Comments on Solutions for Nonsingular Currents in Open String Field Theories

Isao Kishimoto

I. K., Y. Michishita, arXiv:0706.0409 [hep-th], to be published in PTP

Introduction

- Witten's bosonic open string field theory (d=26):

$$S[\Psi] = -\frac{1}{g^2}\left(\frac{1}{2}\langle\Psi, Q_{\mathrm{B}}\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right).$$

- There were various attempts to prove Sen's conjecture since around 1999 using the above.
- Numerically, it has been checked with "level truncation approximation." [c.f. … Gaiotto-Ratelli "Experimental string field theory"(2002)]
- Analytically, some solutions have been constructed.
- Here, we generalize "Schnabl's analytical solutions" (2005, 2007) which include "tachyon vacuum solution" in Sen's conjecture and "marginal solutions."

- In Berkovits' WZW-type superstring field theory (d=10) the action in the NS sector is given by

$$S_{\rm NS}[\Phi] = -\frac{1}{g^2}\int_0^1 dt \langle\langle (\eta_0\Phi)(e^{-t\Phi}Q_{\rm B}e^{t\Phi})\rangle\rangle \,.$$

- There were some attempts to solve the equation of motion.
- Numerically, tachyon condensation was examined using level truncation. [Berkovits(-Sen-Zwiebach)(2000),…]

- Analytically, some solutions have been constructed.
- Recently [April (2007)], Erler / Okawa constructed some solutions, which are generalization of Schnabl / Kiermaier-Okawa-Rastelli-Zwiebach's marginal solution (2007) in bosonic SFT. We consider generalization of their solutions and examine gauge transformations.

Main claim

bosonic SFT

Suppose that $\hat{\psi}$ **is BRST invariant and nilpotent**:

$$Q_{\rm B}\hat{\psi}=0,\quad \hat{\psi}*\hat{\psi}=0.$$ **Then,**

$$\Psi^{(\alpha,\beta)}=P_\alpha*\frac{1}{1+\hat{\psi}*A^{(\alpha+\beta)}}*\hat{\psi}*P_\beta$$

gives a solution to the EOM: $Q_{\rm B}\Psi^{(\alpha,\beta)}+\Psi^{(\alpha,\beta)}*\Psi^{(\alpha,\beta)}=0,$

where

$$Q_{\rm B}P_\alpha=0,\quad P_\alpha*P_\beta=P_{\alpha+\beta}\,,\quad P_{\alpha=0}=I\,,$$
$$Q_{\rm B}A^{(\gamma)}=I-P_\gamma\,.$$

In the case $|r=\alpha+1\rangle=P_\alpha$: wedge state, we have $A^{(\gamma)}=\frac{\pi}{2}\int_0^\gamma d\alpha B_1^L P_\alpha.$

$\hat{\psi}=U_1^\dagger U_1\lambda cJ(0)|0\rangle,$ $\alpha=\beta=1/2$: Schnabl / Kiermaier-Okawa-Rastelli-Zwiebach's marginal solution for nonsingular current is reproduced.

$\hat{\psi}=\hat{\lambda}Q_{\rm B}U_1^\dagger U_1 B_1^L c_1|0\rangle,$ $\alpha=\beta=1/2,\ \hat{\lambda}=\infty$: Schnabl's tachyon vacuum solution is reproduced.

super SFT

Suppose that $\hat{\phi}$ satisfies following conditions:

$$\eta_0 Q_{\rm B}\hat{\phi} = 0, \quad \hat{\phi} * \hat{\phi} = 0, \quad \hat{\phi} * \eta_0\hat{\phi} = 0, \quad \hat{\phi} * Q_{\rm B}\hat{\phi} = 0 \,.$$

Then,

$$\Phi^{(\alpha,\beta)}_{(1)} = \log(1 + P_\alpha * f_{(1)} * P_\beta), \qquad f_{(1)} = \frac{1}{1 - \eta_0\hat{\phi} * Q_{\rm B}\hat{A}^{(\alpha+\beta)}} * \hat{\phi},$$

$$\Phi^{(\alpha,\beta)}_{(2)} = \log(1 + P_\alpha * f_{(2)} * P_\beta), \qquad f_{(2)} = \hat{\phi} * \frac{1}{1 - \eta_0\hat{A}^{(\alpha+\beta)} * Q_{\rm B}\hat{\phi}},$$

$$\Phi^{(\alpha,\beta)}_{(3)} = -\log(1 - P_\alpha * f_{(3)} * P_\beta), \qquad f_{(3)} = \frac{1}{1 - Q_{\rm B}\hat{\phi} * \eta_0\hat{A}^{(\alpha+\beta)}} * \hat{\phi},$$

$$\Phi^{(\alpha,\beta)}_{(4)} = -\log(1 - P_\alpha * f_{(4)} * P_\beta), \qquad f_{(4)} = \hat{\phi} * \frac{1}{1 - Q_{\rm B}\hat{A}^{(\alpha+\beta)} * \eta_0\hat{\phi}},$$

give solutions to the EOM: $\eta_0(e^{-\Phi^{(\alpha,\beta)}_{(i)}} Q_{\rm B} e^{\Phi^{(\alpha,\beta)}_{(i)}}) = 0\,, \quad (i = 1,2,3,4)$

where $\eta_0 P_\alpha = 0, \quad Q_{\rm B} P_\alpha = 0, \quad P_\alpha * P_\beta = P_{\alpha+\beta}\,, \quad P_{\alpha=0} = I\,,$

$$\eta_0 Q_{\rm B}\hat{A}^{(\gamma)} = I - P_\gamma \,.$$

In the case P_α : wedge state, we find $\hat{A}^{(\gamma)} = \int_0^\gamma d\alpha \log\left(\frac{\alpha}{\gamma}\right)\left(\frac{\pi}{2} J_1^{--L} + \alpha\frac{\pi^2}{4}\tilde{G}_1^{-L} B_1^L\right) P_\alpha.$

$$\hat{\phi} = \zeta_a U_1^\dagger U_1 c\xi e^{-\phi}\psi^a(0)|0\rangle \,, \qquad \zeta_a\zeta_b\Omega^{ab} = 0\,, \qquad \alpha = \beta = 1/2$$

: Erler / Okawa’s marginal solutions for nonsingular supercurrents are reproduced.

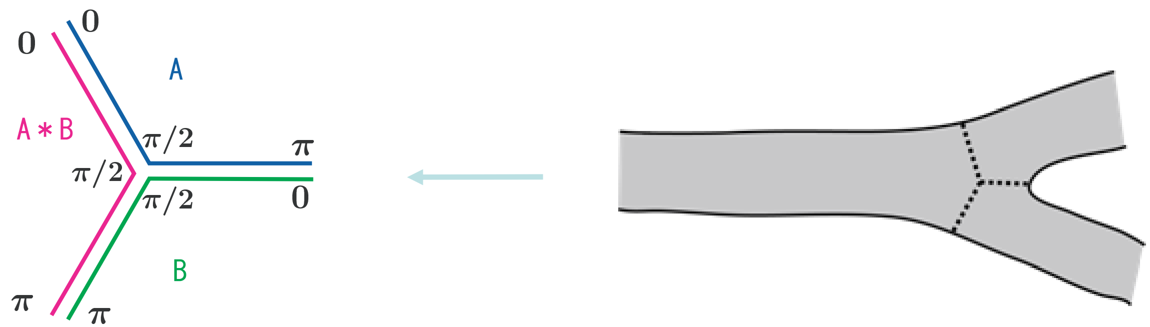
Witten's bosonic open string field theory

Action: $$S[\Psi] = -\frac{1}{g^2}\left(\frac{1}{2}\langle\Psi, Q_{\rm B}\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right)$$

String field: $$|\Psi\rangle = \phi(x)c_1|0\rangle + A_\mu(x)\alpha^\mu_{-1}c_1|0\rangle + iB(x)c_0|0\rangle + \cdots$$

BRST operator: $$Q_{\rm B} = \oint \frac{dz}{2\pi i}\left(cT^{\rm m} + bc\partial c + \frac{3}{2}\partial^2 c\right)$$

Witten star product:



Equation of motion: $$Q_{\rm B}\Psi + \Psi * \Psi = 0$$

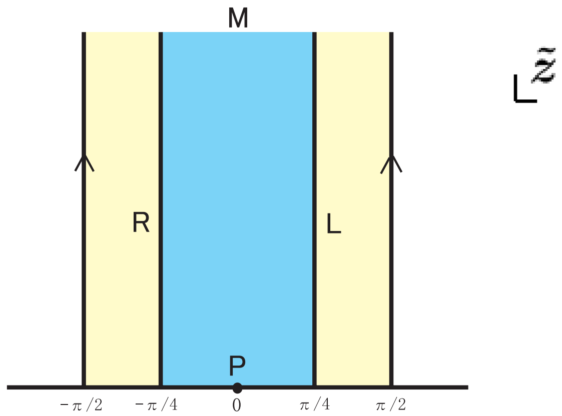
Gauge transformation: $$\delta_\Lambda\Psi = Q_{\rm B}\Lambda + \Psi * \Lambda - \Lambda * \Psi$$

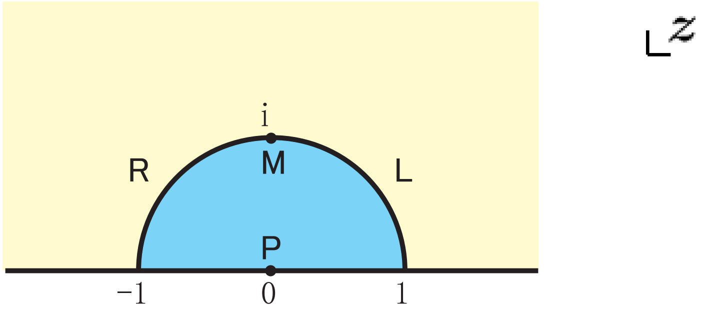
Preliminary

- “sliver frame”: $\tilde{z} = \arctan z$ (z :UHP)

For a primary field ϕ of dim=h,

$$\tilde{\phi}(\tilde{z}) = \left(\frac{dz}{d\tilde{z}}\right)^h \phi(z) = (\cos\tilde{z})^{-2h}\phi(\tan\tilde{z})$$





In particular, we often use

$$\mathcal{L}_0 \equiv \tilde{L}_0 = L_0 + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{4k^2-1} L_{2k}, \qquad K_1 \equiv \tilde{L}_{-1} = L_1 + L_{-1},$$

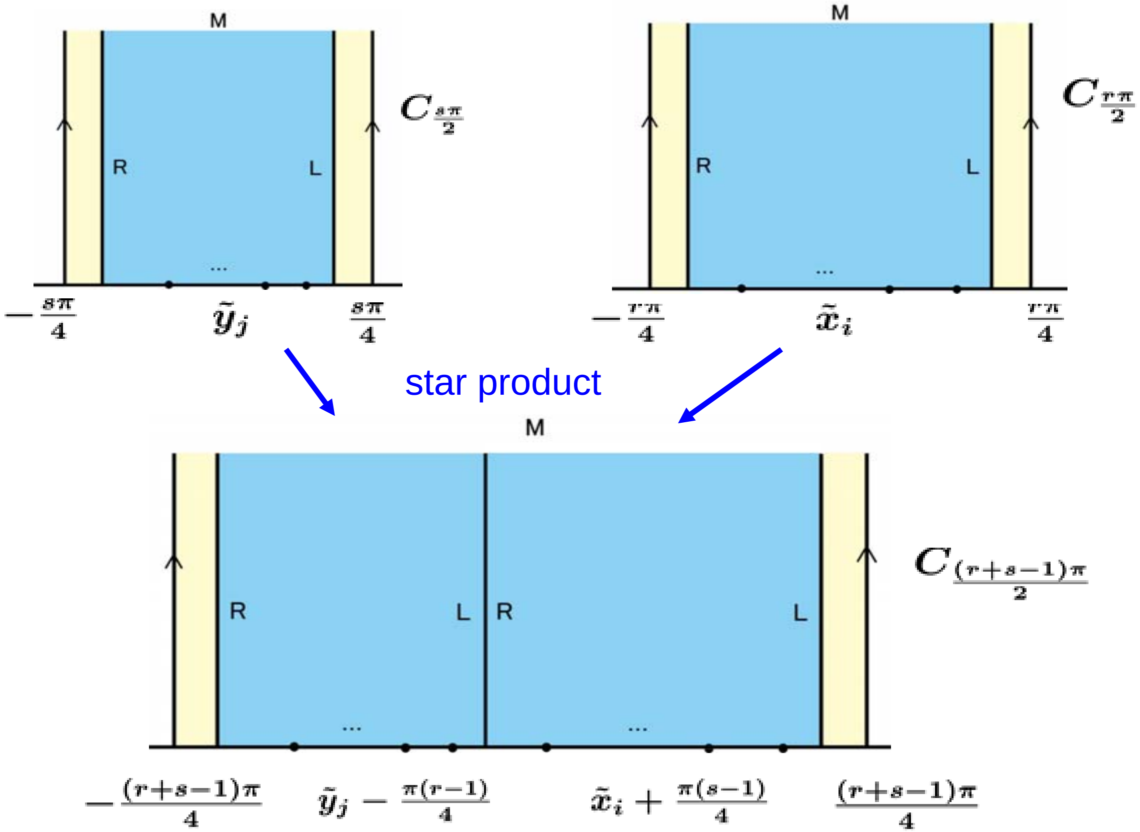
$$\mathcal{B}_0 \equiv \tilde{b}_0 = b_0 + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{4k^2-1} b_{2k}, \qquad B_1 \equiv \tilde{b}_{-1} = b_1 + b_{-1},$$

and

$$\hat{\mathcal{L}} = \mathcal{L}_0 + \mathcal{L}_0^\dagger, \quad K_1^{L/R} = \frac{1}{2}K_1 \pm \frac{1}{\pi}\hat{\mathcal{L}}, \quad \hat{\mathcal{B}} = \mathcal{B}_0 + \mathcal{B}_0^\dagger, \quad B_1^{L/R} = \frac{1}{2}B_1 \pm \frac{1}{\pi}\hat{\mathcal{B}}.$$

Using $U_r = \left(\frac{2}{r}\right)^{\mathcal{L}_0} = \left(\frac{2}{r}\right)^{L_0} e^{-\frac{r^2-4}{3r^2}L_2+\frac{r^4-16}{30r^4}L_4+\cdots}$ we have a $*$ product formula:

$$U_r^\dagger U_r \tilde{\phi}_1(\tilde{x}_1)\cdots\tilde{\phi}_n(\tilde{x}_n)|0\rangle * U_s^\dagger U_s \tilde{\psi}_1(\tilde{y}_1)\cdots\tilde{\psi}_m(\tilde{y}_m)|0\rangle$$
$$= U_{r+s-1}^\dagger U_{r+s-1} \tilde{\phi}_1(\tilde{x}_1+\frac{\pi}{4}(s-1))\cdots\tilde{\phi}_n(\tilde{x}_n+\frac{\pi}{4}(s-1))\tilde{\psi}_1(\tilde{y}_1-\frac{\pi}{4}(r-1))\cdots\tilde{\psi}_m(\tilde{y}_m-\frac{\pi}{4}(r-1))|0\rangle$$



For the wedge state: $|r=\alpha+1\rangle = U_{\alpha+1}^\dagger U_{\alpha+1}|0\rangle = P_\alpha$, we have $P_\alpha * P_\beta = P_{\alpha+\beta}$.

- Associated with the wedge states, we have

$$A^{(\gamma)} = \frac{\pi}{2}\int_0^\gamma d\alpha\, B_1^L P_\alpha$$ such as $$Q_B A^{(\gamma)} = I - P_\gamma$$.

[Ellwood-Schnabl]

With BRST invariant and nilpotent $\hat{\psi}$:

$$Q_B \hat{\psi} = 0, \quad \hat{\psi} * \hat{\psi} = 0,$$

we have a solution to the equation of motion

$$\Psi^{(\alpha,\beta)} = P_\alpha * \frac{1}{1 + \hat{\psi} * A^{(\alpha+\beta)}} * \hat{\psi} * P_\beta$$

$$= \sum_{k=0}^{\infty} (-1)^k P_\alpha * (\hat{\psi} * A^{(\alpha+\beta)})^k * \hat{\psi} * P_\beta \,.$$

$$\because \quad Q_B\Psi^{(\alpha,\beta)} = P_\alpha * Q_B\left(\frac{1}{1+\hat\psi * A^{(\alpha+\beta)}}\right) * \hat\psi * P_\beta$$

$$= -P_\alpha * \frac{1}{1+\hat\psi * A^{(\alpha+\beta)}} * (Q_B(I+\hat\psi * A^{(\alpha+\beta)})) * \frac{1}{1+\hat\psi * A^{(\alpha+\beta)}} * \hat\psi * P_\beta$$

$$= P_\alpha * \frac{1}{1+\hat\psi * A^{(\alpha+\beta)}} * \hat\psi * (Q_B A^{(\alpha+\beta)}) * \frac{1}{1+\hat\psi * A^{(\alpha+\beta)}} * \hat\psi * P_\beta$$

$$= P_\alpha * \frac{1}{1+\hat\psi * A^{(\alpha+\beta)}} * \hat\psi * (I - P_{\alpha+\beta}) * \frac{1}{1+\hat\psi * A^{(\alpha+\beta)}} * \hat\psi * P_\beta$$

$$= P_\alpha * \frac{1}{1+\hat\psi * A^{(\alpha+\beta)}} * \underbrace{\hat\psi * \hat\psi}_{=0} * \frac{1}{1+A^{(\alpha+\beta)} * \hat\psi} * P_\beta$$

$$-P_\alpha * \frac{1}{1+\hat\psi * A^{(\alpha+\beta)}} * \hat\psi * P_\beta * P_\alpha * \frac{1}{1+\hat\psi * A^{(\alpha+\beta)}} * \hat\psi * P_\beta$$

$$= -\Psi^{(\alpha,\beta)} * \Psi^{(\alpha,\beta)}.$$

Note 1. $\lambda\hat\psi$ is also BRST invariant and nilpotent.

$\rightarrow$ $\Psi^{(\alpha,\beta)}$ can naturally include 1-parameter.

Note 2.

In general, for $$\Psi^{(\alpha,\beta)}(\psi) \equiv P_\alpha * \frac{1}{1+\psi * A^{(\alpha+\beta)}} * \psi * P_\beta$$

we have $$Q_\mathrm{B}\Psi^{(\alpha,\beta)}(\psi) + \Psi^{(\alpha,\beta)}(\psi) * \Psi^{(\alpha,\beta)}(\psi)$$
$$= P_\alpha * \frac{1}{1+\psi * A^{(\alpha+\beta)}} * (Q_\mathrm{B}\psi + \psi * \psi) * \frac{1}{1+A^{(\alpha+\beta)} * \psi} * P_\beta \,.$$

We can regard $$\psi \mapsto \Psi^{(\alpha,\beta)}(\psi) = P_\alpha * \frac{1}{1+\psi * A^{(\alpha+\beta)}} * \psi * P_\beta$$

as a map *from a solution to another solution*:

$$Q_\mathrm{B}\psi + \psi * \psi = 0$$
$$\rightarrow \qquad Q_\mathrm{B}\Psi^{(\alpha,\beta)}(\psi) + \Psi^{(\alpha,\beta)}(\psi) * \Psi^{(\alpha,\beta)}(\psi) = 0$$

Composition of maps forms a commutative monoid:

$$\Psi^{(\alpha,\beta)}(\Psi^{(\alpha',\beta')}(\psi)) = \Psi^{(\alpha+\alpha',\beta+\beta')}(\psi), \quad (\alpha,\beta,\alpha',\beta' \geq 0)$$

$$\Psi^{(0,0)}(\psi) = \psi \,.$$

- <u>Example</u> of BRST invariant and nilpotent $\hat{\psi}$

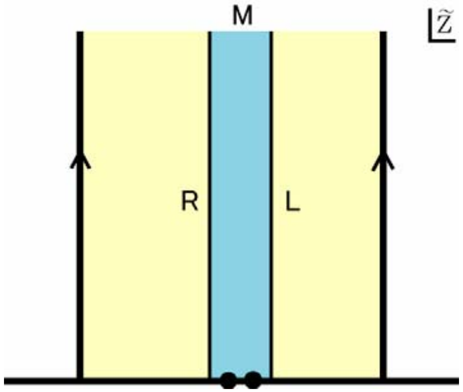
$$\hat{\psi} = \lambda_{\rm s}\hat{\psi}_{\rm s} + \lambda_{\rm m}\hat{\psi}_{\rm m} \,,$$
$$\hat{\psi}_{\rm s} = Q_{\rm B}\hat{\Lambda}_0 \,, \qquad \hat{\Lambda}_0 \equiv U_1^\dagger U_1 B_1^L c_1|0\rangle \,,$$
$$\hat{\psi}_{\rm m} = U_1^\dagger U_1 cJ(0)|0\rangle \,.$$

where $J(z) = \zeta_a J^a(z)$ is "nonsingular" matter primary of dimension 1:

$$\zeta_a\zeta_b g^{ab} = 0 \,, \qquad J^a(y)J^b(z) \sim \frac{g^{ab}}{(y-z)^2} + \frac{1}{y-z} i f^{ab}{}_c J^c(z) + \cdots \,.$$

In particular, $\lambda_{\rm s} = 0$ ⟹ marginal solution

$\lambda_{\rm m} = 0$ ⟹ tachyon solution



$c\zeta_a J^a(\epsilon)\, c\zeta_b J^b(0) \sim 0$

Due to the nonsingular condition for the current,
we find nilpotency :

Marginal solution

From a BRST invariant, nilpotent $\hat{\psi}_{\rm m} = U_1^\dagger U_1 cJ(0)|0\rangle$ which satisfies

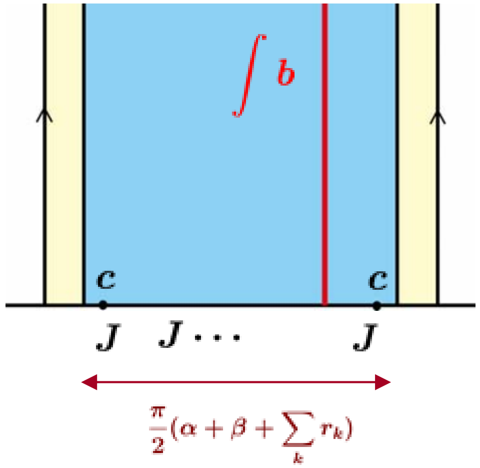
$(\mathcal{B}_0 - \mathcal{B}_0^\dagger)\hat{\psi}_{\rm m} = 0$, we can generate a solution

$$\Psi^{(\alpha,\beta)} = \sum_{k=0}^{\infty}(-1)^k \lambda_{\rm m}^{k+1} P_\alpha * (\hat{\psi}_{\rm m} * A^{(\alpha+\beta)})^k * \hat{\psi}_{\rm m} * P_\beta = \sum_{n=1}^{\infty} \lambda_{\rm m}^n \psi_{{\rm m},n} \,,$$

$$\psi_{{\rm m},1} = U^\dagger_{\alpha+\beta+1} U_{\alpha+\beta+1} \tilde{c}\tilde{J}(\frac{\pi}{4}(\beta-\alpha))|0\rangle \,,$$

$$\begin{aligned}\psi_{{\rm m},k+1} = {} & \left(-\frac{\pi}{2}\right)^k \int_0^{\alpha+\beta} dr_1 \cdots \int_0^{\alpha+\beta} dr_k \, U^\dagger_{\alpha+\beta+1+\sum_{l=1}^k r_l} U_{\alpha+\beta+1+\sum_{l=1}^k r_l} \prod_{m=0}^{k} \tilde{J}\left(\frac{\pi}{4}(\beta-\alpha-\sum_{l=1}^{m} r_l + \sum_{l=m+1}^{k} r_l)\right) \\ & \times \left[-\frac{1}{\pi}\hat{\mathcal{B}}\tilde{c}(\frac{\pi}{4}(\beta-\alpha+\sum_{l=1}^k r_l))\tilde{c}(\frac{\pi}{4}(\beta-\alpha-\sum_{l=1}^k r_l)) + \frac{1}{2}\left(\tilde{c}(\frac{\pi}{4}(\beta-\alpha+\sum_{l=1}^k r_l)) + \tilde{c}(\frac{\pi}{4}(\beta-\alpha-\sum_{l=1}^k r_l))\right)\right]|0\rangle.\end{aligned}$$

$$\Psi^{(\alpha,\beta)} \sim \sum \lambda_{\rm m}^n \int dr_k$$



Tachyon solution

- From a BRST invariant, nilpotent $\hat{\psi}_{\rm s} = Q_{\rm B} U_1^\dagger U_1 B_1^L c_1 |0\rangle$ which satisfies $(\mathcal{B}_0 - \mathcal{B}_0^\dagger)\hat{\psi}_{\rm s} = 0$, we can generate a solution:

$$\Psi^{(\alpha,\beta)} = \sum_{k=0}^{\infty} (-1)^k \lambda_{\rm s}^{k+1} P_\alpha * \hat{\psi}_{\rm s} * (A^{(\alpha+\beta)} * \hat{\psi}_{\rm s})^k * P_\beta = \sum_{n=1}^{\infty} \lambda_{\rm s}^n \psi_{{\rm s},n} .$$

Each term is computed as

$$\psi_{{\rm s},n} = P_\alpha * (Q_{\rm B}\hat{\Lambda}_0) * P_\beta * (P_\alpha * \hat{\Lambda}_0 * P_\beta - I)^{n-1} = -\sum_{l=0}^{n-1} \frac{(-1)^{n-1-l}(n-1)!}{l!(n-1-l)!} \partial_t \psi_{t,l}^{(\alpha,\beta)}|_{t=0} ,$$

$$\begin{aligned} \psi_{t,n}^{(\alpha,\beta)} = {} & \frac{2}{\pi} U^\dagger_{n(\alpha+\beta)+t+\alpha+\beta+1} U_{n(\alpha+\beta)+t+\alpha+\beta+1} \Big[\\ & -\frac{1}{\pi} \hat{\mathcal{B}} \tilde{c}(\frac{\pi}{4}(\beta-\alpha+t+n(\alpha+\beta))) \tilde{c}(\frac{\pi}{4}(\beta-\alpha-t-n(\alpha+\beta))) \\ & + \frac{1}{2} \Big\{ \tilde{c}(\frac{\pi}{4}(\beta-\alpha+t+n(\alpha+\beta))) + \tilde{c}(\frac{\pi}{4}(\beta-\alpha-t-n(\alpha+\beta))) \Big\} \Big] |0\rangle . \end{aligned}$$

Then, we can re-sum the above as

$$\Psi^{(\alpha,\beta)} = -\sum_{l=0}^{\infty} \lambda_S^{l+1} \partial_t \psi_{t,l}^{(\alpha,\beta)}|_{t=0} .$$

Here, expansion parameter is redefined as

$$\lambda_S \equiv \frac{\lambda_{\rm s}}{\lambda_{\rm s}+1} .$$

The solution can be rewritten as $\Psi^{(\alpha,\beta)} = e^{\frac{\pi}{4}(\beta-\alpha)K_1}(\alpha+\beta)^{\frac{D}{2}}\Psi^{(1/2,1/2)}$,

where $K_1 = L_1 + L_{-1}, \quad D = \mathcal{L}_0 - \mathcal{L}_0^\dagger$ are BPZ odd and derivations w.r.t. $*$,

and $\Psi^{(1/2,1/2)}$ is the Schnabl's solution for tachyon condensation at

$$\lambda_S = 1 \quad \leftrightarrow \quad \lambda_s = \infty .$$

By <u>regularizing</u> it as

$$\Psi^{(\alpha,\beta)}|_{\lambda_S=1} = \lim_{N\to\infty}\left(\frac{1}{\alpha+\beta}\psi^{(\alpha,\beta)}_{t=0,N} - \sum_{n=0}^{N}\partial_t\psi^{(\alpha,\beta)}_{t,n}|_{t=0}\right),$$

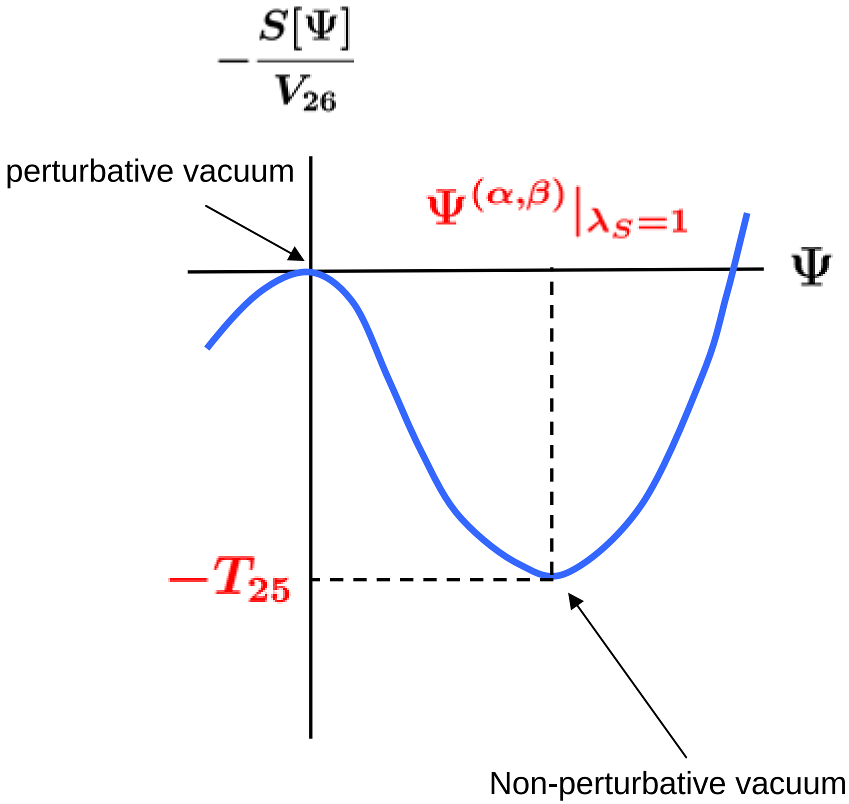
the new BRST operator around the solution $Q'_{\rm B}$ satisfies

$$Q'_{\rm B}A^{(\alpha+\beta)} \equiv Q_{\rm B}A^{(\alpha+\beta)} + \Psi^{(\alpha,\beta)}|_{\lambda_S=1} * A^{(\alpha+\beta)} + A^{(\alpha+\beta)} * \Psi^{(\alpha,\beta)}|_{\lambda_S=1} = I,$$

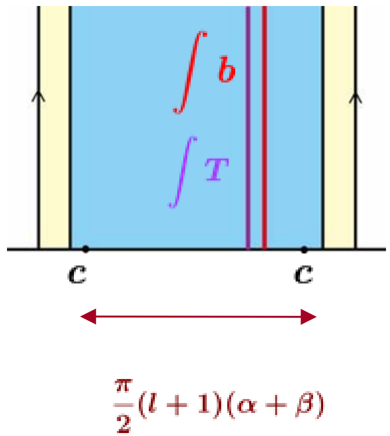
which implies vanishing cohomology and

$$S[\Psi^{(\alpha,\beta)}|_{\lambda_S=1}]/V_{26} = \frac{1}{2\pi^2 g^2} = T_{25}.$$

This result is (α,β)-independent.



$$\Psi^{(\alpha,\beta)} \sim \sum \lambda_S^{l+1}$$



Note

We can evaluate the action as $S[\Psi^{(\alpha,\beta)}]/V_{26} = 0 \quad (|\lambda_S| < 1)\,.$

In fact, the solution can be rewritten as pure gauge form by evaluating the infinite summation formally

$$\Psi^{(\alpha,\beta)} = Q_{\mathrm{B}}(\lambda_S P_\alpha * \hat{\Lambda}_0 * P_\beta) * \frac{1}{1 - \lambda_S P_\alpha * \hat{\Lambda}_0 * P_\beta}\,.$$

Berkovits' WZW-type super SFT

The action for the NS sector is $S_{\rm NS}[\Phi] = -\frac{1}{g^2}\int_0^1 dt\langle\langle(\eta_0\Phi)(e^{-t\Phi}Q_{\rm B}e^{t\Phi})\rangle\rangle.$

String field Φ : ghost number 0, picture number 0, Grassmann even,
expressed by matter and ghosts b, c, ϕ, ξ, η $(\beta = e^{-\phi}\partial\xi, \gamma = \eta e^{\phi})$:

$$Q_{\rm B} = \oint \frac{dz}{2\pi i}(c(T^{\rm m} - \frac{1}{2}(\partial\phi)^2 - \partial^2\phi + \partial\xi\eta) + bc\partial c + \eta e^{\phi}G^{\rm m} - \eta\partial\eta e^{2\phi}b)(z),$$

$$\eta_0 = \oint \frac{dz}{2\pi i}\eta(z).$$

Equation of motion: $\eta_0(e^{-\Phi}Q_{\rm B}e^{\Phi}) = 0 \quad \leftrightarrow \quad Q_{\rm B}(e^{\Phi}\eta_0 e^{-\Phi}) = 0$

Gauge transformation: $\delta e^{\Phi} = \Xi_1 * e^{\Phi} + e^{\Phi} * \Xi_2, \;\; Q_{\rm B}\Xi_1 = 0, \;\; \eta_0\Xi_2 = 0.$

Using the wedge states $|r = \alpha + 1\rangle = P_\alpha$ as in bosonic SFT, we have

$$Q_{\rm B}P_\alpha = 0\,, \quad \eta_0 P_\alpha = 0\,, \quad P_\alpha * P_\beta = P_{\alpha+\beta}\,, \quad P_{\alpha=0} = I.$$

Corresponding to the wedge states, we have constructed $\hat{A}^{(\gamma)}$:

$$\hat{A}^{(\gamma)} = \int_0^\gamma d\alpha \log\left(\frac{\alpha}{\gamma}\right)\left(\frac{\pi}{2}J_1^{--L} + \alpha\frac{\pi^2}{4}\tilde{G}_1^{-L}B_1^L\right)P_\alpha\,,$$

such as $\eta_0\hat{A}^{(\gamma)} = -\frac{\pi}{2}\int_0^\gamma d\alpha B_1^L P_\alpha,\quad Q_{\rm B}\hat{A}^{(\gamma)} = -\frac{\pi}{2}\int_0^\gamma d\alpha \tilde{G}_1^{-L}P_\alpha,\quad \eta_0 Q_{\rm B}\hat{A}^{(\gamma)} = I - P_\gamma.$

$J^{--}(z) = \xi b(z),\ \ \tilde{G}^- = [Q_{\rm B}, J^{--}(z)]$ ⇨ $J_1^{--L},\ \tilde{G}_1^{-L}$ are defined in the same way as B_1^L.

Then, we find that

$$\Phi_{(1)}^{(\alpha,\beta)}(\phi) = \log(1 + P_\alpha * f_{(1)} * P_\beta),\qquad f_{(1)} = \frac{1}{1 + (e^\phi\eta_0 e^{-\phi})Q_{\rm B}\hat{A}^{(\alpha+\beta)}}(e^\phi - 1)\,,$$

$$\Phi_{(2)}^{(\alpha,\beta)}(\phi) = \log(1 + P_\alpha * f_{(2)} * P_\beta),\qquad f_{(2)} = (e^\phi - 1)\frac{1}{1 - \eta_0\hat{A}^{(\alpha+\beta)}(e^{-\phi}Q_{\rm B}e^\phi)}\,,$$

$$\Phi_{(3)}^{(\alpha,\beta)}(\phi) = -\log(1 - P_\alpha * f_{(3)} * P_\beta),\qquad f_{(3)} = \frac{1}{1 - (e^{-\phi}Q_{\rm B}e^\phi)\eta_0\hat{A}^{(\alpha+\beta)}}(1 - e^{-\phi})\,,$$

$$\Phi_{(4)}^{(\alpha,\beta)}(\phi) = -\log(1 - P_\alpha * f_{(4)} * P_\beta),\qquad f_{(4)} = (1 - e^{-\phi})\frac{1}{1 + Q_{\rm B}\hat{A}^{(\alpha+\beta)}(e^\phi\eta_0 e^{-\phi})}\,,$$

map solutions to other solutions because

$$e^{\Phi_{(1)}^{(\alpha,\beta)}}\eta_0 e^{-\Phi_{(1)}^{(\alpha,\beta)}} = e^{\Phi_{(4)}^{(\alpha,\beta)}}\eta_0 e^{-\Phi_{(4)}^{(\alpha,\beta)}} = P_\alpha\frac{1}{1 + (e^\phi\eta_0 e^{-\phi})Q_{\rm B}\hat{A}^{(\alpha+\beta)}}(e^\phi\eta_0 e^{-\phi})P_\beta,$$

$$e^{-\Phi_{(2)}^{(\alpha,\beta)}}Q_{\rm B}e^{\Phi_{(2)}^{(\alpha,\beta)}} = e^{-\Phi_{(3)}^{(\alpha,\beta)}}Q_{\rm B}e^{\Phi_{(3)}^{(\alpha,\beta)}} = P_\alpha(e^{-\phi}Q_{\rm B}e^\phi)\frac{1}{1 - \eta_0\hat{A}^{(\alpha+\beta)}(e^{-\phi}Q_{\rm B}e^\phi)}P_\beta\,.$$

If $\hat{\phi}$ satisfies $\eta_0 Q_{\mathrm{B}}\hat{\phi}=0,\quad \hat{\phi}*\hat{\phi}=0,\quad \hat{\phi}*\eta_0\hat{\phi}=0,\quad \hat{\phi}*Q_{\mathrm{B}}\hat{\phi}=0,$

$\hat{\phi}$ is a solution: $\eta_0(e^{-\hat{\phi}}Q_{\mathrm{B}}e^{\hat{\phi}})=0.$

⇨ $\Phi^{(\alpha,\beta)}_{(i)}(\hat{\phi}),\ (i=1,2,3,4)$ are also solutions.

Example of $\hat{\phi}$ using nonsingular matter supercurrent:

$$\mathrm{J}^a(z,\theta)=\psi^a(z)+\theta J^a(z)$$

$$\hat{\phi}=\zeta_a U_1^\dagger U_1 c\xi e^{-\phi}\psi^a(0)|0\rangle\,,\qquad \zeta_a\zeta_b\Omega^{ab}=0\,,$$

where we suppose

$$\begin{aligned}\psi^a(y)\psi^b(z) &\sim (y-z)^{-1}\Omega^{ab}\,,\\ J^a(y)\psi^b(z) &\sim (y-z)^{-1}if^{ab}{}_c\psi^c(z)\,,\\ J^a(y)J^b(z) &\sim (y-z)^{-2}\Omega^{ab}+(y-z)^{-1}if^{ab}{}_cJ^c(z)\,.\end{aligned}$$

More explicitly, on the flat background, we can take

$$\mathrm{J}^\mu(z,\theta)=\psi^\mu(z)+\theta i\partial X^\mu(z),\qquad \zeta_\mu\zeta_\nu\eta^{\mu\nu}=0\,.$$

Gauge transformations

Using path-ordering, we found

$$\Psi^{(\alpha,\beta)} = V^{(\alpha,\beta)-1} * \psi * V^{(\alpha,\beta)} + V^{(\alpha,\beta)-1} * Q_{\mathrm{B}} * V^{(\alpha,\beta)},$$

$$V^{(\alpha,\beta)} = \mathrm{P}\exp\int_0^1 dt G^{(\alpha,\beta)}(t)\,,$$

$$G^{(\alpha,\beta)}(t) \equiv \frac{-\pi}{2}\left(\alpha(B_1^L P_{t\alpha}) * \frac{1}{1+\psi * A^{(t(\alpha+\beta))}} * \psi * P_{t\beta} + \beta P_{t\alpha} * \frac{1}{1+\psi * A^{(t(\alpha+\beta))}} * \psi * B_1^R P_{t\beta}\right),$$

for bosonic SFT.

(In the case $\alpha = \beta$, this form coincides with Ellwood's one.)

In this sense, $\Psi^{(\alpha,\beta)} \sim \hat{\psi}$

Without the identity state, including Schnabl's marginal and scalar solutions

Based on the identity state, BRST inv. and nilpotent

- Similarly, in super SFT, we have found

$$e^{\Phi^{(\alpha,\beta)}_{(3)}} = W_1 * e^{\phi} * W_2\,, \qquad Q_{\rm B} W_1 = 0, \qquad \eta_0 W_2 = 0\,,$$

$$W_1 \equiv {\rm P}' \exp \int_0^1 dt G_1^{(\alpha,\beta)}(t)\,, \qquad W_2 \equiv {\rm P} \exp \int_0^1 dt G_2^{(\alpha,\beta)}(t)\,,$$

$$G_1^{(\alpha,\beta)}(t) \equiv \frac{\pi}{2}\left[-\alpha K_1^L I + (\alpha+\beta) Q_{\rm B} B_1^R \left(P_{t\alpha} \frac{1}{1 - Q_{\rm B}((e^{\phi}-1)\eta_0 \hat{A}^{(t(\alpha+\beta))})}(e^{\phi}-1) P_{t\beta}\right)\right],$$

$$G_2^{(\alpha,\beta)}(t) \equiv \frac{\pi}{2}\left[\alpha K_1^L I + (\alpha+\beta) B_1^R \left(P_{t\alpha}(e^{-\phi} Q_{\rm B} e^{\phi}) \frac{1}{1 - \eta_0 \hat{A}^{(t(\alpha+\beta))}(e^{-\phi} Q_{\rm B} e^{\phi})} P_{t\beta}\right)\right],$$

$$e^{\Phi^{(\alpha,\beta)}_{(1)}} = W_3 * e^{\phi} * W_4\,, \qquad Q_{\rm B} W_3 = 0, \qquad \eta_0 W_4 = 0,$$

$$W_3 \equiv {\rm P}' \exp \int_0^1 dt G_4^{(\alpha,\beta)}(t)\,, \qquad W_4 \equiv {\rm P} \exp \int_0^1 dt G_3^{(\alpha,\beta)}(t)\,,$$

$$G_3^{(\alpha,\beta)}(t) \equiv \frac{\pi}{2}\left[\alpha K_1^L I - (\alpha+\beta) \eta_0 \tilde{G}_1^{-R} \left(P_{t\alpha} \frac{1}{1 - \eta_0((1-e^{-\phi}) Q_{\rm B} \hat{A}^{(t(\alpha+\beta))})}(1-e^{-\phi}) P_{t\beta}\right)\right],$$

$$G_4^{(\alpha,\beta)}(t) \equiv \frac{\pi}{2}\left[-\alpha K_1^L I + (\alpha+\beta) \tilde{G}_1^{-R} \left(P_{t\alpha} \frac{1}{1 + (e^{\phi}\eta_0 e^{-\phi}) Q_{\rm B} \hat{A}^{(t(\alpha+\beta))}}(e^{\phi}\eta_0 e^{-\phi}) P_{t\beta}\right)\right].$$

$$e^{\Phi^{(\alpha,\beta)}_{(2)}} = U_{23} * e^{\Phi^{(\alpha,\beta)}_{(3)}}\,, \qquad e^{\Phi^{(\alpha,\beta)}_{(1)}} = e^{\Phi^{(\alpha,\beta)}_{(4)}} * V_{41}\,,$$

$$U_{23} \equiv 1 - Q_{\rm B}\left(P_\alpha (e^{\phi}-1) \frac{1}{1 - \eta_0 \hat{A}^{(\alpha+\beta)}(e^{-\phi} Q_{\rm B} e^{\phi})} \eta_0 \hat{A}^{(\alpha+\beta)} (1-e^{-\phi}) P_\beta\right),$$

$$V_{41} \equiv 1 + \eta_0 \left(P_\alpha (1-e^{-\phi}) \frac{1}{1 + Q_{\rm B} \hat{A}^{(\alpha+\beta)}(e^{\phi}\eta_0 e^{-\phi})} Q_{\rm B} \hat{A}^{(\alpha+\beta)} (e^{\phi}-1) P_\beta\right).$$

In this sense, $\Phi_{(i)}^{(\alpha,\beta)} \quad \sim \quad \hat{\phi}$

$i=1,2,3,4$

Without the identity state, including Erler / Okawa's marginal solutions

Based on the identity state,

$$\eta_0 Q_{\rm B}\hat{\phi}=0,\ \ \hat{\phi}*\hat{\phi}=0,$$
$$\hat{\phi}*\eta_0\hat{\phi}=0,\ \ \hat{\phi}*Q_{\rm B}\hat{\phi}=0.$$

Note:

The above gauge equivalence relations seem to be ***formal*** and might not be well-defined.
The gauge parameter string fields might become “singular,” as well as Schnabl or Takahashi-Tanimoto's tachyon solution.

Future problems

- How about general (super)currents? Namely, $\zeta_a\zeta_b g^{ab} \neq 0, \quad \zeta_a\zeta_b\Omega^{ab} \neq 0.$

C.f. [KORZ], [Fuchs-Kroyter-Potting], [Fuchs-Kroyter], [Kiermaier-Okawa]

In [Takahashi-Tanimoto, Kishimoto-Takahashi] some solutions based on the identity state for general (super)current were already constructed.

At least formally, $\Psi^{(\alpha,\beta)}(\Psi^{\mathrm{TT}})$ and $\Phi^{(\alpha,\beta)}(\Phi^{\mathrm{KT}})$ with $\alpha, \beta > 0$

give solutions which are not based on the identity state!

⟶ Zeze's talk!

So far, various computations seem to be rather formal.

- Definition of the "regularity" of string fields?

It is very important in order to investigate "regular solutions," gauge transformations among them and cohomology around them.

理研シンポジウム

弦の場の理論 07

10月6日（土）, 7日（日）

埼玉県和光市理化学研究所
大河内記念ホール

http://www.riken.jp/lab-www/theory/sft/