Non-equilibrium Thermodynamics of Gravitational Screens

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[arXiv:1405.4881]
([arXiv: 1312.1538] is a complementary discussion.)

A key question for quantum gravity is…

- What are the fundamental space-time degrees of freedom?
- What are the quantum space-time constituents if any?

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In the semi-classical region

- What is the entropy of a gravitational system?

Example: Black-hole entropy

- In the case where the space-time is static and contains BHs, the Bekenstein-Hawking entropy

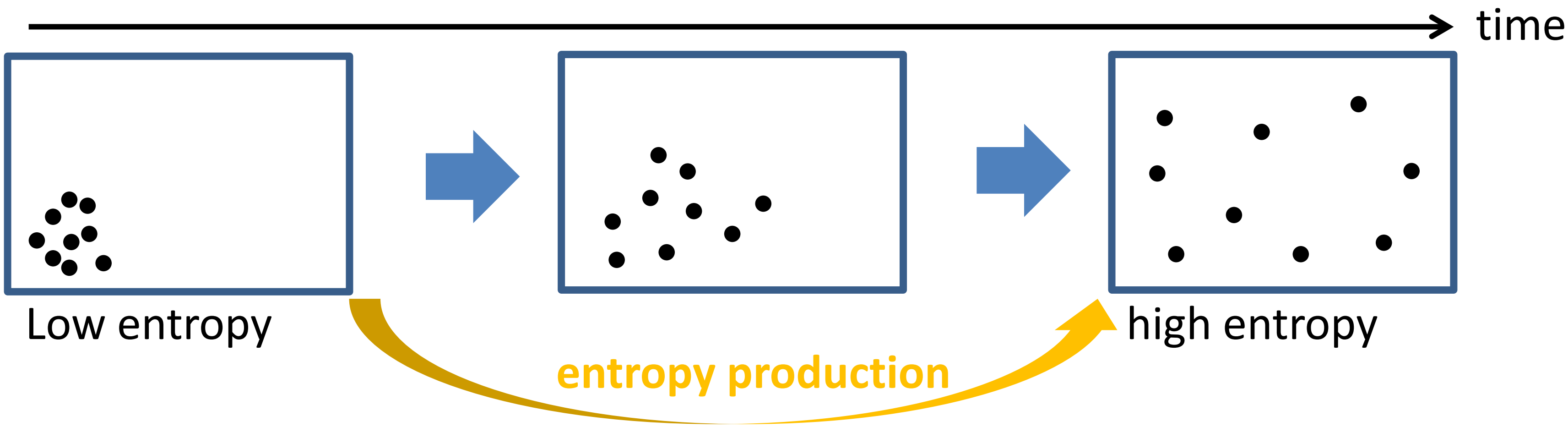
$$S_{BH} = \frac{A}{4l_p^2}$$

should provide some gravitational entropy to the system.

⇒Can we extend this formula to arbitrary spacetime which is dynamical and out of equilibrium?

Out of equilibrium

- Time evolution of a non-gravitational system

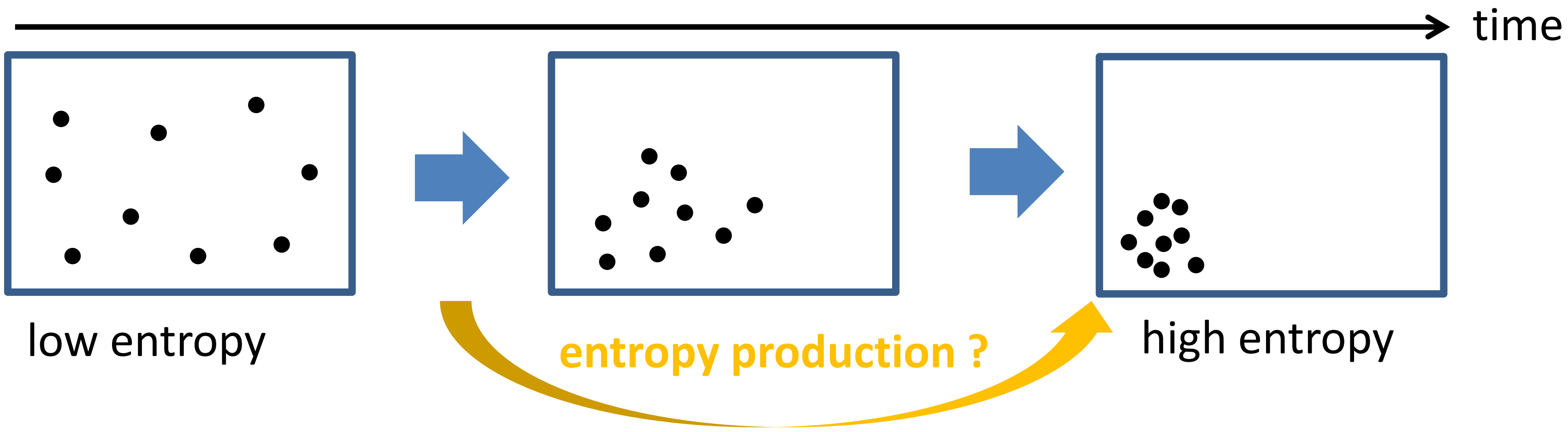


- Arrow of time follows the 2nd law:

$$S = k_B \ln W$$

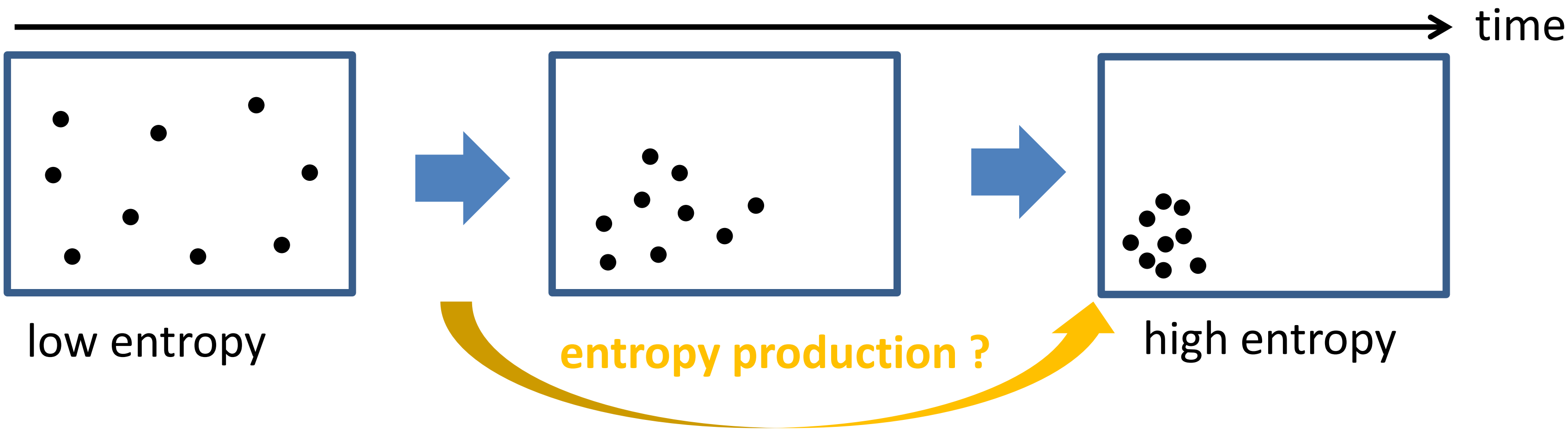
Out of equilibrium

- Time evolution of a gravitational system



Out of equilibrium

- Time evolution of a gravitational system

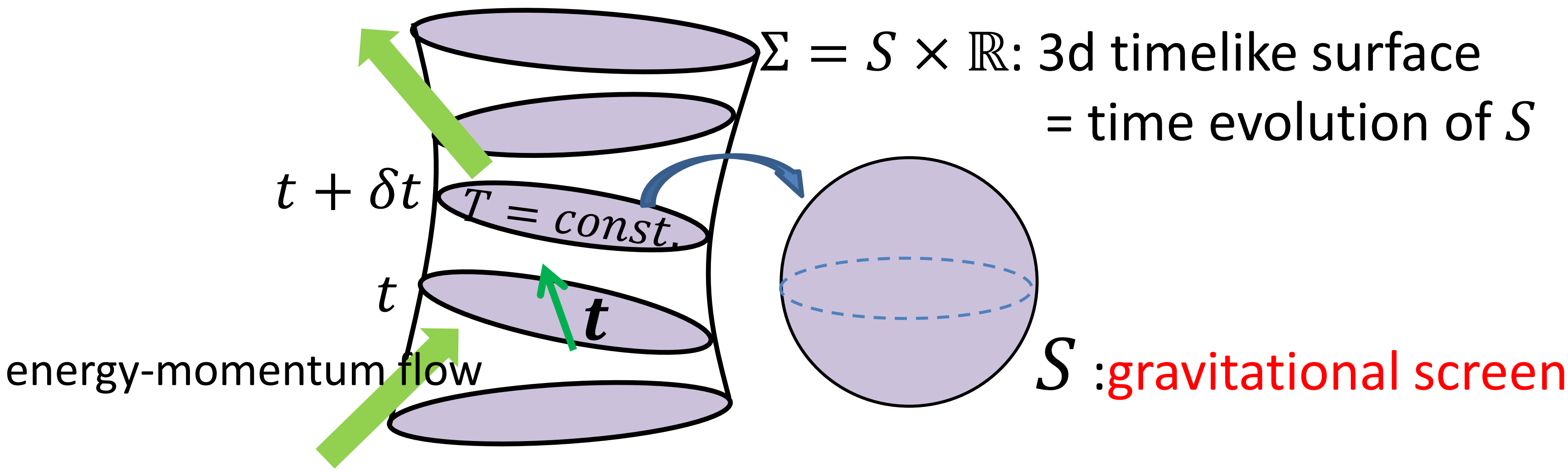


- How can we understand this process as entropy production?
- Why is gravity so special?
- What is the difference that makes the difference?

Setup of a gravitational system

- We want to construct a quasi-local description of gravitational physics without referring to the end of time or space.
- We choose a domain, and call its boundary a gravitational screen.
- This is generally an open system.
- Description of time evolution of this system

$\Rightarrow$ time vector $\boldsymbol{t}$ and time foliation



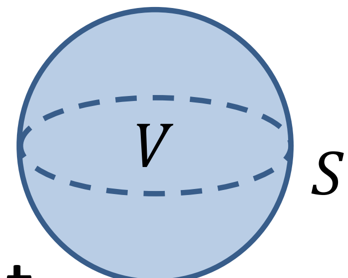
$\Rightarrow$How about the Hamiltonian?

Gravity is fundamentally holographic.

- The Hamiltonian of a 3d gravitational system is provided only by the boundary:

$$H_{\boldsymbol{t}} = \int_V dV\,\big[N\underset{=0}{\mathcal{H}} + N^i\underset{=0}{\mathcal{H}_i}\big] + \int_S dA\,\frac{\kappa_{\boldsymbol{t}}}{8\pi G}$$

diffeomorphism constraints



- The equivalence principle means that gravitational field can be eliminated locally.
- BH membrane paradigm gives a holographic view to dynamics of gravity.

Challenge:

Can we understand time evolution of the inside bulk spacetime as entropy production on the screen in a holographic manner?

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Can we understand time evolution of the inside bulk spacetime as entropy production on the screen in a holographic manner?

Today's results:

- The Einstein eqs projected on the gravitational screen reduce to the thermodynamic equations for a 2d viscous bubble.
- Such a screen has a surface tension and an internal energy.
- Obtain a complete dictionary between bubble thermodynamics and gravity.

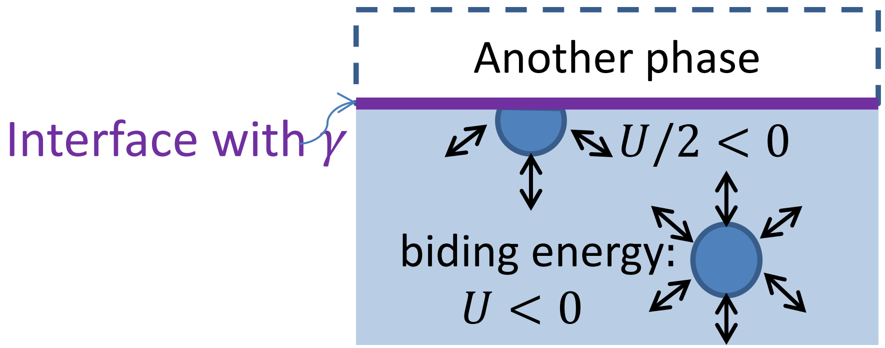
Note:

This analysis is still classical and just an analogy, and temperature and entropy are not identified yet.

What is the surface tension?

- Surface tension γ = energy cost needed to increase the interface area by one unit, due to excess free energy between two phases

$$\delta W = \gamma dA$$



- Thermodynamic relation

$$dU^i = TdS^i + \gamma dA + \mu_i dN^i$$

$$\gamma = -\mathrm{p}_{2\mathrm{d}}$$

- Marangoni flow eq

$$d_t \pi_A = d_A \gamma$$

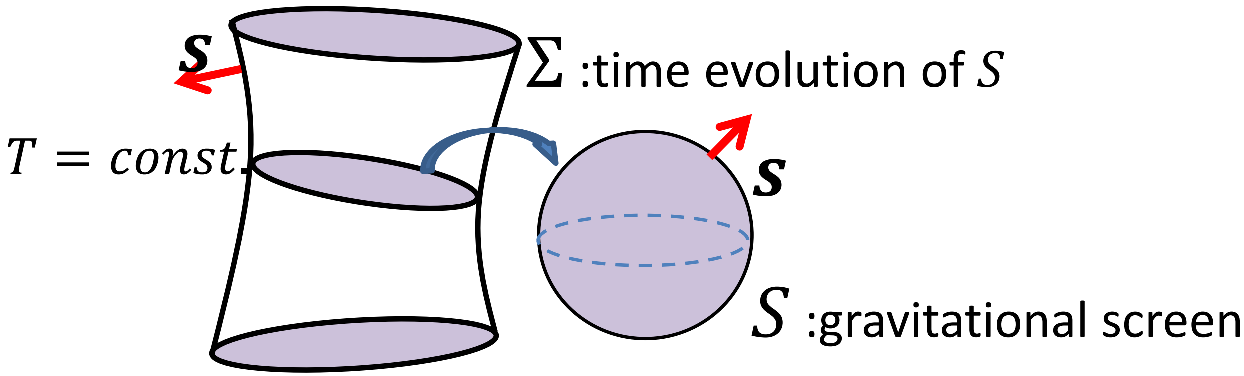
- Young-Laplace law

$$\Delta p + \gamma\theta = 0$$

$$\theta = \frac{1}{R_1} + \frac{1}{R_2} \qquad \Delta p = p_{out} - p_{in}, \qquad \theta = \nabla_{\mathrm{i}} \mathrm{s}^{\mathrm{i}}$$

Screen energy-momentum tensor

- Outward unit normal $\boldsymbol{s}$ embedding Σ in 4d spacetime.
- Induced metric on Σ: $h_{ab} = g_{ab} - s_a s_b$
- Extrinsic curvature of Σ: $H_{ab} = h_a^c h_b^d \nabla_c s_d$



- Such a screen has a screen energy momentum tensor

$$S_{ab} = \frac{1}{8\pi G}(H h_{ab} - H_{ab})$$

- This can be justified by Israel's junction condition in the spirit of the BH membrane paradigm.
- This is the same as the Brown-York tensor.

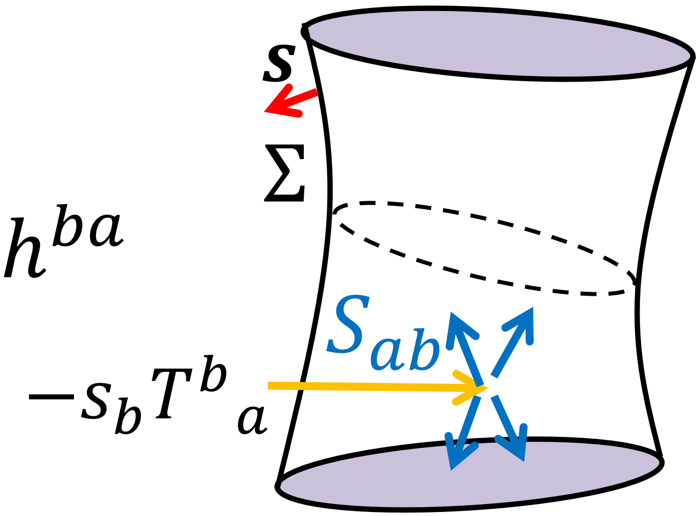
Einstein eq projected on Σ

Einstein eqs projected on Σ can be expressed by the Codazzi and Gauss eqs as

- Conservation eqs on Σ

$$D_b S^{ba} = -T_{\boldsymbol{s}b} h^{ba}$$

$$D_a V^b = h_a^c h_d^b \nabla_c V^d$$



- Radial constraint eq

$$R(h) + H\!:\!H - H^2 = -16\pi G\, T_{\boldsymbol{ss}}$$

⇒Let's express these eqs in terms of physics of the screen S.

2+2 decomposition (1)

- unit timelike normal to the slices ($\mathbf{n}^2 = -1$):

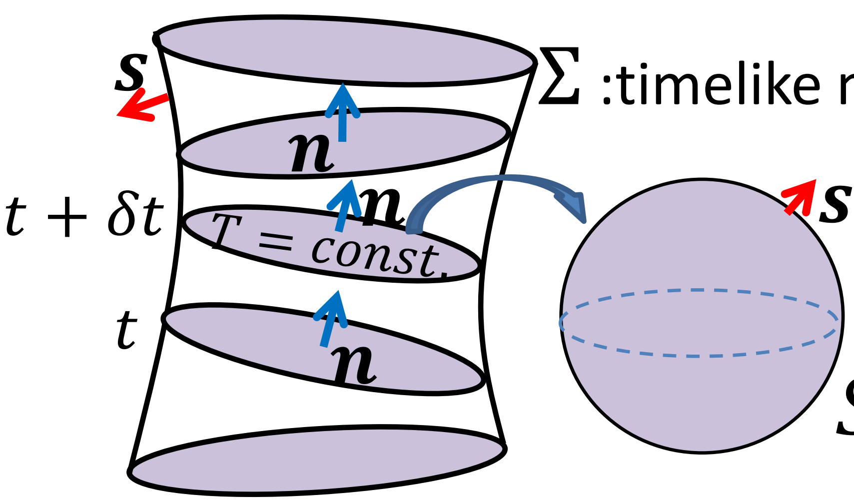
$$\mathbf{n} \mathrel{\widehat{=}} -\rho dt \quad (\rho\text{: screen lapse})$$

- 2d metric on S: $q_{ab} = h_{ab} + n_a n_b$

⇒2+2 decomposition of the metric:

$$\boxed{g_{ab} = -\mathrm{n_a n_b} + \mathrm{s_a s_b} + \mathrm{q_{ab}}}$$

$$0 = \mathrm{q_{ab}} n^b = q_{ab} s^b = \boldsymbol{n} \cdot \boldsymbol{s}$$



Σ :timelike membrane= time evolution of S

S :gravitational screen

2+2 decomposition (2)

The following quantities decompose the 4d spacetime into the 2d surface S and the 2d normal surface $\mathrm{T}^{\perp}\mathrm{S}$:

(1) the extrinsic geometry of S is determined by

- extrinsic curvature tensor:

$$\Theta_{\boldsymbol{n}AB} \equiv q_A^a q_B^b \nabla_a n_b, \qquad \Theta_{\boldsymbol{s}AB} \equiv q_A^a q_B^b \nabla_a s_b$$

$$\theta_{\boldsymbol{n}} \equiv q^{AB}\Theta_{\boldsymbol{n}AB}$$
$$\theta_{\boldsymbol{s}} \equiv q^{AB}\Theta_{\boldsymbol{s}AB}$$

- normal one-form:

$$\omega_A \equiv q_A^a(\boldsymbol{s}\cdot\nabla_a\boldsymbol{n})$$

(2) The normal geometry $\mathrm{T}^{\perp}\mathrm{S}$ is encoded by

- tangential acceleration:

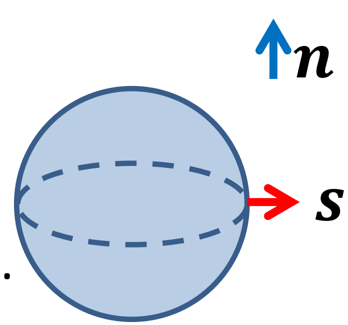
$$a_{\boldsymbol{n}}^A \equiv q_a^A(\nabla_{\boldsymbol{n}}\boldsymbol{n})^a,\ a_{\boldsymbol{s}}^A \equiv -q_a^A(\nabla_{\boldsymbol{s}}\boldsymbol{s})^a, a_{\boldsymbol{ns}}^A \equiv \frac{1}{2}q_a^A(\nabla_{\boldsymbol{s}}\boldsymbol{n}+\nabla_{\boldsymbol{n}}\boldsymbol{s})^a$$

- Twist vector:

$$j^{\mathrm{A}} \equiv q_a^A[\boldsymbol{n},\boldsymbol{s}]^a$$

- normal acceleration:

$$\gamma_{\boldsymbol{n}} \equiv \boldsymbol{s}\cdot\nabla_{\boldsymbol{n}}\boldsymbol{n}, \qquad \gamma_{\boldsymbol{s}} \equiv -\boldsymbol{n}\cdot\nabla_{\boldsymbol{s}}\boldsymbol{s}$$



Note: γ_n is the radial acceleration of an observer flowing the screen.

bubble-fluid eqs from gravity (1)

(1) Momentum conservation

$$\boxed{(D_b S^b_a) q^a_A = -T_{\boldsymbol{s}a} q^a_A}$$

(2) Energy conservation

$$\boxed{(D_b S^b_a) n^a = -T_{\boldsymbol{sn}}}$$

bubble-fluid eqs from gravity (1)

$$(D_b S^b_a) q^a_A = -T_{sa} q^a_A$$

(1) Momentum conservation

$$\boxed{\mathcal{L}_{\boldsymbol{t}}\boldsymbol{\omega} + \theta_t \boldsymbol{\omega} = \boldsymbol{d}\gamma_{\boldsymbol{t}} + \boldsymbol{d} \cdot \widetilde{\boldsymbol{\Theta}}_{\boldsymbol{t}^*} - \theta_{\boldsymbol{t}^*} \boldsymbol{d}\phi_N + (8\pi G) T_{\boldsymbol{s}}.}$$

$$(D_b S^b_a) n^a = -T_{\boldsymbol{sn}}$$

(2) Energy conservation

$$\boxed{\begin{aligned}&\mathcal{L}_{\boldsymbol{t}}\theta_{\boldsymbol{t}^*} + \theta_{\boldsymbol{t}}\theta_{\boldsymbol{t}^*} \\ &\qquad = \gamma_{\boldsymbol{t}}\theta_{\boldsymbol{t}} + \widetilde{\boldsymbol{\Theta}}_{\boldsymbol{t}^*} : \boldsymbol{\Theta}_{\boldsymbol{t}} - \boldsymbol{d} \cdot (\rho^2 \boldsymbol{\omega}) + \theta_{\boldsymbol{t}^*} \mathcal{L}_{\boldsymbol{t}}\phi_N - (8\pi G) T_{\boldsymbol{sn}}\end{aligned}}$$

bubble-fluid eqs from gravity (1)

(1) Momentum conservation

$$\boxed{\mathcal{L}_{\boldsymbol{t}}\boldsymbol{\omega} + \theta_{\boldsymbol{t}}\boldsymbol{\omega} = \boldsymbol{d}\gamma_{\boldsymbol{t}} + \boldsymbol{d}\cdot\widetilde{\boldsymbol{\Theta}}_{\boldsymbol{t}^*} - \theta_{\boldsymbol{t}^*}\boldsymbol{d}\phi_N + (8\pi G)T_{\boldsymbol{s}}.}$$

$\times \frac{-1}{8\pi G}$

$$d_t\boldsymbol{\pi} + \sigma\boldsymbol{\pi} = \boldsymbol{d}\gamma + \boldsymbol{d}\cdot\boldsymbol{\Theta} - \rho\boldsymbol{d}\phi + \boldsymbol{f}$$

Tangential momentum conservation

(2) Energy conservation:

$$\boxed{\begin{aligned}&\mathcal{L}_{\boldsymbol{t}}\theta_{\boldsymbol{t}^*} + \theta_{\boldsymbol{t}}\theta_{\boldsymbol{t}^*}\\ &\qquad = \gamma_{\boldsymbol{t}}\theta_{\boldsymbol{t}} + \widetilde{\boldsymbol{\Theta}}_{\boldsymbol{t}^*}\!:\boldsymbol{\Theta}_{\boldsymbol{t}} - \boldsymbol{d}\cdot(\rho^2\boldsymbol{\omega}) + \theta_{\boldsymbol{t}^*}\mathcal{L}_{\boldsymbol{t}}\phi_N - (8\pi G)T_{\boldsymbol{sn}}\end{aligned}}$$

$\times \frac{-1}{8\pi G}$

$$d_t(u+\rho\phi) + \sigma(u+\rho\phi) = \gamma\sigma + \boldsymbol{\Theta}\!:\boldsymbol{\Sigma} - \boldsymbol{d}\cdot\boldsymbol{q} + \rho d_t\phi + \dot{Q}^{rad}$$

Generalized 1st law = 1st law + eq of ϕ

Note: This system is relativistic.

bubble-fluid eqs from gravity (2)

$$\boxed{R(h) + H{:}H - H^2 = -16\pi G\, T_{ss}}$$

(3) Radial constraint equation

$$\boxed{\begin{aligned} &-(\mathcal{L}_{\boldsymbol{t}} + \theta_{\boldsymbol{t}})\theta_{\boldsymbol{n}} \\ &\qquad = (8\pi G)\left[\bar{T}_{\boldsymbol{t}^*\boldsymbol{s}} + \frac{-\gamma_{\boldsymbol{t}}}{8\pi G}\theta_{\boldsymbol{s}}\right] - \rho\omega^2 - \frac{1}{2}\widetilde{\boldsymbol{\Theta}}_{\boldsymbol{t}}{:}\boldsymbol{\Theta}_{\boldsymbol{n}} - \Delta\rho \end{aligned}}$$

Here $\bar{T}_{\boldsymbol{ss}} \equiv T_{\boldsymbol{ss}} - \frac{G_{\boldsymbol{ss}}(\bar{h})}{8\pi G}$, $\bar{h}_{ab} \equiv s_a s_b + q_{ab}$

bubble-fluid eqs from gravity (2)

(3) Radial constraint equation

$$\boxed{\begin{aligned}-(\mathcal{L}_{\boldsymbol{t}}+\theta_{\boldsymbol{t}})\theta_{\boldsymbol{n}} \\ &= (8\pi G)\left[\bar{T}_{\boldsymbol{t}^*\boldsymbol{s}}+\frac{-\gamma_{\boldsymbol{t}}}{8\pi G}\theta_{\boldsymbol{s}}\right]-\rho\omega^2-\frac{1}{2}\widetilde{\boldsymbol{\Theta}}_{\boldsymbol{t}}:\boldsymbol{\Theta}_{\boldsymbol{n}}-\Delta\rho\end{aligned}}$$

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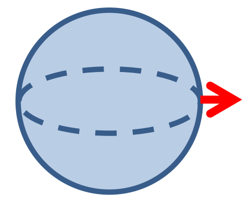

$\boldsymbol{s}$

Equilibrium case $\quad 0 = \bar{T}_{\boldsymbol{t}^*\boldsymbol{s}} + \frac{-\gamma_{\boldsymbol{t}}}{8\pi G}\theta_{\boldsymbol{s}}$

$$\boxed{\gamma = \frac{-\gamma_t}{8\pi G}}$$

Young-Laplace law: $0 = \Delta p + \gamma\theta$

bubble-fluid eqs from gravity (2)

(3) Radial constraint equation

$$\boxed{-(\mathcal{L}_{\boldsymbol{t}}+\theta_{\boldsymbol{t}})\theta_{\boldsymbol{n}} = (8\pi G)\left[\bar{T}_{\boldsymbol{t}^*\boldsymbol{s}}+\frac{-\gamma_{\boldsymbol{t}}}{8\pi G}\theta_{\boldsymbol{s}}\right]-\rho\omega^2-\frac{1}{2}\widetilde{\boldsymbol{\Theta}}_{\boldsymbol{t}}:\boldsymbol{\Theta}_{\boldsymbol{n}}-\Delta\rho}$$

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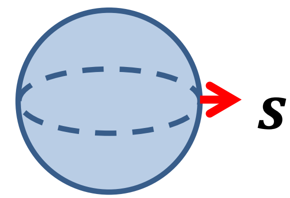

$$\boxed{\gamma = \frac{-\gamma_t}{8\pi G}}$$

Spherical symmetric cases

$$(\mathcal{L}_{\boldsymbol{t}}+\frac{3}{4}\theta_{\boldsymbol{t}})\frac{\theta_{\boldsymbol{n}}}{8\pi G} = -\left[\bar{T}_{\boldsymbol{t}^*\boldsymbol{s}}+\frac{-\gamma_{\boldsymbol{t}}}{8\pi G}\theta_{\boldsymbol{s}}\right]$$

dynamical Young-Laplace law: $d_t\bar{\pi}_r+\sigma\bar{\pi}_r = -(\Delta p+\gamma\theta)$

gravity-thermodynamics dictionary (1)

Bubble thermodynamics		Gravity
specific volume: $v = \frac{1}{\rho}$	$\leftrightarrow$	$\sqrt{q}$: 2d measure
compressibility: $\sigma = \boldsymbol{d} \cdot \boldsymbol{v}$	$\leftrightarrow$	$\theta_{\boldsymbol{t}}$: expansion
Internal + gravitational energy: $u + \rho\phi$	$\leftrightarrow$	$-\frac{\theta_{\boldsymbol{t}^*}}{8\pi G}$:inward radial expansion
surface tension: γ	$\leftrightarrow$	$\frac{-\gamma_t}{8\pi G}$: inward radial acceleration
Newtonian mass: ρ	$\leftrightarrow$	$-\frac{\theta_{\boldsymbol{t}^*}}{8\pi G}$:inward radial expansion
Newton potential: ϕ	$\leftrightarrow$	$\phi_N = \ln \rho$: screen time lapse
momentum: $\boldsymbol{\pi} = \rho \boldsymbol{v}$	$\leftrightarrow$	$-\frac{\boldsymbol{\omega}}{8\pi G}$: normal one-form
rate of strain tensor: $\Sigma_{ij} = \partial_i v_j$	$\leftrightarrow$	$\boldsymbol{\Theta}_{\boldsymbol{t}}$: temporal extrinsic curvature

Note: This system is relativistic.

gravity-thermodynamics dictionary (2)

Bubble Thermodynamics		Gravity
• viscous stress: $\boldsymbol{\Theta}$	$\leftrightarrow$	• $\frac{-\widetilde{\boldsymbol{\Theta}}_{\boldsymbol{t}^*}}{8\pi G}$:twisted radial extrinsic curvature
• heat flux: $\boldsymbol{q}$	$\leftrightarrow$	• $\frac{-\rho^2\boldsymbol{\omega}}{8\pi G}$: rescaled normal one-form
• external force: $\boldsymbol{f}$	$\leftrightarrow$	• $-T_{\boldsymbol{t}^*A}$: tangential matter stress
• radiative heat : $\dot{Q}^{rad}$	$\leftrightarrow$	• $T_{\boldsymbol{t}^*\boldsymbol{t}}$: matter radial flux
• outside pressure: P_{out}	$\leftrightarrow$	• $T_{\boldsymbol{t}^*\boldsymbol{t}}$: matter pressure
• Inside pressure: P_{in}	$\leftrightarrow$	• $\frac{G_{\boldsymbol{t}^*\boldsymbol{s}}(\bar{h})}{8\pi G}$: radial 3d Einstein tensor
• radial momentum: $\bar{\pi}_r$	$\leftrightarrow$	• $\frac{\theta_{\boldsymbol{n}}}{8\pi G}$: timelike expansion

Thermodynamics of interface and the BH thermodynamics

- Gibbs equation

$$\boxed{s^i = -\left(\frac{\partial \gamma}{\partial T}\right)_A}$$

- Thermodynamic equation of state:

$$\boxed{u^i = \gamma - T\left(\frac{\partial \gamma}{\partial T}\right)_A}$$

Thermodynamics of interface and the BH thermodynamics

- Gibbs equation

$$\boxed{s^i = -\left(\frac{\partial \gamma}{\partial T}\right)_A}$$

$\Rightarrow$If $T = T_{BH} = \frac{\hbar \gamma_t}{2\pi}$ is assumed, we have

$$s^i = \frac{1}{4\hbar G}$$

$$\boxed{\gamma \leftrightarrow \frac{-\gamma_t}{8\pi G}}$$

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- Thermodynamic equation of state:

$$\boxed{u^i = \gamma - T\left(\frac{\partial \gamma}{\partial T}\right)_A}$$

⇒If the above result is assumed, we have

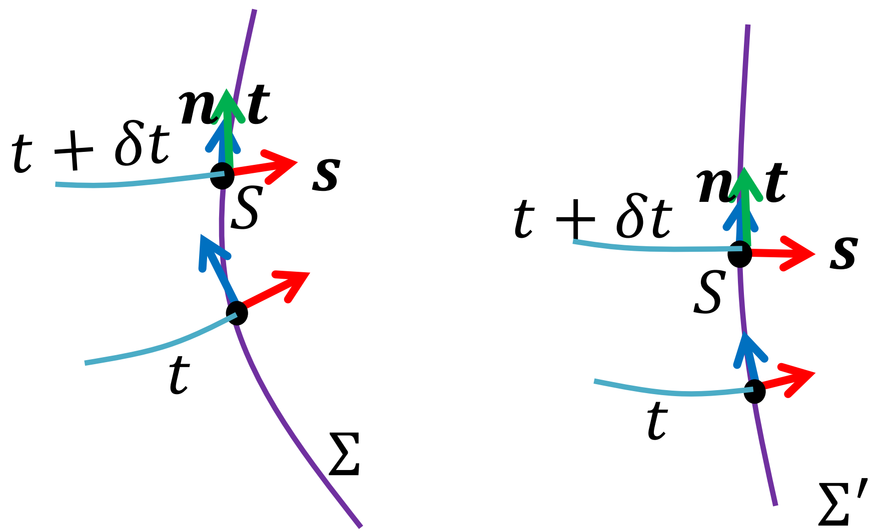
$$u^i = 0$$

$\leftrightarrow \theta = 0$:trapped surface

$$\boxed{u \leftrightarrow \frac{-\theta_{t^*}}{8\pi G}}$$

Constituent eqs

- Choice of time evolution of the screen corresponds to choice of $(\boldsymbol{n}, \boldsymbol{s})$ to the screen.



⇒change constituent eq $\widetilde{\boldsymbol{\Theta}}_{\boldsymbol{s}} = \widetilde{\boldsymbol{\Theta}}_{\boldsymbol{s}}(\boldsymbol{\Sigma})$.

- An example: null time evolution ($\boldsymbol{n} = \boldsymbol{s}$)

$$\Theta \equiv \frac{-\widetilde{\boldsymbol{\Theta}}_{\boldsymbol{t}^*}}{8\pi G} = \frac{-\widetilde{\boldsymbol{\Theta}}_{\boldsymbol{t}}}{8\pi G} = -\frac{1}{16\pi G}\theta_{\boldsymbol{t}}\boldsymbol{q} + \frac{1}{16\pi G}2\sigma_{\boldsymbol{t}},$$

which is the same as BH membrane paradigm.

⇒Our formulation is more fundamental than the BH thermodynamics.

Summery and discussions

- The Einstein eqs projected on the gravitational screen
  reduce the thermodynamic equation
  for a viscous bubble in Newtonian gravity.
- Such a screen has a surface tension
  and an internal energy.
- Obtain a complete dictionary between bubble non-equilibrium thermodynamics and gravity.
- Discuss the usual BH thermodynamics from this point of view.

<Open questions>

- How can we identify temperature independently?

⇒Unruh-like discussion?

- What is the constituent eqs satisfying 2nd law?

⇒Relativistic hydrodynamic formulation?

- How does this view relate to the fluid/gravity correspondence?

⇒More general view?

Thank you very much!