



# Some Properties of Classical Solutions in CSFT

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Isao Kishimoto  
(Univ. of Tokyo, Hongo)

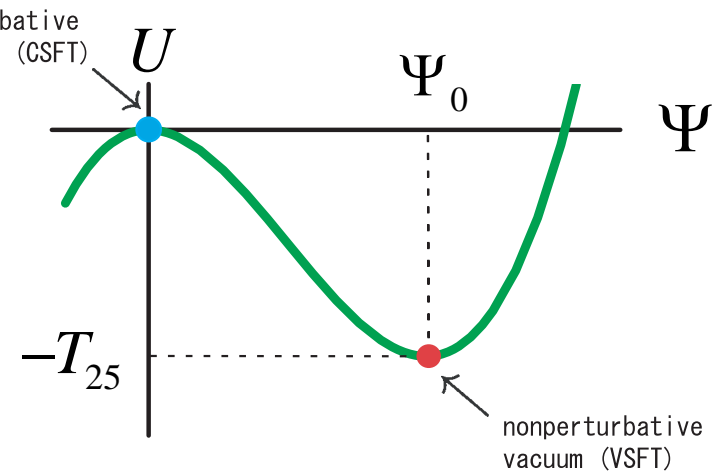
I.K.-K.Ohmori, hep-th/0112169  
I.K.-T.Takahashi, hep-th/0205nnn

# Introduction

- Sen's conjecture

There is a tachyon vacuum in bosonic open string theory. D25-brane vanishes on it and there is no open string excitation.

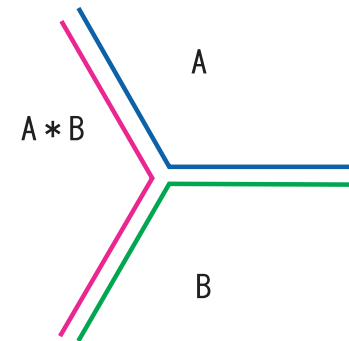
open string (D25-brane)



- Cubic String Field Theory (CSFT) [Witten(1986)]

Here we consider CSFT to investigate Sen's conjecture.

$$S = -\frac{1}{g^2} \left[ \frac{1}{2} \langle \Psi, Q_B \Psi \rangle + \frac{1}{3} \langle \Psi, \Psi * \Psi \rangle \right]$$



Sen's conjecture can be rephrased in CSFT :

- There is a solution  $\Psi_0$  of equation of motion:

$$Q_B \Psi + \Psi * \Psi = 0.$$

- The potential height equals to D25-brane tension:

$$-S |_{\Psi_0} / V_{26} = T_{25}.$$

- The cohomology of new BRST  $Q'_B$  operator around it is trivial:  $Q'_B \psi = 0 \Rightarrow \psi = Q'_B \phi, \quad \exists \phi.$

There are some numerical evidences, but an exact solution is necessary to prove them.

# ■ Another approach: Vacuum String Field Theory (VSFT) [(G)RSZ(2001)]

(Gaiotto-)Rastelli-Sen-Zwiebach proposed SFT around tachyon vacuum:

$$S_V = -\kappa \left[ \frac{1}{2} \langle \Psi, Q_{\text{GRSZ}} \Psi \rangle + \frac{1}{3} \langle \Psi, \Psi * \Psi \rangle \right],$$
$$Q_{\text{GRSZ}} = \frac{1}{2i} (c(i) - c(-i)) = c_0 + \sum_{n \geq 1} (-1)^n (c_{2n} + c_{-2n}).$$

This  $Q_{\text{GRSZ}}$  has trivial cohomology.

There are some solutions of equation of motion which are constructed by projectors with respect to  $*$ .

## Can VSFT reconstruct CSFT?

Recently, Okawa proved D25-brane tension can be reproduced.



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# Some Solutions in CSFT

Horowitz et al.(1988) discussed rather formal solutions in the context of purely CSFT.

$$|\Psi_0\rangle = -Q_L |I\rangle + C_L(f) |I\rangle, \quad f(\pi - \sigma) = f(\sigma), f(\pi / 2) = 0.$$

In particular it can be used to derive VSFT action from CSFT:

$$f_{\text{GRSZ}}(\sigma) = \lim_{\varepsilon \rightarrow 0} (\delta(\sigma - (\pi / 2 - \varepsilon)) + \delta((\pi / 2 + \varepsilon) - \sigma)).$$

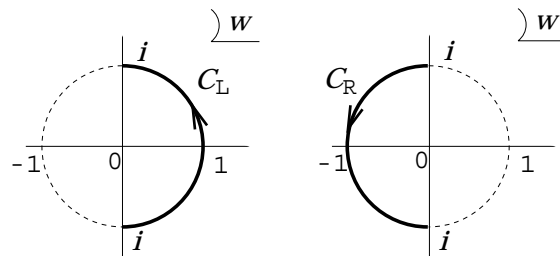
The corresponding solution is **singular**:

$$\begin{aligned} |\Psi_0^{\text{VSFT}}\rangle &= -Q_L |I\rangle + \lim_{\varepsilon \rightarrow 0} \frac{(g^2 \kappa)^{1/3}}{4i} (e^{-i\varepsilon} c(i e^{i\varepsilon}) - e^{i\varepsilon} c(-i e^{-i\varepsilon})) |I\rangle \\ &= \frac{2}{\pi} \sum_{m \geq 0} \frac{(-1)^m}{2m+1} Q_{-(2m+1)} |I\rangle + \frac{(g^2 \kappa)^{1/3}}{2} \lim_{\varepsilon \rightarrow 0} \left( \left( 1 + 2 \sum_{k \geq 1} \cos 2k\varepsilon \right) c_0 - \left( \sum_{k \geq 0} \sin(2k+1)\varepsilon \right) (c_1 - c_{-1}) \right) |I\rangle \end{aligned}$$

- There are some subtleties about identity string field, but here we treat  $|I\rangle$  as the identity with respect to  $*$  :

$$A * I = I * A = A, \quad Q_R I * A = -I * Q_L A = -Q_L A, \dots$$

Notation:



$$C(f) = \oint \frac{dw}{2\pi i} f(w)c(w), \quad C_L(f) = \int_{C_L} \frac{dw}{2\pi i} f(w)c(w),$$

$$Q(f) = \oint \frac{dw}{2\pi i} f(w)j_{\text{BRST}}(w), \quad Q_L(f) = \int_{C_L} \frac{dw}{2\pi i} f(w)j_{\text{BRST}}(w),$$

$$q(f) = \oint \frac{dw}{2\pi i} f(w)j_{\text{gh}}(w), \quad q_L(f) = \int_{C_L} \frac{dw}{2\pi i} f(w)j_{\text{gh}}(w), \dots$$

$$j_{\text{BRST}}(w) = cT^X + :bc\partial c: + \frac{3}{2}\partial^2 c = \sum_n Q_n w^{-n-1}, \quad j_{\text{gh}}(w) = -:bc: = \sum_n q_n w^{-n-1}, \dots$$

- Takahashi and Tanimoto proposed a new regular solution of CSFT which might represent tachyon vacuum. (2002)

First, they found solutions of CSFT:

$$|\Psi_0\rangle = Q_L (e^h - 1) |I\rangle - C_L ((\partial h)^2 e^h) |I\rangle,$$

where  $h(w)$  is some function such that

$$h(w) = \sum h_n (w^n + (-w^{-1})^n), \quad h(\pm i) = 0, \partial h(\pm i) = 0.$$

But these solutions can be rewritten as pure gauge ones **at least formally**.

$$\Psi_0 = e^{q_L(h)I} * Q_B e^{-q_L(h)I}$$

In fact, new BRST operator  $Q'_B = Q(e^h) - C((\partial h)^2 e^h)$  is **formally** rewritten as

$$Q'_B = e^{q(h)} Q_B e^{-q(h)}.$$

However they might become nontrivial solutions at some limits.



## Example

$$h_a(w) = \log \left( 1 + \frac{a}{2} (w + w^{-1})^2 \right)$$

Noting

$$q(h_a) = -q_0 \log(1 - Z(a))^2 + q^{(+)}(h_a) + q^{(-)}(h_a), \quad Z(a) = \frac{1 + a - \sqrt{1 + 2a}}{a},$$

and 
$$\left[ q^{(+)}(h_a), q^{(-)}(h_a) \right] = -2 \log(1 - Z(a))^2,$$

$$\exp(\pm q(h_a)) = (1 - Z(a))^2 \exp(\mp q_0 \log(1 - Z(a))^2) e^{\pm q^{(-)}(h_a)} e^{\pm q^{(+)}(h_a)}$$

is well-defined for  $a > -1/2$ .

Because  $Z(a = -1/2) = -1$ ,  $a = -1/2$  case:

$$h_{a=-1/2}(w) = \log \left( -\frac{1}{4} (w - w^{-1})^2 \right) \quad \text{might be nontrivial.}$$

## More Examples

$$h_a(w^{2k-1}), \quad h_a(w^{2k}) - \log(2a)$$

Noting  $Z(a = -1/2) = -1$ ,  $Z(a = \infty) = 1$ ,

in the same sense, we have nontrivial limit at  $a = -1/2$ ,  $a = \infty$ .  
respectively:

$$h^{(l)}(w) = \log \left( \frac{(-1)^l}{4} (w^l + (-w^{-1})^l) \right).$$

There is well-defined oscillator representation:

$$\begin{aligned} |\Psi_0^{(l)}\rangle &= Q_L \left( -\frac{1}{2} + \frac{(-1)^l}{4} (w^{2l} + w^{-2l}) \right) |I\rangle + C_L \left( l^2 w^{-2} (2 - (-1)^l (w^{2l} + w^{-2l})) \right) |I\rangle \\ &= \sum_{m \geq 0} \frac{\sqrt{\pi} \cdot (-1)^m \cdot 8l^2 \cdot 4}{(2m+1)(2m+1-2l)(2m+1+2l)} \left( -Q_{-(2m+1)} + 4l^2 c_{-(2m+1)} \right) |I\rangle \\ &\quad + \frac{l^2}{\pi} \left( \gamma + 2 \log 2 + \underbrace{\psi\left(\frac{1}{2} - l\right)}_{\frac{1}{2}} + \underbrace{\psi\left(\frac{1}{2} + l\right)}_{\frac{1}{2}} \right) (c_1 - c_{-1}) |I\rangle. \end{aligned}$$

# Potential Height

When we naively compute the value of the action at the solutions constructed on identity string field:

$$|I\rangle = \frac{1}{4i} b\left(\frac{\pi}{2}\right) b\left(-\frac{\pi}{2}\right) \exp\left(\sum_{n \geq 1} \left( \frac{-(-1)^n}{2n} \alpha_{-n} \cdot \alpha_{-n} + (-1)^n c_{-n} b_{-n} \right)\right) c_0 c_1 |0\rangle,$$



we encounter following **divergence**:

$$\langle I | (\dots) | I \rangle \sim \left( \det_{n,m \geq 1} (\delta_{n,m} - \delta_{n,m}) \right)^{-26/2+1} = \infty.$$

It is necessary to regularize  $|I\rangle$  appropriately.

On the other hand, potential height is zero if we use e.o.m. naively in the action:

$$Q_B \Psi_0 + \Psi_0 * \Psi_0 = 0,$$

$$Q_L I * Q_L I = 0, \quad C_L(f)I * C_L(f)I = 0,$$

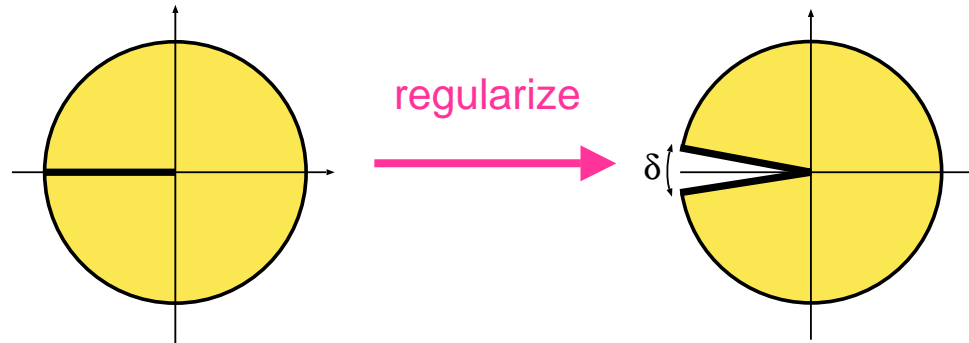
$$\Rightarrow S|_{\Psi_0^{\text{VSFT}}} = -\frac{1}{6g^2} \langle \Psi_0, Q_B \Psi_0 \rangle = \frac{1}{6g^2} \langle \Psi_0, \Psi_0 * \Psi_0 \rangle = 0.$$

Takahashi-Tanimoto solutions give also zero naively, because they are pure gauge formally.

There is a possibility of  $\infty \times 0 = \text{finite} (!?)$

# • Pacman regularization

It is technically easy to use pacman regularization in the computation using LPP+GGRT. [I.K.-K.Ohmori]



But we could not get the definite value at  $\delta = 0$  ,  
for example,

$$\left\langle \frac{1}{2i} \left( e^{-i\varepsilon} c(i e^{i\varepsilon}) - e^{i\varepsilon} c(-i e^{-i\varepsilon}) \right) I_\delta, Q_B \frac{1}{2i} \left( e^{-i\varepsilon} c(i e^{i\varepsilon}) - e^{i\varepsilon} c(-i e^{-i\varepsilon}) \right) I_\delta \right\rangle$$

$$= -\delta^2 \sin^2 \varepsilon \left[ \frac{1}{2} \left\{ \left( \tan \frac{\varepsilon}{2} \right)^{\frac{2}{\delta}} + \left( \tan \frac{\varepsilon}{2} \right)^{-\frac{2}{\delta}} \right\} + 3 \right] V_{26}.$$

- Regularization in oscillator language

If we multiply **damping factor**  $\exp(-\pi t L_0)$  on  $|I\rangle$ , we get a formula for oscillator modes:

$$\langle I | q^{L_0} c_{m_1} c_{m_2} c_{m_3} q^{L_0} | I \rangle = \frac{i^{m_1+m_2+m_3} q^{-2} f(q^4)^{-12} V_{26}}{\sinh(2m_1\pi t) \sinh(2m_2\pi t) \sinh(2m_3\pi t)} \det \begin{pmatrix} 1 & 1 & 1 & 1 \\ (-q)^{m_1} & q^{m_1} & q^{-m_1} & (-q)^{-m_1} \\ (-q)^{m_2} & q^{m_2} & q^{-m_2} & (-q)^{-m_2} \\ (-q)^{m_3} & q^{m_3} & q^{-m_3} & (-q)^{-m_3} \end{pmatrix},$$

$$q := e^{-\pi t}, \quad f(q^2) = \prod_{n=1}^{\infty} (1 - q^{2n}) = (2\pi)^{-1/3} q^{-1/12} \mathfrak{G}'_1(0, it)^{1/3}.$$

Using some relations

$$Q_B |I\rangle = 0, \quad \{Q_B, Q_n\} = 0, \quad \{Q_B, c_n\} = - \sum_{j=-\infty}^{\infty} \left( j + \frac{n}{2} \right) c_{-j} c_{n+j}, \dots,$$

we have the following expression:

$$\begin{aligned}
S |_{\Psi_0^{(1)}} / V_{26} &= \lim_{t \rightarrow 0^+} \frac{-1}{6g^2 V_{26}} \langle I | q^{L_0} C_L (w^{-2} (w + w^{-1})^2) Q_B C_L (w^{-2} (w + w^{-1})^2) q^{L_0} | I \rangle \\
&= \lim_{t \rightarrow 0^+} \frac{-1}{6g^2} \int_{-p/2}^{p/2} dx \int_{-p/2}^{p/2} dy \cos^2 x \cos^2 y \frac{ip^2}{(J_1'(0, 2it))^4} \\
&\cdot \det \begin{pmatrix} 1 & 1 & 1 & 1 \\ \frac{J_4'(x/2 + p/2 - ipt/4, 2it)}{J_4(x/2 + p/2 - ipt/4, 2it)} & \frac{J_3'}{J_3}(\dots) & \frac{J_2'}{J_2}(\dots) & \frac{J_1'}{J_1}(\dots) \\ \frac{J_4'(y/2 + p/2 - ipt/4, 2it)}{J_4(y/2 + p/2 - ipt/4, 2it)} & \frac{J_3'}{J_3}(\dots) & \frac{J_2'}{J_2}(\dots) & \frac{J_1'}{J_1}(\dots) \\ \partial_y \left( \frac{J_4'(y/2 + p/2 - ipt/4, 2it)}{J_4(y/2 + p/2 - ipt/4, 2it)} \right) & \partial_y \left( \frac{J_3'}{J_3} \right) (\dots) & \partial_y \left( \frac{J_2'}{J_2} \right) (\dots) & \partial_y \left( \frac{J_1'}{J_1} \right) (\dots) \end{pmatrix} .
\end{aligned}$$

We do not have definite (or exact) value of the potential height for our solutions yet.

Numerically, it tends to be divergent.(?)

# Cohomology of new BRST operator

## ■ Kinetic term around a solution

$$S|_{\Psi_0+\psi} = -\frac{1}{g} \left[ \frac{1}{2} \langle \psi, Q'_B \psi \rangle + \frac{1}{3} \langle \psi, \psi * \psi \rangle \right] + S|_{\Psi_0},$$

$$Q'_B \psi = Q_B \psi + \Psi_0 * \psi - (-1)^{|\psi|} \psi * \Psi_0.$$

The new BRST operator around our solution  $\Psi_0^{(1)}$  is

$$Q'_B = \frac{1}{2} Q_0 - \frac{1}{4} Q_{-2} - \frac{1}{4} Q_2 + 2c_0 + c_{-2} + c_2 = R_2 + R_0 + R_{-2}.$$

$$R_{\pm 2} := -\frac{1}{4} Q_{\pm 2} + c_{\pm 2}, \quad R_0 := \frac{1}{2} Q_0 + 2c_0 \quad \text{satisfy following relations:}$$

$$R_{\pm 2}^2 = 0, \quad R_{\pm 2} R_0 + R_0 R_{\pm 2} = 0, \quad R_0^2 + R_2 R_{-2} + R_{-2} R_2 = 0.$$



# Claim

$Q'_B$  cohomology is trivial in ghost number 1 states.

Proof

We consider the equation

$$Q'_B \mathbf{y} = 0$$

for the ghost number 1 state

$$\mathbf{y} = \sum_{N \geq h} \mathbf{y}_{-N}.$$

In other words, we solve the following equations:

$$R_2 \mathbf{y}_{-h-k} = 0,$$

$$k = 0, 1,$$

$$R_2 \mathbf{y}_{-h-k-2} = -R_0 \mathbf{y}_{-h-k},$$

$$R_2 \mathbf{y}_{-h-k-2l} = -R_0 \mathbf{y}_{-h-k-2(l-1)} - R_{-2} \mathbf{y}_{-h-k-2(l-2)}, \quad l \geq 2.$$

$$R_2 \quad \text{is rewritten as} \quad R_2 = -\frac{1}{4}Q_2 + c_2 = -\frac{1}{4}\widetilde{Q}_0 \quad .$$

$\widetilde{Q}_0$  is given by replacing  $c_n, b_n$  with  $\widetilde{c}_n := c_{n+2}, \widetilde{b}_n := b_{n-2}$  in  $Q_0 = Q_B$ .

Solutions of  $Q_B \mathbf{y} = 0$  are [Kato-Ogawa, M. Henneaux, ...]

$$|\mathbf{y}\rangle = A_{\text{DDF}} |0, p_0\rangle + B_{\text{DDF}} c_0 |0, p_0\rangle + Q_B |\mathbf{f}\rangle$$

where  $|0, p_0\rangle = e^{ip_0 x} |\Omega\rangle, \quad p_0^\pm = \pm \frac{1}{\sqrt{2a}}, \quad p_0^i = 0,$

$$|\Omega\rangle := c_1 |0\rangle, \quad c_n |\Omega\rangle = 0, n \geq 1, \quad b_n |\Omega\rangle = 0, n \geq 0,$$

and  $A_{\text{DDF}}, B_{\text{DDF}}$  are generated by DDF operators  $A_n^i :$

$$[A_m^i, A_n^j] = m \mathbf{d}_{m+n,0} \mathbf{d}^{i,j}, \quad [L_m^X, A_n^i] = 0.$$

Similarly, solutions of  $\widetilde{Q}_0 \mathbf{y} = 0$  are given by

$$|\mathbf{y}\rangle = A_{\text{DDF}} |\widetilde{0}, p_0\rangle + B_{\text{DDF}} \widetilde{c}_0 |\widetilde{0}, p_0\rangle + \widetilde{Q}_0 |\mathbf{f}\rangle$$

$$|\widetilde{0}, p_0\rangle := b_{-2} b_{-1} |0, p_0\rangle, \quad \widetilde{c}_n |\widetilde{0}, p_0\rangle = 0, n \geq 1, \quad \widetilde{b}_n |\widetilde{0}, p_0\rangle = 0, n \geq 0.$$

Here  $|\mathbf{y}_{-h-k}\rangle$  is **ghost number 1**,

$$R_2 |\mathbf{y}_{-h-k}\rangle = 0 \quad \Rightarrow \quad |\mathbf{y}_{-h-k}\rangle = R_2 |\mathbf{f}_{-h-k-2}\rangle, \quad \exists |\mathbf{f}_{-h-k-2}\rangle.$$

Then  $R_2 |\mathbf{y}_{-h-k-2}\rangle = -R_0 |\mathbf{y}_{-h-k}\rangle = -R_0 R_2 |\mathbf{f}_{-h-k-2}\rangle = R_2 R_0 |\mathbf{f}_{-h-k-2}\rangle,$

$$\Rightarrow |\mathbf{y}_{-h-k-2}\rangle = R_0 |\mathbf{f}_{-h-k-2}\rangle + R_2 |\mathbf{f}_{-h-k-4}\rangle, \quad \exists |\mathbf{f}_{-h-k-4}\rangle.$$

Suppose,  $|\psi_{-h-k-2l}\rangle = R_{-2}|\phi_{-h-k-2(l-1)}\rangle + R_0|\phi_{-h-k-2l}\rangle + R_2|\phi_{-h-k-2(l+1)}\rangle$ ,  $l = m, m-1$

then

$$\begin{aligned} R_2|\psi_{-h-k-2(m+1)}\rangle &= -R_0|\psi_{-h-k-2m}\rangle - R_{-2}|\psi_{-h-k-2(m-1)}\rangle \\ &= -R_0\left(R_{-2}|\phi_{-h-k-2(m-1)}\rangle + R_0|\phi_{-h-k-2m}\rangle + R_2|\phi_{-h-k-2(m+1)}\rangle\right) \\ &\quad - R_{-2}\left(R_{-2}|\phi_{-h-k-2(m-2)}\rangle + R_0|\phi_{-h-k-2(m-1)}\rangle + R_2|\phi_{-h-k-2m}\rangle\right) \\ &= R_2\left(R_{-2}|\phi_{-h-k-2m}\rangle + R_0|\phi_{-h-k-2(m+1)}\rangle\right), \end{aligned}$$

$\Rightarrow$

$$|\psi_{-h-k-2(m+1)}\rangle = R_{-2}|\phi_{-h-k-2m}\rangle + R_0|\phi_{-h-k-2(m+1)}\rangle + R_2|\phi_{-h-k-2(m+2)}\rangle, \quad \exists|\phi_{-h-k-2(m+2)}\rangle$$

By induction, we conclude

$$|\psi_{-h-k-2l}\rangle = R_{-2}|\phi_{-h-k-2(l-1)}\rangle + R_0|\phi_{-h-k-2l}\rangle + R_2|\phi_{-h-k-2(l+1)}\rangle, \quad l \geq 0,$$

namely,  $|\psi\rangle = Q'_B|\phi\rangle$ ,  $\exists|\phi\rangle = \sum_{k=0,1,l \geq 1} |\phi_{-h-k-2l}\rangle$ .

- We have similar arguments for **other solution**  $\Psi_0^{(l)}$  by using  $R_{2l}$  instead of  $R_2$ .

New BRST charge around it is

$$Q_B^{(l)} = \frac{1}{2} Q_0 + 2l^2 c_0 - (-1)^l (R_{2l} + R_{-2l}),$$

where 
$$R_{\pm 2l} = -\frac{1}{4} Q_{\pm 2l} + c_{\pm 2l} = -\frac{1}{4} \tilde{Q}_0^{(\pm l)}.$$

$\tilde{Q}_0^{(l)}$  is given by replacing  $c_n, b_n$  with  $\tilde{c}_n := c_{n+2l}, \tilde{b}_n := b_{n-2l}$  in  $Q_B$ .

We can prove

$Q_B^{(l)}$  cohomology is trivial in ghost number 1 states.

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- There are **nontrivial** solutions for  $Q_B^{(l)} |\psi^{(l)}\rangle = 0$  of the form:

$$|\psi^{(l)}\rangle = \exp(-q(f^{(l)})) \cdot \left( A_{\text{DDF}} b_{-2l} b_{-2l+1} \cdots b_{-2} b_{-1} |0, p_0\rangle + B_{\text{DDF}} b_{-2l+1} \cdots b_{-2} b_{-1} |0, p_0\rangle \right),$$

where  $q(f^{(l)}) = 2 \sum_{n \geq 1} \frac{(-1)^{n(1+l)}}{n} q_{-2ln}$

in **other ghost number sector**.

This follows from the identity:

$$\exp(q(f^{(l)})) Q_B^{(l)} \exp(-q(f^{(l)})) = -(-1)^l R_{2l}.$$



# Summary and Discussion

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- We have investigated whether some solutions of CSFT using identity string field can be tachyon vacuum or not.
- Evaluation of potential height at the solutions expressed by identity string field is rather difficult by our two regularization methods. **It is divergent at least naively.**
- The new BRST charge cohomology around Takahashi-Tanimoto limit solutions is **trivial in ghost number 1 states.** **This suggests they are nontrivial solutions of CSFT** although we do not know yet that they are the tachyon vacuum in Sen's conjecture.

- In the context of Takahashi-Tanimoto solutions, the solution which plugs **CSFT into VSFT** directly is **very singular limit (?)**:

$$|\Psi_0\rangle = Q_L (e^h - 1) |I\rangle - C_L ((\partial h)^2 e^h) |I\rangle$$

$$h(w) \rightarrow -\infty \quad |\Psi_0\rangle = -Q_L |I\rangle + C_L(f) |I\rangle.$$

There is subtlety about  $Q_L I$ :

By CFT calculation, we have  $\langle \phi, Q_L I * Q_L I \rangle = \langle \phi, Q_L^2 I \rangle = 0$ ,

but by oscillator calculation, we have

$$Q_L^2 |I\rangle = -\frac{2\zeta(0)}{\pi^2} (1 + 2\zeta(0)) Q_B c_0 |I\rangle = 0. \quad (?)$$



- Is the identity string field well-defined?

Mystery on  $c_0 I$  [Rastelli-Zwiebach(2000)]

$$c_0 A = c_0 (I * A) = (c_0 I) * A + I * (c_0 A) = (c_0 I) * A + c_0 A,$$
$$\therefore (c_0 I) * A = 0, \forall A.$$

If we take  $A = I$ ,  $0 \neq c_0 I = (c_0 I) * I = 0$  (??)

- Is there another exact solution which do **not** use identity string field in **CSFT**?

# Solutions in CSFT

CSFT

$$Q_B$$

pure gauge

$$Q_B^{(l)}$$

$$\Psi_0^{(l)}$$

singular

VSFT

$$Q_{\text{GRSZ}}$$

$$\Psi_0^{\text{VSFT}}$$

Where is the tachyon vacuum solution?

numerical solution  
in the Siegel gauge