'}{g^3r}\right]X^+\\
&\quad+\mathrm{i}\left[\frac{2f'(\omega-m\Omega)}{f^2g}+\frac{h\Omega'}{fg}\left(\frac{m}{h}-\frac{2h}{r^2}\right)\right]W^++\mathrm{i}\left[-\frac{h\Omega'(\omega-m\Omega)}{f^2g}+\frac{2mh'}{h^2g}-\frac{4h}{r^2g}\left(\frac{h'}{h}-\frac{2}{r}\right)\right]Z^+\\
&\quad+\frac{1}{r^2g\sqrt{\lambda}}\left[(\lambda-4(N+1)-2m(N+2))H^{++}+\frac{(N-1)(\lambda+2mN)}{N}H^{+-}-2\lambda H_L\right]\\
&\quad+\frac{2\sqrt{\lambda}}{r^2g}f_{11}+\frac{\mathrm{i}2h\sqrt{\lambda}}{r^3}f_{12}=-k^2X^+\,,
\end{aligned}$$

$$\begin{aligned}
&-\left[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2N}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda-2(N+1)+2m}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}\right.\\
&\qquad\left.-\frac{1}{g^2}\left(\left(\frac{f'}{f}\right)^2+\left(\frac{h'}{h}\right)^2-\frac{1}{2}\left(\frac{h\Omega'}{f}\right)^2+\frac{2N+3}{r^2}\right)-\frac{2h^2}{r^4}-\frac{2g'}{g^3r}\right]X^-\\
&\quad+\mathrm{i}\left[\frac{2f'(\omega-m\Omega)}{f^2g}+\frac{h\Omega'}{fg}\left(\frac{m}{h}+\frac{2h}{r^2}\right)\right]W^-+\mathrm{i}\left[-\frac{h\Omega'(\omega-m\Omega)}{f^2g}+\frac{2mh'}{h^2g}+\frac{4h}{r^2g}\left(\frac{h'}{h}-\frac{2}{r}\right)\right]Z^-\\
&\quad+\frac{1}{r^2g\sqrt{\lambda}}\left[(\lambda-4(N+1)+2m(N+2))H^{--}+\frac{(N-1)(\lambda-2mN)}{N}H^{+-}-2\lambda H_L\right]\\
&\quad+\frac{2\sqrt{\lambda}}{r^2g}f_{11}-\frac{\mathrm{i}2h\sqrt{\lambda}}{r^3}f_{12}=-k^2X^-\,,
\end{aligned}$$

$$-\left[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2N}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}\right.$$

… and more

[Dias-Figueras- Monteiro-Reall-Santos (2010)]

PERTURBATION EQ

- at "rest frame", perturbation eqs. become one of Schwarzschild BH **only at leading order of $D = \infty$**

$$\delta g_{\mu\nu} = F_{\mu\nu}(r) e^{-i\omega t}\, e^{im\psi} Y_{\ell m}$$

  - perturbation eqs. are decoupling at leading order
  - coupling effects appears as source terms in higher order equation in $1/D$ expansion

    - we can solve equations **analytically**

    - source terms does not have "boost symmetry", which describe the difference from Schwarzschild BH

[Emparan-Suzuki-KT (2014)]

$$\mathsf{R}^2(\mathsf{R}-1)F_{00}^{(k)''}(\mathsf{R}) + \mathsf{R}(2\mathsf{R}-1)F_{00}^{(k)'}(\mathsf{R}) + F_{00}^{(k)}(\mathsf{R}) = \mathcal{S}_{F_{00}}^{(k)},$$

$$\mathsf{R}(\mathsf{R}-1)^2F_{01}^{(k)''}(\mathsf{R}) + (\mathsf{R}-1)(2\mathsf{R}-1)F_{01}^{(k)'}(\mathsf{R}) - F_{01}^{(k)}(\mathsf{R}) = \mathcal{S}_{F_{01}}^{(k)},$$

$$(\mathsf{R}-1)F_{02}^{(k)''}(\mathsf{R}) + 2F_{02}^{(k)'}(\mathsf{R}) = \mathcal{S}_{F_{02}}^{(k)},$$

$$\mathsf{R}(\mathsf{R}-1)^2F_{11}^{(k)''}(\mathsf{R}) + (\mathsf{R}-1)(2\mathsf{R}-1)F_{11}^{(k)'}(\mathsf{R}) - F_{11}^{(k)}(\mathsf{R}) = \mathcal{S}_{F_{11}}^{(k)},$$

$$(\mathsf{R}-1)F_{12}^{(k)''}(\mathsf{R}) + 2F_{12}^{(k)'}(\mathsf{R}) = \mathcal{S}_{F_{12}}^{(k)},$$

$$(\mathsf{R}-1)F_{22}^{(k)''}(\mathsf{R}) + (2\mathsf{R}-1)F_{22}^{(k)'}(\mathsf{R}) = \mathcal{S}_{F_{22}}^{(k)},$$

$$\mathsf{R}(\mathsf{R}-1)^2F_{0}^{(k)''}(\mathsf{R}) + (\mathsf{R}-1)(2\mathsf{R}-1)F_{0}^{(k)'}(\mathsf{R}) - F_{0}^{(k)}(\mathsf{R}) = \mathcal{S}_{F_{0}}^{(k)},$$

$$4\mathsf{R}^2(\mathsf{R}-1)^2F_{1}^{(k)''}(\mathsf{R}) + 4\mathsf{R}(\mathsf{R}-1)(2\mathsf{R}-1)F_{1}^{(k)'}(\mathsf{R}) - F_{1}^{(k)}(\mathsf{R}) = \mathcal{S}_{F_{1}}^{(k)},$$

$$(\mathsf{R}-1)F_{2}^{(k)''}(\mathsf{R}) + 2F_{2}^{(k)'}(\mathsf{R}) = \mathcal{S}_{F_{2}}^{(k)},$$

$$\mathsf{R}(\mathsf{R}-1)H^{+-(k)''}(\mathsf{R}) + (2\mathsf{R}-1)H^{+-(k)'}(\mathsf{R}) = \mathcal{S}_{H^{+-}}^{(k)}\,,$$

WHICH MODE?

QNMs of large D BHs has **two types**

- **non-decoupled QNMs** (universal mode)
  - modes with frequency $\omega = O(D/r_+)$
  - roughly speaking, describes responses to external fields
- **decoupled mode** (confined modes)
  - modes with frequency $\omega = O(D^0/r_+)$
  - capture "internal DoF" of large D BHs

RESULTS: LEADING

- obtain QNM condition of $\delta g_{\mu\nu} = F_{\mu\nu}(r)e^{-i\omega t}\, e^{im\psi}Y_{\ell m}$

$$
\begin{aligned}
0 = \; & \frac{1}{\omega - a(m+2) + i(\ell-2)\sqrt{1-a^2}}\Big[\omega^3 + \omega^2\left(-3am + i\,(3\ell-4)\,\sqrt{1-a^2}\right) \\
& + \omega\left(3a^2\ell^2 - 6iam\sqrt{1-a^2}\,(\ell-1) - 6a^2\ell + 3a^2m^2 - 3\ell^2 + 7\ell - 4\right) \\
& + am\left(2 + (4a^2-5)\ell + 3(1-a^2)\ell^2 - a^2m^2\right) \\
& + i\sqrt{1-a^2}\left(-\left(1-a^2\right)\ell^3 + \left(3-2a^2\right)\ell^2 + \ell\left(3a^2m^2-2\right) - 2a^2m^2\right)\Big].
\end{aligned}
$$

  - **3 modes** as solutions of a cubic equation (explicitly solvable)

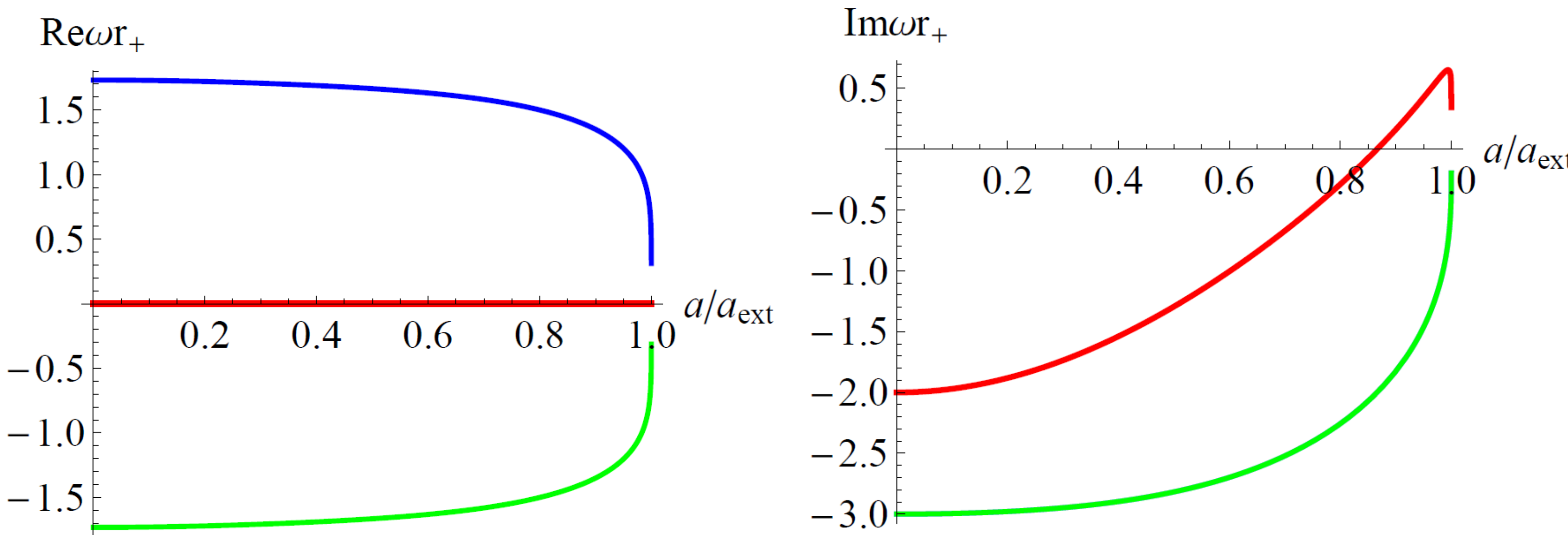
- one mode describes instability at $a > a_c^{\ell m}$

$$
a_c^{\ell \mathrm{m}} = \sqrt{1 - \frac{1}{\ell}} \quad > \quad a_c^{\ell=2, m=2} = \frac{1}{\sqrt{2}}
$$

(dominant instability mode is $\ell = 2, m = 2$)

AXIS MODE

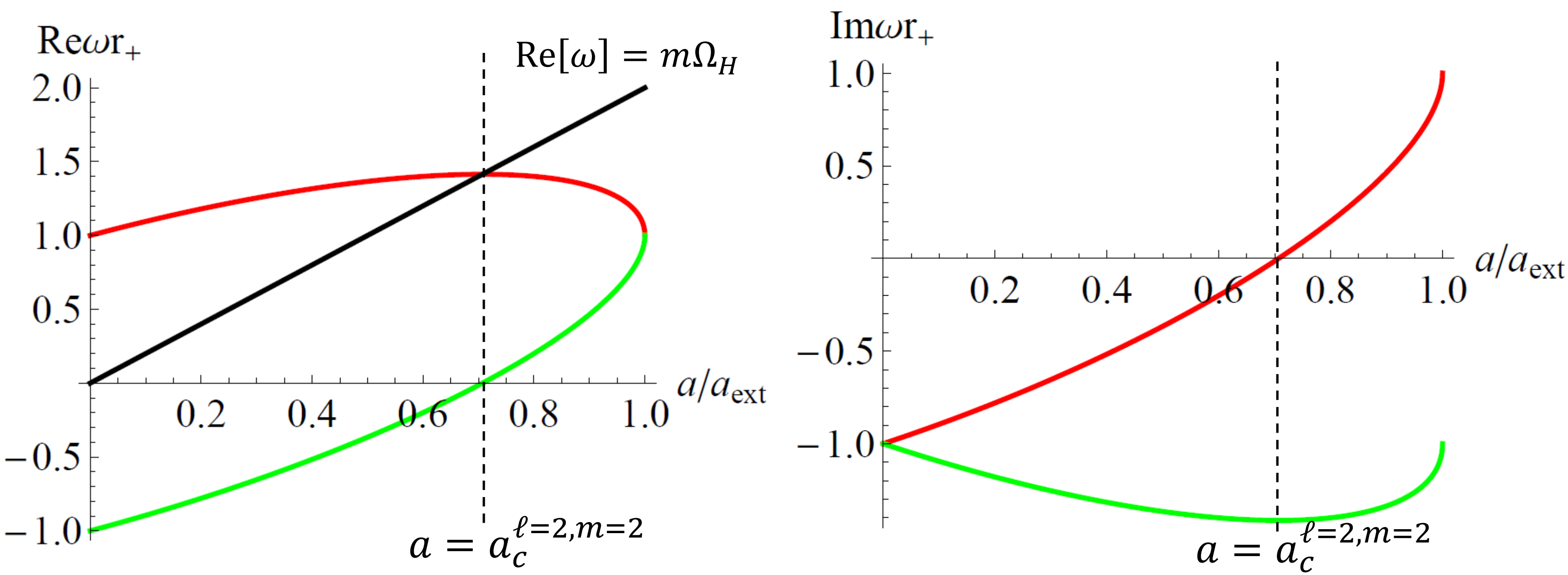
$\ell = 4, m = 0$



- ➢ axisymmetric unstable mode has **pure imaginary frequency**
- ➢ $\ell = 4$ is the dominant unstable mode in axisymmetric perturbation
  - $\ell = 2, m = 0$ is angular momentum perturbation
- ➢ zero mode perturbation at threshold angular momentum
  - appearance of new solution branch

NON-AXIS MODE

$\ell = 2, m = 2$



- non-axisymmetric mode has **complex frequency**
- $\ell = 2, m = 2$ is the dominant unstable mode in perturbations
- all instability modes satisfy superradiance condition

$$\mathrm{Re}[\omega] < m\Omega_H \quad \text{at } a > a_c^{\ell m}$$

RESULTS: NEXT LEADING

- we can obtain $1/D$ correction in higher order solution

$$\omega r_+ = \omega_0 + \frac{\omega_1}{D-3}$$

- threshold angular momentum is corrected

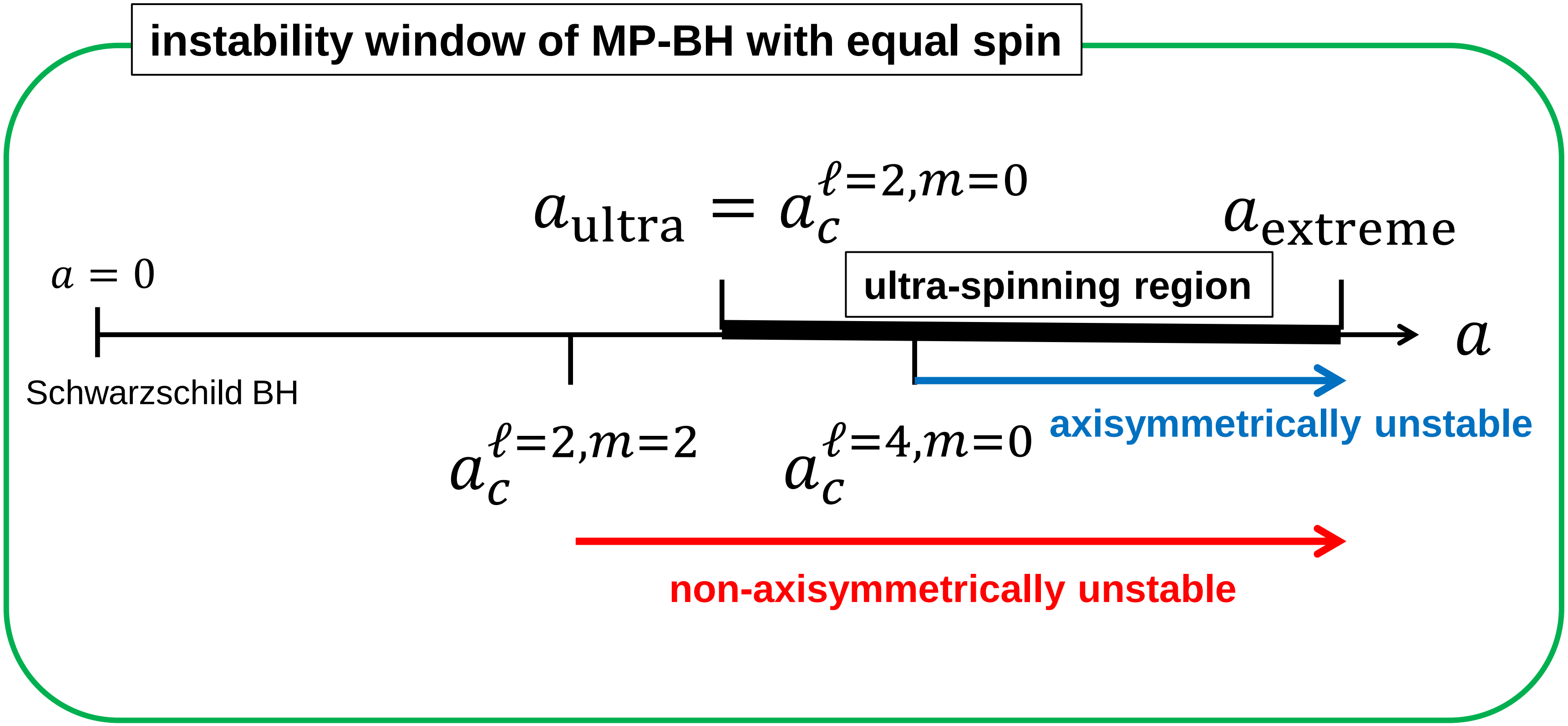
$$\frac{a_c^{\ell m}}{r_+} = \sqrt{1 - \frac{1}{\ell}}\left(1 - \underline{\frac{2}{D-3}\frac{m^2}{4\ell^2}}\right)$$

- dominant unstable mode

$$\frac{a_c^{\ell=2,m=2}}{r_+} < \frac{a_{\mathrm{ultra}}}{r_+} = \frac{1}{\sqrt{2}}$$

INSTABILITY WINDOW

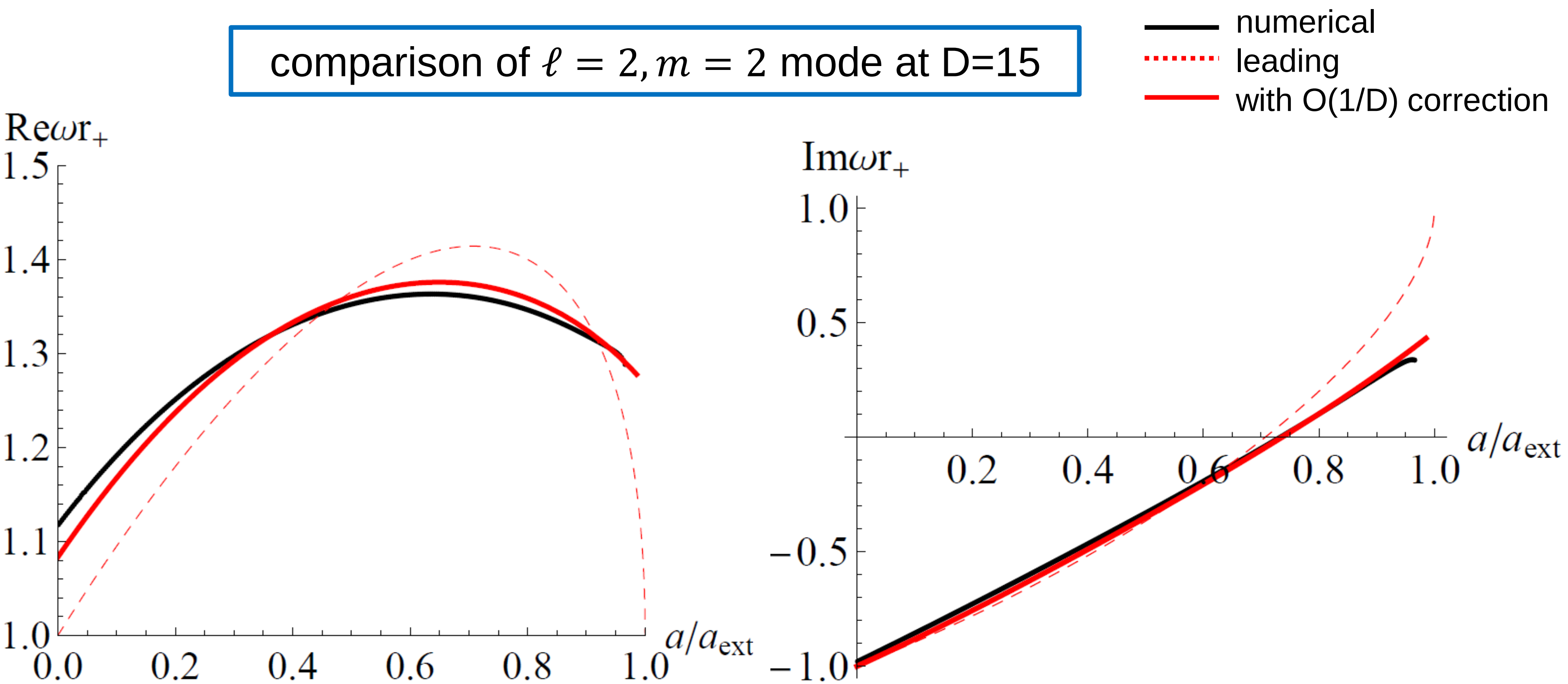
up to $O(1/D)$ correction, we describe instability window



COMPARISON

- comparison with numerical result **[Hartnett-Santos (2013)]**
  - very good agreement if we consider $O(1/D)$ correction

comparison of $\ell = 2, m = 2$ mode at D=15



SUMMARY AND OUTLOOK

- we solved perturbation equations of MP-BH with equal angular momenta by large D expansion
  - found explicit QNM frequency up to $O(1/D)$
  - bar mode ($\ell = 2, m = 2$) is the most dominant unstable mode
    - out of the ultra-spinning region $a_c^{m>0} < a_{\mathrm{ultra}} < a_c^{m=0}$
  - unstable modes always satisfy superradiance condition
- extension to single rotation case
  - consider the dynamics of boost parameter $\alpha(\theta)$