

Twist Field as Three String Interaction Vertex in Light Cone Superstring Field Theory

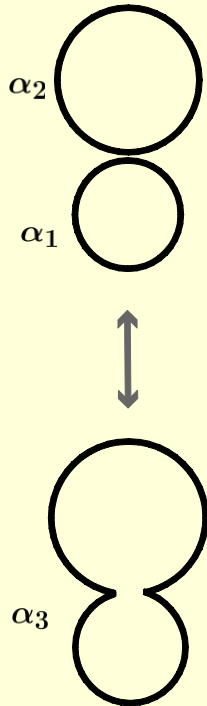
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collaboration with **S. Moriyama, S. Teraguchi**
KMT, NPB744(2006)221; KM, hep-th/061

I. Introduction and motivation

LCSFT and MST seem to be closely related in a sense. We concentrate on their interactions.



Green-Schwarz-Brink's
light-cone superstring field
theory (LCSFT)

3-string vertex

Given by oscillators explicitly:

$$|H_1\rangle = \tilde{Z}^i Z^j v^{ij} (Y) |v_3\rangle$$

$$|Q_1^{\dot{a}}\rangle = \tilde{Z}^i s^{i\dot{a}} (Y) |v_3\rangle,$$

$$|\tilde{Q}_1^{\dot{a}}\rangle = Z^i \tilde{s}^{i\dot{a}} (Y) |v_3\rangle.$$

$$\partial X^i(\sigma) |v_3\rangle \sim |\sigma - \sigma_{\text{int}}|^{-\frac{1}{2}} Z^i |v_3\rangle,$$

$$\bar{\partial} X^i(\sigma) |v_3\rangle \sim |\sigma - \sigma_{\text{int}}|^{-\frac{1}{2}} \tilde{Z}^i |v_3\rangle,$$

$$\lambda^a(\sigma) |v_3\rangle \sim |\sigma - \sigma_{\text{int}}|^{-\frac{1}{2}} Y^a |v_3\rangle.$$

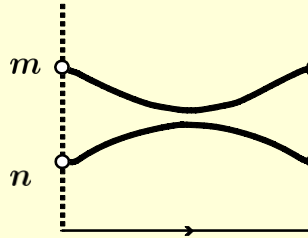
Joining/splitting interaction

With an identification: $|v_3\rangle \leftrightarrow \sigma \tilde{\sigma}$, $|R\rangle \leftrightarrow 1$,

in the bosonic sector, we expect the OPE correspond to

$$\langle R | v_3 \rangle |v_3\rangle \sim (\dots) |R\rangle |R\rangle,$$

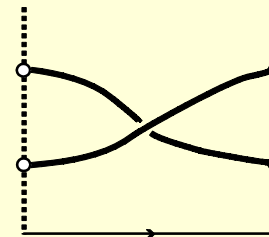
$$\langle R | \langle R | v_3 \rangle |v_3\rangle \sim (\dots) |R\rangle.$$



Dijkgraaf-Verlinde-Verlinde's
matrix string theory (MST)

twist/spin field

$$(\tau^i \Sigma^i \tilde{\tau}^j \tilde{\Sigma}^j)_{mn}$$



Exchange of eigen values

OPEs:

$$\partial X^i(z) \sigma \tilde{\sigma}(0) \sim z^{-\frac{1}{2}} \tau^i \tilde{\sigma}(0),$$

$$\bar{\partial} X^i(\bar{z}) \sigma \tilde{\sigma}(0) \sim \bar{z}^{-\frac{1}{2}} \sigma \tilde{\tau}^i(0),$$

$$\theta^a(z) \Sigma^i(0) \sim z^{-\frac{1}{2}} \frac{1}{\sqrt{2i}} \gamma_{a\dot{a}}^i \Sigma^{\dot{a}}(0),$$

$$\Sigma^i(z) \Sigma^j(0) \sim z^{-1} \delta^{ij},$$

$$\Sigma^{\dot{a}}(z) \Sigma^{\dot{b}}(0) \sim z^{-1} \delta^{\dot{a}\dot{b}},$$

$$\Sigma^i(z) \Sigma^{\dot{a}}(0) \sim z^{-\frac{1}{2}} \frac{1}{\sqrt{2i}} \gamma_{c\dot{a}}^i \theta^c(0), \dots$$

$$\leftarrow \text{?} \rightarrow \sigma \tilde{\sigma}(z, \bar{z}) \cdot \sigma \tilde{\sigma}(0) \sim \left[\frac{1}{|z|^{1/4} (\ln |z|)^{1/2}} \right]^{d-2}$$

Investigations of the correspondence will be useful for developing both LCSFT and MST.

II. LCSFT/MST correspondence

More precise correspondence at least at the linear level with respect to $|v_3\rangle$: [Dijkgraaf-Mott]

$$\begin{array}{ll}
 \text{bosonic sector} & \begin{array}{l} \tilde{Z}^i Z^j |v_3\rangle \leftrightarrow \tau^j \tilde{\tau}^i \\ Z^i |v_3\rangle \leftrightarrow \tau^i \tilde{\sigma} \\ \tilde{Z}^i |v_3\rangle \leftrightarrow \sigma \tilde{\tau}^i \end{array} \\
 \text{ferminic sector} & \begin{array}{l} v^{ij}(Y) |v_3\rangle \leftrightarrow \Sigma^j \tilde{\Sigma}^i \\ s^{i\dot{a}}(Y) |v_3\rangle \leftrightarrow \Sigma^{\dot{a}} \tilde{\Sigma}^i \\ \tilde{s}^{i\dot{a}}(Y) |v_3\rangle \leftrightarrow \Sigma^i \tilde{\Sigma}^{\dot{a}} \\ m^{\dot{a}\dot{b}}(Y) |v_3\rangle \leftrightarrow \Sigma^{\dot{a}} \tilde{\Sigma}^{\dot{b}} \end{array}
 \end{array}$$

$$\begin{aligned}
 |v_3\rangle &= (2\pi)^9 \delta(\alpha_1 + \alpha_2 + \alpha_3) \delta^8(p_1^i + p_2^i + p_3^i) \delta^8(\lambda_1^a + \lambda_2^a + \lambda_3^a) \\
 &\times e^{\frac{1}{2}(a^{(r)\dagger} \tilde{N}^{rs} a^{(s)\dagger} + \tilde{a}^{(r)\dagger} \tilde{N}^{rs} \tilde{a}^{(s)\dagger}) + \tilde{N}^r (a^{(r)\dagger} + \tilde{a}^{(r)\dagger}) P - \frac{\tau_0}{\alpha_{123}} P^2} \\
 &\times e^{\sum Q_{-n}^{\text{II}(r)} \alpha_r^{-1} n^{\frac{1}{2}} \tilde{N}_{nm}^{rs} m^{-\frac{1}{2}} Q_{-m}^{\text{I}(s)} - \sqrt{2} \Lambda \sum \alpha_r^{-1} n^{\frac{1}{2}} \tilde{N}_n^r Q_{-n}^{\text{II}(r)}} |0\rangle. \\
 \tilde{Z}^i &= P^i - \alpha_{123} \sum \alpha_r^{-1} n \tilde{N}_n^r \tilde{a}_n^{(r)\dagger i}, \\
 Z^j &= P^j - \alpha_{123} \sum \alpha_r^{-1} n \tilde{N}_n^r a_n^{(r)\dagger j}, \\
 Y^a &= \Lambda^a - \frac{\alpha_{123}}{\sqrt{2}} \sum \alpha_r^{-1} n^{\frac{1}{2}} \tilde{N}_n^r Q_{-n}^{\text{I}(r)a}. \\
 \alpha_{123} &= \alpha_1 \alpha_2 \alpha_3, \quad P^i = \alpha_1 p_2^i - \alpha_2 p_1^i, \quad \Lambda^a = \alpha_1 \lambda_2^a - \alpha_2 \lambda_1^a.
 \end{aligned}$$

We found a concise expression of the prefactors: [KM]

$$\begin{aligned}
 e^{\mathcal{Y}} &= [e^{\mathcal{Y}}]_{(i,\dot{a}),(j,\dot{b})} = \begin{pmatrix} [\cosh \mathcal{Y}]_{ij} & [\sinh \mathcal{Y}]_{i\dot{b}} \\ [\sinh \mathcal{Y}]_{\dot{a}j} & [\cosh \mathcal{Y}]_{\dot{a}\dot{b}} \end{pmatrix} = \begin{pmatrix} v^{ij}(Y) & i(-\alpha_{123})^{-\frac{1}{2}} s^{i\dot{b}}(Y) \\ (-\alpha_{123})^{-\frac{1}{2}} \tilde{s}^{j\dot{a}}(Y) & m^{\dot{b}\dot{a}}(Y) \end{pmatrix}, \\
 \mathcal{Y} &\equiv \left(\frac{2i}{-\alpha_{123}} \right)^{\frac{1}{2}} Y^a \hat{\gamma}^a, \quad \hat{\gamma}^a = (\hat{\gamma}^a)_{(i,\dot{a}),(j,\dot{b})} = \begin{pmatrix} 0 & \gamma_{\dot{a}\dot{b}}^i \\ \gamma_{\dot{a}\dot{b}}^j & 0 \end{pmatrix}, \quad \hat{\gamma}^a \hat{\gamma}^b + \hat{\gamma}^b \hat{\gamma}^a = 2\delta^{ab} \mathbf{1}_{16}.
 \end{aligned}$$

With this prefactor, the space-time SUSY algebra is satisfied at the linear level:

$$Q_0^{\dot{a}} |Q_1^{\dot{b}}\rangle + Q_0^{\dot{b}} |Q_1^{\dot{a}}\rangle = \tilde{Q}_0^{\dot{a}} |\tilde{Q}_1^{\dot{b}}\rangle + \tilde{Q}_0^{\dot{b}} |\tilde{Q}_1^{\dot{a}}\rangle = 2|H_1\rangle \delta^{\dot{a}\dot{b}}, \quad Q_0^{\dot{a}} \mathcal{P} |\tilde{Q}_1^{\dot{b}}\rangle + \tilde{Q}_0^{\dot{b}} \mathcal{P} |Q_1^{\dot{a}}\rangle = 0.$$

What about the correspondence at the quadratic level with respect to $|v_3\rangle$?

III. Contractions in bosonic LCSFT [KMT]

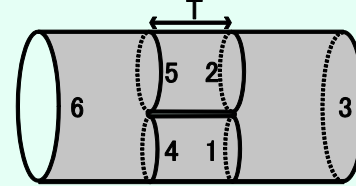
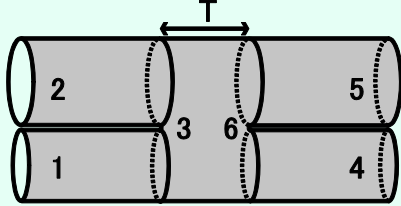
At the quadratic level, we have explicitly computed two types of contractions.

$$\langle \mathbf{R} | v_3 \rangle | v_3 \rangle \propto | \mathbf{R} \rangle | \mathbf{R} \rangle$$

$$\langle \mathbf{R} | \langle \mathbf{R} | v_3 \rangle | v_3 \rangle \propto | \mathbf{R} \rangle$$

can be proved by $\sum_{l,t} \tilde{N}_{nl}^{rt} \tilde{N}_{lm}^{ts} = \delta^{nm} \delta^{rs}$, $\sum_{l,t} \tilde{N}_{nl}^{rt} \tilde{N}_l^t = -\tilde{N}_n^r$, $\sum_{l,t} \tilde{N}_l^t \tilde{N}_l^t = (\alpha_{123})^{-1} 2\tau_0$.

In order to evaluate the divergent coefficients, we regularize them by T :



Using the Cremmer-Gervais identity, we evaluated as

$$\left| \det^{-\frac{d-2}{2}} (1 - \tilde{N}_{T/2}^{33} \tilde{N}_{T/2}^{33}) \right|^2 \sim T^{-\frac{d-2}{4}}$$

$$e^{-\alpha_3^2 \tilde{N}_{T/2}^3 (1 - \tilde{N}_{T/2}^{33} \tilde{N}_{T/2}^{33})^{-1} \tilde{N}_{T/2}^3 (p_1 + p_4)^2} \sim (\log T)^{-\frac{d-2}{2}} \delta^{d-2} (p_1 + p_4)$$

We computed the 2-tachyon diagram at the one loop level using the Mandelstam map: $\rho(u) = |\alpha_3| (\log \vartheta_1(u - U_6|\tau) - \log \vartheta_1(u - U_3|\tau)) - 2\pi i \alpha_1 u$,
 $T = \rho(u_-) - \rho(u_+)$, $\frac{d\rho}{du}(u_{\pm}) = 0$,

where the torus modulus is $e^{-\frac{i\pi}{\tau}} \sim \frac{T}{8|\alpha_3| \sin(\pi\alpha_1/|\alpha_3|)}$, ($T \rightarrow +0$).

The results:

$$\langle \mathbf{R} | e^{-\frac{T}{|\alpha_3|} (L_0^{(3)} + \tilde{L}_0^{(3)})} | v_3 \rangle | v_3 \rangle \sim [T (\log T)^2]^{-\frac{d-2}{4}} | \mathbf{R} \rangle | \mathbf{R} \rangle$$

$$\langle \mathbf{R} | \langle \mathbf{R} | e^{-\frac{T}{\alpha_1} (L_0^{(1)} + \tilde{L}_0^{(1)})} e^{-\frac{T}{\alpha_2} (L_0^{(2)} + \tilde{L}_0^{(2)})} | v_3 \rangle \sim [T (\log T)^2]^{-\frac{d-2}{4}} | \mathbf{R} \rangle$$

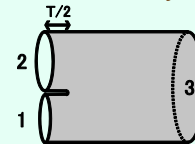
They correspond to the OPE of the twist field!

Note : We fixed α_r ($\alpha_4 = -\alpha_1, \alpha_5 = -\alpha_2$). \longleftrightarrow In OPEs in MST, we fixed (m, n) (label of string bits).

Comment : In the HIKKO closed SFT (d=26), the coefficient of the idempotency relation for the boundary states is roughly a square root of the above:

[I.K.-Matsuo-Watanabe, I.K.-Matsuo]

$$| \mathbf{B} \rangle_{\alpha_1} * T | \mathbf{B} \rangle_{\alpha_2} \sim |\alpha_{123}| T^{-3} | \mathbf{B} \rangle_{\alpha_1 + \alpha_2}$$



IV. Contractions in GSB LCSFT [KM]

Here we consider the fermionic sector of GSB LCSFT.

Without the prefactors, $\langle R|v_3\rangle|v_3\rangle \propto |R\rangle|R\rangle$, $\langle R|\langle R|v_3\rangle|v_3\rangle \propto |R\rangle$ can be shown similarly *except for fermion zero mode*.

For computation of the prefactors including fermion zero mode, we have used Fourier transform and Fierz identities such as

$$[\cosh Y]_{ij}[\cosh Y]_{lk} = 2^{-4} \sum_{p=0}^4 \frac{(-1)^p}{(2p)!} \hat{\gamma}_{ik}^{a_1 \dots a_{2p}} (\cosh Y \hat{\gamma}^{a_1 \dots a_{2p}} \cosh Y)_{lj} = \delta_{ik} \delta_{jl} \left(\frac{4}{\alpha_{123}} \right)^4 \delta^8(Y) + \mathcal{O}(Y^6), \dots$$

The results:

$$\begin{aligned}
 \text{"}H_1 H_1\text{"}: \quad & \langle R|e^{-\frac{T}{|\alpha_3|}(L_0^{(3)} + \tilde{L}_0^{(3)})} v^{ij}|v_3\rangle v^{kl}|v_3\rangle \sim \delta^{ik} \delta^{jl} T^{-2} |R\rangle|R\rangle & \longleftrightarrow & \Sigma^j \tilde{\Sigma}^i(z, \bar{z}) \Sigma^l \tilde{\Sigma}^k(0) \sim \frac{\delta^{ik} \delta^{jl}}{|z|^2} \\
 & \langle R|\langle R|e^{-\frac{T}{\alpha_1}(L_0^{(1)} + \tilde{L}_0^{(1)})} e^{-\frac{T}{\alpha_2}(L_0^{(2)} + \tilde{L}_0^{(2)})} v^{ij}|v_3\rangle v^{kl}|v_3\rangle \sim \delta^{ik} \delta^{jl} T^{-2} |R\rangle & & \\
 \text{"}Q_1^{\dot{a}} \tilde{Q}_1^{\dot{b}}\text{"}: \quad & \langle R|e^{-\frac{T}{|\alpha_3|}(L_0^{(3)} + \tilde{L}_0^{(3)})} s^{i\dot{a}}|v_3\rangle s^{j\dot{b}}|v_3\rangle \sim \delta^{ij} \delta^{\dot{a}\dot{b}} T^{-2} |R\rangle|R\rangle & \longleftrightarrow & \Sigma^{\dot{a}} \tilde{\Sigma}^i(z, \bar{z}) \Sigma^{\dot{b}} \tilde{\Sigma}^j(0) \sim \frac{\delta^{ij} \delta^{\dot{a}\dot{b}}}{|z|^2} \\
 & \langle R|\langle R|e^{-\frac{T}{\alpha_1}(L_0^{(1)} + \tilde{L}_0^{(1)})} e^{-\frac{T}{\alpha_2}(L_0^{(2)} + \tilde{L}_0^{(2)})} s^{i\dot{a}}|v_3\rangle s^{j\dot{b}}|v_3\rangle \sim \delta^{ij} \delta^{\dot{a}\dot{b}} T^{-2} |R\rangle & & \\
 \text{"}H_1 Q_1^{\dot{a}}\text{"}: \quad & \langle R|e^{-\frac{T}{|\alpha_3|}(L_0^{(3)} + \tilde{L}_0^{(3)})} v^{ij}|v_3\rangle s^{k\dot{a}}|v_3\rangle \sim \delta^{ik} T^{-\frac{3}{2}} \gamma_{c\dot{a}}^j (\vartheta_{(2)}^c - \vartheta_{(1)}^c) (\sigma_{\text{int}}) |R\rangle|R\rangle & \longleftrightarrow & \Sigma^j \tilde{\Sigma}^i(z, \bar{z}) \Sigma^{\dot{a}} \tilde{\Sigma}^k(0) \sim \frac{1}{z^{\frac{1}{2}} \bar{z}} \frac{\delta^{ik}}{\sqrt{2i}} \gamma_{c\dot{a}}^j \theta^c(0) \\
 \text{"}Q_1^{\dot{a}} \tilde{Q}_1^{\dot{b}}\text{"}: \quad & \langle R|e^{-\frac{T}{|\alpha_3|}(L_0^{(3)} + \tilde{L}_0^{(3)})} s^{i\dot{a}}|v_3\rangle \tilde{s}^{j\dot{b}}|v_3\rangle \sim T^{-1} \gamma_{c\dot{a}}^j (\vartheta_{(2)}^c - \vartheta_{(1)}^c) \gamma_{d\dot{b}}^i (\vartheta_{(2)}^d - \vartheta_{(1)}^d) (\sigma_{\text{int}}) |R\rangle|R\rangle & \longleftrightarrow & \Sigma^{\dot{a}} \tilde{\Sigma}^i(z, \bar{z}) \Sigma^j \tilde{\Sigma}^{\dot{b}}(0) \sim \frac{1}{2|z|} \gamma_{c\dot{a}}^j \theta^c \gamma_{d\dot{b}}^i \tilde{\theta}^d(0)
 \end{aligned}$$

The correspondence is consistent with OPEs of the spin fields!

V. Future directions

- More detailed correspondence? $(\alpha_r, \mathcal{P}_r) \leftrightarrow (m, n, \int d\sigma, N), \dots$
- Higher order terms of GSB SFT and MST? $H = H_0 + g_s H_1 + g_s^2 H_2 + \dots$, $Q^{\dot{a}} = Q_0^{\dot{a}} + g_s Q_1^{\dot{a}} + g_s^2 Q_2^{\dot{a}} + \dots$,
 $\{Q^{\dot{a}}, Q^{\dot{b}}\} = \{\tilde{Q}^{\dot{a}}, \tilde{Q}^{\dot{b}}\} = 2H \delta^{\dot{a}\dot{b}}$, $[Q^{\dot{a}}, H] = [\tilde{Q}^{\dot{a}}, H] = \{Q^{\dot{a}}, \tilde{Q}^{\dot{b}}\} = 0$.
- pp-wave background?
- Covariantized GSB SFT ? (using "pure spinor"?), ...