Gauge Invariant overlaps for Classical Solutions in Open String Field Theory

Isao Kishimoto

References:

T. Kawano, I. K., T. Takahashi,

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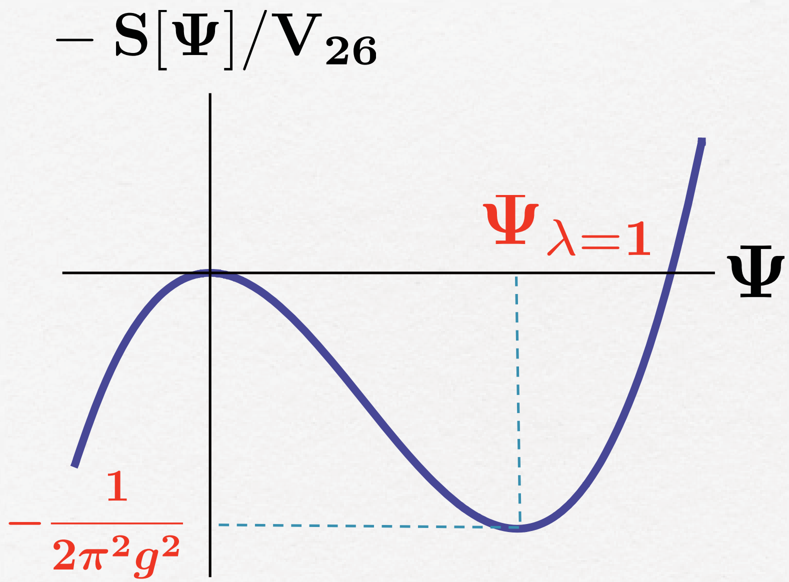
Contents

Non-perturbative vacuum in OSFT

- **Schnabl's solution** (2005): $\Psi_{\lambda=1}$
  - Non-perturbative vacuum in OSFT
  - Potential height=D25-brane tension
  - Vanishing cohomology around $\Psi_{\lambda=1}$

    [Ellwood-Schnabl(2006)]

  - Phantom term (?)
  - Closed string?

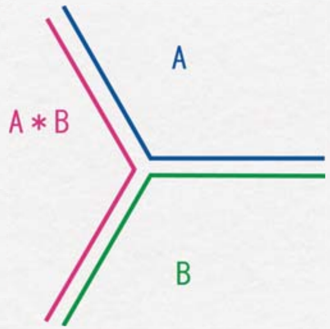


Witten's open string field theory

- Action: $S[\Psi] = -\frac{1}{g^2}\left(\frac{1}{2}\langle\Psi, Q\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right)$

$$|\Psi\rangle = \phi(x)c_1|0\rangle + A_\mu(x)\alpha^\mu_{-1}c_1|0\rangle + iB(x)c_0|0\rangle + \cdots$$

$$Q = \oint \frac{dz}{2\pi i}\left(cT^{\rm m} + bc\partial c + \frac{3}{2}\partial^2 c\right)$$



- Equation of motion: $Q\Psi + \Psi * \Psi = 0$
- Gauge transformation: $\delta_\Lambda \Psi = Q\Lambda + \Psi * \Lambda - \Lambda * \Psi$

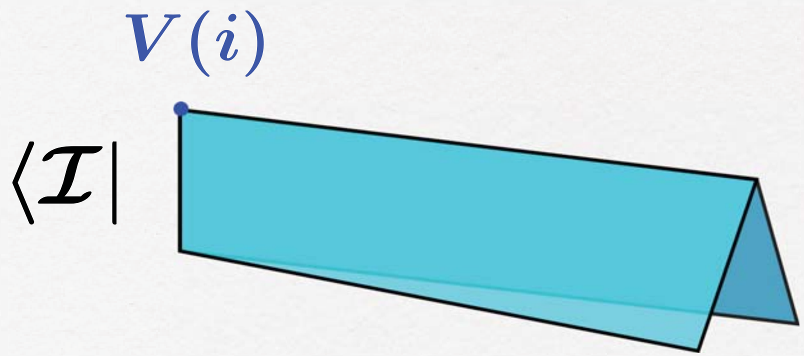
Gauge invariant overlap

$$\mathcal{O}_V(\Psi) = \langle \mathcal{I}|V(i)|\Psi\rangle = \langle \Phi_V, \Psi\rangle$$

[Zwiebach(1992),...]

$$V(i) = c(i)c(-i)V_{\rm m}(i,-i)$$

matter primary, dim (1,1)



On-shell closed string state: $\Phi_V = V(i)|\mathcal{I}\rangle$

Gauge invariance of $\mathcal{O}_V(\Psi)$

$$Q\Phi_V = 0, \quad \langle \Phi_V, \Psi * \Lambda \rangle = \langle \Phi_V, \Lambda * \Psi \rangle$$

on-shell $\qquad$ V: dim (0,0), midpoint

$$\therefore \mathcal{O}_V(\delta_\Lambda \Psi) = 0$$

- In particular, it vanishes for pure gauge solution:

$$\mathcal{O}_V(e^{-\Lambda} Q e^{\Lambda}) = 0$$

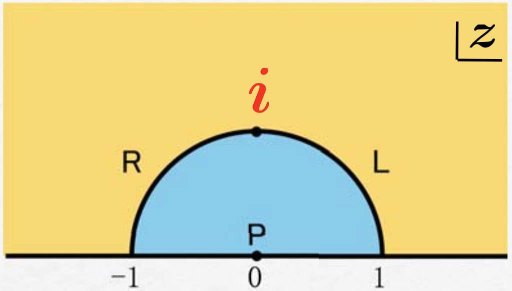
Schnabl's solution

$$
\begin{aligned}
\Psi_\lambda &= \frac{\lambda\partial_r}{\lambda e^{\partial_r}-1}\psi_r|_{r=0} = \sum_{n=0}^{\infty}\frac{f_n(\lambda)}{n!}\partial_r^n\psi_r|_{r=0} \\
&= \begin{cases} \lim_{N\to\infty}\left(\psi_{N+1}-\sum_{n=0}^{N}\partial_r\psi_r|_{r=n}\right) & (\lambda=1) \\ -\sum_{n=0}^{\infty}\lambda^{n+1}\partial_r\psi_r|_{r=n} & (\lambda\neq 1) \end{cases}
\end{aligned}
$$

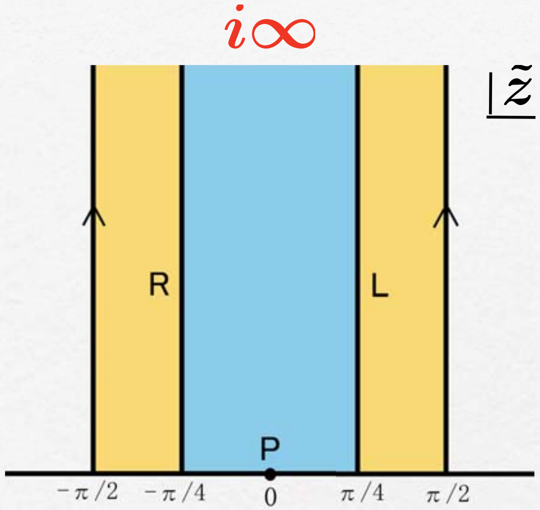
$$
\psi_r \equiv \frac{2}{\pi}U_{r+2}^\dagger U_{r+2}\left[-\frac{1}{\pi}(\mathcal{B}_0+\mathcal{B}_0^\dagger)\tilde{c}(\frac{\pi r}{4})\tilde{c}(-\frac{\pi r}{4})+\frac{1}{2}(\tilde{c}(-\frac{\pi r}{4})+\tilde{c}(\frac{\pi r}{4}))\right]|0\rangle
$$

$$
U_{r+2} = \left(\frac{2}{r+2}\right)^{\mathcal{L}_0}
$$

Sliver frame



$$\arctan z = \tilde{z}$$



$\tilde{\phi}(\tilde{z}) = (\cos\tilde{z})^{-2h}\phi(\tan\tilde{z})$ **for primary with dim *h***

$$\mathcal{B}_0 = b_0 + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{4k^2-1} b_{2k} \qquad \mathcal{L}_0 = \{Q, \mathcal{B}_0\} = L_0 + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{4k^2-1} L_{2k}$$

Evaluation of the action

$$S[\Psi_\lambda]/V_{26} = \begin{cases} \dfrac{1}{2\pi^2 g^2} & (\lambda = 1) \\ 0 & (|\lambda| < 1) \end{cases}$$

- Analytically evaluated using the "regularized" expression of the solution [Schnabl (2005), Okawa / Fuchs-Kroyter (2006)]
- "Phantom term" gives finite contribution

Analytic evaluation of $\mathcal{O}_V(\Psi)$ for Schnabl's solution

- It is convenient to compute in the sliver frame:

$$\mathcal{O}_V(\Psi_\lambda) = \langle \Phi_V, \Psi_\lambda \rangle = \langle \mathcal{I} | \Phi_V * \Psi_\lambda \rangle$$

Written in terms of sliver frame

Simple formulas in sliver frame

On-shell closed string states in sliver frame

$$\Phi_V = \sum_{m,n} \zeta_{mn} c(i) V_m(i) c(-i) V_n(-i) |\mathcal{I}\rangle$$

$$= \sum_{m,n} \zeta_{mn} U_1^\dagger U_1 \tilde{c}(i\infty) \tilde{V}_m(i\infty) \tilde{c}(-i\infty) \tilde{V}_n(-i\infty) |0\rangle$$

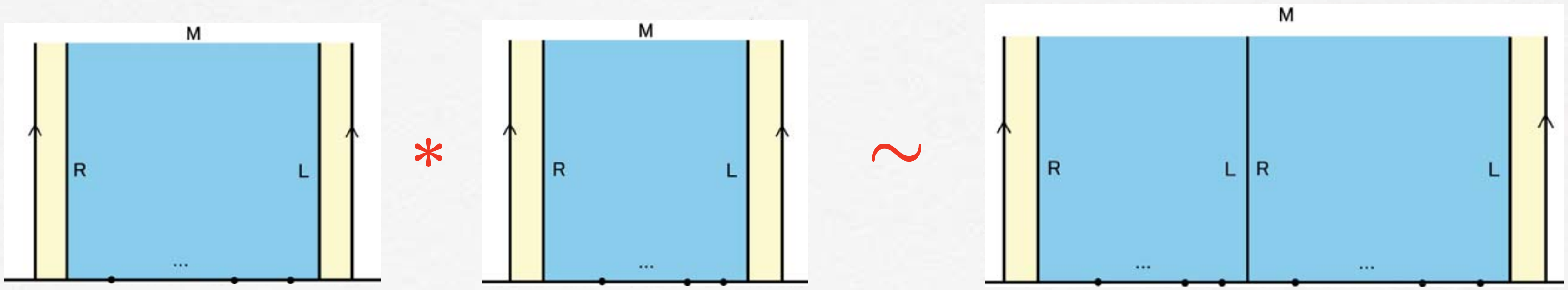
- "regularization" $\pm i\infty \longrightarrow \pm iM$

$$\Phi_{V,M} \equiv \sum_{m,n} \zeta_{mn} U_1^\dagger U_1 \tilde{c}(iM) \tilde{V}_m(iM) \tilde{c}(-iM) \tilde{V}_n(-iM) |0\rangle$$

Star product formula

- Formal algebraic computation can be performed by

$$U_r^\dagger U_r \tilde{\phi}_1(\tilde{x}_1)\cdots\tilde{\phi}_n(\tilde{x}_n)|0\rangle * U_s^\dagger U_s \tilde{\psi}_1(\tilde{y}_1)\cdots\tilde{\psi}_m(\tilde{y}_m)|0\rangle$$
$$= U_{r+s-1}^\dagger U_{r+s-1}\tilde{\phi}_1(\tilde{x}'_1)\cdots\tilde{\phi}_n(\tilde{x}'_n)\tilde{\psi}_1(\tilde{y}'_1)\cdots\tilde{\psi}_m(\tilde{y}'_m)|0\rangle$$



$$((\mathcal{B}_0+\mathcal{B}_0^\dagger)\Psi_1)*\Psi_2 = (\mathcal{B}_0+\mathcal{B}_0^\dagger)(\Psi_1*\Psi_2)+(-1)^{|\Psi_1|}\frac{\pi}{2}\Psi_1*B_1\Psi_2\,,\cdots$$

[Schnabl(2005)]

Computation and result

- OPE: $$V_m(y)V_n(z) \sim \frac{v_{mn}}{(y-z)^2} + \text{finite} \quad (y \to z)$$
- Computation for ψ_r

$$\langle \Phi_{V,M}, \psi_r \rangle = \frac{C_V}{2\pi i}\left(\sinh\frac{4M}{r+1} - \frac{4M}{\pi}\sin\frac{\pi}{r+1}\right)\left(\cosh\frac{4M}{r+1} - \cos\frac{\pi}{r+1}\right)\left(\sinh\frac{4M}{r+1}\right)^{-2},$$

$$C_V = {}_{\text{mat}}\langle 0|0\rangle_{\text{mat}} \sum_{m,n} \zeta_{mn} v_{mn} .$$

$$\therefore \langle \Phi_V, \psi_r \rangle = \lim_{M\to+\infty} \langle \Phi_{V,M}, \psi_r \rangle = \frac{C_V}{2\pi i}$$ **independent of *r***

- Result for Schnabl's solution

$$\mathcal{O}_V(\Psi_\lambda) = \sum_{k=0}^{\infty} \frac{f_k(\lambda)}{k!} \partial_r^k \langle \Phi_V, \psi_r \rangle|_{r=0} = f_0(\lambda)\langle \Phi_V, \psi_0 \rangle = \begin{cases} \frac{C_V}{2\pi i} & (\lambda = 1) \\ 0 & (\lambda \neq 1) \end{cases}$$

Subtleties

- Only the phantom term contributes if the regularized expression for $\lambda = 1$ is used:

$$\mathcal{O}_V(\Psi_{\lambda=1}) = \lim_{N\to\infty}\left(\langle \Phi_V, \psi_{N+1}\rangle - \sum_{n=0}^{N} \partial_r \langle \Phi_V, \psi_r\rangle|_{r=n}\right)$$

- The order of the limits is not exchangeable:

$$\lim_{N\to\infty}\left(\lim_{M\to+\infty}\langle \Phi_{V,M}, \psi_{N+1}\rangle\right) \neq \lim_{M\to+\infty}\left(\lim_{N\to\infty}\langle \Phi_{V,M}, \psi_{N+1}\rangle\right) = 0$$

Absorption of the phantom term

- For $\lambda = 1$, the phantom term can be eliminated:

$$\begin{aligned}
\Psi_{\lambda=1} &= \lim_{N\to\infty}\left(\psi_{N+1} - \sum_{n=0}^{N} \partial_r \psi_r|_{r=n}\right) \\
&= \lim_{N\to\infty}\left(\psi_0 + \sum_{n=0}^{N} (\psi_{n+1} - \psi_n - \partial_r \psi_r|_{r=n})\right) \\
&= \psi_0 + \sum_{n=0}^{\infty} (\psi_{n+1} - \psi_n - \partial_r \psi_r|_{r=n})
\end{aligned}$$

On-shell closed tachyon state

$$\Phi_k = \frac{1}{4i}\lim_{\theta\to\frac{\pi}{2}} c(e^{i\theta})c(e^{-i\theta}) :e^{ik\cdot X(e^{i\theta},e^{-i\theta})}: |\mathcal{I}\rangle$$
$$= \frac{1}{4} e^{E_{\rm m}+E_{\rm gh}} c_0 c_1 |0\rangle$$

$$E_{\rm m} = -\sum_{n=1}^{\infty}\frac{(-1)^n}{2n}\alpha_{-n}\cdot\alpha_{-n} - \sum_{n=1}^{\infty}\frac{2i\sqrt{2\alpha'}(-1)^n}{2n-1}k_i\alpha^i_{-2n+1},$$
$$E_{\rm gh} = \sum_{n=1}^{\infty}(-1)^n c_{-n}b_{-n}$$

k^i : Dirichlet direction

$k^i k^i = 4/\alpha' \longleftrightarrow Q|\Phi_k\rangle = 0$

Zero momentum dilaton state

$$
\begin{aligned}
\Phi_\eta &= \frac{1}{52\alpha' i}\eta_{\mu\nu} \lim_{\theta\to\frac{\pi}{2}} c(e^{i\theta})\partial X^\mu(e^{i\theta})c(e^{-i\theta})\partial X^\nu(e^{-i\theta})|\mathcal{I}\rangle \\
&= \left(\frac{1}{4} - \frac{2}{13}\sum_{n,m=1}^{\infty} mn\cos\frac{(m-n)\pi}{2}\,\alpha_{-m}\cdot\alpha_{-n}\right) e^E c_0 c_1|0\rangle\,, \\
E &= \sum_{n=1}^{\infty}(-1)^n\left(-\frac{1}{2n}\alpha_{-n}\cdot\alpha_{-n} + c_{-n}b_{-n}\right)
\end{aligned}
$$

- BRST invariance can be checked by explicit computation. $Q|\Phi_\eta\rangle = 0$

Map to get on-shell closed string states

$$|V_c\rangle = c\bar{c}V_{\rm m}|0\rangle \quad \longmapsto \quad \langle\hat{\gamma}(1_c,2)|V_c\rangle_{1_c} = {}_2\langle\Phi_V|$$

open-closed string vertex

Examples:

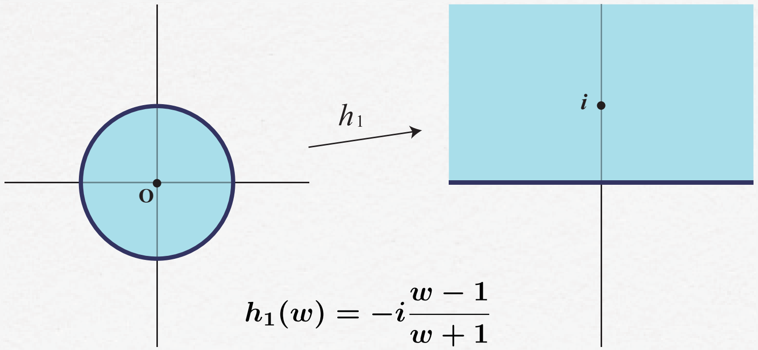
$$V_{\rm m} = e^{ik_iX^i} \quad \longmapsto \quad \Phi_k \qquad \text{closed tachyon state}$$

$$V_{\rm m} = \frac{1}{26}\partial X\cdot\bar{\partial}X \quad \longmapsto \quad \Phi_\eta \qquad \text{zero momentum dilaton state}$$

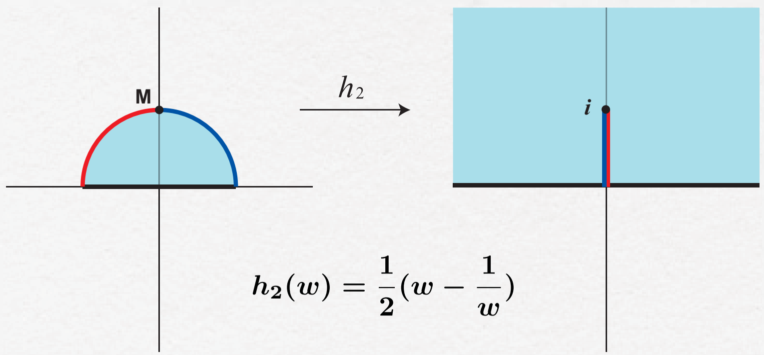
Shapiro-Thorn vertex

$$\langle \hat{\gamma}(1_c, 2)|\phi_c\rangle_{1_c}|\psi\rangle_2 = \langle h_1[\phi_c(0,0)] h_2[\psi(0)]\rangle$$

CFT correlator on UHP [LPP]



closed



open

- BRST invariance:

$$\langle\hat{\gamma}(1_c, 2)|(Q_c^{(1)} + \bar{Q}_c^{(1)} + Q^{(2)}) = 0$$

- Relations among Virasoro generators:

[Rastelli-Zwiebach]

$$\langle\hat{\gamma}(1_c, 2)|(L_{2m-1}^{(2)} + L_{-2m+1}^{(2)})$$
$$= \langle\hat{\gamma}(1_c, 2)|\left(4(2m-1)i(-1)^m(L_0^{(1)} - \bar{L}_0^{(1)}) - \sum_{k=2}^{\infty}(f_{2m-1,k}L_{k-1}^{(1)} + \bar{f}_{2m-1,k}\bar{L}_{k-1}^{(1)}\right)$$

$$\langle\hat{\gamma}(1_c, 2)|(L_{2m}^{(2)} - L_{-2m}^{(2)})$$
$$= \langle\hat{\gamma}(1_c, 2)|\left((-1)^m m\frac{c}{2} - 8m(-1)^m(L_0^{(1)} + \bar{L}_0^{(1)}) - \sum_{k=2}^{\infty}(f_{2m,k}L_{k-1}^{(1)} + \bar{f}_{2m,k}\bar{L}_{k-1}^{(1)}\right)$$

Level expansion of Schnabl's solution

- Derived from expansion of ψ_r

$$\psi_{r-2} = \left[\prod_{k=1,\leftarrow}^{\infty} e^{u_{2k}(r)L_{-2k}}\right]\left[\frac{1}{\pi}\sin\frac{2\pi}{r}\left(1-\frac{r}{2\pi}\sin\frac{2\pi}{r}\right)\sum_{p\geq -1;p:\mathrm{odd}}\left(\frac{2}{r}\cot\frac{\pi}{r}\right)^p c_{-p}|0\rangle\right.$$
$$\left.+\frac{r}{2\pi^2}\left(\sin\frac{2\pi}{r}\right)^2\sum_{s\geq 2;s:\mathrm{even}}\frac{(-1)^{\frac{s}{2}+1}}{s^2-1}\left(\frac{2}{r}\right)^s\sum_{p,q\geq -1;p+q:\mathrm{odd}}(-1)^q\left(\frac{2}{r}\cot\frac{\pi}{r}\right)^{p+q} b_{-s}c_{-p}c_{-q}|0\rangle\right]$$

$$u_2(r) = -\frac{r^2-4}{3r^2},\quad u_4(r) = \frac{r^4-16}{30r^4},\quad u_6(r) = -\frac{16(r^2-4)(r^2-1)(r^2+5)}{945r^6},\ldots$$

From its coefficients, $\psi_{N+1} = O(N^{-3}) \quad (N\to\infty)$

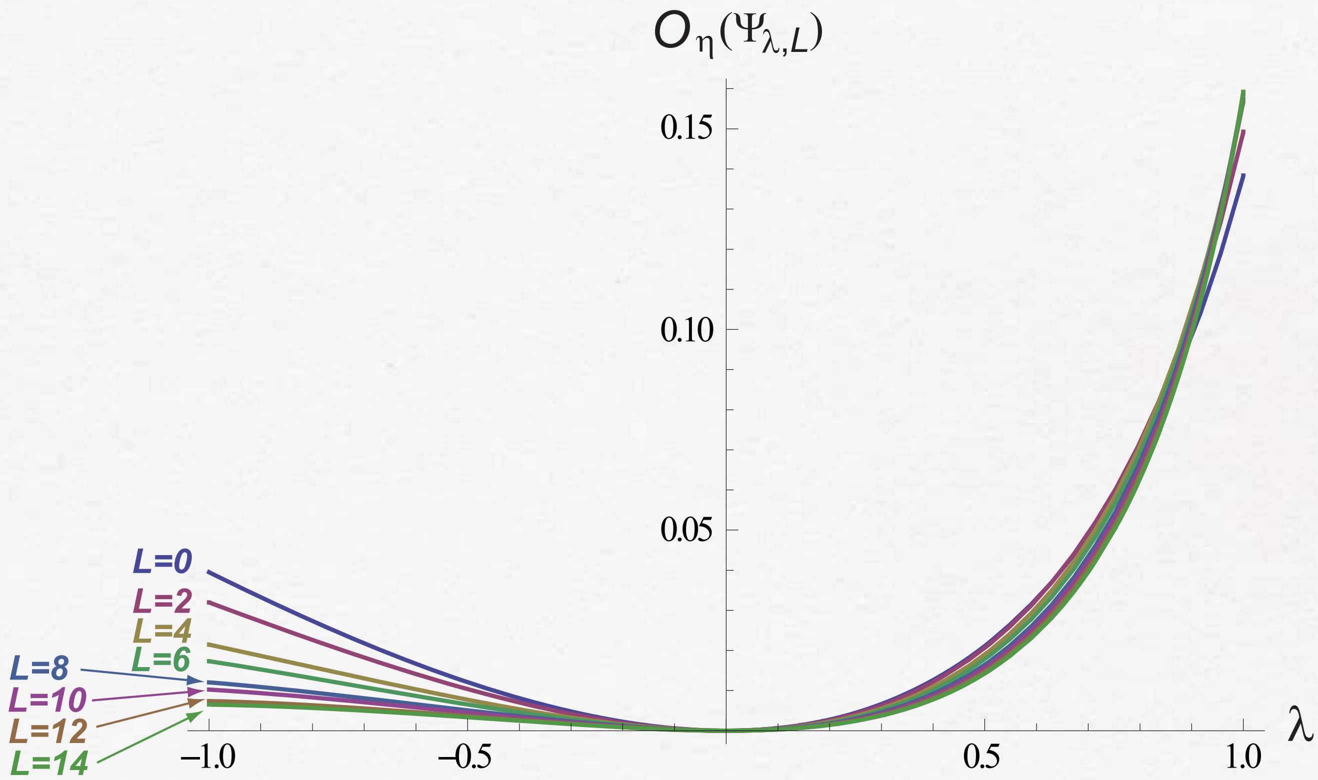
Level truncation for the gauge invariant overlap

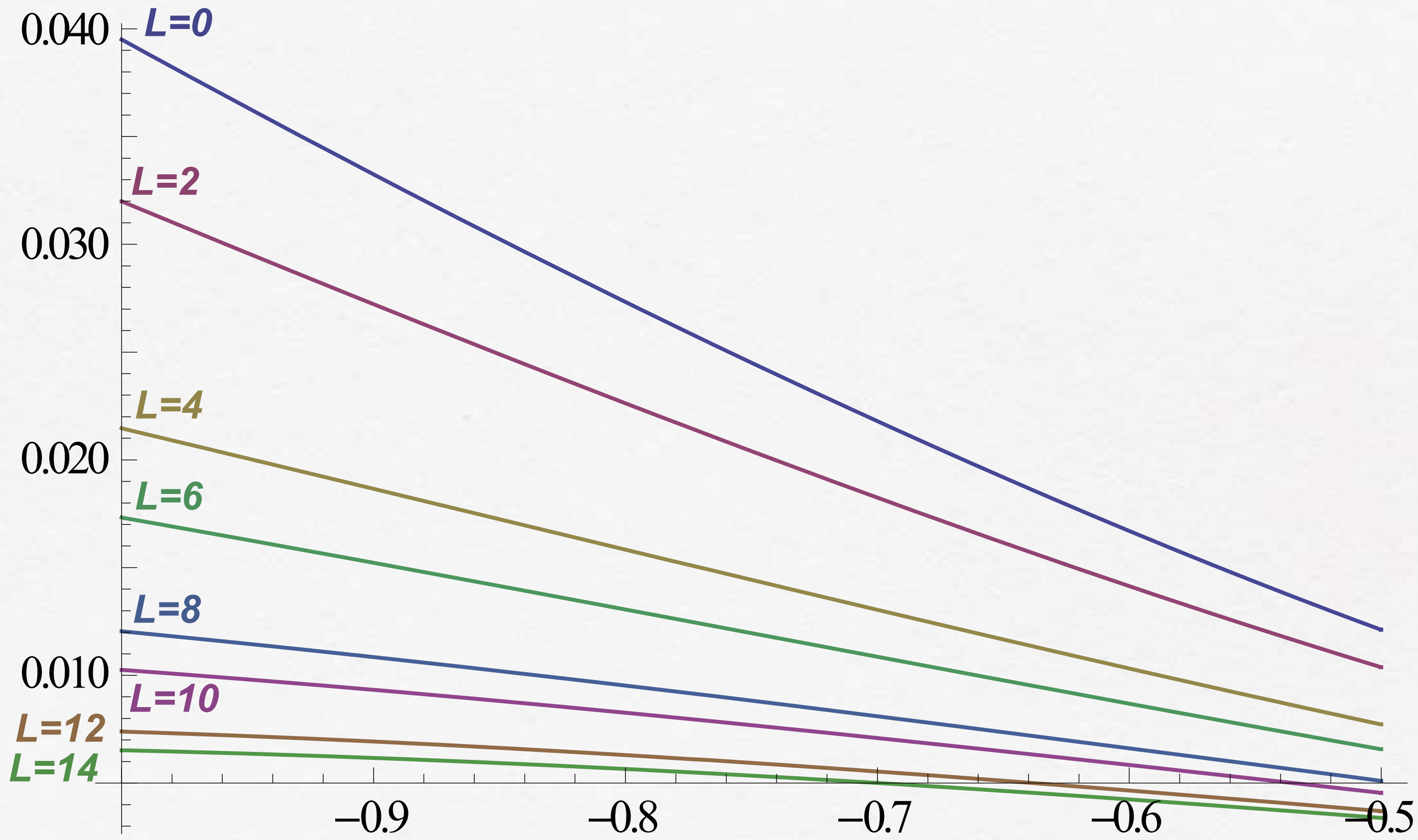
- Up to level ***L***, using ***Mathematica***, we have evaluated

$$\mathcal{O}_\eta(\Psi_{\lambda,L}) = -\sum_{n=0}^{\infty} \lambda^{n+1}\partial_r \langle \Phi_\eta, \psi_{r,L}\rangle|_{r=n}$$

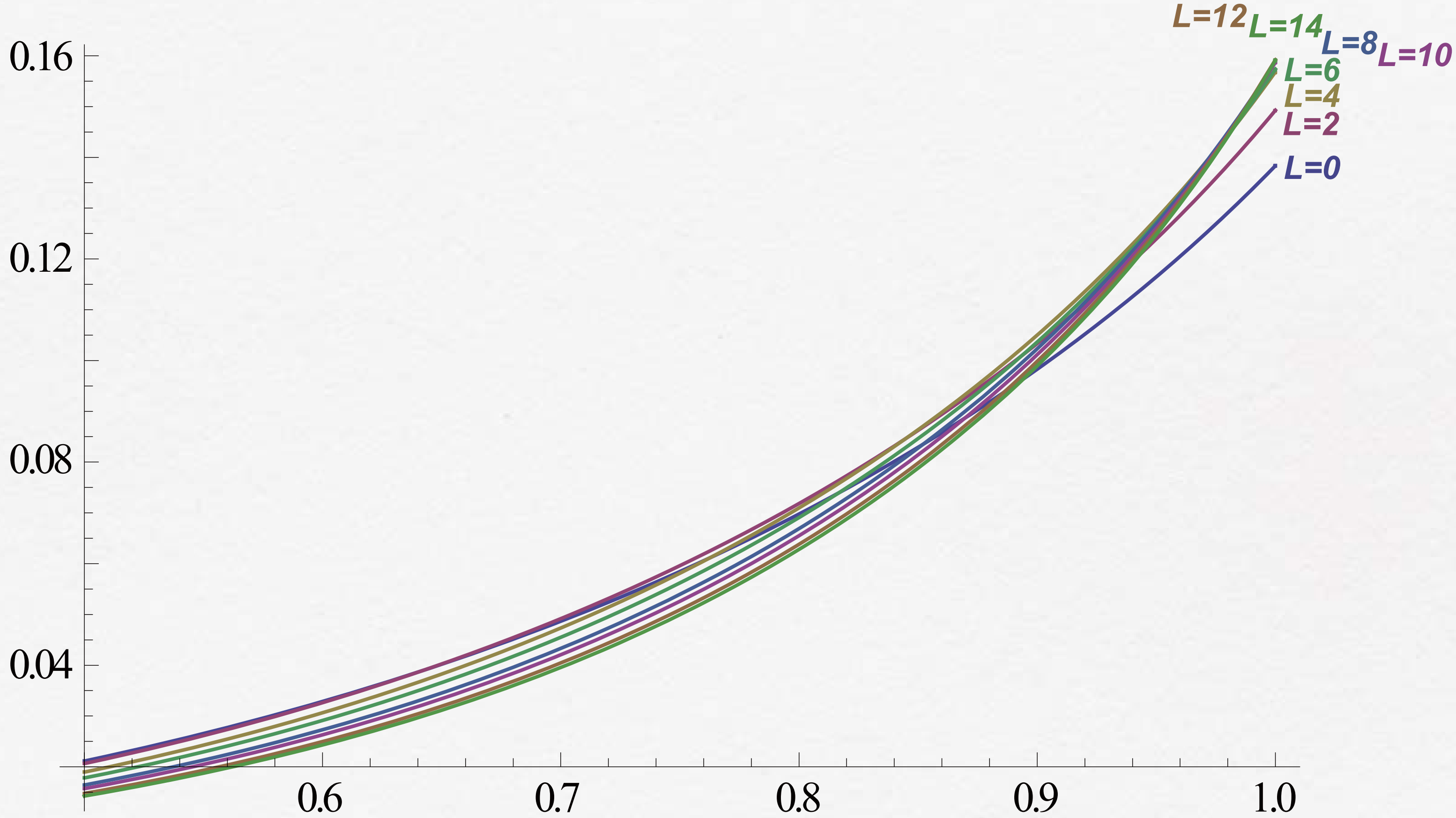
- Convergent for $-1 \le \lambda \le 1$
- The phantom term for $\lambda = 1$ doesn't contribute.

Result by level truncation





L=0
L=2
L=4
L=6
L=8
L=10
L=12
L=14
0.040
0.030
0.020
0.010
–0.9
–0.8
–0.7
–0.6
–0.5



0.16
0.12
0.08
0.04
0.6
0.7
0.8
0.9
1.0
L=12
L=14
L=8
L=10
L=6
L=4
L=2
L=0

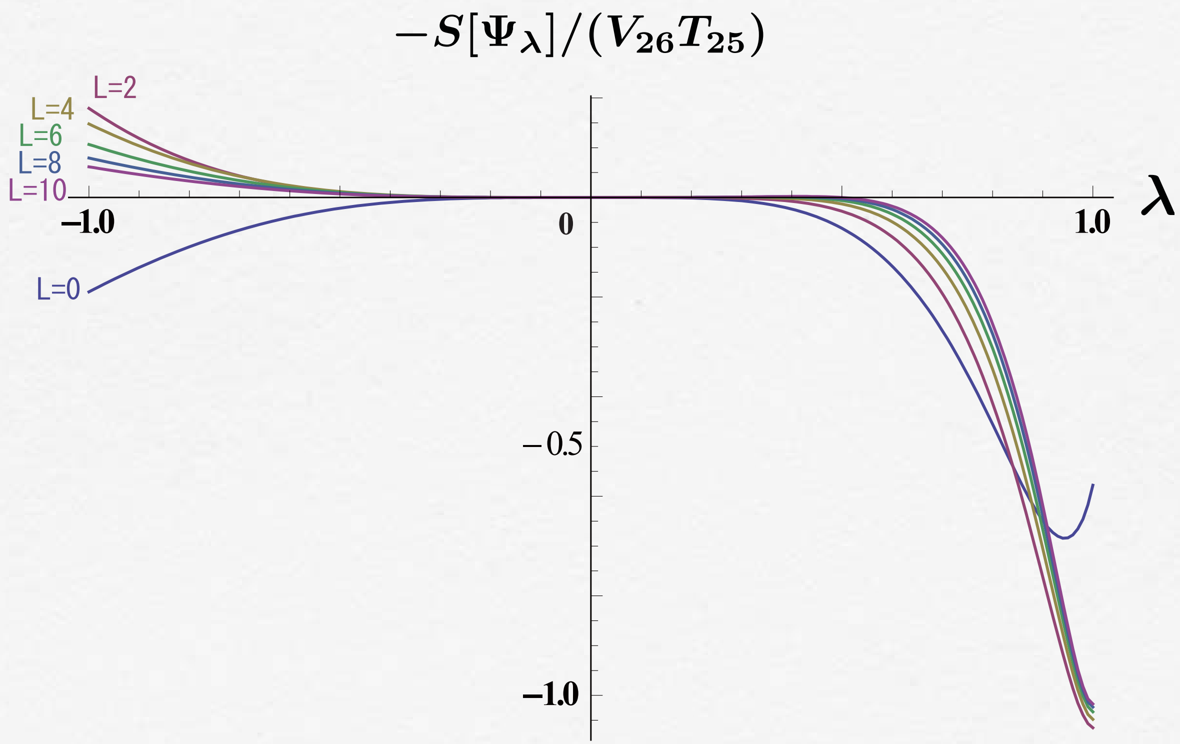
- Approaches to the analytical result for $L \longrightarrow \infty$

$$\mathcal{O}_\eta(\Psi_\lambda) = \begin{cases} \dfrac{1}{2\pi} \simeq 0.159155 & (\lambda = 1) \\ 0 & (\lambda \neq 1) \end{cases}$$

L	$\mathcal{O}_\eta(\Psi_{\lambda=1,L})$
0	0.13837
2	0.14928
4	0.15686
6	0.15740
8	0.15880
10	0.15877
12	0.15922
14	0.15916

Evaluation of the action by level truncation

- Schnabl(2005), Takahashi(2007)



L	$-S[\Psi_{\lambda=1}]/(V_{26}T_{25})$
0	-0.57792
2	-1.06518
4	-1.04798
6	-1.03287
8	-1.02326
10	-1.01705

"(L,3L)" approximation

Gauge invariant overlap for the numerical solution in Siegel gauge

$b_0|\Psi_{\rm N}\rangle = 0$ [Sen-Zwiebach(1999),...,Gaiotto-Rastelli(2002)]

L	$\mathcal{O}_\eta(\Psi_{\rm N})$
0	0.114044
2	0.139790
4	0.147931
6	0.151225
8	0.152887
10	0.154029
12	0.154750

(L,2L) approximation

L	$\mathcal{O}_\eta(\Psi_{\rm N})$
0	0.114044
2	0.141626
4	0.148325
6	0.151369
8	0.152976
10	0.154080
12	-

(L,3L) approximation

- 97.2% of $\dfrac{1}{2\pi} = 0.159155\cdots$ $\mathcal{O}_\eta(\Psi_{\rm N}) \simeq \mathcal{O}_\eta(\Psi_{\lambda=1})$

Evaluation of the action for numerical solution

- Gaiotto-Rastelli(2002), **Table 1**

L	$-2\pi^2 g^2 S[\Psi_N]/V_{26}$
0	-0.6846161
2	-0.9485534
4	-0.9864034
6	-0.9947727
8	-0.9977795
10	-0.9991161
12	-0.9997907
14	-1.0001580
16	-1.0003678
18	-1.00049

(L,2L) approximation

L	$-2\pi^2 g^2 S[\Psi_N]/V_{26}$
0	-0.6846161
2	-0.9593766
4	-0.9878218
6	-0.9951771
8	-0.9979302
10	-0.9991825
12	-0.9998223
14	-1.0001737
16	-1.0003754
18	-1.0004937

(L,3L) approximation

Equivalence of gauge invariant overlaps in universal space

- For any L_0-level L,

$$\mathcal{O}_k(\Psi_\lambda|_L) = \mathcal{O}_\eta(\Psi_\lambda|_L) \qquad \mathcal{O}_k(\Psi_{\rm N}|_L) = \mathcal{O}_\eta(\Psi_{\rm N}|_L)$$

- In general, $\mathcal{O}_k(\psi_L) = \mathcal{O}_\eta(\psi_L)$ for

$$\psi_L = \sum_{\substack{p,q\geq 0, n_i\geq 2, j_i\geq 1, k_i\geq 0 \\ n_1+\cdots+n_p+j_1+\cdots+j_l+k_1+\cdots+k_q=L}} c^{(L)}_{n_i,j_i,k_i} L^{(\rm m)}_{-n_1}\cdots L^{(\rm m)}_{-n_p} b_{-j_1}\cdots b_{-j_q} c_{-k_1}\cdots c_{-k_q} c_1|0\rangle$$

because on-shell closed string states satisfy

$$(L^{(\rm m)}_{2n} - L^{(\rm m)}_{-2n})|\Phi_{c\bar{c}V_{\rm m}}\rangle = (-1)^n 3n|\Phi_{c\bar{c}V_{\rm m}}\rangle, \quad (L^{(\rm m)}_{2n-1} + L^{(\rm m)}_{-2n+1})|\Phi_{c\bar{c}V_{\rm m}}\rangle = 0$$

Summary

- Schnabl's solution Ψ_λ takes nontrivial values for gauge invariants only for $\lambda = 1$:

$$S[\Psi_\lambda]/V_{26} = \begin{cases} \frac{1}{2\pi^2 g^2} & (\lambda = 1) \\ 0 & (|\lambda| < 1) \end{cases} \qquad \mathcal{O}_{k/\eta}(\Psi_\lambda) = \begin{cases} \frac{1}{2\pi} & (\lambda = 1) \\ 0 & (-1 \leq \lambda < 1) \end{cases}$$

- The results is consistent with the interpretation that
  - $\Psi_{\lambda=1}$:nontrivial solution, $\Psi_{|\lambda|<1}$:pure gauge solution
- $\Psi_{\lambda=1}$ takes (almost) the same values for gauge invariants as the numerical solution in the Siegel gauge Ψ_{N}:

$$S[\Psi_{\lambda=1}] \simeq S[\Psi_{\mathrm{N}}] \qquad \mathcal{O}_{k/\eta}(\Psi_{\lambda=1}) \simeq \mathcal{O}_{k/\eta}(\Psi_{\mathrm{N}})$$

- The above suggests the gauge equivalence: $\Psi_{\lambda=1} \sim \Psi_{\mathrm{N}}$

Discussion

- What is the physical meaning of the gauge invariant overlaps?

$$\mathcal{O}_V(\Psi_{\lambda=1}) = \langle\hat{\gamma}(1_c, 2)|\psi_0\rangle_2\, c_1^{(1)}\bar{c}_1^{(1)}|V_m\rangle_{1_c} \propto \langle B_N|c_0^- c_1\bar{c}_1|V_m\rangle$$

  - More directly,

$$\langle\hat{\gamma}(1_c, 2)|\psi_0\rangle_2\mathcal{P}_{1_c} = \frac{1}{2\pi}\langle B_N|c_0^-$$

- Back reaction in open-closed string field theory?

Ellwood proposed [arXiv:0804.1131]

$$\mathcal{O}_V(\Psi) = \mathcal{A}_\Psi^{\text{disk}}(V) - \mathcal{A}_0^{\text{disk}}(V)$$

Off-shell extension of boundary state?

$$\langle\hat{\gamma}(1_c,2)|\Psi_{\lambda=1}\rangle_2\mathcal{P}b_0^- = \frac{1}{2\pi}\langle B_N| + \langle\hat{\gamma}(1_c,2)|\chi\rangle_2\mathcal{P}b_0^-$$

↑ level matching projection in closed string Hilbert space

Here, the Schnabl's solution is given by

$$\begin{aligned}\Psi_{\lambda=1} &= \psi_0 + \sum_{n=0}^{\infty}(\psi_{n+1} - \psi_n - \partial_r\psi_r|_{r=n}) \\ &\equiv \psi_0 + \chi\end{aligned}$$

which satisfies $\mathcal{Q}(-\Psi_{\lambda=1}) + (-\Psi_{\lambda=1}) * (-\Psi_{\lambda=1}) = 0$

$\mathcal{Q} \equiv Q + \mathrm{ad}_{\Psi_{\lambda=1}}$:BRST operator around $\Psi_{\lambda=1}$