Recent developments on analytic solutions in open string field theories

Isao Kishimoto

References

- I. K.,

  "Recent development on analytic solutions in string field theory,"

  Soryushiron Kenkyu 114-6,F-13 (2007-3) (in Japanese)
- I. K., Y. Michishita,

  "Comments on Solutions for Nonsingular Currents in Open String Field Theories,"

  Prog.Theor.Phys.118(2007)347 [arXiv:0706.0409]

INTRODUCTION

- 1999-

  There are various attempts to prove Sen's conjecture using Witten's open string field theory:

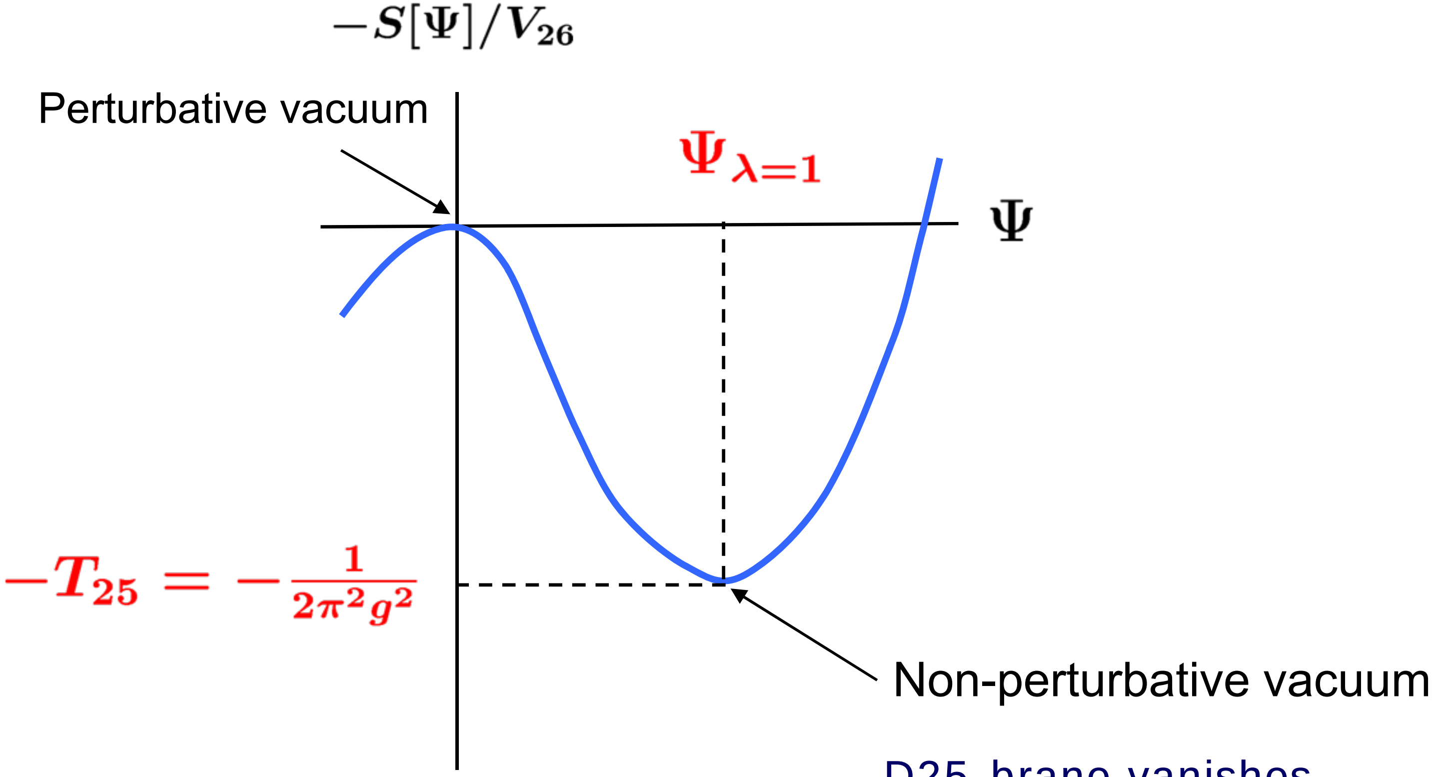
$$S[\Psi] = -\frac{1}{g^2}\left(\frac{1}{2}\langle\Psi, Q\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right)$$

$\rightarrow$ Equation of motion $Q\Psi + \Psi * \Psi = 0$

Numerical solutions using level truncation "approximation,"
Analytic solutions using the identity state,
…

- Schnabl's solution for tachyon condensation $\Psi_{\lambda=1}$

Adv.Theor.Math.Phys.10(2006)433[hep-th/0511286]



D25-brane vanishes.
No open strings.

- No BRST cohomology around Schnabl's solution proved by Ellwood-Schnabl JHEP02(2007)096[hep-th/0606142]

$$S[\Psi_{\lambda=1}+\Psi'] = S[\Psi']|_{Q\to Q'} + S[\Psi_{\lambda=1}]$$

$$A \equiv \frac{\pi}{2} B_1^L \int_1^2 dr |r\rangle$$

$$Q'A = QA + \Psi_{\lambda=1} * A + A * \Psi_{\lambda=1} \ = \ \mathcal{I}$$

In fact, using the above,

$$Q'B = 0$$

$$\Rightarrow \quad B = \mathcal{I} * B = (Q'A) * B = Q'(A*B) + A*(Q'B) = Q'(A*B)$$

- In 2007, new solutions for deformations by nonsingular marginal operator

Schnabl, hep-th/0701248; Kiermaier-Okawa-Rastelli-Zwiebach,hep-th/0701249

Solutions to the EOM: $Q\Psi + \Psi * \Psi = 0$

Extension of Schnabl/KORZ's marginal solutions to Berkovits' superstring field theory

Erler, JHEP07(2007)050[arXiv:0704.0930]; Okawa, arXiv:0704.0936, arXiv:0704.3612

Solutions to the EOM: $\eta_0(e^{-\Phi}Qe^{\Phi}) = 0$

These solutions are all generated from simple solutions based on the identity state.

I.K.-Y.Michishita, PTP118(2007)347[arXiv:0706.0409]

(Furthermore, we can generalize the above solutions.)

- Different type of new solutions for deformations by marginal operator:

Fuchs-Kroyter-Potting, arXiv:0704.2222 (bosonic SFT)

Fuchs-Kroyter, arXiv:0706.0717 (super SFT) $\quad J = i\lambda_\mu \partial X^\mu$

Generalization

Kiermaier-Okawa, arXiv:0707.4472 (bosonic SFT),
arXiv:0708.3394 (super SFT)

$\longrightarrow$ Okawa's talk in "String field theory 07" (Oct. 6, RIKEN)

Witten's bosonic string field theory

- Action: $$S[\Psi] = -\frac{1}{g^2}\left(\frac{1}{2}\langle\Psi, Q\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right)$$
- String field: $$|\Psi\rangle = \phi(x)c_1|0\rangle + A_\mu(x)\alpha^\mu_{-1}c_1|0\rangle + iB(x)c_0|0\rangle + \cdots$$

$$X^\mu(z) = x^\mu - i\sqrt{2\alpha'}\alpha_0^\mu \log z + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^\mu z^{-n},$$

$$[\alpha_n^\mu, \alpha_m^\nu] = n\delta_{n+m,0}\eta^{\mu\nu}, \qquad [x^\mu, \alpha_0^\nu] = i\sqrt{2\alpha'}\eta^{\mu\nu},$$

$$c(z) = \sum_n c_n z^{-n+1}, \qquad b(z) = \sum_n b_n z^{-n-2}, \qquad \{b_n, c_m\} = \delta_{n+m,0},$$

- BRST operator : $$Q = \oint \frac{dz}{2\pi i}\left(cT^{\mathrm{m}} + bc\partial c + \frac{3}{2}\partial^2 c\right)$$

  (Kato-Ogawa)

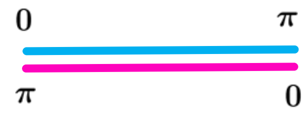
$$T^{\mathrm{m}} = -\frac{1}{4\alpha'} :\partial X_\mu \partial X^\mu:$$

- Inner product (BPZ): $\langle\cdot,\cdot\rangle : \mathcal{H}\otimes\mathcal{H}\to\mathbb{C}$

$$\langle\Psi,\Phi\rangle = \langle R(1,2)|\Psi\rangle_1|\Phi\rangle_2$$

Reflector:

$$\langle R(1,2)|(X^{\mu(1)}(\pi-\sigma)-X^{\mu(2)}(\sigma))=0,\quad\cdots$$



$$X^{\mu(r)}(\sigma_r)=x^{\mu(r)}+i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^{\mu(r)}\cos n\sigma_r,\quad \ldots$$

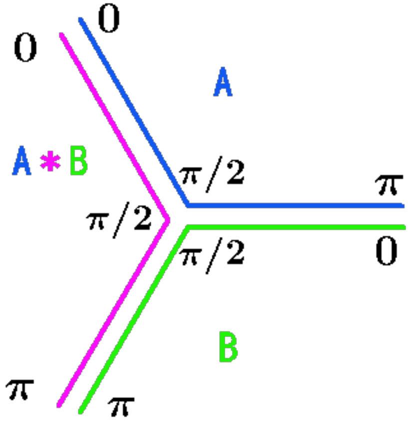
The Kinetic term is computed as

$$\begin{aligned}&\langle\Psi,Q\Psi\rangle\\&=\int d^{26}x\left(\phi(-\alpha'\partial^2-1)\phi-\alpha' A_\mu\partial^2 A^\mu+2\sqrt{2\alpha'}B\partial_\mu A^\mu+2B^2+\cdots\right)\end{aligned}$$

- Star product: $* : \mathcal{H} \otimes \mathcal{H} \to \mathcal{H}$

$$|A * B\rangle_4 = \langle V_3(1,2,3)|R(4,1)\rangle |A\rangle_2 |B\rangle_3$$

$$\langle R(1,2)|R(2,3)\rangle = \mathrm{id}_{31}$$

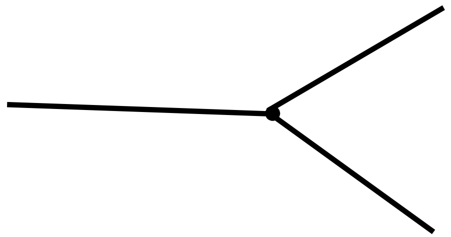


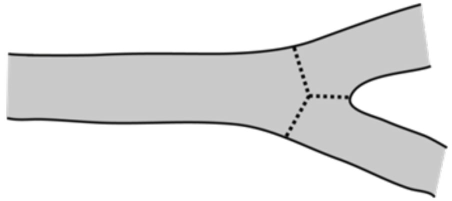
3-string vertex:

$$\langle V_3(1,2,3)|(X^{\mu(r)}(\pi-\sigma) - X^{\mu(r+1)}(\sigma)) = 0, \quad \cdots$$
$$0 \le \sigma \le \pi/2$$

- The interaction term is given by the delta functional:

$$\int dx\,(\phi(x))^3$$
$$= \int dx_1 dx_2 dx_3 \delta(x_1 - x_2)\delta(x_2 - x_3)$$
$$\phi(x_1)\phi(x_2)\phi(x_3)$$





$$\langle \Psi, \Psi * \Psi \rangle$$
$$= \langle V_3(1,2,3)|\Psi\rangle_1 |\Psi\rangle_2 |\Psi\rangle_3$$
$$\sim \int \prod_{0\leq\sigma\leq\pi/2} (\delta(X^{(1)}(\pi-\sigma) - X^{(2)}(\sigma))\delta(X^{(2)}(\pi-\sigma) - X^{(3)}(\sigma))$$
$$\times\, \delta(X^{(3)}(\pi-\sigma) - X^{(1)}(\sigma))(bc \text{ ghost} \cdots))$$
$$\times\, \Psi[X^{(1)}(\sigma),\cdots]\Psi[X^{(2)}(\sigma),\cdots]\Psi[X^{(3)}(\sigma),\cdots]$$

$$\Psi[X(\sigma),\cdots] = \langle X(\sigma),\cdots|\Psi\rangle$$

- Equation of moiton:

$$Q\Psi + \Psi * \Psi = 0$$

The action $S[\Psi]$ has gauge invariance.

- Gauge transformation (infinitesimal):

$$\delta_\Lambda \Psi = Q\Lambda + \Psi * \Lambda - \Lambda * \Psi$$

- Gauge transformation (finite)

$$\Psi' = e^{-\Lambda} * \Psi * e^{\Lambda} + e^{-\Lambda} * Qe^{\Lambda}$$

- In principle, we can compute the star product using explicit oscillator representation:

$$|R(1,2)\rangle = \int \frac{d^dp_1}{(2\pi)^d}\int \frac{d^dp_2}{(2\pi)^d}(2\pi)^d\delta^d(p_1+p_2)(c_0^{(1)}+c_0^{(2)}) \times e^{-\sum_{n\geq 1}\frac{(-1)^n}{n}\alpha_{-n}^{(1)}\alpha_{-n}^{(2)}+\sum_{n\geq 1}(-1)^n(c_{-n}^{(1)}b_{-n}^{(2)}+c_{-n}^{(1)}b_{-n}^{(2)})}c_1^{(1)}|p_1\rangle_1 c_1^{(2)}|p_2\rangle_2$$

$$\langle V_3(1,2,3)| = K^3\int \frac{d^dp_1}{(2\pi)^d}\int \frac{d^dp_2}{(2\pi)^d}\int \frac{d^dp_3}{(2\pi)^d}(2\pi)^d\delta^d(p_1+p_2+p_3) \times {}_1\langle p_1|c_{-1}^{(1)}c_0^{(1)}\;{}_2\langle p_2|c_{-1}^{(2)}c_0^{(2)}\;{}_3\langle p_3|c_{-1}^{(3)}c_0^{(3)}e^{E(1,2,3)},$$

$$E(1,2,3) = \frac{1}{2}\sum_{r,s=1,2,3}\sum_{n,m\geq 0}\alpha_n^{(r)}N_{nm}^{rs}\alpha_m^{(s)}+\sum_{r,s=3,4,5}\sum_{n\geq 1,m\geq 0}c_n^{(r)}X_{nm}^{rs}b_m^{(s)},$$

$$\langle p|c_{-1}c_0c_1|p'\rangle = (2\pi)^d\delta^d(p-p'),$$
$$\alpha_n|p\rangle = 0 \quad (n\geq 1),$$
$$\alpha_0|p\rangle = (\sqrt{2\alpha'}p)|p\rangle,$$
$$c_n|p\rangle = 0, \quad (n\geq 2), \qquad b_n|p\rangle = 0, \quad (n\geq -1), \cdots$$

- Neumann coefficients are explicitly given by:

$$K = \frac{3\sqrt{3}}{4}, \qquad \left(\frac{1+x}{1-x}\right)^k \equiv \sum_{n=0}^{\infty} \eta_n^k x^n$$

$$N^{rr}_{2n,2m} = \frac{(-1)^{n+m}}{6}\left(\frac{\eta_{2n}^{1/3}\eta_{2m}^{2/3}+\eta_{2n}^{2/3}\eta_{2m}^{1/3}}{n+m}+\frac{\eta_{2n}^{1/3}\eta_{2m}^{2/3}-\eta_{2n}^{2/3}\eta_{2m}^{1/3}}{n-m}\right),$$

$$(n,m\geq 1, n\neq m),$$

$$N^{rr}_{2n+1,2m+1} = \frac{-(-1)^{n+m}}{6}\left(\frac{\eta_{2n+1}^{1/3}\eta_{2m+1}^{2/3}+\eta_{2n+1}^{2/3}\eta_{2m+1}^{1/3}}{n+m+1}+\frac{\eta_{2n+1}^{1/3}\eta_{2m+1}^{2/3}-\eta_{2n+1}^{2/3}\eta_{2m+1}^{1/3}}{n-m}\right),$$

$$\cdots$$

Note: The Neumann matrices are essentially constructed by

$$T_{2n,2m+1} = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} d\sigma \cos(2n\sigma)\cos((2m+1)\sigma) = \frac{-4(2m+1)(-1)^{m+n}}{\pi((2n)^2-(2m+1)^2)},$$

$$(n\geq 1, m\geq 0).$$

There are some nonlinear relations among them.

However, it seems quite difficult to solve the EOM explicitly using the above.

- Using the LPP (LeClair-Peskin-Preitshopf) prescription, the reflector and the 3-string vertex are obtained by CFT correlator.

$$
\begin{aligned}
\langle R(1,2)|A\rangle_1|B\rangle_2 &= \langle A, B\rangle \\
&= \langle I \circ A(0)\, B(0)\rangle\ , \\
\langle V_3(1,2,3)|A\rangle_1|B\rangle_2|C\rangle_3 &= \langle A, B * C\rangle \\
&= \left\langle f_1^{(3)} \circ A(0)\, f_2^{(3)} \circ B(0)\, f_3^{(3)} \circ C(0)\right\rangle .
\end{aligned}
$$

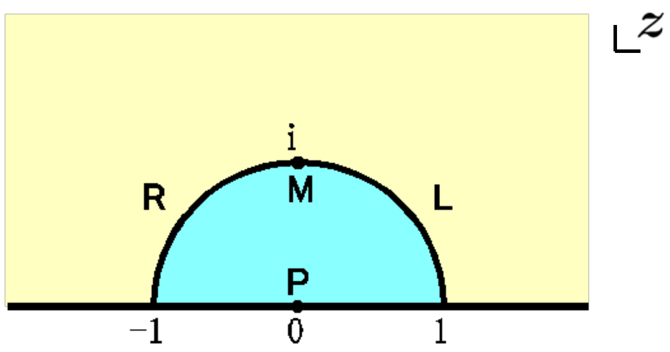
Conventionally, they are defined on UHP.
The conformal maps from half unit disk to UHP are given by:

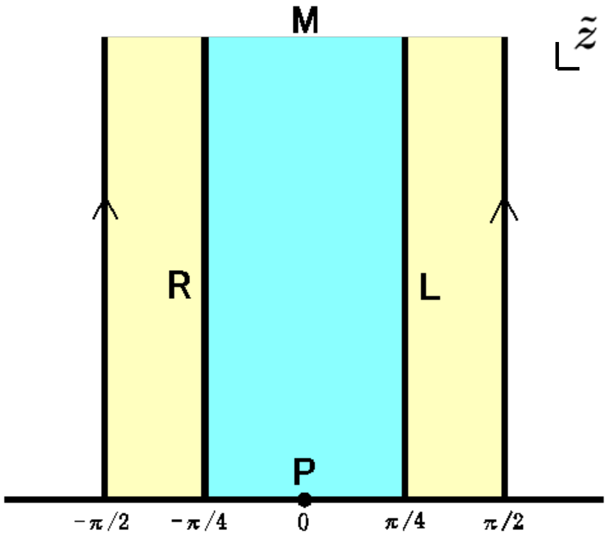
$$
I(z) = -1/z
$$

$$
\begin{aligned}
f_k^{(3)}(z) &= h^{-1}(e^{\frac{2i\pi}{3}}(h(z))^{\frac{2}{3}}) , \\
h(z) &= \frac{1+iz}{1-iz}
\end{aligned}
$$

SLIVER FRAME

From UHP z to semi-infinite cylinder $\tilde{z} = \arctan z$





Primary field $\phi(z)$ $\longrightarrow$ $\tilde{\phi}(\tilde{z}) = \left(\frac{dz}{d\tilde{z}}\right)^h \phi(z) = (\cos\tilde{z})^{-2h}\phi(\tan\tilde{z})$

In the sliver frame, new oscillators can be written by linear combinations of the conventional ones. For example,

$$\mathcal{L}_0 \equiv \tilde{L}_0 = L_0 + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{4k^2-1} L_{2k}, \qquad K_1 \equiv \tilde{L}_{-1} = L_1 + L_{-1},$$

$$\mathcal{B}_0 \equiv \tilde{b}_0 = b_0 + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{4k^2-1} b_{2k}, \qquad B_1 \equiv \tilde{b}_{-1} = b_1 + b_{-1},$$

$$\cdots$$

We define:

$$\hat{\mathcal{L}} = \mathcal{L}_0 + \mathcal{L}_0^\dagger, \quad K_1^{L/R} = \frac{1}{2} K_1 \pm \frac{1}{\pi} \hat{\mathcal{L}},$$

$$\hat{\mathcal{B}} = \mathcal{B}_0 + \mathcal{B}_0^\dagger, \quad B_1^{L/R} = \frac{1}{2} B_1 \pm \frac{1}{\pi} \hat{\mathcal{B}}$$

Using, $$U_r = \left(\frac{2}{r}\right)^{\mathcal{L}_0} = \left(\frac{2}{r}\right)^{L_0} e^{-\frac{r^2-4}{3r^2}L_2 + \frac{r^4-16}{30r^4}L_4 + \cdots}$$

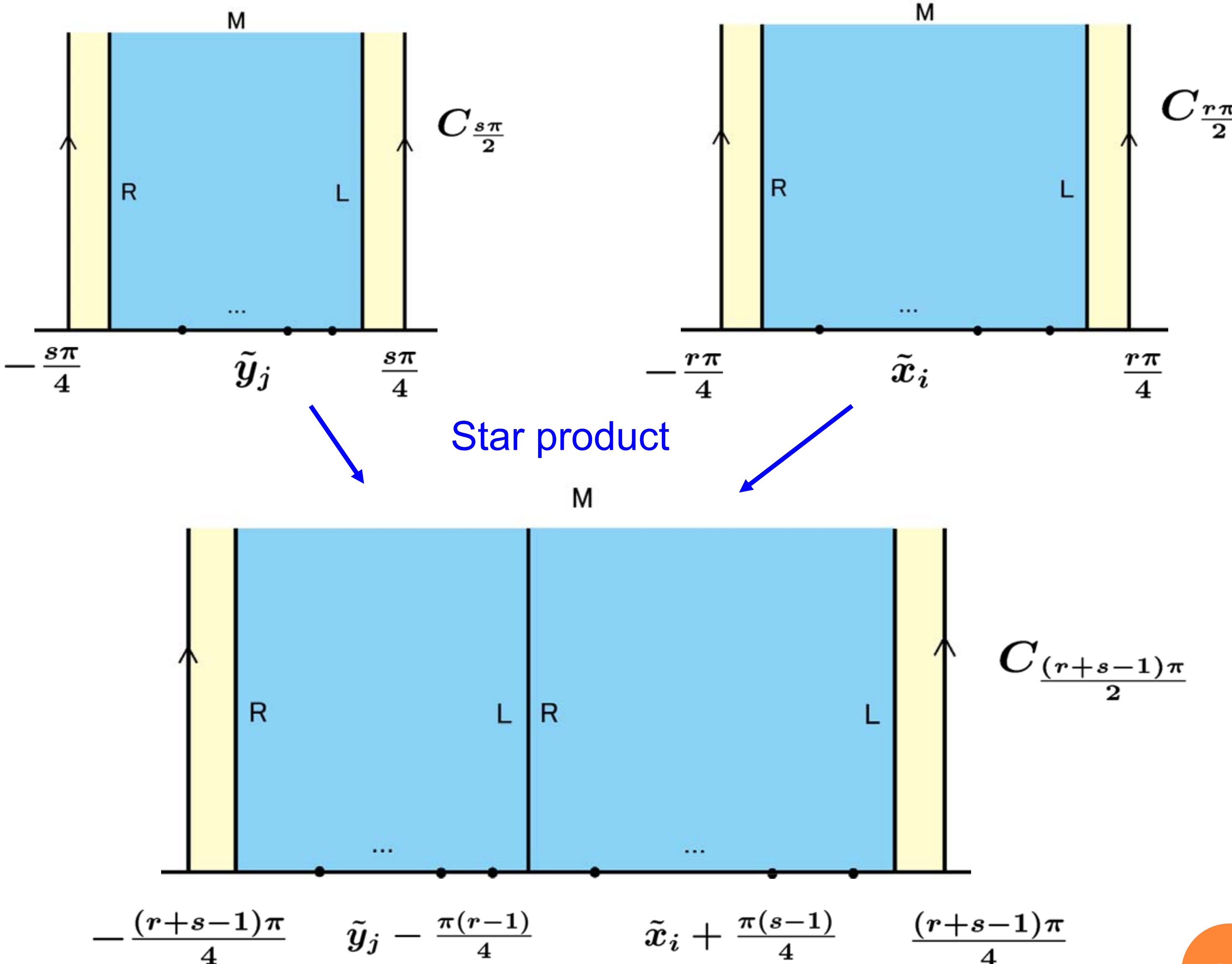
we have a "simple" star product formula:

$$U_r^\dagger U_r \tilde{\phi}_1(\tilde{x}_1)\cdots\tilde{\phi}_n(\tilde{x}_n)|0\rangle * U_s^\dagger U_s \tilde{\psi}_1(\tilde{y}_1)\cdots\tilde{\psi}_m(\tilde{y}_m)|0\rangle =$$
$$U_{r+s-1}^\dagger U_{r+s-1}\tilde{\phi}_1(\tilde{x}_1+\frac{\pi}{4}(s-1))\cdots\tilde{\phi}_n(\tilde{x}_n+\frac{\pi}{4}(s-1))\tilde{\psi}_1(\tilde{y}_1-\frac{\pi}{4}(r-1))\cdots\tilde{\psi}_m(\tilde{y}_m-\frac{\pi}{4}(r-1))|0\rangle$$

In the case of no insertion, a commutative algebra for wedge states is reproduced.

$$|r=\alpha+1\rangle = U_{\alpha+1}^\dagger U_{\alpha+1}|0\rangle = P_\alpha \qquad P_\alpha * P_\beta = P_{\alpha+\beta}$$

$|r=1\rangle = U_1^\dagger U_1|0\rangle = \mathcal{I}$ is the identity state.


M
R
L
$C_{\frac{s\pi}{2}}$
$-\frac{s\pi}{4}$
$\tilde{y}_j$
$\frac{s\pi}{4}$
M
R
L
$C_{\frac{r\pi}{2}}$
$-\frac{r\pi}{4}$
$\tilde{x}_i$
$\frac{r\pi}{4}$
Star product
M
R
L
R
L
$C_{\frac{(r+s-1)\pi}{2}}$
$-\frac{(r+s-1)\pi}{4}$
$\tilde{y}_j - \frac{\pi(r-1)}{4}$
$\tilde{x}_i + \frac{\pi(s-1)}{4}$
$\frac{(r+s-1)\pi}{4}$

SCHNABL'S SOLUTION FOR TACHYON CONDENSATION

- Noting $\{Q, \tilde{c}(\tilde{z})\} = \tilde{c}\tilde{\partial}\tilde{c}(\tilde{z}), \quad \{Q, \hat{\mathcal{B}}\} = \hat{\mathcal{L}}$

$$\hat{\mathcal{L}}^n \tilde{c}_{p_1} \tilde{c}_{p_2} \cdots \tilde{c}_{p_N} |0\rangle, \quad \hat{\mathcal{B}} \hat{\mathcal{L}}^m \tilde{c}_{q_1} \tilde{c}_{q_2} \cdots \tilde{c}_{q_M} |0\rangle,$$

generate an algebra with the star product and derivation Q .

※ $\mathcal{L}_0$-levels（eigenvalue） of the above states are
$n - p_1 - p_1 \cdots - p_N, \; 1 + m - q_1 - q_2 \cdots - q_M$, respectively.

※The star product of terms with $\mathcal{L}_0$-levels h_1, h_2 yields terms with $\mathcal{L}_0$-level h_{12} such as $h_{12} \geq h_1 + h_2$.

Ansatz for solutions with ghost number 1:

$$\Psi = \sum_{\substack{n\geq 0 \\ p\leq 1}} f_{n,p}\hat{\mathcal{L}}^n \tilde{c}_p|0\rangle + \sum_{\substack{n\geq 0 \\ p,q\leq 1}} f_{n,p,q}\hat{\mathcal{B}}\hat{\mathcal{L}}^n \tilde{c}_p\tilde{c}_q|0\rangle\,.$$

Similarly to the conventional Siegel gauge condition: $b_0\Psi = 0$,
we impose the Schnabl gauge condition:

$$\mathcal{B}_0\Psi = 0 \quad \leftrightarrow \quad 2f_{n,p,0} + (n+1)f_{n+1,p} = 0\,.$$

Furthermore, we impose twist symmetry: $(-1)^{L_0+1}\Psi = \Psi$

$$\leftrightarrow \quad f_{n,p} = 0,\ (p:\ \text{even}),\ f_{n,p,q} = 0,\ (p+q:\ \text{even})$$

In the case p, $p+q$: odd, we take the coefficients as

$$f_{n,p} = \frac{(-1)^n \pi^{-p}}{2^{n-2p+1} n!} f_{n-p+1} \,, \qquad f_{n,p,q} = \frac{(-1)^{n+q} \pi^{-p-q}}{2^{n-2(p+q)+3} n!} f_{n-p-q+2}$$

which is compatible with the gauge condition, and substitute the ansatz to the EOM. Its coefficient for $\hat{\mathcal{L}}^N \tilde{c}_1 \tilde{c}_0 |0\rangle$ implies

$$-\frac{2}{\pi}\left(-\frac{1}{2}\right)^N \left((N-1)\frac{f_N}{N!} + \sum_{n=0}^{N} \sum_{m=0}^{N-n} \frac{f_n f_m}{n! m! (N-n-m)!} \right) = 0$$

$\rightarrow$ a differential equation for the generating function:

$$\left(x\frac{d}{dx} - 1 \right) f(x) + e^x f(x)^2 = 0 \,.$$

$$f(x) \equiv \sum_{n=0}^{\infty} \frac{f_n}{n!} x^n$$

Solutuion to the diff. eq.: $f(x) = \dfrac{\lambda x}{\lambda e^x - 1}$

⇩

"Candidate" for the solution to EOM

$$\Psi_\lambda = \sum_{n=0}^{\infty} \sum_{p \geq -1,\, p:\mathrm{odd}} \frac{(-1)^n \pi^p}{n!\, 2^{n+2p+1}} f_{n+p+1} \hat{\mathcal{L}}^n \tilde{c}_{-p} |0\rangle$$
$$+ \sum_{n=0}^{\infty} \sum_{p,q \geq -1,\, p+q:\mathrm{odd}} \frac{(-1)^{n+q} \pi^{p+q}}{n!\, 2^{n+2(p+q)+3}} f_{n+p+q+2} \hat{\mathcal{B}} \hat{\mathcal{L}}^n \tilde{c}_{-p} \tilde{c}_{-q} |0\rangle,$$

$$f_n = \begin{cases} B_n & (\lambda = 1) \\ -n\lambda\, \mathrm{Li}_{1-n}(\lambda) - \delta_{n,0}\lambda & (\lambda \neq 1) \end{cases}$$

← Bernoulli number

↖ polylogarithmic function

We have checked several hundred terms of the EOM other than $\hat{\mathcal{L}}^N \tilde{c}_1 \tilde{c}_0|0\rangle$ using *Mathematica*.

However, it seems to be difficult to prove all terms directly.

$$
\begin{aligned}
&Q\Psi_\lambda + \Psi_\lambda * \Psi_\lambda \\
&= \sum_{\substack{N\geq 0;p,q\leq 1\\ p+q:\mathrm{odd}}} \frac{(-1)^N \pi^{-p-q}}{N!2^{N+2-2(p+q)}}\Bigg[(p-q-(-1)^q N)f_{N+1-p-q} \\
&+\sum_{\substack{k=0\\ k+l:\mathrm{odd}}}^{1-p}\sum_{l=0}^{1-q}\binom{1-p}{k}\binom{1-q}{l}\sum_{j=0}^{N}\sum_{n=0}^{l}\sum_{m=0}^{N-j+k}\binom{N}{j}\binom{l}{n}\binom{N-j+k}{m}(-1)^l f_{n+1+j-p-k}f_{m+1-q-l} \\
&+\sum_{\substack{k=0\\ k+l:\mathrm{even}}}^{1-p}\sum_{l=0}^{1-q}\binom{1-p}{k}\binom{1-q}{l}\sum_{j=0}^{N}\sum_{n=0}^{N-j+k+l}\binom{N}{j}\binom{N-j+k+l}{n}(-1)^{l+q}f_n f_{j+2-k-l-p-q}\Bigg]\hat{\mathcal{L}}^N\tilde{c}_p\tilde{c}_q|0\rangle \\
&+\sum_{\substack{N\geq 0;p,q,r\leq 1\\ p+q+r:\mathrm{odd}}} \frac{(-1)^N \pi^{-p-q-r}}{N!2^{N+4-2(p+q+r)}}\Bigg[-(-1)^r 2(p-q)f_{N+2-p-q-r} \\
&\quad+\sum_{k_1=0}^{1-p}\sum_{k_2=0}^{1-q}\sum_{l=0}^{1-r}\binom{1-p}{k_1}\binom{1-q}{k_2}\binom{1-r}{l}\sum_{j=0}^{N}\sum_{n=0}^{k_2+l}\sum_{m=0}^{N-j+k_1}\binom{N}{j}\binom{k_2+l}{n}\binom{N-j+k_1}{m} \\
&\qquad\times((-1)^{q+l}-(-1)^{r+k_2})f_{n+1+j-p-k_1}f_{m+2-q-r-k_2-l}\Bigg]\hat{\mathcal{B}}\hat{\mathcal{L}}^N\tilde{c}_p\tilde{c}_q\tilde{c}_r|0\rangle \\
&= 0 \quad (?)
\end{aligned}
$$

EOM can be checked using a *different* expression:

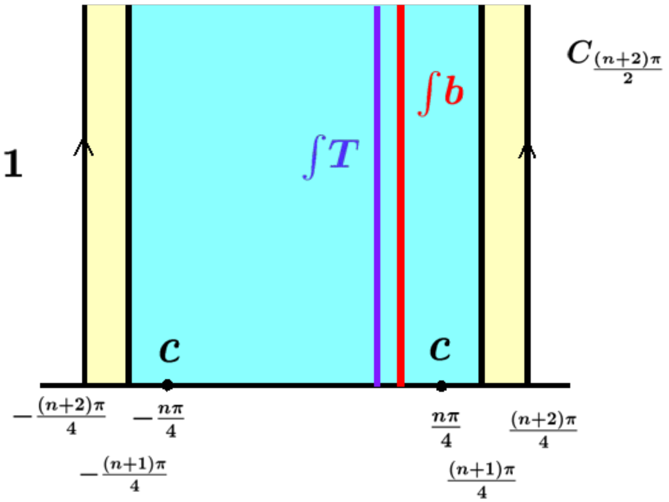
$$\Psi_\lambda = \frac{\lambda\partial_r}{\lambda e^{\partial_r}-1}\psi_r|_{r=0} = \sum_{k=0}^{\infty}\frac{f_k}{k!}\partial_r^k\psi_r|_{r=0}$$

$$\begin{aligned}
\psi_r &\equiv \frac{2}{\pi}U_{r+2}^\dagger U_{r+2}\left[-\frac{1}{\pi}\hat{\mathcal{B}}\tilde{c}(\frac{\pi r}{4})\tilde{c}(-\frac{\pi r}{4}) + \frac{1}{2}(\tilde{c}(-\frac{\pi r}{4}) + \tilde{c}(\frac{\pi r}{4}))\right]|0\rangle \\
&= \frac{2}{\pi}P_{1/2} * U_1^\dagger U_1 c_1|0\rangle * B_1^L P_r * U_1^\dagger U_1 c_1|0\rangle * P_{1/2} \\
&= \sum_{\substack{n\geq 0;p\geq -1 \\ p:\text{odd}}} \frac{(-1)^n\pi^p}{n!2^{n+2p+1}} r^{n+p+1}\hat{\mathcal{L}}^n\tilde{c}_{-p}|0\rangle \\
&\quad + \sum_{\substack{n\geq 0;p,q\geq -1 \\ p+q:\text{odd}}} \frac{(-1)^{n+q}\pi^{p+q}}{n!2^{n+2p+2q+3}} r^{n+p+q+2}\hat{\mathcal{B}}\hat{\mathcal{L}}^n\tilde{c}_{-p}\tilde{c}_{-q}|0\rangle
\end{aligned}$$

Expanding it with respect to λ, we have

$$\Psi_\lambda = -\lambda \sum_{n=0}^{\infty} \lambda^n e^{n\partial_r} \partial_r \psi_r|_{r=0} = -\sum_{n=0}^{\infty} \lambda^{n+1} \partial_r \psi_r|_{r=n}$$

$$= \sum_{n=0}^{\infty} \lambda^{n+1}$$



$$= (Q\lambda\Lambda_0) * \frac{1}{1-\lambda\Lambda_0}, \qquad \Lambda_0 \equiv B_1^L c_1|0\rangle$$

pure gauge form → (trivial) solution to the EOM!

However, if and only if $\lambda = 1$, we have $f_0 = 1(\neq 0)$

→ Euler-Maclaurin expansion

$$\Psi_{\lambda=1} = \psi_\infty - \sum_{n=0}^{\infty} \frac{B_n}{n!}\left(\partial_r^n \psi_r|_{r=\infty} - \partial_r^n \psi_r|_{r=0}\right)$$
$$= \lim_{N\to\infty}\left(\psi_{N+1} - \sum_{n=0}^{N} \partial_r \psi_r|_{r=n}\right)$$

In the last equation, *N* is a "regularization."

The first term goes to zero by L_0-level truncation. (→ Phantom)

$$\psi_{N+1} = \frac{1}{N^3}\frac{4\pi^2}{3}\left[\prod_{k=1,\leftarrow}^{\infty} e^{u_{2k}(\infty)L_{-2k}}\right]\sum_{p\geq -1;p:\mathrm{odd}}\left(\frac{2}{\pi}\right)^p c_{-p}|0\rangle$$
$$+\frac{1}{N^3}\frac{8}{3}\left[\prod_{k=1,\leftarrow}^{\infty} e^{u_{2k}(\infty)L_{-2k}}\right]\sum_{p,q\geq -1;p+q:\mathrm{odd}}(-1)^q\left(\frac{2}{\pi}\right)^{p+q} b_{-2}c_{-p}c_{-q}|0\rangle + \cdots$$
$$= \mathcal{O}(N^{-3})$$

- By ignoring the first term, we can show the EOM using the identity:

$$Q\partial_r\psi_r|_{r=0} = 0\,,$$

$$Q\partial_r\psi_r|_{r=n+1} = \sum_{k=0}^{n} \partial_r\psi_r|_{r=k} * \partial_s\psi_s|_{s=n-k}$$

⇓

$$Q\left(-\sum_{n=0}^{\infty}\lambda^{n+1}\partial_r\psi_r|_{r=n}\right) + \left(-\sum_{n=0}^{\infty}\lambda^{n+1}\partial_r\psi_r|_{r=n}\right) * \left(-\sum_{m=0}^{\infty}\lambda^{m+1}\partial_s\psi_s|_{s=m}\right) = 0\,,$$

$$\forall\lambda$$

- However, the first term (phantom) cannot be ignored when one evaluates the potential height. It gives finite contribution!

- Evaluation of the action

Based on $\langle \tilde{c}(\tilde{x})\tilde{c}(\tilde{y})\tilde{c}(\tilde{z})\rangle / V_{26} = \sin(\tilde{x}-\tilde{y})\sin(\tilde{x}-\tilde{z})\sin(\tilde{y}-\tilde{z})$

we have

$$\langle \psi_n, Q\psi_m\rangle / V_{26} = \frac{1}{\pi^2}\left(1+\cos\left(\frac{\pi(m-n)}{m+n+2}\right)\right)\left(-1+\frac{m+n+2}{\pi}\sin\left(\frac{2\pi}{m+n+2}\right)\right)$$
$$+2\sin^2\left(\frac{\pi}{m+n+2}\right)\left[-\frac{m+n+1}{\pi^2}+\frac{mn}{\pi^2}\cos\left(\frac{\pi(m-n)}{m+n+2}\right)+\frac{(m+n+2)(m-n)}{2\pi^3}\sin\left(\frac{\pi(m-n)}{m+n+2}\right)\right],$$
$$\langle \psi_n, \psi_m * \psi_k\rangle / V_{26} = \frac{(n+m+k+3)^2}{\pi^3}\sin^2\left(\frac{\pi}{n+m+k+3}\right)$$
$$\times\left[\sin\left(\frac{2\pi(n+1)}{n+m+k+3}\right)+\sin\left(\frac{2\pi(m+1)}{n+m+k+3}\right)+\sin\left(\frac{2\pi(k+1)}{n+m+k+3}\right)\right]$$

⇓

$$\sum_{m=0}^{n}\langle \partial_r\psi_r|_{r=m}, Q\partial_s\psi_s|_{s=n-m}\rangle = 0,$$
$$\sum_{m=0}^{n}\sum_{k=0}^{n-m}\langle \partial_r\psi_r|_{r=m}, \partial_s\psi_s|_{s=k} * \partial_t\psi_t|_{t=n-m-k}\rangle = 0$$

$\rightarrow$ $S[\Psi_\lambda]$ should be zero!?

- Naively, the quadratic term of the action can be evaluated as

$$\langle \Psi_\lambda, Q\Psi_\lambda \rangle = \sum_{r=0}^{\infty}\sum_{s=0}^{\infty} \lambda^{r+s+2} \langle \partial_r \psi_r, Q \partial_s \psi_s \rangle$$

$$\Rightarrow \quad \lim_{N\to\infty} \sum_{t=0}^{N} \lambda^{t+2} \sum_{r=0}^{t} \langle \partial_s \psi_s|_{s=r}, Q\partial_u \psi_u|_{u=t-r} \rangle = 0$$

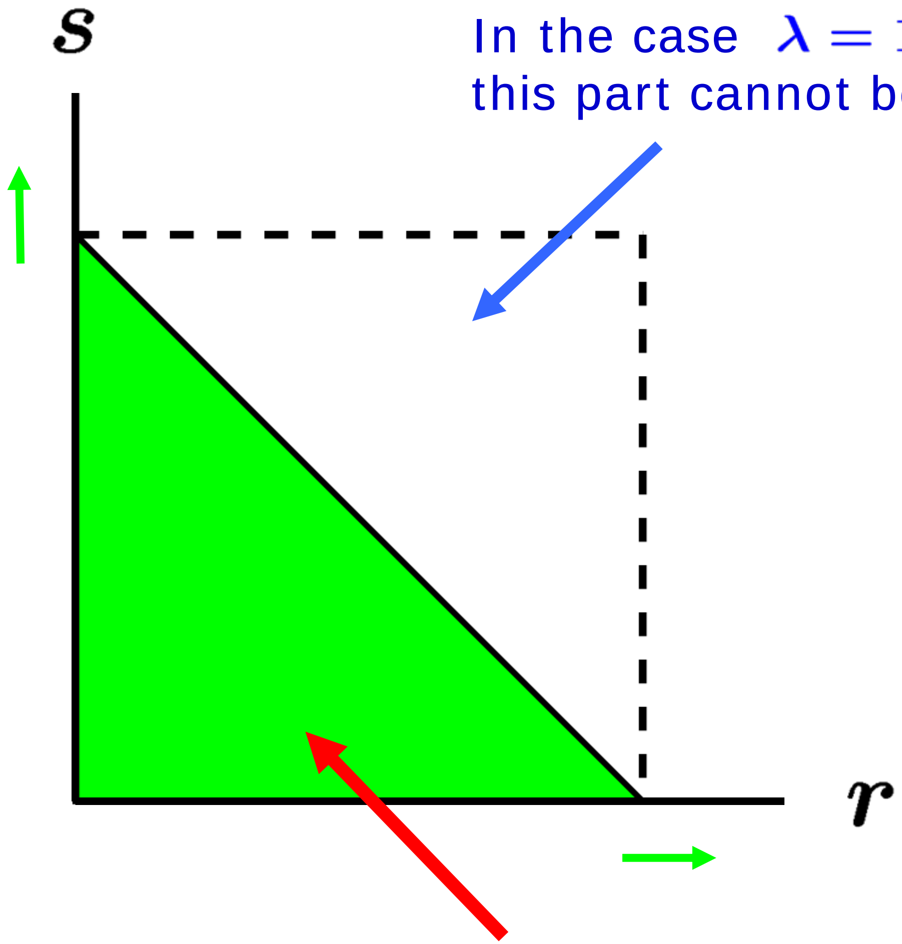
Similarly, the cubic term of the action is

$$\langle \Psi_\lambda, \Psi_\lambda * \Psi_\lambda \rangle = -\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\sum_{t=0}^{\infty} \lambda^{r+s+t+3} \langle \partial_r \psi_r, \partial_s \psi_s * \partial_t \psi_t \rangle$$

$$\Rightarrow \quad -\lim_{N\to\infty} \sum_{n=0}^{N} \lambda^{n+3} \sum_{m=0}^{n} \sum_{k=0}^{n-m} \langle \partial_r \psi_r|_{r=m}, \partial_s \psi_s|_{s=k} * \partial_t \psi_t|_{t=n-m-k} \rangle = 0$$

$$\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\lambda^{r+s+2}\langle\partial_r\psi_r, Q\partial_s\psi_s\rangle \Rightarrow \lim_{N\to\infty}\sum_{r=0}^{N}\sum_{s=0}^{N}\lambda^{r+s+2}\langle\partial_r\psi_r, Q\partial_s\psi_s\rangle$$

In the case $\lambda = 1$,
this part cannot be ignored.

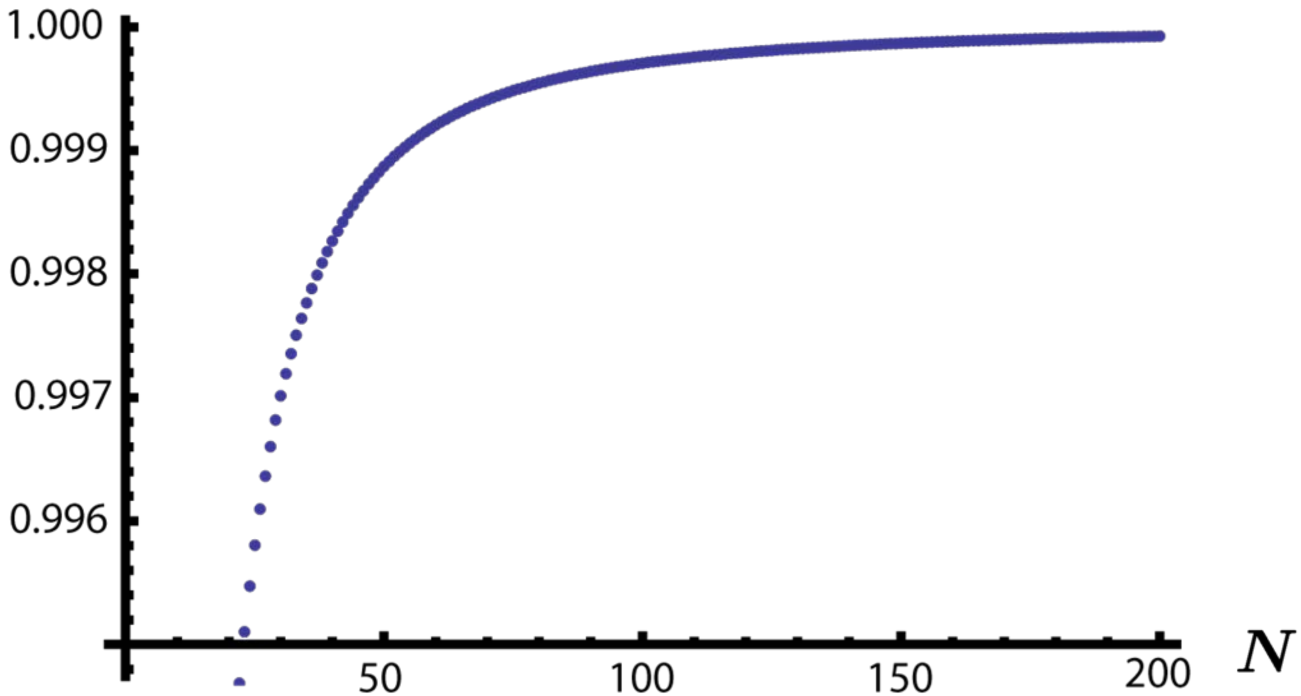


The lower triangle part gives zero.

In the case $\lambda = 1$, using $\Psi^{(N)}_{\lambda=1} \equiv \psi_{N+1} - \sum_{n=0}^{N} \partial_r \psi_r|_{r=n}$

the action is numerically calculated as follows:

$$(2\pi^2 g^2) S[\Psi^{(N)}_{\lambda=1}]/V_{26}$$



The large *N* limit can be evaluated analytically:

$$\lim_{N\to\infty} S[\Psi_{\lambda=1}^{(N)}]/V_{26} = \frac{1}{2\pi^2 g^2}\,.$$

Similarly, with $\Psi_{\lambda\neq 1}^{(N)} \equiv -\sum_{n=0}^{N} \lambda^{n+1} \partial_r \psi_r|_{r=n}$, we can show

$$\lim_{N\to\infty} S[\Psi_{|\lambda|<1}^{(N)}]/V_{26} = 0\,.$$

In the above sense,

$$S[\Psi_\lambda]/V_{26} = \begin{cases} \frac{1}{2\pi^2 g^2} = T_{25} & (\lambda = 1) \\ 0 & (|\lambda| < 1) \end{cases}$$

: tachyon vacuum

: pure gauge

Schnabl / KORZ's marginal solution

- A map from solution to solution

Suppose $\{P_\alpha\}_{\alpha\geq 0}$ such as

$$QP_\alpha = 0\,, \quad P_\alpha * P_\beta = P_{\alpha+\beta}\,, \quad P_{\alpha=0} = \mathcal{I}$$

and associated $A^{(\gamma)}$ such as $QA^{(\gamma)} = \mathcal{I} - P_\gamma$

then

$$\Psi^{(\alpha,\beta)}(\psi) = P_\alpha * \frac{1}{1+\psi * A^{(\alpha+\beta)}} * \psi * P_\beta$$

gives a map form solution to solution.

Because Q is a derivation, we have a relation:

$$Q\Psi^{(\alpha,\beta)}(\psi) + \Psi^{(\alpha,\beta)}(\psi) * \Psi^{(\alpha,\beta)}(\psi)$$
$$= P_\alpha * \frac{1}{1+\psi * A^{(\alpha+\beta)}} * (Q\psi + \psi * \psi) * \frac{1}{1+A^{(\alpha+\beta)} * \psi} * P_\beta\,.$$

Therefore, $Q\psi + \psi * \psi = 0$

$$\Rightarrow \quad Q\Psi^{(\alpha,\beta)}(\psi) + \Psi^{(\alpha,\beta)}(\psi) * \Psi^{(\alpha,\beta)}(\psi) = 0\,.$$

- Explicit example of $\{P_\alpha\}_{\alpha\geq 0}$ and $A^{(\gamma)}$:

$$P_\alpha = |\alpha+1\rangle = U^\dagger_{\alpha+1}U_{\alpha+1}|0\rangle = e^{-\frac{\alpha-1}{2}\hat{\mathcal{L}}}|0\rangle = e^{-\frac{\pi}{2}\alpha K_1^L}\mathcal{I}\,,$$

$$A^{(\gamma)} = \int_0^\gamma d\alpha \frac{\pi}{2} B_1^L P_\alpha\,.$$

- In order to solve the EOM using $\Psi^{(\alpha,\beta)}(\,\cdot\,)$ a solution ψ : $Q\psi + \psi * \psi = 0$ is necessary.

Instead, we impose stronger conditions:

$$Q\hat{\psi} = 0, \quad \hat{\psi} * \hat{\psi} = 0$$

which imply that $\hat{\psi}$ is a solution.

From this <u>simple</u> solution $\hat{\psi}$, we can **generate** complicated solutions by $\Psi^{(\alpha,\beta)}(\hat{\psi})$.

- Example of BRST-invariant and nilpotent string field:

$$\hat{\psi} = \lambda_s \hat{\psi}_s + \lambda_m \hat{\psi}_m \,,$$
$$\hat{\psi}_s = Q\hat{\Lambda}_0 \,, \qquad \hat{\Lambda}_0 \equiv U_1^\dagger U_1 B_1^L c_1 |0\rangle \,,$$
$$\hat{\psi}_m = U_1^\dagger U_1 cJ(0)|0\rangle \,.$$

Here, $J(z)$ is nonsingular marginal operator:

$$J(z)J(0) \sim \textbf{finite}. \quad (z \to 0)$$

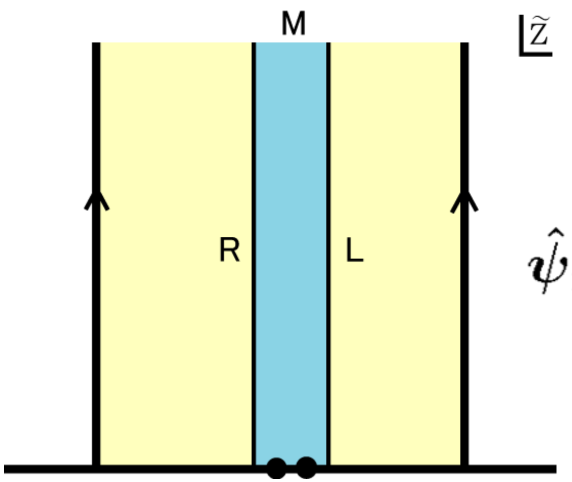
Ex.)

$$J = i\partial X^+$$

Light-cone direction

$$J = :e^{X^0}:$$

Rolling tachyon



$$\hat{\psi}_m * \hat{\psi}_m = 0$$

$$cJ(z)cJ(0) \sim 0. \quad (z \to 0)$$

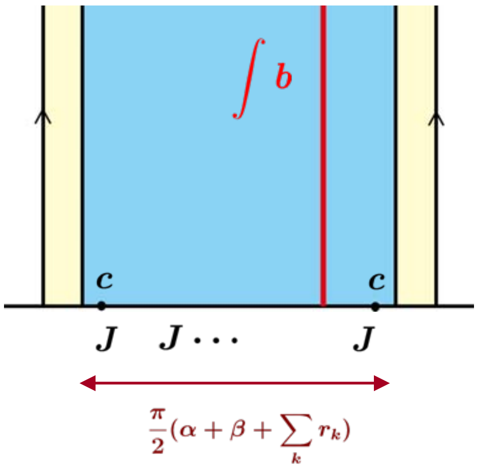
Marginal solution

$$\Psi^{(\alpha,\beta)}(\lambda_{\rm m}\hat{\psi}_{\rm m}) = \sum_{n=1}^{\infty} \lambda_{\rm m}^{n}\psi_{{\rm m},n}\,.$$

In the case $\alpha=\beta=1/2$
Schnabl / KORZ's solution

$$\begin{aligned}
\psi_{{\rm m},1} &= U^{\dagger}_{\alpha+\beta+1}U_{\alpha+\beta+1}\tilde{c}\tilde{J}(\frac{\pi}{4}(\beta-\alpha))|0\rangle\,,\\
\psi_{{\rm m},k+1} &= \left(-\frac{\pi}{2}\right)^{k}\int_0^{\alpha+\beta} dr_1\cdots\int_0^{\alpha+\beta} dr_k\, U^{\dagger}_{\alpha+\beta+1+\sum_{l=1}^{k}r_l}U_{\alpha+\beta+1+\sum_{l=1}^{k}r_l}\prod_{m=0}^{k}\tilde{J}\Big(\frac{\pi}{4}(\beta-\alpha-\sum_{l=1}^{m}r_l+\sum_{l=m+1}^{k}r_l)\Big)\\
&\quad\times\Big[-\frac{1}{\pi}\hat{\mathcal{B}}\tilde{c}(\frac{\pi}{4}(\beta-\alpha+\sum_{l=1}^{k}r_l))\tilde{c}(\frac{\pi}{4}(\beta-\alpha-\sum_{l=1}^{k}r_l))+\frac{1}{2}\Big(\tilde{c}(\frac{\pi}{4}(\beta-\alpha+\sum_{l=1}^{k}r_l))+\tilde{c}(\frac{\pi}{4}(\beta-\alpha-\sum_{l=1}^{k}r_l))\Big)\Big]|0\rangle.
\end{aligned}$$

$$\Psi^{(\alpha,\beta)}(\lambda_{\rm m}\hat{\psi}_{\rm m}) \sim \sum \lambda_{\rm m}^{n}\int dr_k$$



TACHYON SOLUTION (REVISITED)

Let us consider a solution generated from a BRST-inv and nilpotent $\boldsymbol{\lambda}_{\rm s}\hat{\psi}_{\rm s}$:

$$\Psi^{(\alpha,\beta)}(\boldsymbol{\lambda}_{\rm s}\hat{\psi}_{\rm s}) = \sum_{n=1}^{\infty} \boldsymbol{\lambda}_{\rm s}^n \psi_{{\rm s},n}\,.$$

Each terms can be rewritten as :

$$\begin{aligned}
\psi_{{\rm s},n} &= P_\alpha * (Q\hat{\Lambda}_0) * P_\beta * (P_\alpha * \hat{\Lambda}_0 * P_\beta - \mathcal{I})^{n-1} \\
&= -\sum_{l=0}^{n-1} \frac{(-1)^{n-1-l}(n-1)!}{l!(n-1-l)!} \partial_t \psi_{t,l}^{(\alpha,\beta)}|_{t=0}\,, \\
\psi_{t,n}^{(\alpha,\beta)} &= \frac{2}{\pi} U^\dagger_{n(\alpha+\beta)+t+\alpha+\beta+1} U_{n(\alpha+\beta)+t+\alpha+\beta+1} \Big[\\
&\quad -\frac{1}{\pi}\hat{\mathcal{B}}\tilde{c}(\frac{\pi}{4}(\beta-\alpha+t+n(\alpha+\beta)))\tilde{c}(\frac{\pi}{4}(\beta-\alpha-t-n(\alpha+\beta))) \\
&\quad +\frac{1}{2}\Big\{\tilde{c}(\frac{\pi}{4}(\beta-\alpha+t+n(\alpha+\beta))) + \tilde{c}(\frac{\pi}{4}(\beta-\alpha-t-n(\alpha+\beta)))\Big\}\Big]|0\rangle\,.
\end{aligned}$$

Exchanging the order of double sum, we have

$$\Psi^{(\alpha,\beta)}(\lambda_s\hat{\psi}_s) = -\sum_{l=0}^{\infty}\lambda_S^{l+1}\partial_t\psi_{t,l}^{(\alpha,\beta)}|_{t=0}\,.$$

Here,we have redefined the parameter: $\lambda_S \equiv \dfrac{\lambda_s}{\lambda_s+1}\,.$

Furthermore, we can compute as

$$\begin{aligned}\Psi^{(\alpha,\beta)}(\lambda_s\hat{\psi}_s) &= e^{\frac{\pi}{4}(\beta-\alpha)K_1}(\alpha+\beta)^{\frac{D}{2}}\left(-\sum_{l=0}^{\infty}\lambda_S^{l+1}\partial_r\psi_r|_{r=l}\right)\\ &= e^{\frac{\pi}{4}(\beta-\alpha)K_1}(\alpha+\beta)^{\frac{D}{2}}\frac{\lambda_S\partial_r}{\lambda_S e^{\partial_r}-1}\psi_r|_{r=0}\\ &= e^{\frac{\pi}{4}(\beta-\alpha)K_1}(\alpha+\beta)^{\frac{D}{2}}\Psi_{\lambda=\lambda_S}\,.\end{aligned}$$

Schnabl's solution

Note: $K_1 = L_1 + L_{-1},\ D = \mathcal{L}_0 - \mathcal{L}_0^{\dagger}$ are BPZ odd, commutative with Q and derivations.

Using this relation and property of $\boldsymbol{\Psi_\lambda}$, we conclude

$$\boldsymbol{S[\Psi^{(\alpha,\beta)}(\lambda_s\hat{\psi}_s)]/V_{26}} = \boldsymbol{S[\Psi_{\lambda=\lambda_S}]/V_{26}} = \begin{cases} \frac{1}{2\pi^2 g^2} & (\lambda_S = 1) \\ 0 & (|\lambda_S| < 1) \end{cases} .$$

Note: $\boldsymbol{\lambda_S = 1 \leftrightarrow \lambda_s = \infty}$

Formally, the solution has pure gauge form:

$$\Psi^{(\alpha,\beta)}(\lambda_s\hat{\psi}_s) = Q(\lambda_S P_\alpha * \hat{\Lambda}_0 * P_\beta) * \frac{1}{1 - \lambda_S P_\alpha * \hat{\Lambda}_0 * P_\beta} .$$

EXTENSION TO SUPERSTRING FIELD THEORY

Berkovits' WZW-type superstring field theory (NS sector):

$$S_{\rm NS}[\Phi] = -\frac{1}{g^2}\int_0^1 dt \langle\!\langle (\eta_0\Phi)(e^{-t\Phi}Qe^{t\Phi})\rangle\!\rangle$$

$$= -\frac{1}{g^2}\sum_{M,N=0}^{\infty}\frac{(-1)^M}{(M+N+2)(M+N+1)M!N!}\langle\!\langle (\eta_0\Phi)\Phi^M(Q_{\rm B}\Phi)\Phi^N\rangle\!\rangle \,.$$

String feild Φ : #ghost 0, #picture 0, Grassmann even,
written by $b, c, \phi, \xi, \eta \quad (\beta = e^{-\phi}\partial\xi, \gamma = \eta e^{\phi})$

$$Q = \oint \frac{dz}{2\pi i}(c(T^{\rm m} - \tfrac{1}{2}(\partial\phi)^2 - \partial^2\phi + \partial\xi\eta) + bc\partial c + \eta e^{\phi}G^{\rm m} - \eta\partial\eta e^{2\phi}b)(z),$$

$$\eta_0 = \oint \frac{dz}{2\pi i}\eta(z).$$

- n-string vertex is given by CFT correlator in the large Hilbert space.

$$\langle V_n|A_1\rangle\cdots|A_n\rangle = \langle\!\langle A_1\cdots A_n\rangle\!\rangle$$
$$= \left\langle f_1^{(n)}\circ A_1(0)\ \cdots\ f_n^{(n)}\circ A_n(0)\right\rangle$$
$$= \langle A_1, (\cdots(A_2 * A_3) * \cdots * A_{n-1}) * A_n\rangle = \langle A_1, A_2 * \cdots * A_n\rangle$$

$$f_k^{(n)}(z) = h^{-1}(e^{\frac{2i\pi k}{n}}(h(z))^{\frac{2}{n}}), \qquad h(z) = \frac{1+iz}{1-iz}$$

We can use the same techniques (the sliver frame, star product formula, wedge states,…) as the bosonic case.

EOM: $\eta_0(e^{-\Phi}Qe^{\Phi}) = 0 \quad \leftrightarrow \quad Q(e^{\Phi}\eta_0 e^{-\Phi}) = 0$

Gauge tr.: $\delta e^{\Phi} = \Xi_1 * e^{\Phi} + e^{\Phi} * \Xi_2,\ \ Q\Xi_1 = 0,\ \ \eta_0\Xi_2 = 0.$

We have found *a map from solution to solution* similarly to the bosonic case.

Suppose $\{P_\alpha\}_{\alpha\geq 0}$ such as

$$QP_\alpha = 0, \quad \eta_0 P_\alpha = 0,$$
$$P_\alpha * P_\beta = P_{\alpha+\beta}, \quad P_{\alpha=0} = \mathcal{I}$$

and associated $\hat{A}^{(\gamma)}$ which satisfies

$$\eta_0 Q \hat{A}^{(\gamma)} = \mathcal{I} - P_\gamma$$

Then,

$$\Phi^{(\alpha,\beta)}_{(1)}(\phi) = \log(1 + P_\alpha * f_{(1)}(\phi) * P_\beta)\,,$$

$$f_{(1)}(\phi) = \frac{1}{1 + (e^{\phi}\eta_0 e^{-\phi})Q\hat{A}^{(\alpha+\beta)}}(e^{\phi} - 1)\,,$$

$$\Phi^{(\alpha,\beta)}_{(2)}(\phi) = \log(1 + P_\alpha * f_{(2)}(\phi) * P_\beta)\,,$$

$$f_{(2)}(\phi) = (e^{\phi} - 1)\frac{1}{1 - \eta_0 \hat{A}^{(\alpha+\beta)}(e^{-\phi}Qe^{\phi})}\,,$$

$$\Phi^{(\alpha,\beta)}_{(3)}(\phi) = -\log(1 - P_\alpha * f_{(3)}(\phi) * P_\beta)\,,$$

$$f_{(3)}(\phi) = \frac{1}{1 - (e^{-\phi}Qe^{\phi})\eta_0 \hat{A}^{(\alpha+\beta)}}(1 - e^{-\phi})\,,$$

$$\Phi^{(\alpha,\beta)}_{(4)}(\phi) = -\log(1 - P_\alpha * f_{(4)}(\phi) * P_\beta)\,,$$

$$f_{(4)}(\phi) = (1 - e^{-\phi})\frac{1}{1 + Q\hat{A}^{(\alpha+\beta)}(e^{\phi}\eta_0 e^{-\phi})}\,,$$

give maps from solution to solution.

We can check the EOM by using relations:

$$e^{\Phi_{(1)}^{(\alpha,\beta)}(\phi)}\eta_0 e^{-\Phi_{(1)}^{(\alpha,\beta)}(\phi)} = e^{\Phi_{(4)}^{(\alpha,\beta)}(\phi)}\eta_0 e^{-\Phi_{(4)}^{(\alpha,\beta)}(\phi)} = P_\alpha \frac{1}{1+(e^{\phi}\eta_0 e^{-\phi})Q\hat{A}^{(\alpha+\beta)}}(e^{\phi}\eta_0 e^{-\phi})P_\beta,$$

$$e^{-\Phi_{(2)}^{(\alpha,\beta)}(\phi)}Q e^{\Phi_{(2)}^{(\alpha,\beta)}(\phi)} = e^{-\Phi_{(3)}^{(\alpha,\beta)}(\phi)}Q e^{\Phi_{(3)}^{(\alpha,\beta)}(\phi)} = P_\alpha (e^{-\phi}Qe^{\phi})\frac{1}{1-\eta_0\hat{A}^{(\alpha+\beta)}(e^{-\phi}Qe^{\phi})}P_\beta$$

Explicit example of $\{P_\alpha\}_{\alpha\geq 0}$ and $\hat{A}^{(\gamma)}$:

$$P_\alpha = |\alpha+1\rangle = U^\dagger_{\alpha+1}U_{\alpha+1}|0\rangle = e^{-\frac{\alpha-1}{2}\hat{\mathcal{L}}}|0\rangle = e^{-\frac{\pi}{2}\alpha K_1^L}\mathcal{I},$$

$$\hat{A}^{(\gamma)} = \int_0^\gamma d\alpha \log\left(\frac{\alpha}{\gamma}\right)\left(\frac{\pi}{2}J_1^{--L} + \alpha\frac{\pi^2}{4}\tilde{G}_1^{-L}B_1^L\right)P_\alpha .$$

$$J^{--}(z) \equiv \xi b(z), \quad \tilde{G}^{-} \equiv [Q, J^{--}(z)]$$

⟹ J_1^{--L}, $\tilde{G}_1^{-L}$ are defined in the same way as B_1^L.

- To solve the EOM using $\Phi^{(\alpha,\beta)}_{(i)}(\,\cdot\,)$
  a solution $\phi : \eta_0(e^{-\phi}Qe^{\phi}) = 0$ is necessary.

Instead, we impose stronger conditions

$$\eta_0 Q\hat{\phi} = 0, \quad \hat{\phi} * \hat{\phi} = 0, \quad \hat{\phi} * \eta_0\hat{\phi} = 0, \quad \hat{\phi} * Q\hat{\phi} = 0$$

which implies $\hat{\phi}$ is a solution. For example,

$$\hat{\phi} = U_1^{\dagger} U_1 c\xi e^{-\phi}\psi^{+}(0)|0\rangle$$ (light-cone direction)

From a simple solution $\hat{\phi}$, we can generate
  more complicated solution by $\Phi^{(\alpha,\beta)}_{(i)}(\hat{\phi})$

In particular, $\alpha = \beta = 1/2$: Erler / Okawa's solution

SUMMARY AND FUTURE DIRECTIONS

- Since Schnabl's construction of tachyon solution (2005), there have been new developments in open string field theories.
- In this year, new marginal solutions using nonsingular (super)current are constructed in both bosonic and super string field theory.
- They are all generated from simple solutions by maps from solution to solution.
- For more general (super)currents, new marginal solutions are constructed.

- How about other solutions?

  For example, we can generate new "regular" solutions from Takahashi-Tanimoto / Kishimoto-Takahashi's solution (bosonic/super), which are based on the identity state, using maps from solution to solution.

- How about gauge equivalence among solutions?

- Physical meaning of obtained solutions? BRST cohomology around them?

- We should define "regularity" of string fields because some formal treatments might be dangerous.

On gauge equivalence

- Using path-order forms with respect to the star product, "maps from solution to solution" can be rewritten as gauge transformations.

In the case of our explicit examples,

bosonic: $\Psi^{(\alpha,\beta)}(\hat{\psi}) \quad \sim \quad \hat{\psi}$

super: $\Phi^{(\alpha,\beta)}_{(i)}(\hat{\phi}) \quad \sim \quad \hat{\phi}$

Based on wedge states without the identity state (↑ left-hand side)

Based on the identity state (↑ right-hand side)

This may imply the gauge transformations are singular.