Numerical Evaluation of Gauge Invariants for *a*-gauge Solutions in Open String Field Theory

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Collaboration with Tomohiko Takahashi (Nara Women's Univ.)

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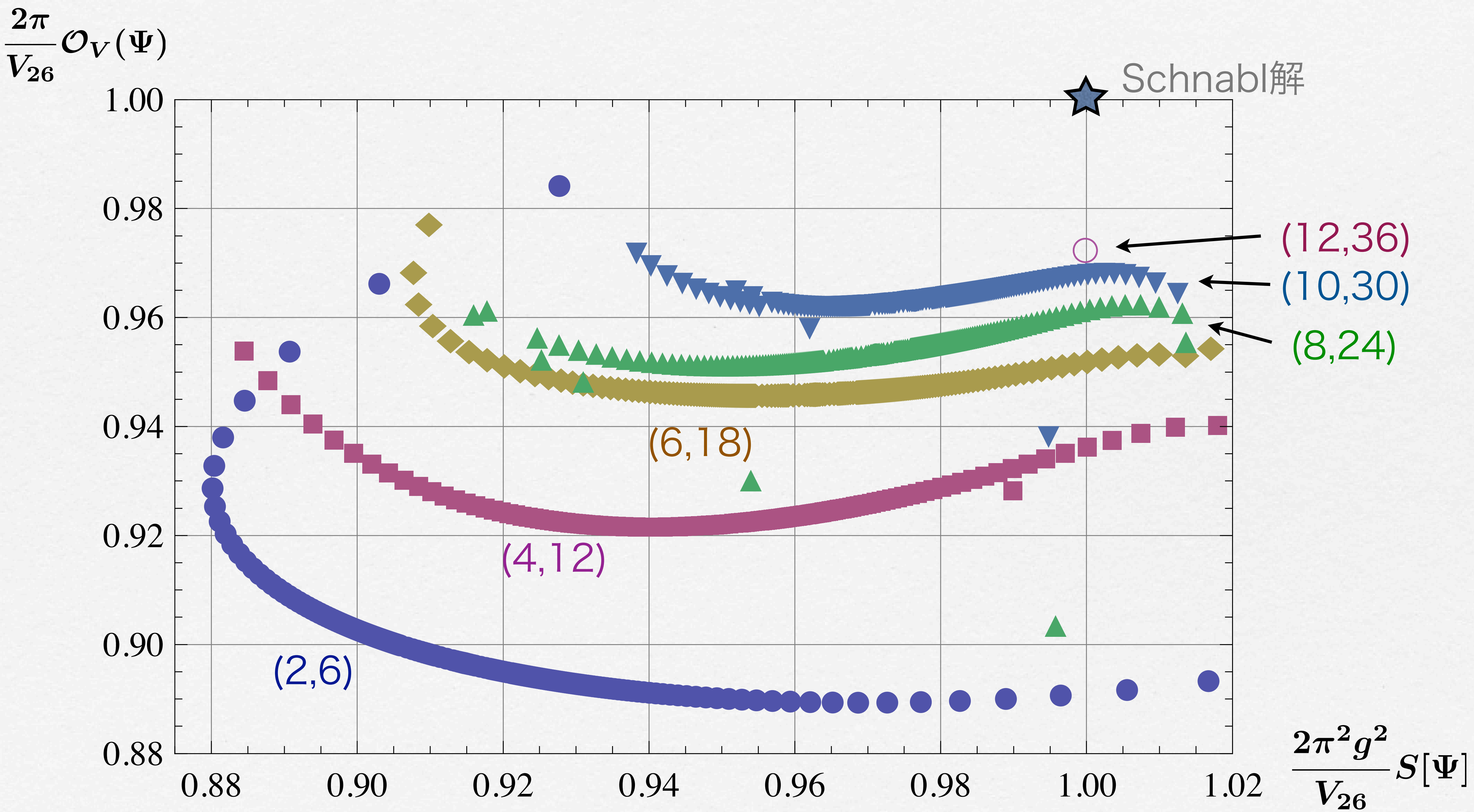
cf. T. Kawano, I.K., T. Takahashi, NPB803(2008)135, arXiv:0804.1541

Result

(L,3L)-truncation



Gauge Invariant Overlap

$$\mathcal{O}_V(\Psi) = \langle \hat{\gamma}(1_c, 2) | \Phi_V \rangle_{1_c} | \Psi \rangle_2$$

$$|\Phi_V\rangle = -\frac{1}{26} \eta_{\mu\nu} \alpha^{\mu}_{-1} \bar{\alpha}^{\nu}_{-1} c_1 \bar{c}_1 |0\rangle$$

: on-shell closed string state

$$\delta_\Lambda \Psi = Q_B \Lambda + \Psi * \Lambda - \Lambda * \Psi$$

$$\longrightarrow \quad \delta_\Lambda \mathcal{O}_V(\Psi) = 0$$

Asano-Kato's *a*-gauge

$$(b_0 M + a\, b_0 c_0 \tilde{Q})|\Psi\rangle = 0$$

$$Q_{\mathrm{B}} = \tilde{Q} + c_0 L_0 + b_0 M$$

In particular,

$a = 0$ equivalent to the Siegel gauge: $b_0|\Psi\rangle = 0$

$$a = \infty \longrightarrow b_0 c_0 \tilde{Q}|\Psi\rangle = 0$$

corresponds to the Landau gauge

Summary

Our numerical results suggest:

$-\infty \leqq a \lesssim 0,\ 1 \ll a \leqq \infty$

$L \to +\infty$

$$S[\Psi_{a,L}]|_L \to S[\Psi_{\rm Schnabl}]$$

$$\mathcal{O}_V(\Psi_{a,L}) \to \mathcal{O}_V(\Psi_{\rm Schnabl})$$

These are consistent with the gauge equivalence:

$$\Psi_a \sim \Psi_{\rm Schnabl}$$

We have also checked
the BRST invariance of the numerical solutions.

Numerical Evaluation of Gauge Invariants for a-gauge Solutions in Open String Field Theory

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Collaboration with T.Takahashi, arXiv:0902.0445

Non-perturbative vacuum in bosonic open string field theory

- Schnabl's solution $\Psi_{\rm Sch}$

Gauge invariants

(1) Action: D-brane tension

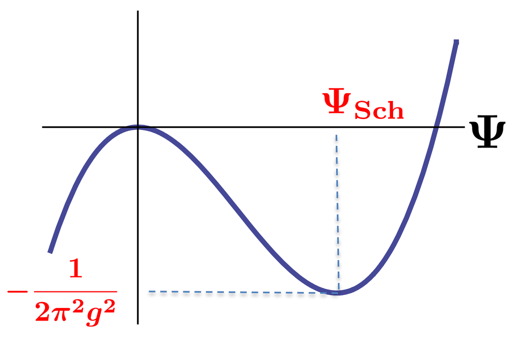
$$S[\Psi_{\rm Sch}]/V_{26} = \frac{1}{2\pi^2 g^2}$$

[Schnabl(2005),Okawa,Fuchs-Kroyter(2006)]

(2) Gauge invariant overlap:

$$\mathcal{O}_V(\Psi_{\rm Sch})/V_{26} = \frac{1}{2\pi}$$

[Ellwood, Kawano-Kishimoto-Takahashi(2008)]



Numerical solution by level truncation

- Numerical solution in the Siegel gauge: $b_0|\Psi_{\rm N}\rangle = 0$

[...,Sen-Zwiebach(1999),...]

(1) $S[\Psi_{\rm N}]/S[\Psi_{\rm Sch}]$

(L,2L)-truncation		(L,3L)-truncation	
(2,4)	0.9485534	(2,6)	0.9593766
(4,8)	0.9864034	(4,12)	0.9878218
(6,12)	0.9947727	(6,18)	0.9951771
(8,16)	0.9977795	(8,24)	0.9979302
(10,20)	0.9991161	(10,30)	0.9991825
(12,24)	0.9997907	(12,36)	0.9998223
(14,28)	1.0001580	(14,42)	1.0001737
(16,32)	1.0003678	(16,48)	1.0003754
(18,36)	1.00049	(18,54)	1.0004937

[Gaiotto-Rastelli(2002)]

(2) $\mathcal{O}_V(\Psi_{\rm N})/\mathcal{O}_V(\Psi_{\rm Sch})$

(L,2L)-truncation		(L,3L)-truncation	
(2,4)	0.8783238	(2,6)	0.8898618
(4,8)	0.9294792	(4,12)	0.9319524
(6,12)	0.9501746	(6,18)	0.9510789
(8,16)	0.9606165	(8,24)	0.9611748
(10,20)	0.9677900	(10,30)	0.9681148
(12,24)	0.9723211	(12,36)	0.9725595
(14,28)	0.9760046	(14,42)	0.9761715

[Kawano-Kishimoto-Takahashi(2008)] and the latest result

Evidence for gauge equivalence: $\Psi_{\rm N} \sim \Psi_{\rm Sch}$

Numerical solutions in a-gauges

- Asano-Kato's a-gauge $(b_0 M + a\, b_0 c_0 \tilde{Q})|\Psi_a\rangle = 0$

$$Q = \tilde{Q} + c_0 L_0 + b_0 M$$

$a = 0$ ⇒ Siegel gauge: $b_0|\Psi_0\rangle = 0$

$a = \infty$ ⇒ Landau gauge: $b_0 c_0 \tilde{Q}|\Psi_\infty\rangle = 0$

(1) For a-gauge solution, (6,18)-trucation $S[\Psi_a]/S[\Psi_{\rm Sch}]$

$a = \infty$	0.9609438
$a = 4.0$	0.9244886
$a = 0.5$	1.0045858
$a = -2.0$	0.9798943
⋮	⋮

[Asano-Kato(2006)]

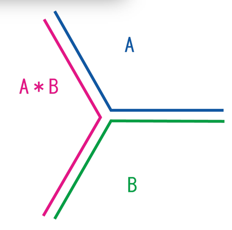
(2) $\mathcal{O}_V(\Psi_a)/\mathcal{O}_V(\Psi_{\rm Sch})$ (?) (and higher level?)

⇒ our computation

Bosonic cubic open string field theory

Action: $S[\Psi] = -\frac{1}{g^2}\left(\frac{1}{2}\langle\Psi, Q\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right)$

$$Q = \oint \frac{dz}{2\pi i}\left(cT^{\rm m} + bc\partial c + \frac{3}{2}\partial^2 c\right)$$



Equation of motion: $Q\Psi + \Psi * \Psi = 0$

Gauge transformation: $\delta_\Lambda \Psi = Q\Lambda + \Psi * \Lambda - \Lambda * \Psi$

→ $\delta_\Lambda S[\Psi] = 0$

Restrict string field to twist even sector in the universal space:

$$\Psi = (t_1 + t_2 b_{-1}c_{-1} + t_3 L^{(\rm m)}_{-2} + \cdots)c_1|0\rangle + (u_1 b_{-2} + \cdots)c_0 c_1|0\rangle$$

Gauge invariant overlap

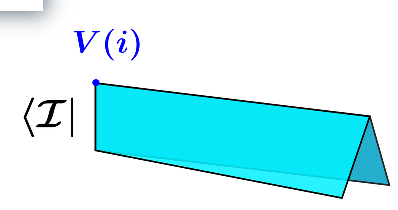
Gauge invariant for on-shell closed string state

$$\mathcal{O}_V(\Psi) = \langle\mathcal{I}|V(i)|\Psi\rangle = \langle\hat{\gamma}(1_{\rm c}, 2)|\Phi_V\rangle_{1_{\rm c}}|\Psi\rangle_2$$

$$|\Phi_V\rangle = c_1\bar{c}_1|V_{\rm m}\rangle$$

$V_{\rm m}$:matter primary with (1,1)-dim.

→ $\delta_\Lambda \mathcal{O}_V(\Psi) = 0$



Ellwood's proposal (2008): $\mathcal{O}_V(\Psi) = \mathcal{A}^{\rm disk}_\Psi(V) - \mathcal{A}^{\rm disk}_0(V)$

[cf. Kiemaier-Okawa-Zwiebach(2008)]

↑ Disk amplitude for a closed string vertex V specified by a solution Ψ

In particular, $\mathcal{O}_V(\Psi_{\rm Sch}) = 0 - \mathcal{A}^{\rm disk}_0(V)$

$|\Phi_V\rangle = -\frac{1}{26}\eta_{\mu\nu}\alpha^\mu_{-1}\bar{\alpha}^\nu_{-1}c_1\bar{c}_1|0\rangle$ for explicit numerical computation

Construction of numerical solutions

$\Psi_{(0)} = \frac{64}{81\sqrt{3}}c_1|0\rangle$: nontrivial solution for (0,0)-truncation

$(b_0 M + a\, b_0 c_0 \tilde{Q})\Psi_{(n+1)} = 0$: a-gauge condition

$\mathcal{P}(Q_{\Psi_{(n)}}\Psi_{(n+1)} - \Psi_{(n)} * \Psi_{(n)}) = 0$: linear equations!

$\mathcal{P} = c_0 b_0$:a projection to solve equations

$Q_{\Psi_{(n)}}\Phi \equiv Q\Phi + \Psi_{(n)} * \Phi - (-1)^{|\Phi|}\Phi * \Psi_{(n)}$:"BRST op." around $\Psi_{(n)}$

⇒ We can define $\Psi_{(n)} \mapsto \Psi_{(n+1)}$

$$\Psi_{(n+1)} \simeq (Q_{\Psi_{(n)}})^{-1}(\Psi_{(n)} * \Psi_{(n)})$$

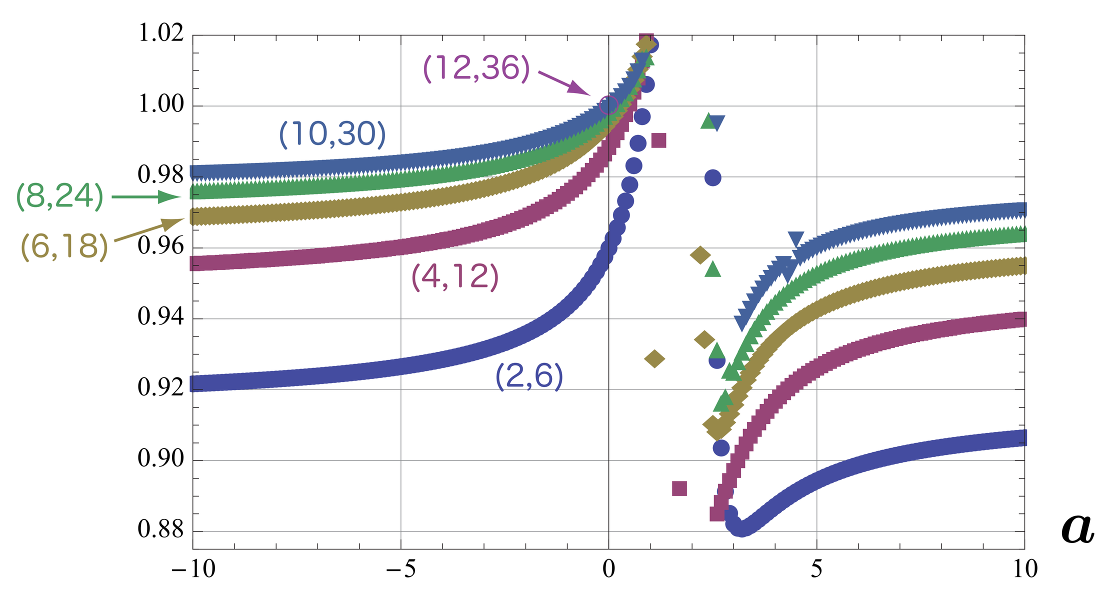
[Gaiotto-Rastelli(2002)]

If it converges for $n \longrightarrow \infty$

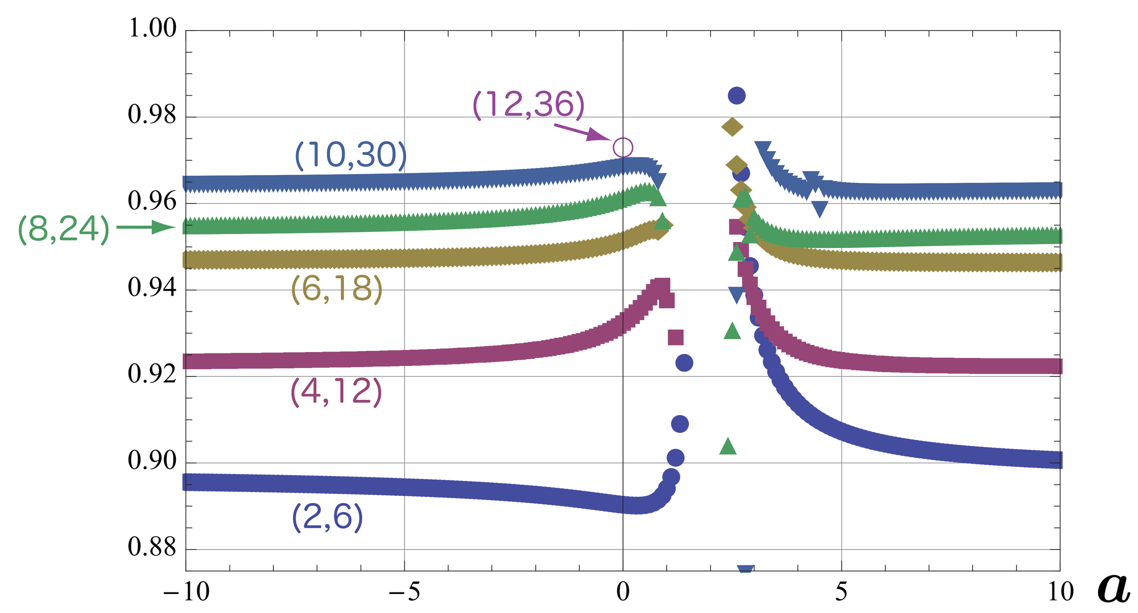
$(b_0 M + a\, b_0 c_0 \tilde{Q})\Psi_{(\infty)} = 0$: a-gauge condition

$\mathcal{P}(Q\Psi_{(\infty)} + \Psi_{(\infty)} * \Psi_{(\infty)}) = 0$: projected part of eq. of motion

$S[\Psi_a]/S[\Psi_{\rm Sch}]$ (L,3L)-truncation



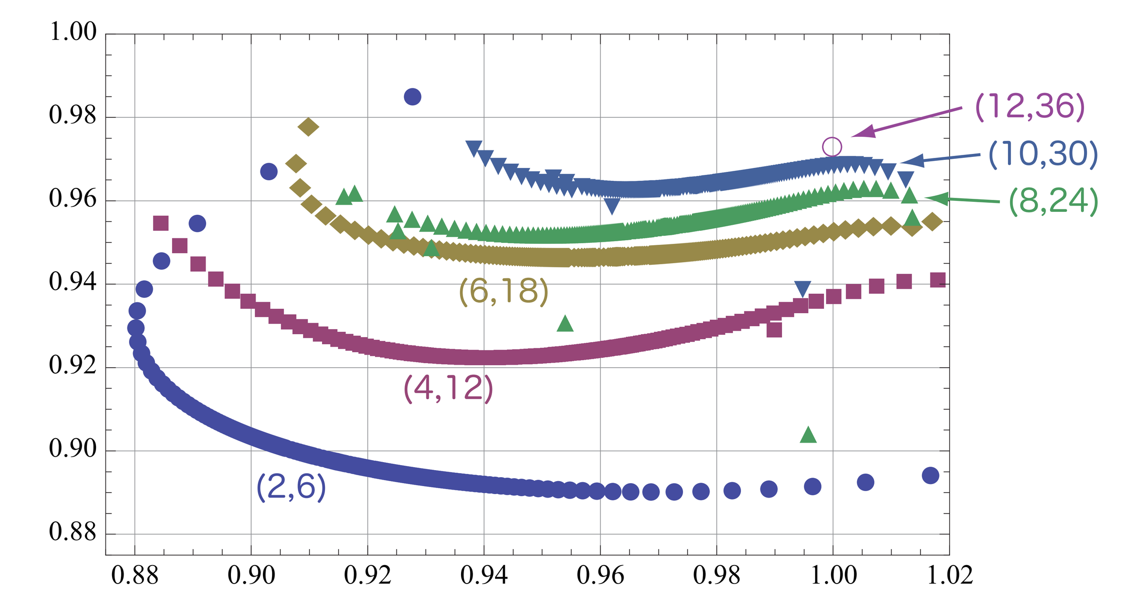
$\mathcal{O}_V(\Psi_a)/\mathcal{O}_V(\Psi_{\rm Sch})$ (L,3L)-truncation



Gauge invariants for various a-gauge solutions

(L,3L)-truncation

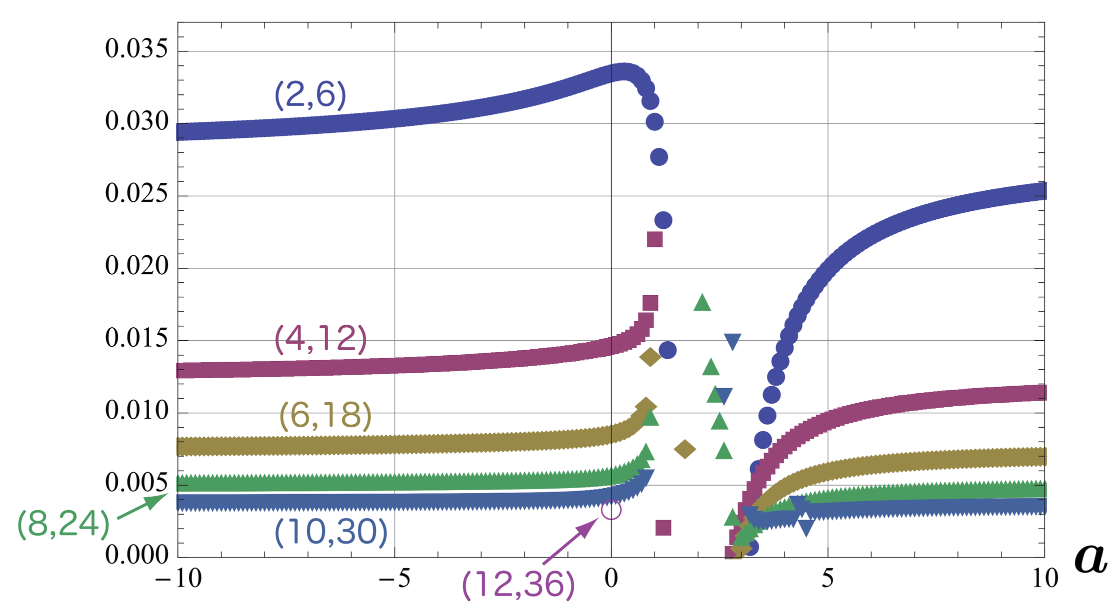
$\mathcal{O}_V(\Psi_a)/\mathcal{O}_V(\Psi_{\rm Sch})$



$S[\Psi_a]/S[\Psi_{\rm Sch}]$

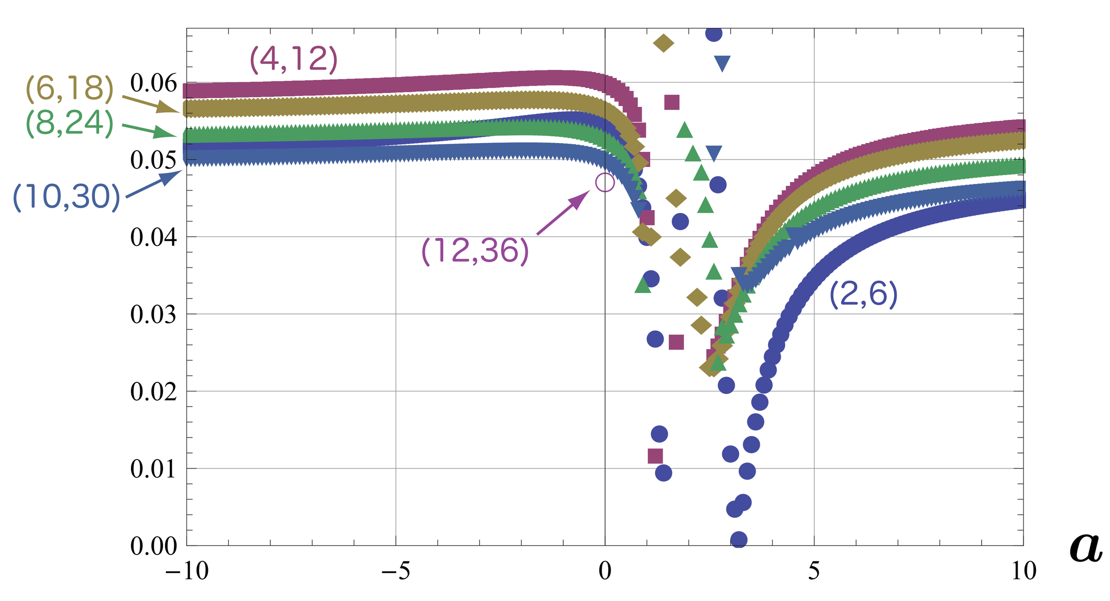
Coefficient of $c_{-2}c_1|0\rangle \in (1-\mathcal{P})(Q\Psi_a + \Psi_a * \Psi_a)$

(L,3L)-truncation



$\frac{\|(1-\mathcal{P})(Q\Psi_a + \Psi_a * \Psi_a)\|}{\|\Psi_a\|}$

(L,3L)-truncation



Summary

- We have evaluated gauge invariants (action and gauge invariant overlap) for numerical solutions in a-gauges by level truncation ((L,2L) and (L,3L)-method).
- We have checked "BRST invariance" of solutions.
- Our numerical results suggest:

$$-\infty \leqq a \lesssim 0,\ 1 \ll a \leqq \infty$$

$$L \to +\infty \qquad S[\Psi_{a,L}]|_L \to S[\Psi_{\rm Sch}] \qquad \mathcal{O}_V(\Psi_{a,L}) \to \mathcal{O}_V(\Psi_{\rm Sch})$$

- These are consistent with the gauge equivalence: $\Psi_a \sim \Psi_{\rm Sch}$

※Perturbatively, $a = 1$-gauge is ill-defined.