超弦の場の理論の古典解とその性質

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refs. I. Kishimoto, T. Takahashi, JHEP11(2005)051 [hep-th/0506240],
I. Kishimoto, T. Takahashi, hep-th/0510224

Introduction

- String Field Theory (SFT) is a candidate for nonperturbative formulation of string theory.
- In bosonic SFT, some phenomena such as tachyon condensation have been investigated extensively using *level truncation numerically and exact solutions analytically*.
- In super SFT, similar works are done although concrete and detailed analysis is less developed than bosonic case.
- *We have constructed a class of exact classical solutions to super SFT and studied their properties.*

Witten's bosonic cubic SFT

Action:

$$S = -\frac{1}{g^2}\left(\frac{1}{2}\langle\Psi, Q_{\rm B}\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right)$$

String field:

$$|\Psi\rangle = \phi(x)c_1|0\rangle + A_\mu(x)\alpha^\mu_{-1}c_1|0\rangle + iB(x)c_0|0\rangle + \cdots$$

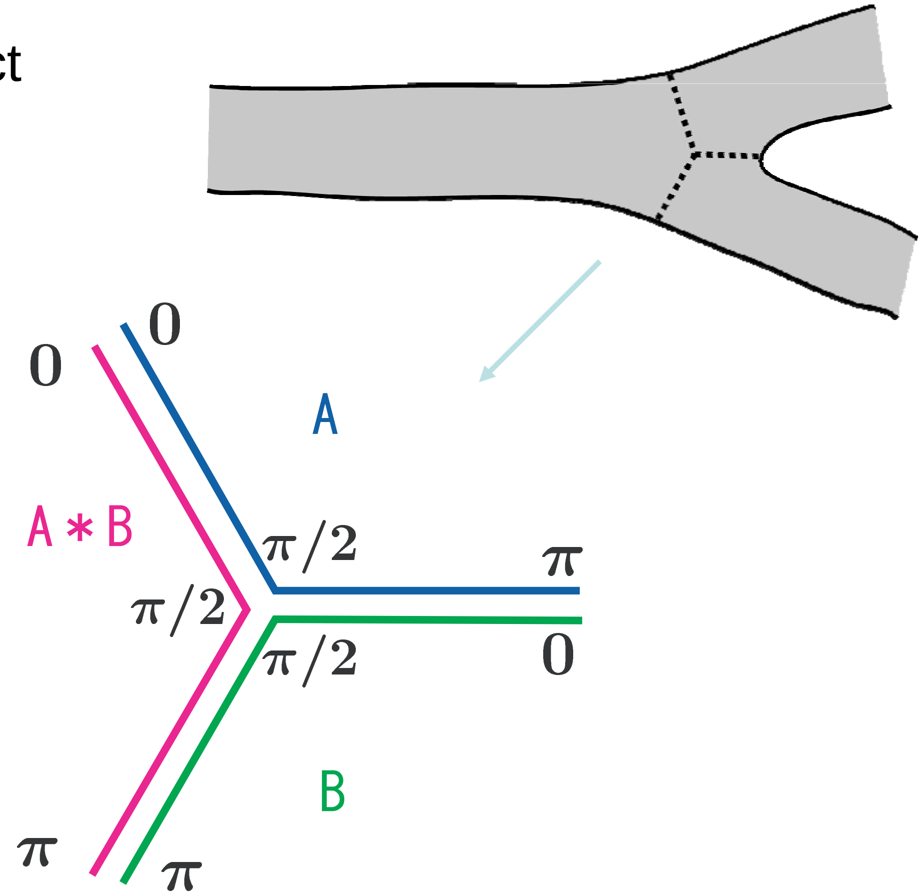
$$X^\mu(\sigma) = x^\mu + i\sqrt{\alpha'/2}\sum_{n\neq 0}\frac{1}{n}\alpha^\mu_n \cos n\sigma, \quad \ldots$$

Kinetic term: BRST operator

$$Q_{\rm B} = \oint\frac{dz}{2\pi i}j_{\rm B}(z) = \oint\frac{dz}{2\pi i}\left(cT^{\rm m} + bc\partial c + \frac{3}{2}\partial^2 c\right)$$

$$\langle\Psi, Q_{\rm B}\Psi\rangle$$
$$= \int d^{26}x\left(\phi(-\alpha'\partial^2 - 1)\phi - \alpha' A_\mu\partial^2 A^\mu + 2\sqrt{2\alpha'}B\partial_\mu A^\mu + 2B^2 + \cdots\right)$$

Interaction term: the Witten $*$ product



$$\int dx\,(\phi(x))^3$$
$$= \int dx_1 dx_2 dx_3 \delta(x_1 - x_2)\delta(x_2 - x_3)\, \phi(x_1)\phi(x_2)\phi(x_3)$$

$$\langle \Psi, \Psi * \Psi \rangle$$
$$= \langle V_3(1,2,3)|\Psi\rangle_1|\Psi\rangle_2|\Psi\rangle_3$$
$$\sim \int \prod_{0\le\sigma\le\pi/2} \left(\delta(X^{(1)}(\pi-\sigma) - X^{(2)}(\sigma))\delta(X^{(2)}(\pi-\sigma) - X^{(3)}(\sigma)) \times \delta(X^{(3)}(\pi-\sigma) - X^{(1)}(\sigma))(bc\ \text{ghost} \cdots)\right)$$
$$\times \Psi[X^{(1)}(\sigma),\cdots]\Psi[X^{(2)}(\sigma),\cdots]\Psi[X^{(3)}(\sigma),\cdots]$$

equation of motion: $$Q_{\mathrm{B}}|\Psi\rangle + |\Psi * \Psi\rangle = 0$$

gauge transformation: $$\delta_{\Lambda}\Psi = Q_{\mathrm{B}}\Lambda + \Psi * \Lambda - \Lambda * \Psi$$

$$\longrightarrow \qquad \delta_{\Lambda} S = 0$$

(※) $$Q_{\mathrm{B}}^2 = 0, \quad \langle A, Q_{\mathrm{B}} B\rangle = -(-1)^{|A|}\langle Q_{\mathrm{B}} A, B\rangle,$$

$$Q_{\mathrm{B}}(A * B) = (Q_{\mathrm{B}} A) * B + (-1)^{|A|} A * (Q_{\mathrm{B}} B),$$

$$\langle A, B\rangle = \langle B, A\rangle, \qquad \langle A, B * C\rangle = \langle B, C * A\rangle,$$

$$(A * B) * C = A * (B * C)$$: associative

Note : $A * B \neq B * A$ in general.

Known classical solutions to the equation of motion in (Witten's bosonic) SFT

- Numerical solutions in the Siegel gauge: $b_0|\Psi\rangle = 0$ with level truncation method.
  (Sen-Zwiebach,…,Gaiotto-Rastelli,….)

- Exact solutions using identity string field: $|I\rangle$ $\quad (A * I = I * A = A)$

$$\Psi_0 = -Q_L I := -\int_{C_{\rm left}} j_{\rm B}\, I$$

: derives purely cubic SFT (Horowitz,…)

$$\Psi_0 = -Q_L I + \frac{a}{4i}(e^{-i\epsilon}c(ie^{i\epsilon}) - e^{i\epsilon}c(-ie^{-i\epsilon}))I$$

:derives VSFT (Kishimoto-Ohmori)

$$\Psi_0 = Q_L(e^h - 1)I - C_L((\partial h)^2 e^h)I$$

: universal solution (Takahashi-Tanimoto)

$$\Psi_0 = -V_L(F)I - \frac{1}{4}C_L(F^2)I$$

:marginal solution (Takahashi-Tanimoto)

⇨ We consider super version of this type solution.

- Other (regular) solutions using butterfly, wedge states, or other method
  (Okawa, Schnabl, Kluson, Lechtenfeld et al., Michishita, …)

Contents

A brief review of super SFT

- We use Berkovits' open super SFT.

The action for NS(+) sector is given by

WZW type

$$S[\Phi]$$
$$= \frac{1}{2g^2}\langle\langle (e^{-\Phi}Q_{\rm B}e^{\Phi})(e^{-\Phi}\eta_0 e^{\Phi}) - \int_0^1 dt (e^{-t\Phi}\partial_t e^{t\Phi})\{(e^{-t\Phi}Q_{\rm B}e^{t\Phi}),(e^{-t\Phi}\eta_0 e^{t\Phi})\}\rangle\rangle$$
$$= -\frac{1}{g^2}\int_0^1 dt \langle\langle (\eta_0\Phi)(e^{-t\Phi}Q_{\rm B}e^{t\Phi})\rangle\rangle$$

[Berkovits-Okawa-Zwiebach(2004)]

$$= -\frac{1}{g^2}\sum_{M,N=0}^{\infty}\frac{(-1)^M}{(M+N+2)(M+N+1)M!N!}\langle\langle(\eta_0\Phi)\Phi^M(Q_{\rm B}\Phi)\Phi^N\rangle\rangle\,.$$

String field Φ : ghost number 0, picture number 0, Grassmann even, represented by $X^\mu, \psi^\mu, b, c, \phi, \xi, \eta \quad (\beta = e^{-\phi}\partial\xi, \gamma = \eta e^{\phi})$

$$Q_{\rm B} = \oint \frac{dz}{2\pi i}(c(T^{\rm m} - \tfrac{1}{2}(\partial\phi)^2 - \partial^2\phi + \partial\xi\eta) + bc\partial c + \eta e^{\phi}G^{\rm m} - \eta\partial\eta e^{2\phi}b)(z)$$

$$\eta_0 = \oint\frac{dz}{2\pi i}\eta(z)$$

Q_B, η_0 such as $Q_B^2 = 0,\ \eta_0^2 = 0,\ \{Q_B, \eta_0\} = 0$

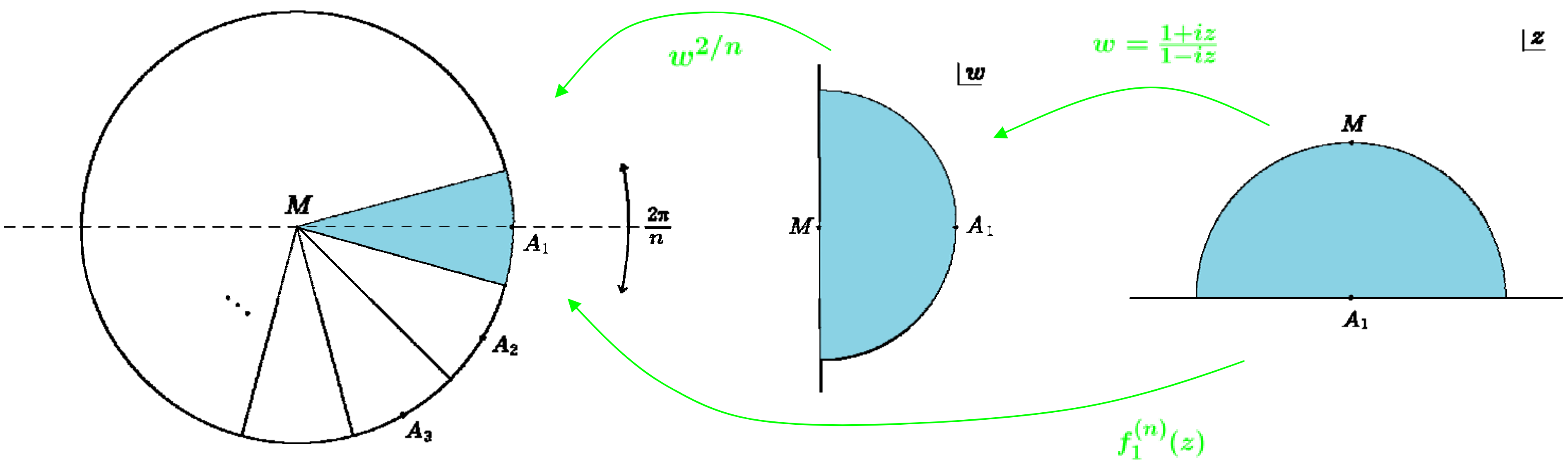
are derivations with respect to the star product:

$$Q_B(A * B) = Q_B A * B + (-1)^{|A|} A * Q_B B, \quad \eta_0(A * B) = \eta_0 A * B + (-1)^{|A|} A * \eta_0 B$$

The star product is given by 3-string vertex: $\langle A * B| = \langle V_3|A\rangle|B\rangle$.

n-string vertex is defined using CFT correlator in the *large* Hilbert space:

$$\langle V_n|A_1\rangle \cdots |A_n\rangle = \langle\langle A_1 \cdots A_n \rangle\rangle := \left\langle f_1^{(n)}[\mathcal{O}_{A_1}] \cdots f_n^{(n)}[\mathcal{O}_{A_n}] \right\rangle$$
$$= \langle A_1|(\cdots(A_2 * A_3) * \cdots * A_{n-1}) * A_n\rangle = \langle A_1|A_2 * \cdots * A_n\rangle$$



Some formulae:

$$\langle\langle A_1 \cdots A_{n-1} \Phi \rangle\rangle = \langle\langle \Phi A_1 \cdots A_{n-1} \rangle\rangle \,,$$
$$\langle\langle A_1 \cdots A_{n-1} (Q_{\rm B}\Phi) \rangle\rangle = -\langle\langle (Q_{\rm B}\Phi) A_1 \cdots A_{n-1} \rangle\rangle \,,$$
$$\langle\langle A_1 \cdots A_{n-1} (\eta\Phi) \rangle\rangle = -\langle\langle (\eta\Phi) A_1 \cdots A_{n-1} \rangle\rangle \,,$$
$$\langle\langle Q_{\rm B}(\cdots) \rangle\rangle = \langle\langle \eta(\cdots) \rangle\rangle = 0 \,.$$

（※） In 2 dimension, the WZW action is given by:

$$S = \frac{1}{2g^2}\int d^2z \mathrm{Tr}(\bar{A}_z \bar{A}_{\bar{z}}) + \frac{1}{2g^2}\int d^2z \int_0^1 dt \mathrm{Tr}(A_t[A_z, A_{\bar{z}}]) \,, \qquad (\bar{A}_{z,\bar{z}} \equiv A_{z,\bar{z}}|_{t=1})$$
$$= \frac{1}{g^2}\int d^2z \int_0^1 dt \, \mathrm{Tr}((\partial_z A_t) A_{\bar{z}}), \qquad (A_i \equiv e^{-\Phi}(\partial_i e^{\Phi}) \,, \quad i = t, z, \bar{z}).$$

We obtain Berkovits' super SFT action by an appropriate replacement:

$$\text{product} \quad \rightarrow \quad \text{Witten's} * \text{product},$$
$$\partial_z, \partial_{\bar{z}} \quad \rightarrow \quad \eta_0, Q_B : \text{derivation w.r.t.} * \text{product and nilpotent},$$
$$\int d^2z \mathrm{Tr}(\cdots) \quad \rightarrow \quad \langle\langle \cdots \rangle\rangle : \text{CFT correlator in the } \textit{large} \text{ Hilbert space}.$$

- Variation of the action: $\delta S = \frac{1}{g^2}\langle\langle e^{-\Phi}\delta e^{\Phi}\,\eta_0(e^{-\Phi}Q_{\rm B}e^{\Phi})\rangle\rangle$

- Equation of motion: $\eta_0(e^{-\Phi}Q_{\rm B}e^{\Phi}) = 0$

- Gauge transformation: $\delta e^{\Phi} = Q_{\rm B}\Lambda_0 * e^{\Phi} + e^{\Phi} * \eta_0\Lambda_1$

- Re-expansion of the action around a classical solution Φ_0 :

$$S[\Phi] = S[\Phi_0] + S'[\Phi'] \qquad (e^{\Phi} = e^{\Phi_0}e^{\Phi'})$$

where $S'[\Phi'] = S[\Phi']|_{Q_{\rm B}\to Q'_{\rm B}}$

New BRST operator $Q'_{\rm B}$ is a derivation such as

$$Q'_{\rm B}A = Q_{\rm B}A + e^{-\Phi_0}Q_{\rm B}e^{\Phi_0} * A - (-1)^{|A|}A * e^{-\Phi_0}Q_{\rm B}e^{\Phi_0}$$

which satisfies $Q'^2_{\rm B} = 0,\ \{Q'_{\rm B}, \eta_0\} = 0$

A class of classical solutions

- We find a class of classical solutions to EOM:

$$\Phi_0 = -\tilde{V}_L(F)I$$ where

$$\tilde{V}_L(F) \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i} F(z)\tilde{v}(z), \quad F(-1/z) = z^2 F(z), \quad \tilde{v}(z) \equiv \frac{1}{\sqrt{2}} c\xi e^{-\phi}\psi(z),$$

and $|I\rangle$ is the identity string field.

In fact, we can compute $e^{-\Phi_0} Q_{\rm B} e^{\Phi_0} = -V_L(F)I + \frac{1}{4} C_L(F^2)I$ where

$$V_L(F) \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i} F(z) v(z), \quad v(z) = \frac{i}{2\sqrt{\alpha'}} c\partial X(z) + \frac{1}{\sqrt{2}} \eta e^{\phi}\psi(z), \quad C_L(F^2) \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i} F(z)^2 c(z).$$

⇨ $\eta_0(e^{-\Phi_0} Q_{\rm B} e^{\Phi_0}) = 0$ due to $\eta_0 |I\rangle = 0$

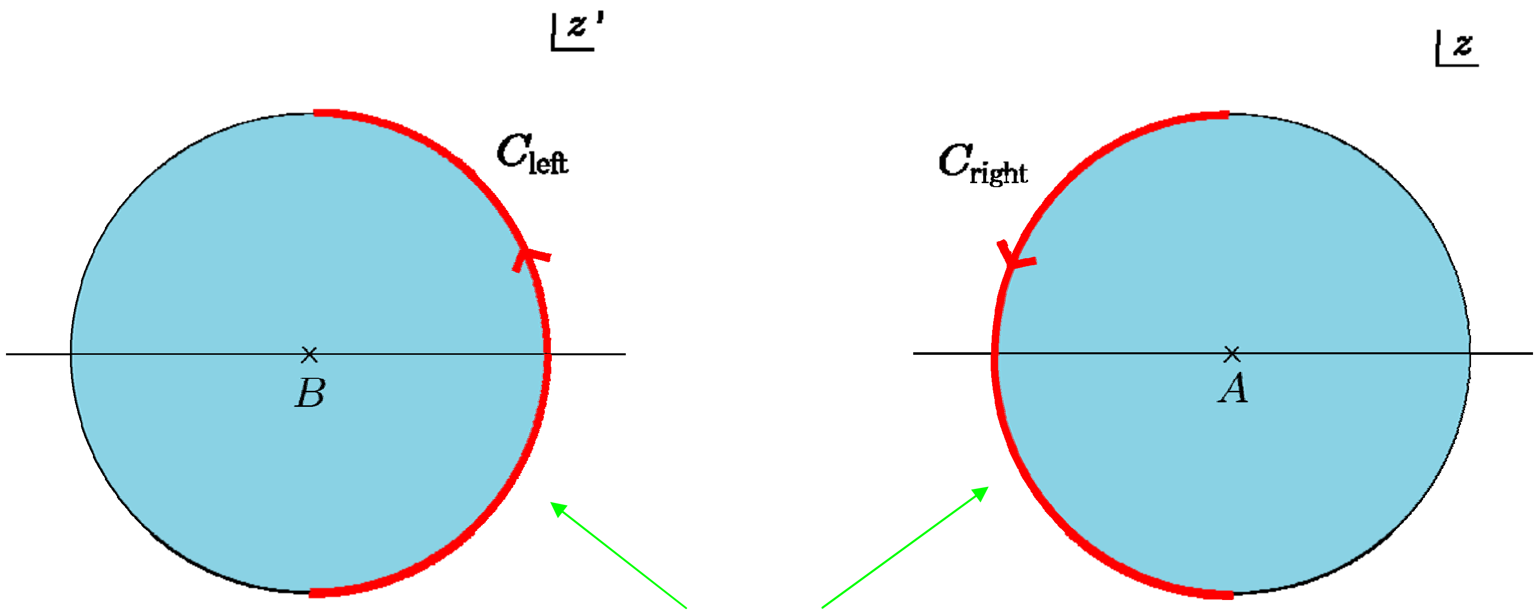
We have used following properties in calculations:

$$\Sigma_R(F)A * B = -(-1)^{|\sigma||A|}A * \Sigma_L(F)B,$$

$$\Sigma_R(F)I = -\Sigma_L(F)I, \quad \Sigma_L(F)I * A = \Sigma_L(F)A,$$ where

$$\Sigma_{L/R}(F) \equiv \int_{C_{\mathrm{left/right}}} \frac{dz}{2\pi i} F(z)\sigma(z), \quad F(-1/z) = z^{2(1-h)}F(z)$$

and $\sigma(z)$ is a primary field with conformal dimension h.



identified by $zz' = -1$ in $A * B$

Properties of our classical solutions

- Vacuum energy vanishes at our solution: $\Phi_0 = -\tilde{V}_L(F)I.$

By replacing F with tF, we have $\eta_0(e^{-t\Phi_0}Q_{\rm B}e^{t\Phi_0}) = 0.$

Therefore we can estimate as

$$S[\Phi_0] = \frac{1}{g^2}\int_0^1 dt \langle\langle \Phi_0\, \eta_0(e^{-t\Phi_0}Q_{\rm B}e^{t\Phi_0})\rangle\rangle = 0.$$

※ We have used Berkovits-Okawa-Zwiebach's expression of the action.

※ This derivation is rather direct than counterpart in *bosonic* SFT.

- $\Phi_0 = -\tilde{V}_L(F)I$ has well-defined oscillator expression in the sense that each coefficient is convergent.

Explicitly, the solution can be expanded as

$$
\begin{aligned}
&|\Phi_0\rangle \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d\sigma}{\sqrt{2}\pi} e^{i\sigma} F(e^{i\sigma}) \Big[c_1 (2\cos\sigma)^{-1} + c_0 i \tan\sigma + c_{-1} \left(1 + (2\cos\sigma)^{-1}\right) \\
&\qquad\qquad + 2 \sum_{m\geq 1} \left(c_{-2m} i \sin 2m\sigma + c_{-(2m+1)} \cos(2m+1)\sigma \right) \Big] \\
&\quad \times \Big[\xi_0 + 2 \sum_{l\geq 1} \left(\xi_{-2l} \cos 2l\sigma + \xi_{-(2l-1)} i \sin(2l-1)\sigma \right) \Big] \\
&\quad \times \exp\Big[\sum_{p\geq 1} \left(\frac{\cos 2p\sigma}{p} j_{-2p} + \frac{2i \sin(2p-1)\sigma}{2p-1} j_{-(2p-1)} \right) \Big] e^{-\hat{\phi}_0} \\
&\quad \times \sum_{k=0}^{\infty} \Big[\psi_{-(2k+\frac{1}{2})} \sum_{q=0}^{k} \frac{(-1)^{k-q} (2(k-q))!}{2^{2(k-q)} ((k-q)!)^2 (2(k-q)-1)} \cos(2q+1)\sigma \\
&\qquad\qquad + \psi_{-(2k+\frac{3}{2})} \sum_{q=0}^{k} \frac{(-1)^{k-q} (2(k-q))!}{2^{2(k-q)} ((k-q)!)^2 (2(k-q)-1)} i \sin 2(q+1)\sigma \Big] |I\rangle \\
&= \frac{1}{\sqrt{2}} \Big(\int_{C_{\text{left}}} \frac{dz}{2\pi i} F(z) c_1 \xi_0 \psi_{-\frac{1}{2}} \\
&\quad + \int_{C_{\text{left}}} \frac{dz}{2\pi i} F(z) (z^{-1} - z) \left((c_0 \xi_0 + c_1 \xi_{-1} + c_1 \xi_0 j_{-1}) \psi_{-\frac{1}{2}} + c_1 \xi_0 \psi_{-\frac{3}{2}} \right) + \cdots \Big) e^{-\hat{\phi}_0} |I\rangle .
\end{aligned}
$$

The divergent from $(\cos\sigma)^{-1}$ at the midpoint is cancelled by $F(-1/z) = z^2 F(z)$.

The identity string field $|I\rangle$ can be expressed using oscillators:

$$|I\rangle = e^{E_{Xbc}+E_{\psi\phi\xi\eta}}|p^\mu = 0, q = 0\rangle$$ where

$$E_{Xbc} = \sum_{n\geq 1} \frac{-(-1)^n}{2n}\alpha^\mu_{-n}\alpha_{-n\mu} + \sum_{n\geq 2}(-1)^n c_{-n}b_{-n} - \sum_{k\geq 1}(-1)^k(2c_0 b_{-2k} + (c_1 - c_{-1})b_{-2k-1})$$

$$E_{\psi\phi\xi\eta} = \sum_{r,s\geq 1/2} \frac{I_{rs}}{2}\psi^\mu_{-r}\psi_{-s\mu} + \sum_{n\geq 1}\frac{(-1)^n}{2n}(j_{-n})^2 - \sum_{k\geq 1}\frac{(-1)^k}{k}j_{-2k} + \sum_{n\geq 1}(-1)^n\eta_{-n}\xi_{-n}$$

$$I_{rs} = \begin{cases} -\frac{r(2s-1)}{r^2-s^2}\left(\frac{-1}{4}\right)^{\frac{r+s}{2}}\frac{(r-\frac{1}{2})!(s-\frac{3}{2})!}{[(\frac{1}{2}(r-\frac{1}{2}))!(\frac{1}{2}(s-\frac{3}{2}))!]^2} & (r-\frac{1}{2}:\mathrm{even}; s-\frac{1}{2}:\mathrm{odd}) \\ -\frac{s(2r-1)}{r^2-s^2}\left(\frac{-1}{4}\right)^{\frac{r+s}{2}}\frac{(r-\frac{3}{2})!(s-\frac{1}{2})!}{[(\frac{1}{2}(r-\frac{3}{2}))!(\frac{1}{2}(s-\frac{1}{2}))!]^2} & (r-\frac{1}{2}:\mathrm{odd}; s-\frac{1}{2}:\mathrm{even}) \\ 0 & (\mathrm{otherwise}) \end{cases},$$

The above Neumann coefficients are computed from $h_I(z) = 2z/(1-z^2)$ as 1-string LPP vertex: $\langle I|A\rangle = \langle h_I[\mathcal{O}_A(0)]\rangle$ in the large Hilbert space.
In particular, $Q_\mathrm{B}|I\rangle = 0$, $\eta_0|I\rangle = 0$.

- New BRST operator around this solution:

$$Q'_{\rm B} = Q_{\rm B} - V_L(F) - V_R(F) + \frac{1}{4}(C_L(F^2) + C_R(F^2))$$
$$= e^{\frac{i}{2\sqrt{\alpha'}}(X_L(F)+X_R(F))} Q_{\rm B}\, e^{-\frac{i}{2\sqrt{\alpha'}}(X_L(F)+X_R(F))},$$

$$X_{L/R}(F) \equiv \int_{C_{\rm left/right}} \frac{dz}{2\pi i} F(z) X(z).$$

Therefore, noting $[X_{\rm L/R}(F), \eta_0] = 0,$ a field redefinition

$$\Phi'' = e^{-\frac{i}{2\sqrt{\alpha'}}(X_L(F)+X_R(F))}\Phi' = e^{-\frac{i}{2\sqrt{\alpha'}}X_L(F)I} * \Phi' * e^{\frac{i}{2\sqrt{\alpha'}}X_L(F)I}$$

reproduces the original action in the sense that

$$S[Q_{\rm B}; \Phi] = S[Q_{\rm B}; \Phi_0] + S'[Q'_B; \Phi'] = S[Q_{\rm B}; \Phi''].$$

where $S[Q_{\rm B}; \Phi_0] = 0$ $\quad (e^{\Phi} = e^{\Phi_0} e^{\Phi'})$

By introducing the Chan-Paton factor i,j, this field redefinition becomes

$$\begin{aligned}\Phi''_{ij} &= e^{-\frac{i}{2\sqrt{\alpha'}}X_L(F_i)I} * \Phi'_{ij} * e^{\frac{i}{2\sqrt{\alpha'}}X_L(F_j)I} \\ &= e^{-\frac{i}{2\sqrt{\alpha'}}(X_L(F_i)+X_R(F_j))}\Phi'_{ij} = e^{-\frac{i}{2\sqrt{\alpha'}}(f_i-f_j)\hat{x}+\cdots}\Phi'_{ij}\end{aligned}$$

where $f_i = \int_{C_{\rm left}} \frac{dz}{2\pi i} F_i(z) = -\int_{C_{\rm right}} \frac{dz}{2\pi i} F_i(z), \quad X(z) = \hat{x} + \cdots .$

Namely, it induces a momentum shift: $p \;\rightarrow\; p - \frac{1}{2\sqrt{\alpha'}}(f_i - f_j).$

This effect is just the same as background Wilson lines.

- Our solution can be rewritten as a *locally* pure gauge form:

$$e^{\Phi_0} = \exp\left\{Q_{\rm B}\left(-\frac{1}{2\sqrt{\alpha'}}\Omega_L(F)I\right)\right\} * \exp\left\{\eta_0\left(-\frac{i}{2\sqrt{\alpha'}}\xi_0 X_L(F)I\right)\right\},$$

$$\Omega_L(F) \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i} F(z)\, i\, c\xi\partial\xi e^{-2\phi}\, X(z),$$

which becomes <u>nontrivial</u> when the direction of X is compactified.

Comment on Ramond sector and supersymmetry

- For string fields (Φ, Ψ) in (NS(+),R(+)) sector, (which have (gh#,pic#)=(0,0),(0,1/2), respectively,) the equations of motion are given by [Berkovits]

$$
\begin{aligned}
f_1 &\equiv \eta_0(e^{-\Phi}Q_{\mathrm{B}}e^{\Phi}) + (\eta_0\Psi)^2 = 0, \\
f_2 &\equiv e^{-\Phi}(Q_{\mathrm{B}}(e^{\Phi}(\eta_0\Psi)e^{-\Phi}))e^{\Phi} = 0\,.
\end{aligned}
$$

Under the gauge transformation

$$
\begin{cases}
\delta e^{\Phi} = e^{\Phi}(\eta_0\Lambda_1 - \{\eta_0\Psi, \Lambda_{\frac{1}{2}}\}) + (Q_{\mathrm{B}}\Lambda_0)e^{\Phi}\,, \\
\delta\Psi = \eta_0\Lambda_{\frac{3}{2}} + [\Psi, \eta_0\Lambda_1] + Q_{\mathrm{B}}\Lambda_{\frac{1}{2}} + \{e^{-\Phi}Q_{\mathrm{B}}e^{\Phi}, \Lambda_{\frac{1}{2}}\}\,,
\end{cases}
$$

the equations of motion transform covariantly:

$$
\begin{aligned}
\delta f_1 &= [f_1, \eta_0\Lambda_1] - \eta_0[f_2, \Lambda_{\frac{1}{2}}]\,, \\
\delta f_2 &= [f_1, Q_{\mathrm{B}}\Lambda_{\frac{1}{2}} + \{e^{-\Phi}Q_{\mathrm{B}}e^{\Phi}, \Lambda_{\frac{1}{2}}\}] + [f_2, \eta_0\Lambda_1] - \{[f_2, \Lambda_{\frac{1}{2}}], \eta_0\Psi\}\,.
\end{aligned}
$$

Λ_P : gauge parameter with pic# P.

Let us consider a particular parameter in the above. $\Lambda_{\frac{1}{2}} = \epsilon_\alpha \int_{C_{\rm left}} \frac{dz}{2\pi i} \xi S^\alpha_{(-1/2)}(z) I$

(This is an analogy with counterpart in Witten's *cubic* super SFT which has space-time SUSY at least formally.)

Then, we *define* a global space-time SUSY transformation:

$$\delta_\epsilon e^\Phi = -e^\Phi \mathcal{S}(\epsilon)\eta_0\Psi, \quad \delta_\epsilon(\eta_0\Psi) = \eta_0\mathcal{S}(\epsilon)(e^{-\Phi}Q_{\rm B}e^\Phi),$$

$$\mathcal{S}(\epsilon) \equiv \epsilon_\alpha \oint \frac{dz}{2\pi i}\xi S^\alpha_{(-1/2)}(z).$$

The equations of motion transform as

$$\begin{aligned}\delta_\epsilon f_1 &= \eta_0\mathcal{S}(\epsilon)f_2\,,\\ \delta_\epsilon f_2 &= -\{Q_{\rm B},\mathcal{S}(\epsilon)\}f_1 + [f_1,\mathcal{S}(\epsilon)(e^{-\Phi}Q_{\rm B}e^\Phi)] + \{\mathcal{S}(\epsilon)f_2,\eta_0\Psi\},\end{aligned}$$

which preserve EOMs: $(f_1,f_2)=(0,0) \quad \Rightarrow \quad (\delta_\epsilon f_1, \delta_\epsilon f_2) = (0,0).$

⇩

Our solution $(\Phi,\Psi) = (-\tilde{V}_L(F)I, 0)$ is invariant under this transformation.

Note: δ_ϵ reproduces usual SUSY transformation in 10d super Yang-Mills theory at linearized level and on-shell.

Concretely, for massless fields, we expand string fields as

$$|\Phi_A\rangle = \int \frac{d^{10}p}{(2\pi)^{10}}(\tilde{A}_\mu(p)c\xi e^{-\phi}\psi^\mu(0) + \tilde{B}(p)c\partial c\xi\partial\xi e^{-2\phi}(0))|p^\mu, q=0\rangle,$$
$$|\Psi_\lambda\rangle = \int \frac{d^{10}p}{(2\pi)^{10}}\tilde{\lambda}_\alpha(p)\xi S^\alpha_{(-1/2)}c(0)|p^\mu, q=0\rangle\,,$$

and we have calculated $\delta_\epsilon|\Phi_A\rangle,\ \delta_\epsilon(\eta_0|\Psi_\lambda\rangle)$ up to linear terms.

Using linearized equation of motion:

$$Q_{\rm B}\eta_0|\Phi_A\rangle = 0, \quad Q_{\rm B}\eta_0|\Psi_\lambda\rangle = 0,$$

we have obtained a transformation for component fields:

$$\delta_\epsilon A_\mu = -i\epsilon\Gamma_\mu C\lambda, \quad \delta_\epsilon\lambda = \frac{i}{2}\sqrt{\frac{\alpha'}{2}}F_{\mu\nu}(\epsilon\Gamma^{\mu\nu}), \quad (F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu).$$

Generalization

- In the construction of our solutions, we have used U(1) supercurrent:

$$\mathbf{J}(z,\theta) = \psi(z) + \theta \frac{i}{\sqrt{2\alpha'}} \partial X(z).$$

⇓

It can be generalized to supercurrent associated with G

$$\mathbf{J}^a(z,\theta) = \psi^a(z) + \theta J^a(z) \qquad (a = 1, \cdots, \mathrm{dim}G)$$

such as

$$\psi^a(y)\psi^b(z) \sim \frac{1}{y-z}\frac{1}{2}\Omega^{ab},$$
$$J^a(y)\psi^b(z) \sim \frac{1}{y-z} f^{ab}{}_c \psi^c(z),$$
$$J^a(y)J^b(z) \sim \frac{1}{(y-z)^2}\frac{1}{2}\Omega^{ab} + \frac{1}{y-z} f^{ab}{}_c J^c(z),$$

where

$$f^{ab}{}_c = -f^{ba}{}_c, \qquad f^{ab}{}_d f^{cd}{}_e + f^{bc}{}_d f^{ad}{}_e + f^{ca}{}_d f^{bd}{}_e = 0,$$
$$\Omega^{ab} = \Omega^{ba}, \qquad f^{ab}{}_c \Omega^{cd} + f^{ad}{}_c \Omega^{cb} = 0.$$

Suppose $\exists\ \Omega_{ab}$ such as $\Omega^{ac}\Omega_{cb} = \delta^a_b$. Then, matter super Virasoro operators are given by Sugawara construction:

$$T^{\rm m}(z) \;=\; \Omega_{ab}\!:\!(J^aJ^b+\partial\psi^a\psi^b)\!:\!(z) + \frac{2}{3}\Omega_{ad}\Omega_{be}f^{de}{}_c\!:\!(J^a\!:\!\psi^b\psi^c\!:+\psi^a\!:(\psi^bJ^c - J^b\psi^c)\!:)\!:\!(z),$$

$$G^{\rm m}(z) \;=\; 2\Omega_{ab}:\!J^a\psi^b\!:\!(z) + \frac{4}{3}\Omega_{ad}\Omega_{be}f^{de}{}_c\!:\!\psi^a\!:\!\psi^b\psi^c\!::\!(z)\,,$$

where the central charge is $c^{\rm m} = \frac{3}{2}{\rm dim}G - f^{ac}{}_d f^{bd}{}_c \Omega_{ab}.$ [Mohammedi(1994)]

We suppose $c^{\rm m} = 15$ for super SFT.

In this case, we have similarly confirmed that

$$\Phi_0 = -\tilde{V}^a_L(F_a)I\,,$$

$$\tilde{V}^a_L(F_a) \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i}F_a(z)\tilde{v}^a(z), \quad F_a(-1/z) = z^2 F_a(z)\,,$$

$$\tilde{v}^a(z) \equiv \frac{1}{\sqrt{2}}c\xi e^{-\phi}\psi^a(z)\,,$$

satisfies equation of motion: $\eta_0(e^{-\Phi_0}Q_{\rm B}e^{\Phi_0}) = 0.$

It corresponds to a marginal deformation by J^a.

- Inclusion of GSO(−) sector

Super SFT on a non-BPS brane [Berkovits,Berkovits-Sen-Zwiebach(2000)]

$$S[\hat{\Phi}] = -\frac{1}{2g^2}\int_0^1 dt\, \mathrm{Tr}\langle\langle(\hat{\eta}_0\hat{\Phi})(e^{-t\hat{\Phi}}\hat{Q}_{\mathrm{B}}e^{t\hat{\Phi}})\rangle\rangle\,,$$

$$\hat{Q}_{\mathrm{B}} = Q_{\mathrm{B}}\otimes\sigma_3, \qquad \hat{\eta}_0 = \eta_0\otimes\sigma_3\,,$$

$$\hat{\Phi} = \Phi_+\otimes 1 + \Phi_-\otimes\sigma_1\,,$$

where $\Phi_+ : \mathrm{GSO}(+),\ \ \Phi_- : \mathrm{GSO}(-).$

(※) Algebraic property is almost the same.

Equation of motion: $\hat{\eta}_0(e^{-\hat{\Phi}}\hat{Q}_{\mathrm{B}}e^{\hat{\Phi}}) = 0\,.$

The same form as that in super SFT on BPS (GSO-projected) brane!

⇓

We can also construct marginal solutions in the GSO（－）sector if there exists a supercurrent with GSO（－）components.

A class of GSO(−) solutions

Let us compactify X^9 direction to S^1 with the critical radius $R=\sqrt{2\alpha'}$.

Then, we find an SU(2) supercurrent $\mathbf{J}^a(z,\theta)=\psi^a(z)+\theta J^a(z)$ as

$$
\begin{aligned}
\mathrm{J}^1(z,\theta) &= \sqrt{2}\sin\left(\frac{X^9}{\sqrt{2\alpha'}}\right)(z)\otimes\sigma_2+\theta(-\sqrt{2})\psi^9\cos\left(\frac{X^9}{\sqrt{2\alpha'}}\right)(z)\otimes\sigma_1\,,\\
\mathrm{J}^2(z,\theta) &= \sqrt{2}\cos\left(\frac{X^9}{\sqrt{2\alpha'}}\right)(z)\otimes\sigma_2+\theta\sqrt{2}\psi^9\sin\left(\frac{X^9}{\sqrt{2\alpha'}}\right)(z)\otimes\sigma_1\,,\\
\mathrm{J}^3(z,\theta) &= \psi^9(z)\otimes\sigma_3+\theta\frac{i}{\sqrt{2\alpha'}}\partial X^9(z)\otimes 1\,.
\end{aligned}
$$

Note: $e^{in\frac{X^9}{\sqrt{2\alpha'}}}$ (n :odd) should be treated as “fermion.”

We have assigned cocycle factors (Pauli matrices) to each component appropriately.

The above is an analogy with the SU(2) current in bosonic string theory:

$$
J^1=\sqrt{2}\cos\left(\frac{X^{25}}{\sqrt{\alpha'}}\right),\quad J^2=\sqrt{2}\sin\left(\frac{X^{25}}{\sqrt{\alpha'}}\right),\quad J^3=\frac{i}{\sqrt{2\alpha'}}\partial X^{25}\quad (R=\sqrt{\alpha'})
$$

Actually, we can check the SU(2) supercurrent algebra with

$\Omega^{ab} = 2\delta_{a,b}, \quad f^{ab}_{\ \ c} = -i\epsilon_{abc}$ and we have

$$\begin{aligned} T^9(z) &= \frac{1}{2} : (J^a J^a + \partial\psi^a\psi^a) : (z) - \frac{i}{6}\epsilon_{abc} : (J^a : \psi^b\psi^c : + \psi^a : (\psi^b J^c - J^b\psi^c)) : (z) \\ &= \left(-\frac{1}{4\alpha'}(\partial X^9)^2(z) - \frac{1}{2}\psi^9\partial\psi^9(z)\right) \otimes 1\,, \\ G^9(z) &= : J^a\psi^a : (z) - \frac{i}{3}\epsilon_{abc} : \psi^a : \psi^b\psi^c :: (z) = \frac{i}{\sqrt{2\alpha'}}\psi^9\partial X^9(z) \otimes \sigma_3\,. \end{aligned}$$

which satisfies super Virasoro algebra with $c = \frac{3}{2}$.

Then, we can take the BRST operator $\hat{Q}_{\mathrm{B}}$ as

$$\begin{aligned} &\hat{Q}_{\mathrm{B}} = Q_{\mathrm{B}}\sigma_3 \\ &= \oint \frac{dz}{2\pi i}(c\sigma_3(T^{\mathrm{m}} - \frac{1}{2}(\partial\phi)^2 - \partial^2\phi + \partial\xi\eta) + bc\partial c\sigma_3 + \eta e^{\phi}G^{\mathrm{m}} - \eta\partial\eta e^{2\phi}b\sigma_3) \end{aligned}$$

where $T^{\mathrm{m}} = \sum_{\mu=0}^{9}\left(-\frac{1}{4\alpha'}\partial X_\mu\partial X^\mu - \frac{1}{2}\psi^\mu\partial\psi_\mu\right), \quad G^{\mathrm{m}} = \sum_{\mu=0}^{9}\frac{i}{\sqrt{2\alpha'}}\psi^\mu\sigma_3\partial X^\mu$

We can construct a solution to EOM $\hat{\eta}_0(e^{-\hat{\Phi}}\hat{Q}_{\rm B}e^{\hat{\Phi}}) = 0$:

$$\hat{\Phi}_0 = -\tilde{V}_L^a(F_a)I\,,$$

$$\tilde{V}_L^a(F_a) \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i} F_a(z)\tilde{v}^a(z), \quad F_a(-1/z) = z^2 F_a(z)\,,$$

$$\tilde{v}^a(z) \equiv \frac{1}{\sqrt{2}}(c\xi e^{-\phi}\otimes\sigma_3)\,\psi^a(z)\,, \quad a = 1, 2, 3.$$

Around this solution, new BRST operator is

$$\hat{Q}'_{\rm B}\hat{A}$$
$$= \hat{Q}_{\rm B}\hat{A} + \left[\left(-V_L^a(F_a) + \frac{1}{4}C_L(F_aF_a)\right)I\right] * \hat{A} - (-1)^{{\rm gh}(\hat{A})}\hat{A} * \left[\left(-V_L^a(F_a) + \frac{1}{4}C_L(F_aF_a)\right)I\right]$$
$$= \left((Q_{\rm B} + \frac{1}{4}C(F_aF_a))\sigma_3 - V^3(F_3) - V_L^1(F_1) - V_L^2(F_2) - (-1)^{\hat{F}+\hat{n}}(V_R^1(F_1) + V_R^2(F_2))\right)\hat{A},$$

where

$$V^a_{L/R}(F) = \int \frac{dz}{2\pi i} F(z) v^a(z),$$
$$v^a(z) \equiv [\hat{Q}_{\rm B}, \tilde{v}^a(z)] = \frac{1}{\sqrt{2}} c\sigma_3 J^a(z) + \frac{1}{\sqrt{2}}\eta e^{\phi}\psi^a(z),$$
$$a = 1, 2, 3.$$

$$\hat{n} = \oint \frac{dz}{2\pi i} \frac{i}{\sqrt{2\alpha'}} \partial X^9(z)$$: momentum along the X^9 direction.

$(-1)^{\hat{F}}$: GSO($\pm$) which is given by

$$\hat{F} = \oint \frac{dz}{2\pi i} \left(\sum_{k=1}^{5} :\psi_+^k \psi_-^k:(z) - \partial\phi(z) \right),$$

$$\psi_\pm^1 \equiv \frac{i}{\sqrt{2}}(\psi^0 \pm \psi^1), \quad \psi_\pm^k \equiv \frac{1}{\sqrt{2}}(\psi^{2k-2} \pm i\psi^{2k-1}), \quad k = 2,3,4,5.$$

We can discuss physics around the solution $\hat{\Phi}_0$
by investigating the obtained new BRST operator: $\hat{Q}'_{\rm B}$.

In particular, let us consider a solution given by $F_a(z) = \delta_a^1 F(z)$
and $\tilde{v}^1(z) = -ic\xi e^{-\phi} \sin\left(\frac{X^9}{\sqrt{2\alpha'}}\right)(z) \otimes \sigma_1$ in the following.

Technically, we use fermionization and rebosonization method after Sen's argument in the context of CFT. Namely,

$$e^{\pm\frac{i}{\sqrt{2\alpha'}}X^9(z)} = \frac{1}{\sqrt{2}}(\xi^9(z) \pm i\eta^9(z)) \otimes \tau_1 \quad : \ (\psi^9, X^9) \to (\psi^9, \xi^9, \eta^9)$$

$$\xi^9(z) \pm i\psi^9(z) = \sqrt{2}e^{\pm\frac{i}{\sqrt{2\alpha'}}\phi^9(z)} \otimes \tilde{\tau}_1 \quad : \ (\psi^9, \xi^9, \eta^9) \to (\phi^9, \eta^9)$$

where we introduce Pauli matrices $\tau_i, \tilde{\tau}_i \ (i = 1, 2, 3)$ as cocycle factors.

Then, the new BRST operator can be rewritten as

$$\begin{aligned} \hat{Q}'_{\rm B} &= (Q_{\rm B} + \frac{1}{4}C(F^2))\sigma_3 - V_L^1(F) - (-1)^{\hat{F}+\hat{n}}V_R^1(F) \\ &= \begin{cases} e^{-\frac{i}{2\sqrt{\alpha'}}(\phi_L^9(F)+\phi_R^9(F))\sigma_1\tau_2}\hat{Q}_{\rm B}\, e^{\frac{i}{2\sqrt{\alpha'}}(\phi_L^9(F)+\phi_R^9(F))\sigma_1\tau_2} \\ \qquad \text{on} \quad (-1)^{\hat{F}+\hat{n}} = +1 \\ e^{-\frac{i}{2\sqrt{\alpha'}}(\phi_L^9(F)-\phi_R^9(F))\sigma_1\tau_2}\hat{Q}_{\rm B}\, e^{\frac{i}{2\sqrt{\alpha'}}(\phi_L^9(F)-\phi_R^9(F))\sigma_1\tau_2} \\ \qquad \text{on} \quad (-1)^{\hat{F}+\hat{n}} = -1 \end{cases} \end{aligned}$$

where $\quad \phi^9_{L/R}(F) \equiv \int_{C_{\rm left/right}} \frac{dz}{2\pi i}F(z)\phi^9(z).$

This expression implies that our solution induces a string field redefinition:

$$\hat{\Phi}'' = e^{\frac{i}{2\sqrt{\alpha'}}\phi_L^9(F)I\sigma_1\tau_2} * \hat{\Phi}' * e^{-\frac{i}{2\sqrt{\alpha'}}\phi_L^9(F)I\sigma_1\tau_2}$$

$$= \begin{cases} e^{\frac{i}{2\sqrt{\alpha'}}(\phi_L^9(F)+\phi_R^9(F))\sigma_1\tau_2}\hat{\Phi}' & \text{on} \quad (-1)^{\hat{F}+\hat{n}} = +1 \\ e^{\frac{i}{2\sqrt{\alpha'}}(\phi_L^9(F)-\phi_R^9(F))\sigma_1\tau_2}\hat{\Phi}' & \text{on} \quad (-1)^{\hat{F}+\hat{n}} = -1 \end{cases}$$

in the sense that the action can be rewritten as

$$S[\hat{Q}_{\rm B};\hat{\Phi}] = S[\hat{Q}_{\rm B};\hat{\Phi}_0] + S[\hat{Q}'_{\rm B};\hat{\Phi}'] = S[\hat{Q}_{\rm B};\hat{\Phi}''].$$

$$\underset{0}{\shortparallel} \quad (\, e^{\hat{\Phi}} = e^{\hat{\Phi}_0}e^{\hat{\Phi}'} \,)$$

Due to $\int_{C_{\rm left}}\frac{dz}{2\pi i}F(z) + \int_{C_{\rm right}}\frac{dz}{2\pi i}F(z) = 0$ by construction of the solution: $F(-1/z) = z^2F(z)$

$\phi_L^9(F)+\phi_R^9(F)$ does not include $\hat{\phi}_0^9$: zero mode of ϕ^9.

On the other hand, $\phi_L^9(F)-\phi_R^9(F) = 2f\hat{\phi}_0^9 + \cdots$ where $f \equiv \int_{C_{\rm left}}\frac{dz}{2\pi i}F(z)$.

⇩

ϕ^9 momentum changes by $\pm\frac{f}{\sqrt{\alpha'}}$ in $(-1)^{\hat{F}+\hat{n}} = -1$ sector by the field redefinition.

- Critical value of f

In the case of $f = \frac{2m+1}{\sqrt{2}}, \quad (m \in \mathrm{Z}),$ all states in $(-1)^{\hat{F}+\hat{n}} = -1$ sector changes to $(-1)^{\hat{F}+\hat{n}} = +1$ and all states in $(-1)^{\hat{F}+\hat{n}} = +1$ remain

because $(-1)^{\hat{F}+\hat{n}} \partial\phi^9(z)(-1)^{-(\hat{F}+\hat{n})} = +\partial\phi^9(z)\,,$

$$(-1)^{\hat{F}+\hat{n}} e^{i\frac{2m+1}{\sqrt{2\alpha'}}\hat{\phi}_0^9}(-1)^{-(\hat{F}+\hat{n})} \;=\; -\,e^{i\frac{2m+1}{\sqrt{2\alpha'}}\hat{\phi}_0^9}\,.$$

Furthermore, the redefined string field has the following structure:

$$\hat{\Phi}'' = \Psi_+^{\mathrm{e}} \otimes 1 \otimes 1 \otimes 1 + \Psi_+'^{\mathrm{e}} \otimes 1 \otimes \tau_1 \otimes \tilde{\tau}_1 + \Psi_-^{\mathrm{o}} \otimes \sigma_1 \otimes \tau_2 \otimes \tilde{\tau}_1 + \Psi_-'^{\mathrm{o}} \otimes \sigma_1 \otimes \tau_3 \otimes 1$$

where superscript e/o denotes $(-1)^{\hat{n}}$ and subscript ± denotes $(-1)^{\hat{F}}$.

And in this expression we should represent the derivations as

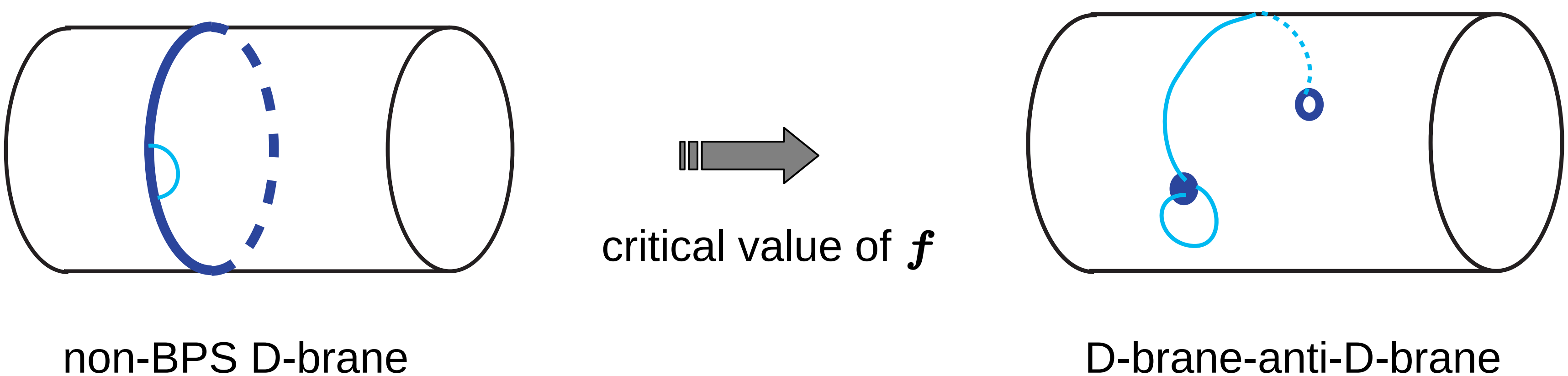
$$\hat{Q}_{\mathrm{B}} = Q_{\mathrm{B}} \otimes \sigma_3 \otimes \tau_3 \otimes \tilde{\tau}_3, \quad \hat{\eta}_0 = \eta_0 \otimes \sigma_3 \otimes \tau_3 \otimes \tilde{\tau}_3.$$

This redefined action $S[\hat{Q}_{\rm B};\hat{\Phi}'']$ has the same structure as

$$\hat{Q}_{\rm B}=Q_{\rm B}\otimes\sigma_3\otimes 1,\quad \hat{\eta}_0=\eta_0\otimes\sigma_3\otimes 1,$$

$$\hat{\Phi}''=\Psi^{\rm e}_{+}\otimes 1\otimes 1+\Psi'^{\rm e}_{+}\otimes 1\otimes\tau_3+\Psi^{\rm o}_{-}\otimes\sigma_1\otimes\tau_1+\Psi'^{\rm o}_{-}\otimes\sigma_1\otimes\tau_2.$$

If we regard σ_i / τ_i as internal / external CP factor, and take T-dual picture (momentum ⟷ winding), this action represents super SFT on a D-brane-anti-D-brane system, in which a D-brane and an anti-D-brane are situated at antipodal points along the circle.



This picture is consistent with Sen's statement (1998) using boundary CFT!

Summary and Discussion

- We have constructed a class of exact classical solutions to Berkovits' super SFT, which have vanishing vacuum energy.
- We find that our solution represents background Wilson line (including Ramond sector).
- We have identified "global space-time SUSY transformaion'' in Berkovits' super SFT and found that our solution is invariant under it.
- We have also construct a class of solutions by supercurrents generally, which correspond to marginal deformations in conformal field theory.
- GSO(−) solutions can be similarly constructed at the critical radius using an SU(2) supercurrent. At the critical value of f of the solution, it represents a process: non-BPS → D-anti-D.

- Our solution corresponds to a super extension of "marginal solution" [Takahashi-Tanimoto(2001)] in bosonic SFT.

$$\Psi_0 = -V_L^a(F_a)I - \frac{1}{4}g^{ab}C_L(F_aF_b)I, \quad V_L^a(f) \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i}\frac{1}{\sqrt{2}}f(z)cJ^a(z)$$

$$\Downarrow$$

$$e^{-\Phi_0}Q_{\rm B}e^{\Phi_0} = -V_L^a(F_a)I + \frac{1}{8}\Omega^{ab}C_L(F_aF_b)I,$$

$$V_L^a(f) \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i}\frac{1}{\sqrt{2}}f(z)(cJ^a + \eta e^{\phi}\psi^a)(z)$$

- Can we construct an exact *universal* solution to super SFT which corresponds to tachyon condensation on non-BPS D9 brane?
- Can we evaluate potential height *if* we obtain a universal solution such as bosonic SFT?
- Can we construct other type of exact solutions? (For example, super version of Schnabl's method?)
- …