Towards the gravity/CYBE correspondence

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References: 1401.4855, 1402.6147, 1404.1838, 1404.3657

Introduction

The AdS/CFT correspondence

type IIB string on $AdS_5 \times S^5$ ⟷ 4D $\mathcal{N}=4$ SU(N) SYM $(N \to \infty)$

A recent progress: the discovery of integrability behind the duality

It provides a powerful tool.

⇒ It enables us to check its conjectured relations without SUSY, even at finite coupling, in spite of severe quantum corrections.

EX anomalous dimensions, amplitudes etc.

(An enormous amount of works have been done with integrable techniques)

Here we are concerned with the classical integrability of the string-theory side.

The classical integrability of the $AdS_5 x S^5$ superstring

The coset structure of AdS_5 x S^5 is closely related to integrability.

$$\boxed{\mathrm{AdS}_5 \times \mathrm{S}^5 = \frac{SO(2,4)}{SO(1,4)} \times \frac{SO(6)}{SO(5)}}$$

: symmetric coset

Z_2-grading $\Rightarrow$ classical integrability

Including fermions

$$\boxed{\frac{PSU(2,2|4)}{SO(1,4) \times SO(5)}}$$

: super coset

[Bena-Polchinski-Roiban, 2003]

Z_4-grading $\Rightarrow$ classical integrability

This fact is the starting point of our later argument.

The next issue:

Integrable deformations of the $AdS_5 x S^5$ superstring

Integrable deformations ⟺ gravity solutions of type IIB SUGRA

deformations of AdS_5 x S^5

Integrability techniques ⟺ solution generation techniques

Integrable twists

TsT transformations

Many integrable deformations are known.

➡ There should be many gravity solutions.

Motive

a classification of gravity solutions based on integrable deformations

A systematic way to consider integrable deformations

Yang-Baxter sigma model approach [Klimcik, 2008]

Integrable defomation!

$$S = \int d^2x \, \mathrm{tr}(JJ) \quad \longrightarrow \quad S^{(\eta)} = \int d^2x \, \mathrm{tr}\left(J \, \underline{\frac{1}{1-\eta R}} \, J \right)$$

η is a constant and *R* is a linear operator fixed by a classical r-matrix.

Integrable deformations are specified by classical r-matrices

⬆

(modified) classical Yang-Baxter equation (CYBE)

This approach has recently been generalized to the $AdS_5 x S^5$ superstring.

[Delduc-Magro-Vicedo, 1309.5850] [Kawaguchi-Matsumoto-KY, 1401.4855]

The gravity/CYBE correspondence

classical r-matrices ⟷ type IIB gravity solutions

A schematic picture

CYBE

↑ a classical r-matrix

Deformed $AdS_5 \times S^5$ superstring action

→ rewrite the action & read off

The metric (string frame) & NS-NS 2-form

↕ compare!

A gravity solution in type IIB SUGRA

Note: The metric & NS-NS 2-form are successfully obtained so far.

However, further studies are necessary for the RR sector.

2 kinds of integrable deformations of the $AdS_5 x S^5$ superstring action

1) Standard *q*-deformation [Delduc-Magro-Vicedo, 1309.5850]

2) Non-standard *q*-deformation [Kawaguchi-Matsumoto-KY, 1401.4855]

The content of my talk

1. Standard *q*-deformation (review)

   An example: η-deformation of $AdS_5 \times S^5$

2. Non-standard *q*-deformation (Our works)

   Some examples

3. Summary & Discussion

1. Standard *q*-deformation

(review)

3 slides $\left\{ \begin{array}{ll} \text{[Metsaev-Tseytlin, hep-th/9805028]} & \text{x 1} \\ \text{[Delduc-Magro-Vicedo, 1309.5850]} & \text{x 1} \\ \text{[Arutyunov-Borsato-Frolov, 1312.3542]} & \text{x 1} \end{array} \right.$

Green-Schwarz action of the $AdS_5 \times S^5$ superstring

[Metsaev-Tseytlin, hep-th/9805028]
[For a nice review, see 0901.4937]

$$\boxed{S = -\frac{1}{2}\int d\tau \int_0^{2\pi} d\sigma \, P_-^{\alpha\beta}\, \mathrm{Str}\left(A_\alpha d \circ A_\beta\right)}$$

Maurer-Cartan 1-form

$$A_\alpha \equiv g^{-1}\partial_\alpha g\,, \quad g \in SU(2,2|4)\,,$$

Projection op. on the supercoset

$$d \equiv P_1 + 2P_2 - P_3$$

Projection on the world-sheet

$$P_\pm^{\alpha\beta} \equiv \frac{1}{2}(\gamma^{\alpha\beta} \pm \epsilon^{\alpha\beta})$$

$$\left[\begin{array}{l} \gamma^{\alpha\beta} = \mathrm{diag}(-1,1) \\ \epsilon^{\alpha\beta} \;:\; \text{anti-symm. tensor} \end{array}\right.$$

NOTE

- a coordinate system is introduced via a parameterization of g
- Lax pair is constructed $\Rightarrow$ classical integrability

q-deformed action (standard q-def.)

[Delduc-Magro-Vicedo, PRL122 (2014) 051601, 1309.5850]

$$S = -\frac{(1+\eta^2)^2}{2(1-\eta^2)} \int d\tau \int_0^{2\pi} d\sigma\, P_-^{\alpha\beta}\, \mathrm{Str}\left[A_\alpha d \circ \boxed{\frac{1}{1-\eta R_g \circ d}}(A_\beta)\right]$$

Difference : η : deformation parameter, $d \equiv P_1 + \boxed{\frac{2}{1-\eta^2}} P_2 - P_3$

$$R_g = \mathrm{Ad}g^{-1} \circ R \circ \mathrm{Ad}g$$

R-operator: $$\boxed{R(M) = \begin{cases} -iM & (\text{if } M \text{ is a positive root}) \\ +iM & (\text{if } M \text{ is a negative root}) \end{cases}}$$

NOTE :

- a subsector ➡ q-deformed SU(2) [Drinfeld-Jimbo]
- the construction of Lax pair ➡ classical integrability
- kappa-symmetry

A q-deformation of $AdS_5 \times S^5$

[Arutyunov-Borsato-Frolov, 1312.3542]

$$ds^2 = ds^2_{\rm AdS} + ds^2_{\rm S}$$

sometimes called η-deformation of $AdS_5 \times S^5$

Singularity!

$$\begin{aligned}
ds^2_{\rm AdS} &= \frac{1}{1-\kappa^2 \sinh^2\rho}\left[-\cosh^2\rho\, dt^2 + d\rho^2\right] \\
&\quad + \sinh^2\rho \left[\frac{1}{1+\kappa^2 \sinh^4\rho\, \sin^2\zeta}\left(d\zeta^2 + \cos^2\zeta\, d\psi_1^2\right) + \sin^2\zeta\, d\psi_2^2\right] \\
ds^2_{\rm S} &= \frac{1}{1+\kappa^2 \sin^2\theta}\left[d\theta^2 + \cos^2\theta\, d\phi^2\right] \\
&\quad + \sin^2\theta \left[\frac{1}{1+\kappa^2 \sin^4\theta\, \sin^2\xi}\left(d\xi^2 + \cos^2\xi\, d\phi_1^2\right) + \sin^2\xi\, d\phi_2^2\right]
\end{aligned}$$

Deformation parameter: $\kappa \equiv \dfrac{2\eta}{1-\eta^2}$

NS-NS 2-form is turned on both AdS and S

NOTE: The other fields in type IIB SUGRA have not been determined yet.

The RR fluxes may be complex. [Hoare-Roiban-Tseytlin, 1403.5517]

2. Non-standard q-deformation

[Kawaguchi-Matsumoto-KY, 1401.4855]

[Kawaguchi-Matsumoto-KY, 1402.6147]

[Matsumoto-KY, 1404.1838, 1404.3657, in preparation]

The relation between deformation and R-operator

deformation

If an R-operator is given, a deformation is specified.

(where different R-ops. may lead to the identical deformation)

$\Updownarrow$

R-operator $\Longleftrightarrow$ skew-symmetric classical r-matrix

$$R \ : \quad \mathfrak{g} \longrightarrow \mathfrak{g} \qquad\qquad r \in \mathfrak{g} \otimes \mathfrak{g}$$

$$R(X) \equiv \langle r_{12}, 1 \otimes X \rangle = \sum_i a_i \langle b_i, X \rangle \qquad \text{for} \quad X \in \mathfrak{g}$$

$$r_{12} = \sum_i a_i \otimes b_i \qquad \text{with} \qquad a_i, b_i \in \mathfrak{g}$$

Two origins of classical r-matrix

1) modified classical Yang-Baxter eq. (mCYBE)

2) classical Yang-Baxter eq. (CYBE)

2 kinds of q-deformations

1) Standard q-deformation $\leftarrow$ mCYBE

classical r-matrix of Drinfeld-Jimbo type [Delduc-Magro-Vicedo, 1309.5850]

2) Non-standard q-deformation (Jordanian deformation) $\leftarrow$ CYBE

Some examples of r-matrices [Kawaguchi-Matsumoto-KY, 1401.4855]

Advantages of Jordanian deformations

i) Partial deformations are possible (only for AdS_5 or S^5)

ii) Some examples of the corresponding gravity solutions

[Kawaguchi-Matsumoto-KY, 1402.6147]
[Matsumoto-KY, 1404.1838, 1404.3657, in preparation]

Jordanian deformed $AdS_5 x S^5$ superstring action

[Kawaguchi-Matsumoto-KY, 1401.4855]

$$S=-\frac{1}{2}\int_{-\infty}^{\infty}d\tau\int_{0}^{2\pi}d\sigma\;P_{-}^{\alpha\beta}\mathrm{Str}\left[A_\alpha d\circ\boxed{\frac{1}{1-\eta\left[R_{\mathrm{Jor}}\right]_g\circ d}}(A_\beta)\right]$$

Maurer-Cartan 1-form

$$A_\alpha\equiv g^{-1}\partial_\alpha g\,,\quad g\in SU(2,2|4)\quad,$$

Projection on the group manifold

$$d\equiv P_1+2P_2-P_3$$

Projection on the world-sheet

$$P_{\pm}^{\alpha\beta}\equiv\frac{1}{2}(\gamma^{\alpha\beta}\pm\epsilon^{\alpha\beta})$$

$$\left\{\begin{array}{l}\gamma^{\alpha\beta}=\mathrm{diag}(-1,1)\\ \epsilon^{\alpha\beta}\;:\;\;\text{anti-symm. tensor}\end{array}\right.$$

- Lax pair is constructed
- kappa-symmetry is preserved

NOTE :

From mCYBE → CYBE,

Lax pair and kappa-trans. are modified.

Examples of Jordanian deformations

i) 3-parameter gamma-deformations of S^5

[Matsumoto-KY, 1404.1838]

Abelian r-matirx: $$r_{\text{Abe}}^{(\mu_1,\mu_2,\mu_3)} = \mu_3\, h_1 \wedge h_2 + \mu_1\, h_2 \wedge h_3 + \mu_2\, h_3 \wedge h_1$$

where μ_i and h_i $(i = 1, 2, 3)$ are deformation parameters and the Cartan generators of $\mathfrak{su}(4)$.

Metric: $$ds^2 = ds^2_{\text{AdS}_5} + \sum_{i=1}^{3}(d\rho_i^2 + G\rho_i^2 d\phi_i^2) + G\rho_1^2\rho_2^2\rho_3^2\left(\sum_{i=1}^{3}\hat{\gamma}_i\, d\phi_i\right)^2 ,$$

B-field: $$B_2 = G\left(\hat{\gamma}_3\, \rho_1^2\rho_2^2\, d\phi_1 \wedge d\phi_2 + \hat{\gamma}_1\, \rho_2^2\rho_3^2\, d\phi_2 \wedge d\phi_3 + \hat{\gamma}_2\, \rho_3^2\rho_1^2\, d\phi_3 \wedge d\phi_1\right)$$

$$\textstyle\sum_{i=1}^{3} \rho_i^2 = 1 \qquad\qquad G^{-1} \equiv 1 + \hat{\gamma}_3^2\, \rho_1^2\rho_2^2 + \hat{\gamma}_1^2\, \rho_2^2\rho_3^2 + \hat{\gamma}_2^2\, \rho_3^2\rho_1^2$$

[Lunin-Maldacena, Frolov, 2005]

A special case: $\hat{\gamma}_1 = \hat{\gamma}_2 = \hat{\gamma}_3 \equiv \hat{\gamma}$ $\Longrightarrow$ Lunin-Maldacena background

The parameter identification: $$8\eta\, \mu_1 = \hat{\gamma}_1\, , \quad 8\eta\, \mu_2 = \hat{\gamma}_2\, , \quad 8\eta\, \mu_3 = \hat{\gamma}_3$$

ii) The gravity dual for non-commutative SYM

[Matsumoto-KY, 1404.3657]

Abelian Jordanian r-matrix: $r_{\rm AJ} = \mu\, p_2 \wedge p_3 + \nu\, p_0 \wedge p_1$

where $p_\mu \equiv \frac{1}{2}\gamma_\mu - m_{\mu 5}\,,\quad m_{\mu 5} = \frac{1}{4}\left[\gamma_\mu, \gamma_5\right],\quad \gamma_\mu$: a basis of $\mathfrak{su}(2,2)$

Metric:

[Hashimoto-Itzhaki, Maldacena-Russo, 1999]

$$ds^2 = \frac{z^2}{z^4 + a'^4}(dx_0^2 + dx_1^2) + \frac{z^2}{z^4 + a^4}(dx_2^2 + dx_3^2) + \frac{dz^2}{z^2} + d\Omega_5^2$$

B-field: $$B_{01} = \frac{a'^2}{z^4 + a'^4}\,,\qquad B_{23} = \frac{a^2}{z^4 + a^4}$$

The parameter identification: $2\eta\,\mu = a^2\,,\qquad 2\eta\,\nu = a'^2$

NOTE: The integrability of this background has not been shown before our work.

Other examples: Dhokarh-Haque-Hashimoto background [0801.3812]

Hubeny-Rangamani-Ross background [hep-th/0504034]

A new background [Kawaguchi-Matsumoto-KY, 1402.6147]

have been obtained from classical r-matrices.

A lot of gravity solutions would be found out!

NOTE

1) The solutions obtained so far can also be reproduced from $AdS_5 \times S^5$ via solution generating techniques in SUGRAs

e.g., TsT transformations, T-dualities with smearing

2) The RR sector and dilaton have not been confirmed yet. However, kappa-symmetry indicates that the RR sector can also be reproduced. It can be directly checked with the supercoset construction in principle, though it would be quite intricate.

3. Summary & Discussion

Summary

q-deformations of the $AdS_5 \times S^5$ superstring

2 kinds:

1. Standard q-deformation (mCYBE) [Delduc-Magro-Vicedo]

   η-deformed background [Arutunov-Borsato-Frolov]

2. Non-standard q-deformation (CYBE) [Kawaguchi-Matsumoto-KY]

   Some examples have been found out.

Discussion

For standard q-deformation

1-loop beta function, SUGRA sol., the gauge-theory counter part?

For non-standard q-deformation

More examples. The RR sector and dilaton.

Thank you!