Vacuum structure around identity based solutions in open string field theory

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Based on

I.K.-Takahashi, arXiv:0904.1095 (to appear in PTP)

(cf. I.K.-Takahashi, arXiv:0902.0445; Kawano-I.K.-Takahashi, arXiv:0804.1541)

Contents

Non-perturbative vacuum in bosonic open string field theory

- Schnabl's solution Ψ_{Sch}

Gauge invariants

(1) Action: D-brane tension

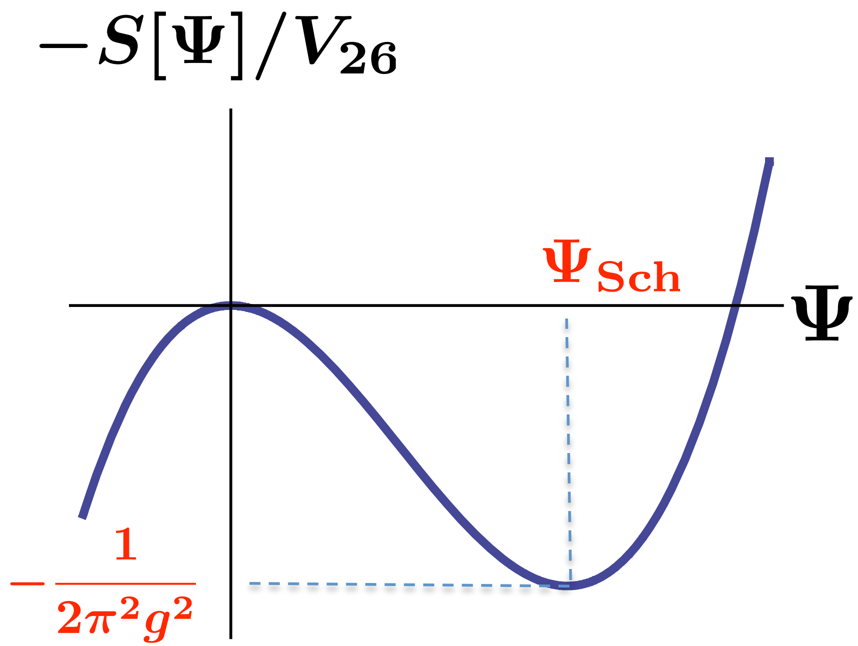
$$S[\Psi_{\text{Sch}}]/V_{26} = \frac{1}{2\pi^2 g^2}$$

[Schnabl(2005),Okawa,Fuchs-Kroyter(2006)]

(2) Gauge invariant overlap:

$$\mathcal{O}_V(\Psi_{\text{Sch}})/V_{26} = \frac{1}{2\pi}$$

[Ellwood, Kawano-Kishimoto-Takahashi(2008)]



Bosonic cubic open string field theory

Action: $$S[\Psi] = -\frac{1}{g^2}\left(\frac{1}{2}\langle\Psi, Q_{\rm B}\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right)$$

$$Q_{\rm B} = \oint \frac{dz}{2\pi i}\left(cT^{(\rm m)} + bc\partial c\right)$$

Equation of motion: $$Q_{\rm B}\Psi + \Psi * \Psi = 0$$

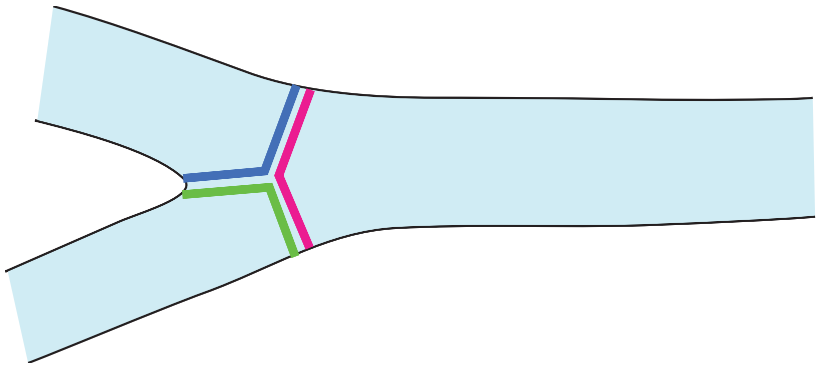
Gauge transformation: $$\delta_\Lambda \Psi = Q_{\rm B}\Lambda + \Psi * \Lambda - \Lambda * \Psi$$

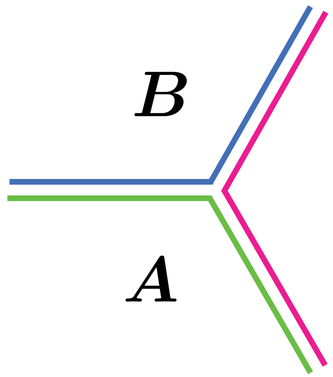
$$\rightarrow \quad \delta_\Lambda S[\Psi] = 0$$

Here, we restrict string fields to twist even sector in the universal space:

$$\Psi = (t_1 + t_2 b_{-1}c_{-1} + t_3 L^{(\rm m)}_{-2} + \cdots)c_1|0\rangle + (u_1 b_{-2} + \cdots)c_0 c_1|0\rangle$$

Witten type interaction





$$|A*B\rangle = \sum_i |\phi^i\rangle\langle f_{(1)}[\phi_i] f_{(2)}[A] f_{(3)}[B]\rangle_{\mathrm{UHP}}$$

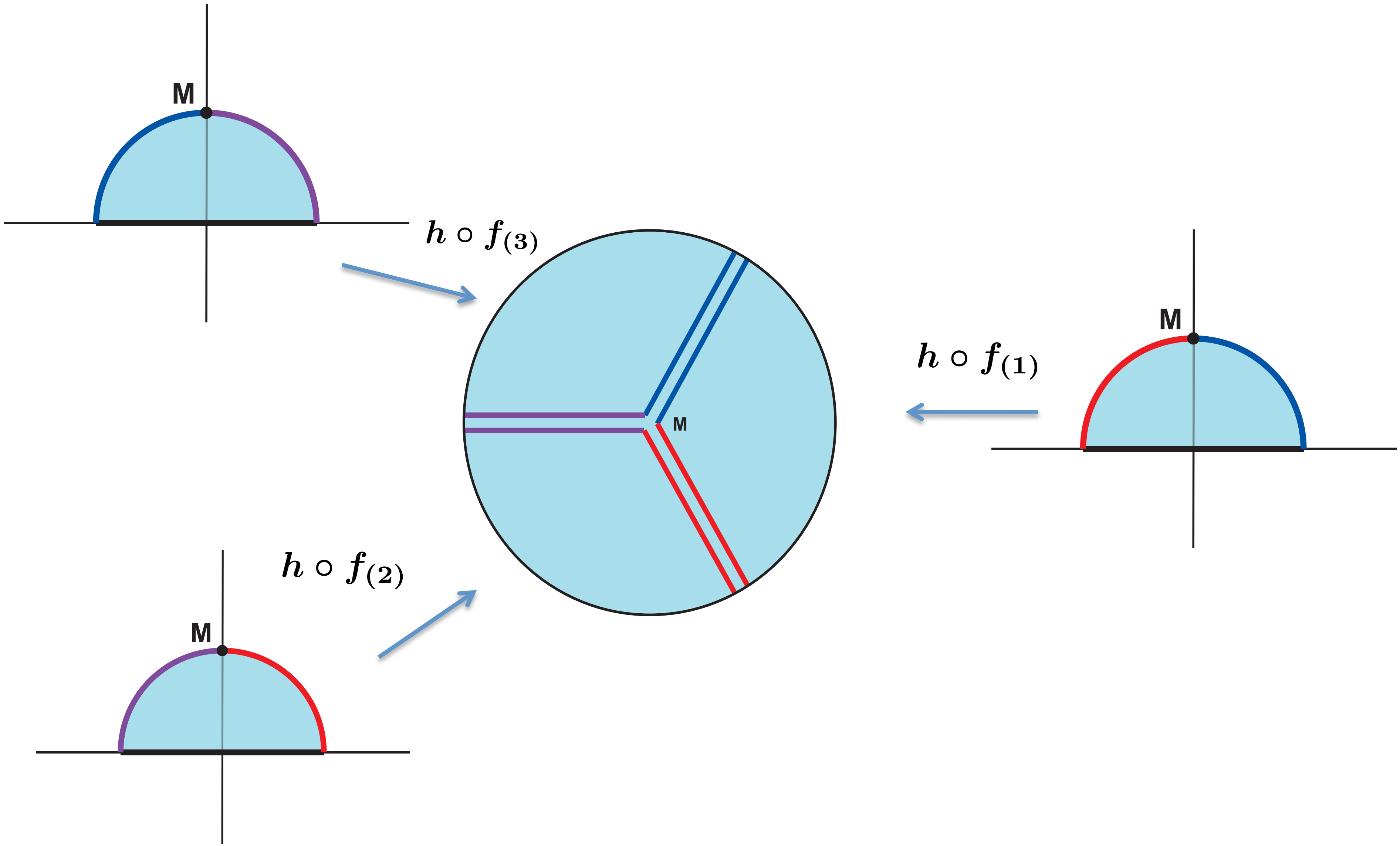
$$f_{(r)}(z) = h^{-1}(e^{(1-r)\frac{2\pi}{3}i} h(z)^{\frac{2}{3}})$$

$$h(z) = \frac{1+iz}{1-iz}$$

ϕ^i :basis of worldsheet fields

$\langle\phi_i, \phi^j\rangle = \delta_i^j$

3 string vertex



Gauge invariant overlap

Gauge invariant for on–shell closed string state

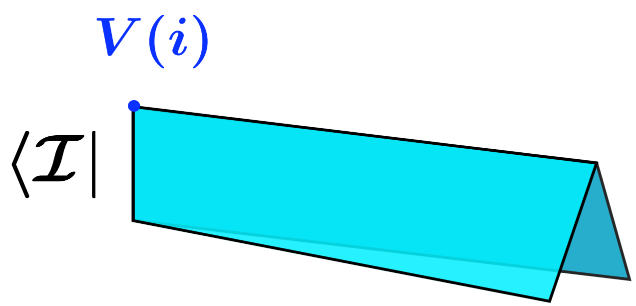
$$\mathcal{O}_V(\Psi) = \langle \mathcal{I}|V(i)|\Psi\rangle = \langle \hat{\gamma}(1_c, 2)|\Phi_V\rangle_{1_c}|\Psi\rangle_2$$

$$|\Phi_V\rangle = c_1\bar{c}_1|V_m\rangle$$

V_m :matter primary with (1,1)-dim.

$$\mathcal{O}_V(Q_B\Lambda) = 0$$

$$\mathcal{O}_V(\Psi * \Lambda) = \mathcal{O}_V(\Lambda * \Psi)$$

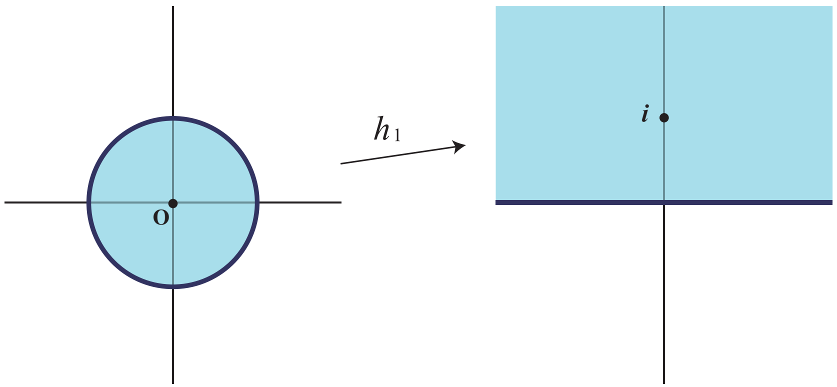


$$\Rightarrow \quad \delta_\Lambda \mathcal{O}_V(\Psi) = 0$$

In particular, it vanishes for pure gauge solutions: $\mathcal{O}_V(e^{-\Lambda}Q_B e^{\Lambda}) = 0$

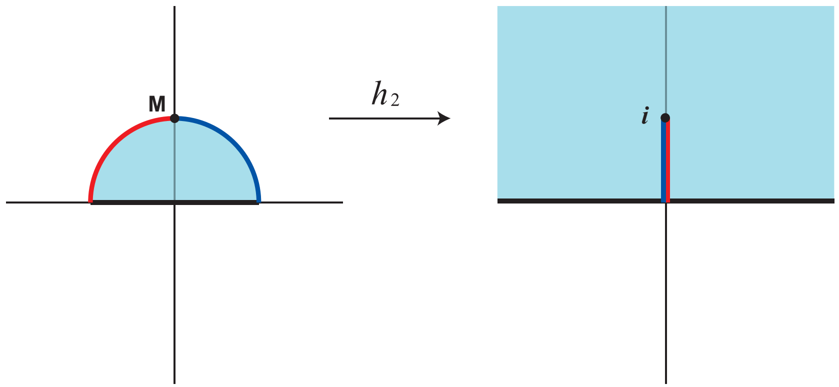
Shapiro-Thorn's vertex

$$\langle \hat{\gamma}(1_c, 2) | \phi_c \rangle_{1_c} | \psi \rangle_2 = \langle h_1[\phi_c(0)] h_2[\psi(0)] \rangle_{\mathrm{UHP}}$$



$$h_1(w) = -i \frac{w-1}{w+1}$$

closed string



$$h_2(w) = I \circ h_{\mathcal{I}}(w) = \frac{1}{2}\left(w - \frac{1}{w}\right)$$

open string

identity state: $\langle \mathcal{I} | \phi \rangle = \langle h_{\mathcal{I}}[\phi(0)] \rangle_{\mathrm{UHP}}$

Gauge invariant overlap for Schnabl's analytic solution

- Schnabl's solution for tachyon condensation

$$\Psi_{\rm Sch} = \frac{\partial_r}{e^{\partial_r}-1}\psi_r|_{r=0} = \sum_{n=0}^{\infty}\frac{B_n}{n!}\partial_r^n\psi_r|_{r=0}$$

$$= \lim_{N\to+\infty}\left(\psi_{N+1} - \sum_{n=0}^{N}\partial_r\psi_r|_{r=n}\right)$$

$$\psi_r \equiv \frac{2}{\pi}U_{r+2}^\dagger U_{r+2}\left[-\frac{1}{\pi}(\mathcal{B}_0+\mathcal{B}_0^\dagger)\tilde{c}(\frac{\pi r}{4})\tilde{c}(-\frac{\pi r}{4}) + \frac{1}{2}(\tilde{c}(-\frac{\pi r}{4})+\tilde{c}(\frac{\pi r}{4}))\right]|0\rangle \qquad U_r \equiv (2/r)^{\mathcal{L}_0}$$

⇨ $\mathcal{O}_V(\psi_r)$:independent of r

[Ellwood, Kawano-Kishimoto-Takahashi (2008)]

$$\mathcal{O}_V(\Psi_{\rm Sch}) = \mathcal{O}_V(\psi_0) = \lim_{N\to\infty}\mathcal{O}_V(\psi_{N+1})$$

Analytic computation of gauge invariant overlap for Schnabl's solution (1)

- Note: $$\psi_r = \frac{2}{\pi} c_1|0\rangle + O(\mathcal{L}_0 - \mathcal{L}_0^\dagger, \mathcal{B}_0 - \mathcal{B}_0^\dagger, c_n + (-1)^n c_{-n})$$

↑ ψ_0

does not contribute to the gauge invariant overlap.

[I.K.(2008)]

$$\langle\hat{\gamma}(1_c,2)|\left((L_n^{(2)} - (-1)^n L_{-n}^{(2)} - (-1)^{\frac{n}{2}}\frac{n}{4}c\,\delta_{n:\mathrm{even}}\right)$$

$$= \langle\hat{\gamma}(1_c,2)|(-2i^n)\sum_{m\geq 0}(-1)^m(\eta^n_{2m+1} - \eta^n_{2m-1})(L_m^{(1)} + (-1)^n\bar{L}_m^{(1)})$$

$$\langle\hat{\gamma}(1_c,2)|(b_n^{(2)} - (-1)^n b_{-n}^{(2)})$$

$$= \langle\hat{\gamma}(1_c,2)|(-2i^n)\sum_{m\geq 0}(-1)^m(\eta^n_{2m+1} - \eta^n_{2m-1})(b_m^{(1)} + (-1)^n\bar{b}_m^{(1)})$$

$$\langle\hat{\gamma}(1_c,2)|(c_m^{(2)} + (-1)^m c_{-m}^{(2)})$$

$$\left(\frac{1+x}{1-x}\right)^k = \sum_{n=0}^{\infty}\eta_n^k x^n$$

$$= \langle\hat{\gamma}(1_c,2)|\frac{-i^m}{4}\sum_{n\geq 1}(-1)^n(\eta^{2n}_{m+1} - \eta^{2n}_{m-1} + \delta_{m,1})(c_n^{(1)} + (-1)^m\bar{c}_n^{(1)})$$

Analytic computation of gauge invariant overlap for Schnabl's solution (2)

- Relation to the boundary state

$$\langle \hat{\gamma}(1_c, 2)|\psi_0\rangle_2 \mathcal{P}_{1_c} = \frac{1}{2\pi}\langle B|c_0^-$$

[Kawano-I.K.-Takahashi(2008)]

generalization

[Kiermaier-Okawa-Zwiebach(2008)]

$$\begin{aligned}
|B_*(\Psi_{\rm Sch})\rangle &\equiv e^{\frac{\pi^2}{s}(L_0+\bar{L}_0)} \oint_s \mathrm{P} e^{-\int_0^s dt[\mathcal{L}_R(t)+\{\mathcal{B}_R(t),\Psi_{\rm Sch}\}]} \\
&= |B\rangle + \sum_{k=1}^{\infty} |B_*^{(k)}(\Psi_{\rm Sch})\rangle \\
&= 0
\end{aligned}$$

Analytic computation of gauge inv. overlap for Schnabl's solution (3)

$$\begin{aligned} \mathcal{O}_V(\Psi_{\mathrm{Sch}}) &= \mathcal{O}_V(\psi_0) = \langle \hat{\gamma}(1_{\mathrm{c}}, 2) | \Phi_V \rangle_{1_{\mathrm{c}}} | \psi_0 \rangle_2 \\ &= \frac{1}{2\pi} \langle B | c_0^- | \Phi_V \rangle \end{aligned}$$

$$\langle B | = \langle 0 | c_{-1} \bar{c}_{-1} c_0^+ \exp\left(- \sum_{n=1}^{\infty} \left(\frac{1}{n} \alpha_n \cdot \bar{\alpha}_n + c_n \bar{b}_n + \bar{c}_n b_n \right) \right)$$

For the Schnabl solution with a parameter $\lambda \quad (\lambda \neq 1)$:

$$\Psi_\lambda = \frac{\lambda \partial_r}{\lambda e^{\partial_r} - 1} \psi_r |_{r=0} = \sum_{n=1}^{\infty} \frac{f_n(\lambda)}{n!} \partial_r^n \psi_r |_{r=0} = - \sum_{n=0}^{\infty} \lambda^{n+1} \partial_r \psi_r |_{r=n}$$

$\Rightarrow$ $$\mathcal{O}_V(\Psi_\lambda) = 0$$

Gauge invariants for Schnabl's solution

Our result: $(\Psi_{\lambda=1} \equiv \Psi_{\rm Sch})$

$$\mathcal{O}_V(\Psi_\lambda) = \begin{cases} 1/(2\pi)\langle B|c_0^-|\Phi_V\rangle & (\lambda = 1) \\ 0 & (|\lambda| < 1) \end{cases}$$

is consistent with

$$S[\Psi_\lambda] = \begin{cases} 1/(2\pi^2 g^2) & (\lambda = 1) \\ 0 & (|\lambda| < 1) \end{cases}$$

[Schnabl(2005),Okawa,Fuchs-Kroyter(2006)]

↓

$\lambda = 1$ → $\Psi_{\rm Sch}$: nontrivial solution

$|\lambda| < 1$ → Ψ_λ : pure gauge solution

Erler-Schnabl's solution

[Erler-Schnabl (2009)]

$$\begin{aligned}\Psi_{\mathrm{ES}} &= \frac{2}{\pi}\frac{1}{\sqrt{1+\frac{\pi}{2}K_1^L}}\mathcal{I} * \hat{U}_1 c_1|0\rangle * \left(1+\frac{\pi}{2}K_1^L\right) B_1^L \hat{U}_1 c_1|0\rangle * \frac{1}{\sqrt{1+\frac{\pi}{2}K_1^L}}\mathcal{I} \\ &= \frac{2}{\pi}\int_0^\infty dt \int_0^\infty ds \frac{e^{-t-s}}{\pi\sqrt{ts}}|t+1\rangle * \hat{U}_1 c_1|0\rangle \left(1+\frac{\pi}{2}K_1^L\right) B_1^L \hat{U}_1 c_1|0\rangle * |s+1\rangle \\ &= \int_0^\infty dt \int_0^\infty ds \frac{e^{-t-s}}{\pi\sqrt{ts}}\hat{U}_{t+s+1}\left(\frac{2}{\pi}\tilde{c}\left(\frac{\pi}{4}(s-t)\right)+\frac{1}{\pi}Q_{\mathrm{B}}\hat{\mathcal{B}}\tilde{c}\left(\frac{\pi}{4}(s-t)\right)\right)|0\rangle\end{aligned}$$

A simple solution ("phantomless" solution)

Note:

$$\frac{1}{\sqrt{1-K}} = \int_0^\infty dt\, \frac{e^{-(1-K)t}}{\sqrt{\pi}\sqrt{t}}$$

Evaluation of gauge invariant overlap for Erler-Schnabl's solution

- Using some relations:

$$\hat{U}_{r+1} = r^{(\mathcal{L}_0 - \mathcal{L}_0^\dagger)/2} r^{-\mathcal{L}_0}, \quad \tilde{c}(x) = e^{xK_1} \tilde{c}(0) e^{-xK_1}$$

$$\int_0^\infty dt \int_0^\infty ds \frac{e^{-t-s}}{\pi\sqrt{ts}}(t+s) = 1$$

we can compute the gauge invariant overlap as follows:

$$\begin{aligned} \mathcal{O}_V(\Psi_{\mathrm{ES}}) &= \langle \hat{\gamma}(1_c, 2) | \Phi_V \rangle_{1_c} | \Psi_{\mathrm{ES}} \rangle_2 = \langle \hat{\gamma}(1_c, 2) | \Phi_V \rangle_{1_c} \frac{2}{\pi} c_1 | 0 \rangle \\ &= \frac{1}{2\pi} \langle B | c_0^- | \Phi_V \rangle \end{aligned}$$

Evaluation of the action for Erler-Schnabl's solution (1)

Note:

$$S[\Psi_{\rm ES}] = \frac{1}{6g^2}\langle \Psi_{\rm ES}, \Psi_{\rm ES} * \Psi_{\rm ES}\rangle = \frac{1}{6g^2}\langle \mathcal{I}|\Psi_{\rm ES} * \Psi_{\rm ES} * \Psi_{\rm ES}\rangle$$

$$\begin{aligned}\Psi_{\rm ES} * \Psi_{\rm ES} * \Psi_{\rm ES} \;\;=\;\; & -\frac{4}{\pi^2}\int_0^\infty dt \int_0^\infty ds' \frac{e^{-t-s'}}{\pi\sqrt{ts}}\int_0^\infty dx e^{-x}\hat{U}_{t+s'+x+1} \\ & \tilde{c}\left(\frac{\pi}{4}(s'-t+x)\right)\tilde{c}\tilde{\partial}\tilde{c}\left(\frac{\pi}{4}(s'-t-x)\right)|0\rangle \\ & +Q_{\rm B}(\cdots)\end{aligned}$$

$$\langle \mathcal{I}|Q_{\rm B} = 0$$

$$\langle \mathcal{I}|\hat{U}_{r+1} = \langle 0|U_r, \quad \langle 0|\tilde{c}(x)\tilde{c}\tilde{\partial}\tilde{c}(y)|0\rangle = -\sin^2(x-y)$$

Evaluation of the action for Erler-Schnabl's solution (2)

$$
\begin{aligned}
\langle \Psi_{\mathrm{ES}}, \Psi_{\mathrm{ES}} * \Psi_{\mathrm{ES}} \rangle &= -\frac{4}{\pi^2} \int_0^\infty dt \int_0^\infty ds' \frac{e^{-t-s'}}{\pi\sqrt{ts}} \int_0^\infty dx \\
& \quad e^{-x} \langle 0 | U_{t+s'+x} \tilde{c} \left(\frac{\pi}{4}(s' - t + x) \right) \tilde{c}\tilde{\partial}\tilde{c} \left(\frac{\pi}{4}(s' - t - x) \right) |0\rangle \\
&= -\frac{4}{\pi^2} \int_0^\infty dt \int_0^\infty ds' \frac{e^{-t-s'}}{\pi\sqrt{ts}} \int_0^\infty dx \frac{d}{dx} e^{-x} \left(\frac{t+s'+x}{2} \right)^2 \\
& \quad \langle 0 | \tilde{c} \left(\frac{\pi}{2} \frac{x}{t+s'+x} \right) \tilde{c}\tilde{\partial}\tilde{c} \left(\frac{\pi}{2} \frac{-x}{t+s'+x} \right) |0\rangle \\
&= -\frac{1}{\pi^2} \int_0^\infty dx \int_0^\infty dy e^{-x-y} (x+y)^2 \left(-\sin^2 \frac{\pi x}{x+y} \right) \\
&= \frac{1}{\pi^2} \int_0^\infty dr e^{-r} r^3 \int_0^1 du \sin^2 \pi u = \frac{3}{\pi^2}
\end{aligned}
$$

$$
S[\Psi_{\mathrm{ES}}] = \frac{1}{2\pi^2 g^2}
$$

Numerical solution by level truncation

- Numerical solution in the Siegel gauge : $b_0|\Psi_{\mathrm{N}}\rangle = 0$

  [...,Sen-Zwiebach (1999),...]

(1) $S[\Psi_{\mathrm{N}}]/S[\Psi_{\mathrm{Sch}}]$

(L,2L)-truncation		(L,3L)-truncation	
(2,4)	0.9485534	(2,6)	0.9593766
(4,8)	0.9864034	(4,12)	0.9878218
(6,12)	0.9947727	(6,18)	0.9951771
(8,16)	0.9977795	(8,24)	0.9979302
(10,20)	0.9991161	(10,30)	0.9991825
(12,24)	0.9997907	(12,36)	0.9998223
(14,28)	1.0001580	(14,42)	1.0001737
(16,32)	1.0003678	(16,48)	1.0003754
(18,36)	1.00049	(18,54)	1.0004937

[Gaiotto-Rastelli (2002)]

(2) $\mathcal{O}_V(\Psi_{\mathrm{N}})/\mathcal{O}_V(\Psi_{\mathrm{Sch}})$

(L,2L)-truncation		(L,3L)-truncation	
(2,4)	0.8783238	(2,6)	0.8898618
(4,8)	0.9294792	(4,12)	0.9319524
(6,12)	0.9501746	(6,18)	0.9510789
(8,16)	0.9606165	(8,24)	0.9611748
(10,20)	0.9677900	(10,30)	0.9681148
(12,24)	0.9723211	(12,36)	0.9725595
(14,28)	0.9760046	(14,42)	0.9761715
(16,32)	0.9785442	(16,48)	0.9786768

[Kawano-Kishimoto-Takahashi (2008)]
and the latest result

Evidence of gauge equivalence: $\Psi_{\mathrm{N}} \sim \Psi_{\mathrm{Sch}}$

On fitting of the value of the action

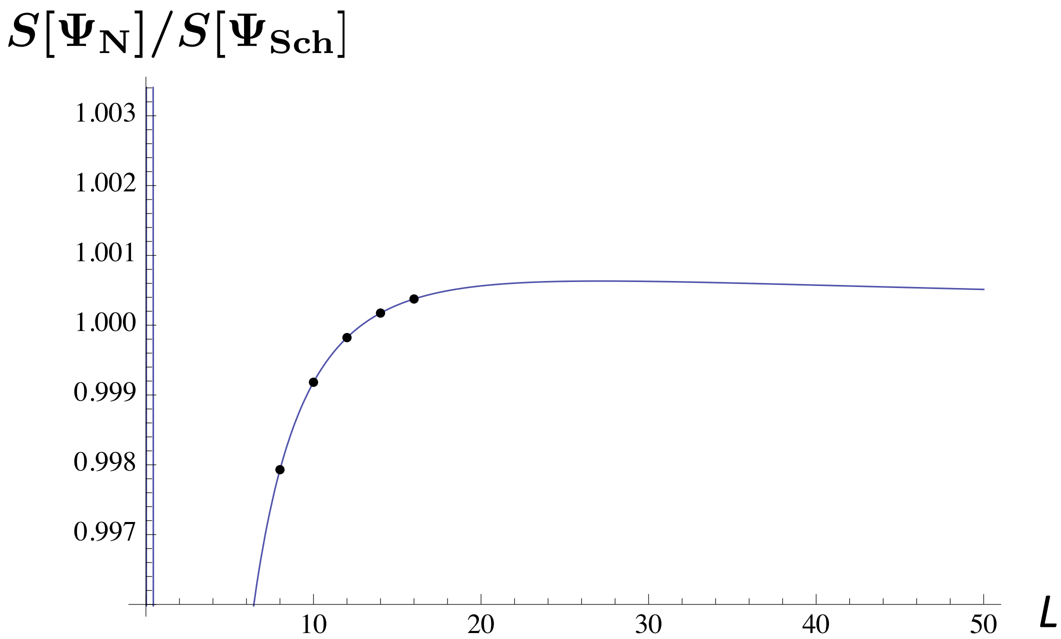
- an extrapolation for the value of the *action*:

[Gaiotto-Rastelli (2002)]

$$F_N(L) = \sum_{n=0}^{N} \frac{a_n}{(L+1)^n}$$

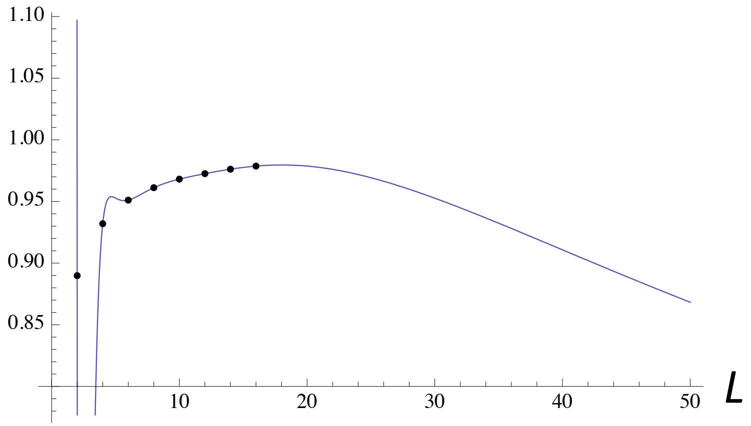
data for (L,3L)-truncation $(L = 0, 2, 4, 6, 8, 10, 12, 14, 16; N = 9)$

$$F_9(\infty) = 1.00003$$



Extrapolation of the gauge invariant overlap?

If we use the same fit function in the same way as the action *naively*, we have

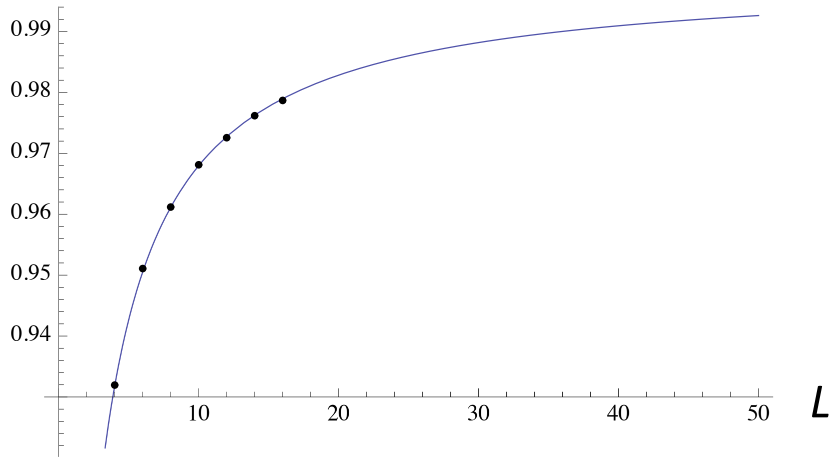


$$F_9(\infty) = 0.442107$$

The fitting does not work well.

However, if we take a fit function: $F_{\rm exp}(L) = a_0 \exp\left(-\frac{a_1}{L+1} - \frac{a_2}{(L+1)^2}\right)$

$\mathcal{O}_V(\Psi_{\rm N})/\mathcal{O}_V(\Psi_{\rm Sch})$



using data for (L,3L)-truncation

$$(L = 0, 2, 4, 6, 8, 10, 12, 14, 16)$$

$$F_{\rm exp}(\infty) = 0.99954$$

A good fitting function (!?)

Numerical solutions in Asano-Kato's *a*-gauges

- Asano-Kato's *a*-gauge $(b_0 M + a\, b_0 c_0 \tilde{Q})|\Psi_a\rangle = 0$

$$Q_{\rm B} = \tilde{Q} + c_0 L_0 + b_0 M$$

$a = 0$ ⇒ Siegel gauge: $b_0|\Psi_0\rangle = 0$

$a = \infty$ ⇒ Landau gauge: $b_0 c_0 \tilde{Q}|\Psi_\infty\rangle = 0$

(1) For *a*-gauge solution, (6,18)-trucation $S[\Psi_a]/S[\Psi_{\rm Sch}]$

$a = \infty$	0.9609438
$a = 4.0$	0.9244886
$a = 0.5$	1.0045858
$a = -2.0$	0.9798943
⋮	⋮

[Asano-Kato(2006)]

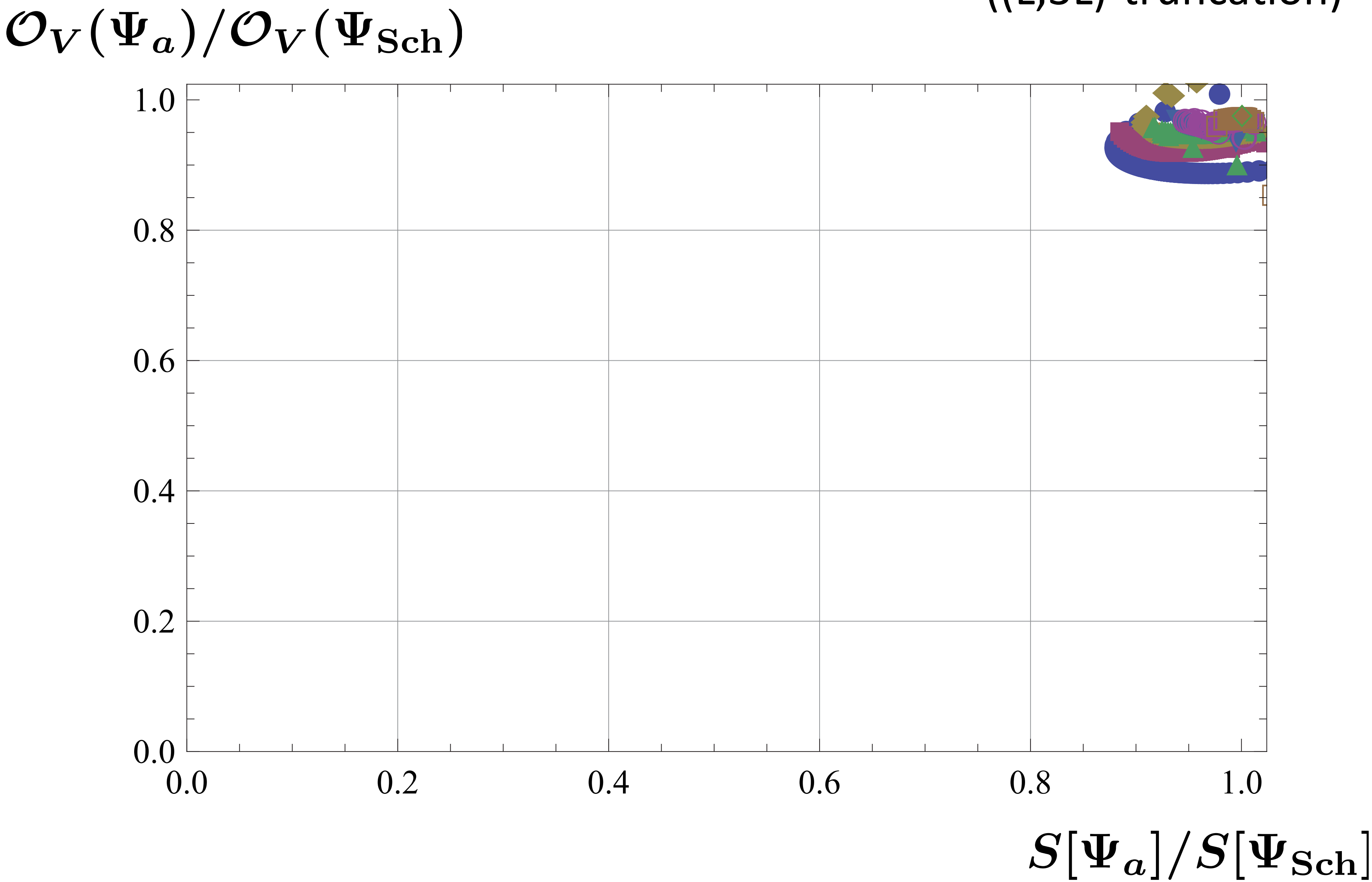
higher level?

(2) $\mathcal{O}_V(\Psi_a)/\mathcal{O}_V(\Psi_{\rm Sch})$ (?)

[I.K.-Takahashi, 0902.0445]

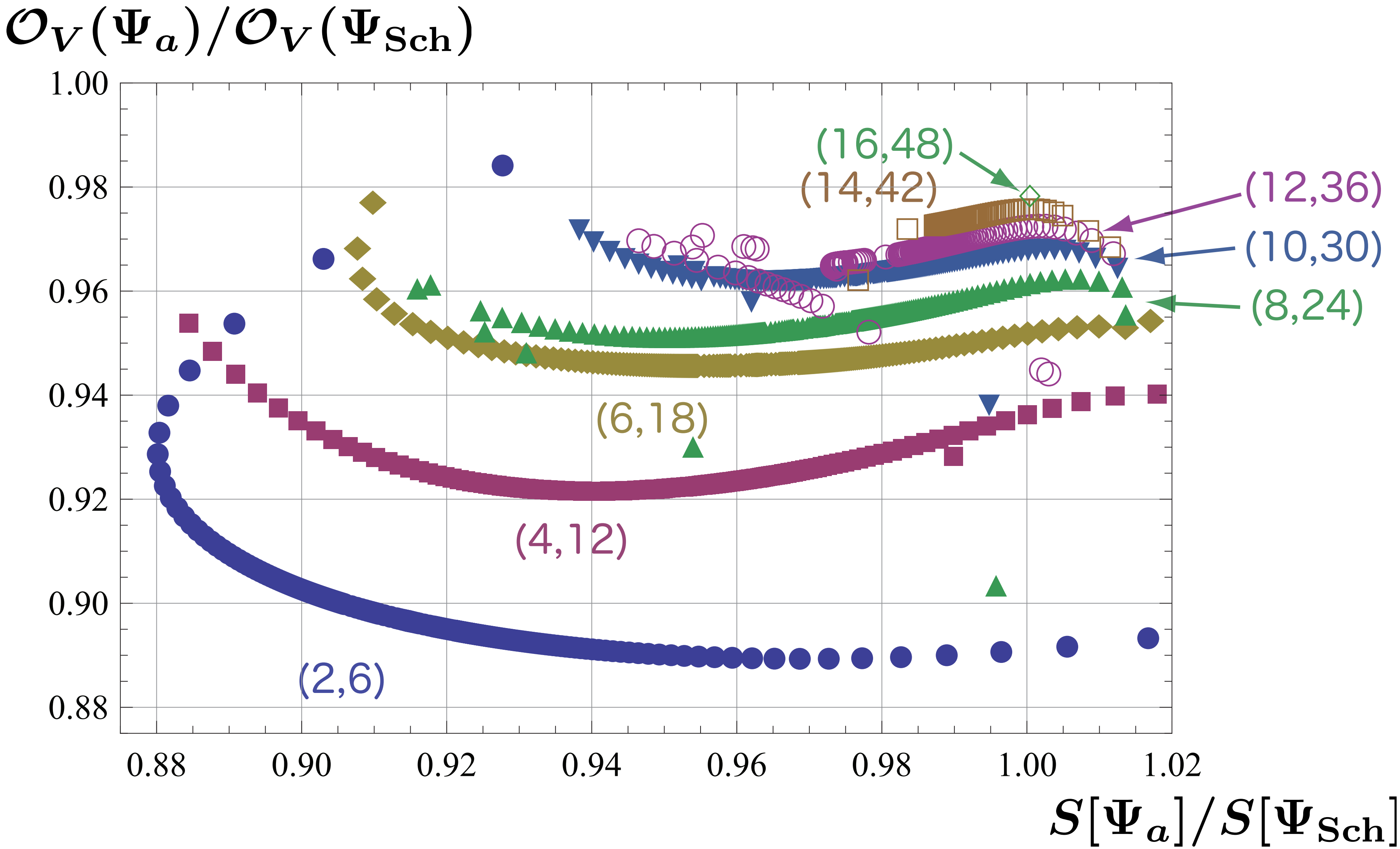
Gauge invariants for AK's *a*-gauge solutions

((L,3L)-truncation)



Gauge invariants for AK's *a*-gauge solutions

((L,3L)-truncation)



Summary (1)

- We have evaluated gauge invariants (action and gauge invariant overlap) for numerical solutions in AK's *a*-gauges by level truncation
- Our numerical results suggest:

$$L \to +\infty \qquad S[\Psi_{a,L}]|_L \to S[\Psi_{\mathrm{Sch}}] = S[\Psi_{\mathrm{ES}}]$$

$$\mathcal{O}_V(\Psi_{a,L}) \to \mathcal{O}_V(\Psi_{\mathrm{Sch}}) = \mathcal{O}_V(\Psi_{\mathrm{ES}})$$

- It is consistent with the gauge equivalence:

$$\Psi_a \sim \Psi_{\mathrm{Sch}} \sim \Psi_{\mathrm{ES}}$$

Takahashi-Tanimoto's solution

- "Identity based solution" [Takahashi-Tanimoto (2002)]

$$\Psi_0 = Q_L(e^h - 1)\mathcal{I} - C_L((\partial h)^2 e^h)\mathcal{I}$$

$$Q_L(f) \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i} f(z) j_{\rm B}(z) \qquad C_L(f) \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i} f(z) c(z)$$

$$h(-1/z) = h(z), \quad h(\pm i) = 0$$

In the following, we take

$$\begin{aligned} h(z) &= \log\left(1 + \frac{a}{2}\left(z + \frac{1}{z}\right)^2\right) \\ &= -\log(1 - Z(a))^2 - \sum_{n=1}^{\infty} \frac{(-1)^n}{n} Z(a)^n (z^{2n} + z^{-2n}) \end{aligned}$$

$$Z(a) = (1 + a - \sqrt{1 + 2a})/a \qquad a \geq -1/2$$

Half-integration and identity state

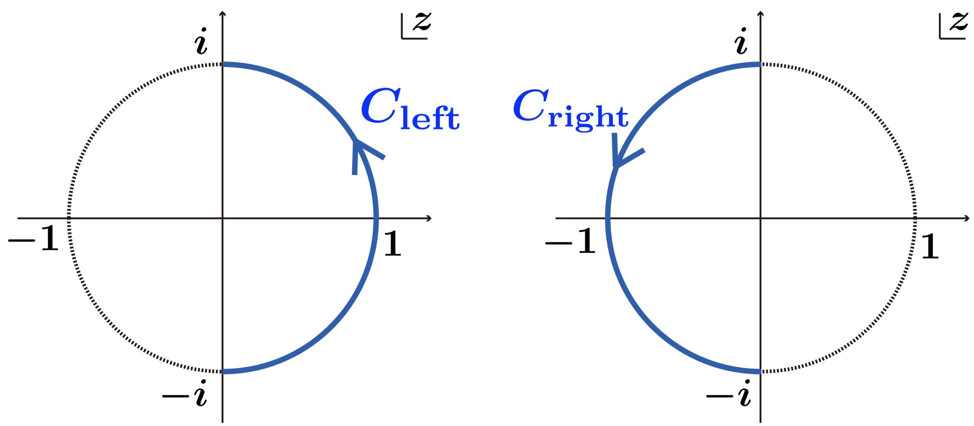
$$(\sigma(z)A) * B = (-1)^{|\sigma||A|} * (z'^{2h}\sigma(z')B) \qquad (zz' = -1,\ |z| = 1,\ \mathrm{Re}\, z \leq 0)$$

primary field (dim *h*)

$$(\Sigma_R(F)A) * B = -(-1)^{|\sigma||A|} A * (\Sigma_L(F)B)$$

$$\Sigma_{L(R)}(F) = \int_{C_{\mathrm{left(right)}}} \frac{dz}{2\pi i} F(z)\,\sigma(z)$$

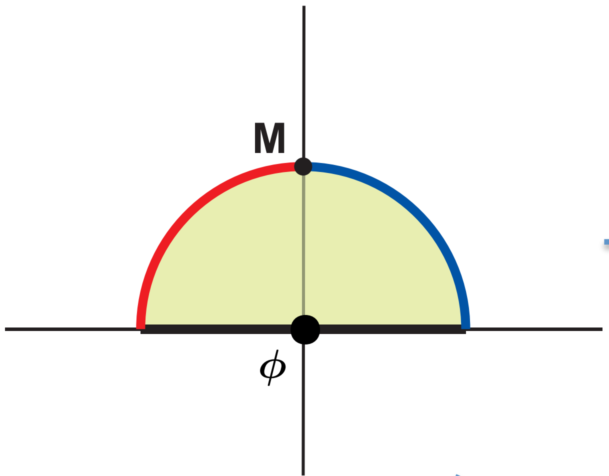
$$F(-1/z) = (z^2)^{1-h} F(z)$$

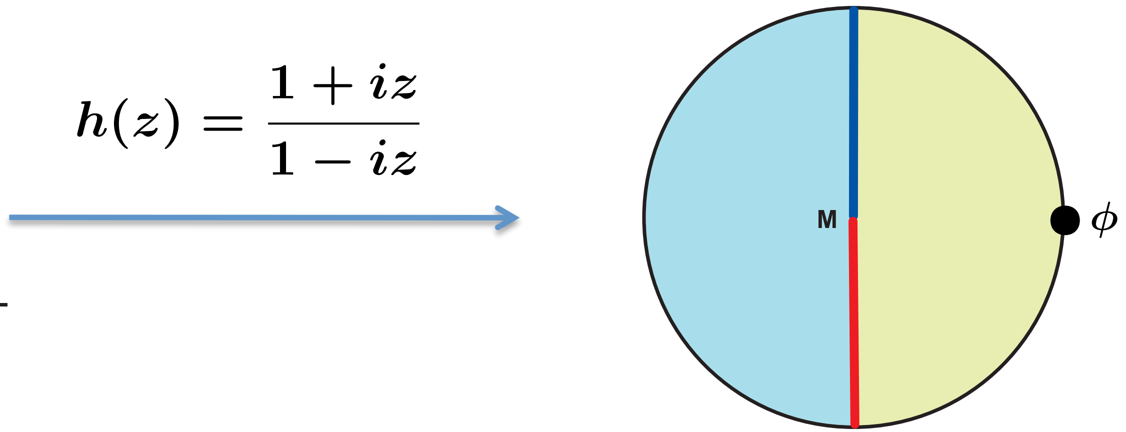


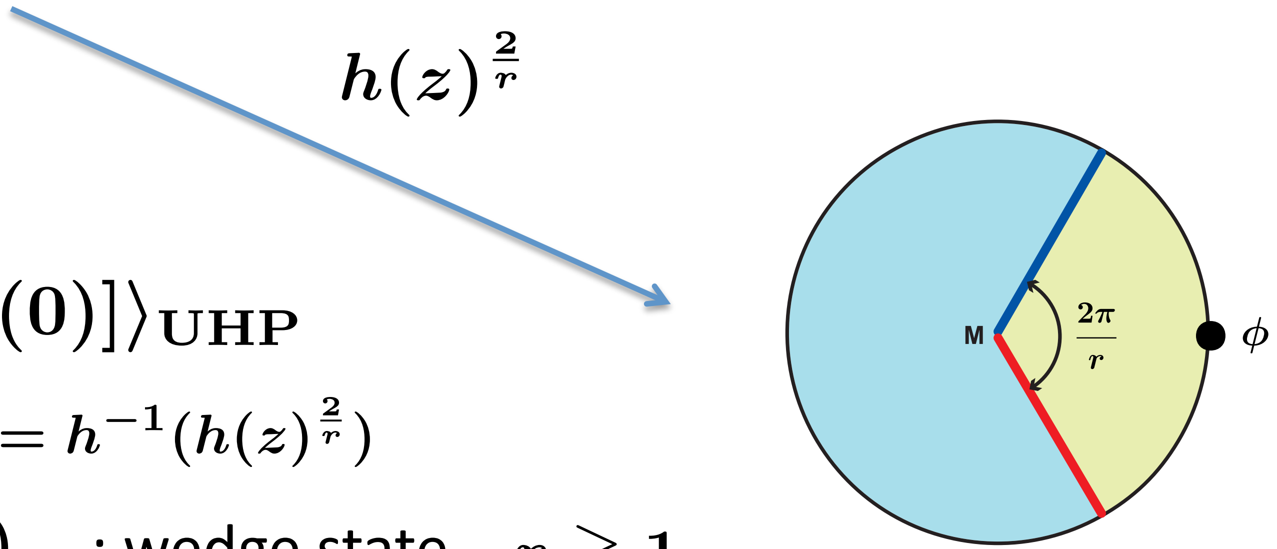
$$\Sigma_L(F)\mathcal{I} = -\Sigma_R(F)\mathcal{I}$$

$$\mathcal{I} * A = A * \mathcal{I} = A$$: identity element with respect to the star product

Wedge state





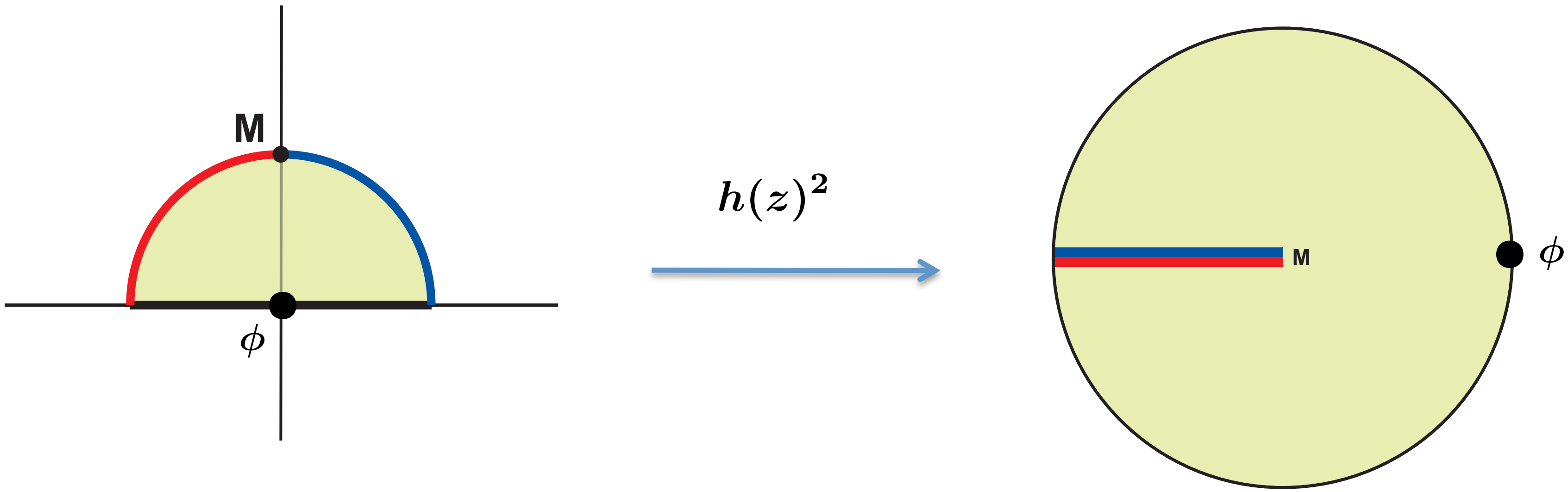


$$\langle r|\phi\rangle = \langle f_r[\phi(0)]\rangle_{\mathrm{UHP}}$$

$$f_r(z) = h^{-1}(h(z)^{\frac{2}{r}})$$

$|r\rangle = \mathrm{bpz}(\langle r|)$: wedge state $\quad r \geqq 1$

Identity state



$$\langle \mathcal{I}|\phi\rangle = \langle h_{\mathcal{I}}[\phi(0)]\rangle_{\rm UHP}$$

$$h_{\mathcal{I}}(z) = h^{-1}(h(z)^2) = \frac{2z}{1-z^2}$$

$$\begin{aligned} |\mathcal{I}\rangle &= |r=1\rangle = 2^{\mathcal{L}_0^\dagger}|0\rangle \\ &= \cdots e^{-\frac{1}{2^{k-1}}L_{-2^k}} \cdots e^{-\frac{1}{8}L_{-16}} e^{-\frac{1}{4}L_{-8}} e^{-\frac{1}{2}L_{-4}} e^{L_{-2}}|0\rangle \end{aligned}$$

$$\mathcal{L}_0^\dagger = L_0 + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{4k^2-1} L_{-2k}$$

On the TT solution

- Formal pure gauge form:

$$\Psi_0 = \exp(q_L(h)\mathcal{I})Q_{\rm B}\exp(-q_L(h)\mathcal{I})$$

Gauge parameter string field:

$$\exp(\pm q_L(h)\mathcal{I}) = \exp(\pm q_L(h))\mathcal{I}$$

$\exp(\pm q_L(h))$: ill-defined for $a = -1/2$

$$q_L(f) \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i} f(z) :cb:(z)$$

Non-trivial solution (?)

On the TT solution (1)

It is difficult to compute $S[\Psi_0], \quad \mathcal{O}_V(\Psi_0)$ *directly*, because $\langle \mathcal{I}|(\cdots)|\mathcal{I}\rangle$ is divergent.

→ Identity based solutions may be "singular."

↓

Alternatively, we investigate *string field theories around* Ψ_0.

SFT around the TT solution

- Expansion around the TT solution:

$$\begin{aligned} S_a[\Phi] &= S[\Psi_0+\Phi]-S[\Psi_0] \\ &= -\frac{1}{g^2}\left[\frac{1}{2}\langle\Phi, Q'\Phi\rangle+\frac{1}{3}\langle\Phi,\Phi*\Phi\rangle\right] \end{aligned}$$

$$\begin{aligned} Q' &= (1+a)Q_{\rm B}+\frac{a}{2}(Q_2+Q_{-2})+4aZ(a)c_0-2aZ(a)^2(c_2+c_{-2}) \\ &\quad -2a(1-Z(a)^2)\sum_{n=2}^{\infty}(-1)^n Z(a)^{n-1}(c_{2n}+c_{-2n}) \end{aligned}$$

$$j_{\rm B}(z)=cT^{\rm m}(z)+:bc\partial c:+\frac{3}{2}\partial^2 c(z)=\sum_{n=-\infty}^{\infty}Q_n z^{-n-1}$$

$$(Q')^2=0$$

$$\delta_\Lambda\Phi=Q'\Lambda+\Phi*\Lambda-\Lambda*\Phi$$

$\Rightarrow$

$$\delta_\Lambda S_a[\Phi]=0$$

$$\delta_\Lambda \mathcal{O}_V(\Phi)=0$$

On the new BRST operator

- cohomology of Q' [I.K.-Takahashi (2002)]

$a > -1/2$ the same as the original Q_{B}

→ Ψ_0 : pure gauge

$a = -1/2$ no cohomology at ghost number 1 sector

→ no open string

Ψ_0 : tachyon vacuum (!?)

Note: vanishing cohomology around Schnabl's solution → [Ellwood-Schnabl (2006)]

Numerical solution in SFT around the TT solution (1)

- We solve the EOM: $Q'\Phi + \Phi * \Phi = 0$

in the Siegel gauge $b_0\Phi = 0$ [cf. Gaiotto-Rastelli(2002)]

by level truncation with the iterative algorithm:

$$c_0 b_0 (c_0 L(a)\Phi^{(n+1)} + \Phi^{(n)} * \Phi^{(n+1)} + \Phi^{(n+1)} * \Phi^{(n)} - \Phi^{(n)} * \Phi^{(n)}) = 0$$

$$\begin{aligned} L(a) &= \{b_0, Q'\} \\ &= (1+a)L_0 + \frac{a}{2}(L_2 + L_{-2}) + a(q_2 - q_{-2}) + 4(1 + a - \sqrt{1+2a}) \end{aligned}$$

Using the above we can define $\Phi^{(n)} \mapsto \Phi^{(n+1)}$

dimension of the truncated space in the Siegel-gauge:

L	0	2	4	6	8	10	12	14	16	18
dim.	1	3	9	26	69	171	402	898	1925	3985

Numerical solution in SFT around the TT solution (2)

If the iteration converges $$c_0 b_0 (Q' \Phi^{(\infty)} + \Phi^{(\infty)} * \Phi^{(\infty)}) = 0$$

Projected part of the equation of motion

We also check the "BRST invariance": [cf. Hata-Shinohara (2000)]

$$\| b_0 c_0 (Q' \Phi^{(\infty)} + \Phi^{(\infty)} * \Phi^{(\infty)}) \| / \| \Phi^{(\infty)} \| \ll 1$$

⇓

We evaluate the gauge invariants:

(1)potential height: $$f_a(\Phi) = 2\pi^2 \left(\frac{1}{2} \langle \Phi, c_0 L(a) \Phi \rangle + \frac{1}{3} \langle \Phi, \Phi * \Phi \rangle \right)$$

(2)gauge invariant overlap: $$\mathcal{O}_V(\Phi) = 2\pi \langle \hat{\gamma}(1_c, 2) | \Phi_V \rangle_{1_c} | \Phi \rangle_2$$

Construction of stable vacuum solution

- The initial configuration for $a = 0 \ (Q' = Q_{\rm B})$

$$\Phi^{(0)} = \frac{64}{81\sqrt{3}} c_1 |0\rangle \quad \xrightarrow{\text{iteration}} \quad \Phi_1\big|_{a=0}$$

conventional tachyon vacuum solution

the nontrivial solution for (0,0) truncation

- The initial configuration for $a = \epsilon \ (0 < |\epsilon| \ll 1)$

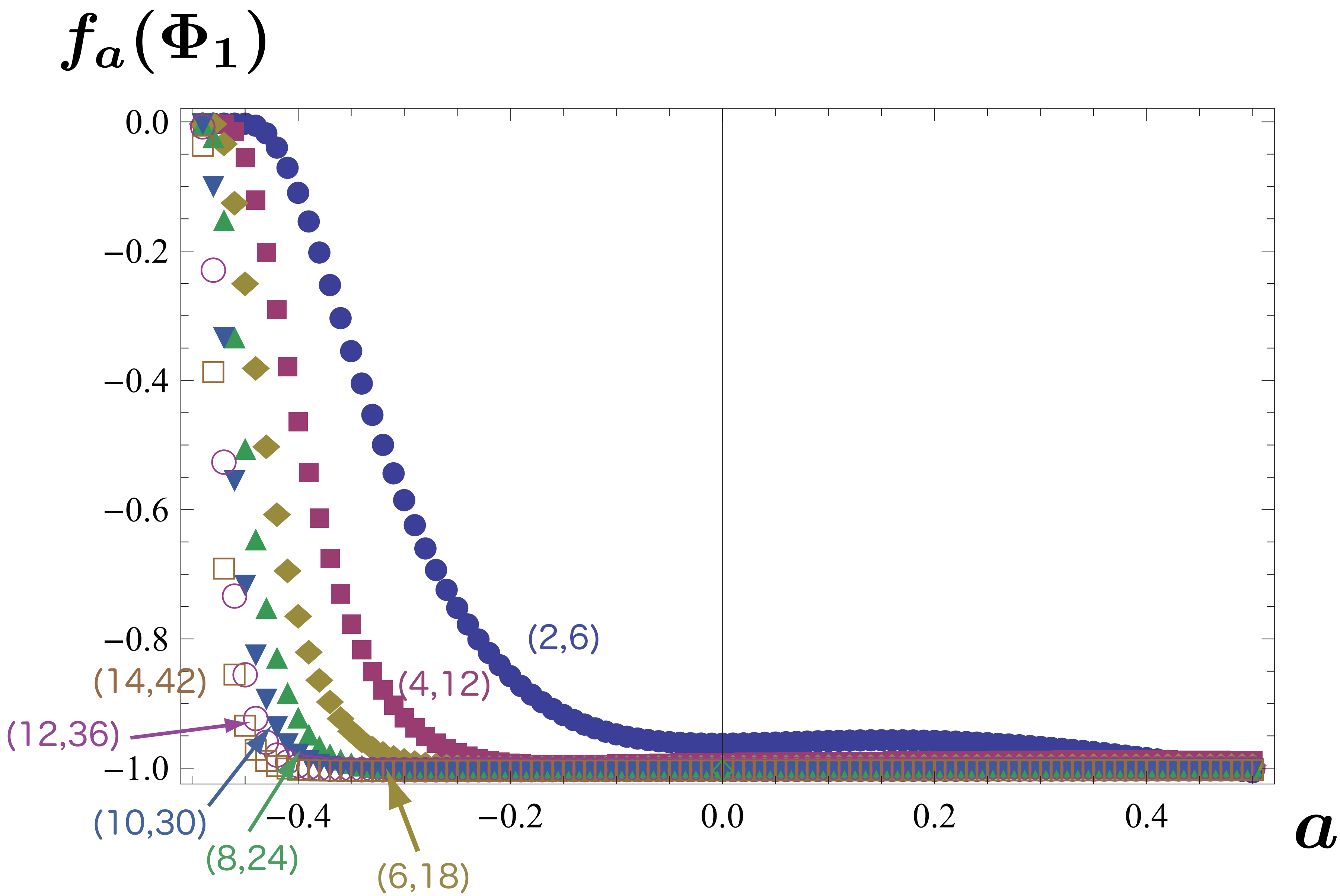
$$\Phi^{(0)} = \Phi_1\big|_{a=0} \quad \xrightarrow{\text{iteration}} \quad \Phi_1\big|_{a=\epsilon}$$

- The initial configuration for $a = 2\epsilon$

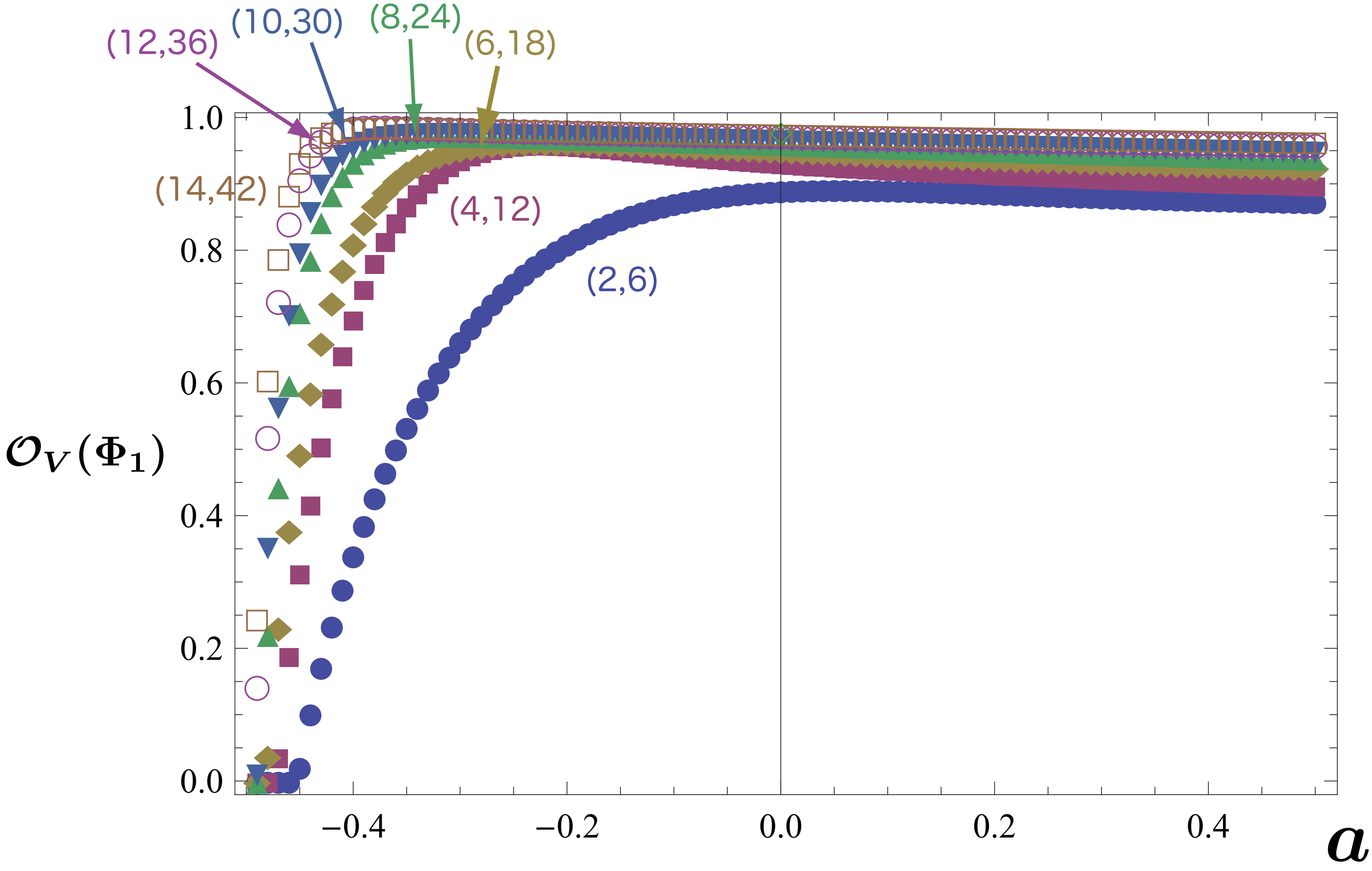
$$\Phi^{(0)} = \Phi_1\big|_{a=\epsilon} \quad \xrightarrow{\text{iteration}} \quad \Phi_1\big|_{a=2\epsilon}$$

$\vdots$

Potential height for Φ_1

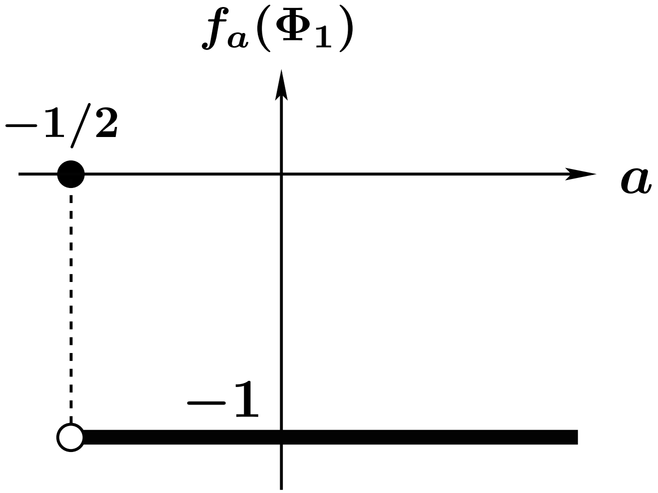


Gauge invariant overlap for Φ_1



Stable vacuum solution

- For $L \to \infty$, numerical results suggest



$a > -1/2$

Φ_1 :nontrivial tachyon vacuum

$a = -1/2$

$\Phi_1 = 0$

$a > -1/2$ ⇒ Ψ_0 : pure gauge

$a = -1/2$ ⇒ Ψ_0 : tachyon vacuum (!?)

Construction of unstable vacuum solution

- The initial configuration for $a = -1/2$

$$\Phi^{(0)} = -\frac{32}{9\sqrt{3}} c_1 |0\rangle \xrightarrow{\text{iteration}} \Phi_2|_{a=-1/2}$$

the nontrivial solution for (0,0) truncation

- The initial configuration for $a = -1/2 + \epsilon \ (0 < \epsilon \ll 1)$

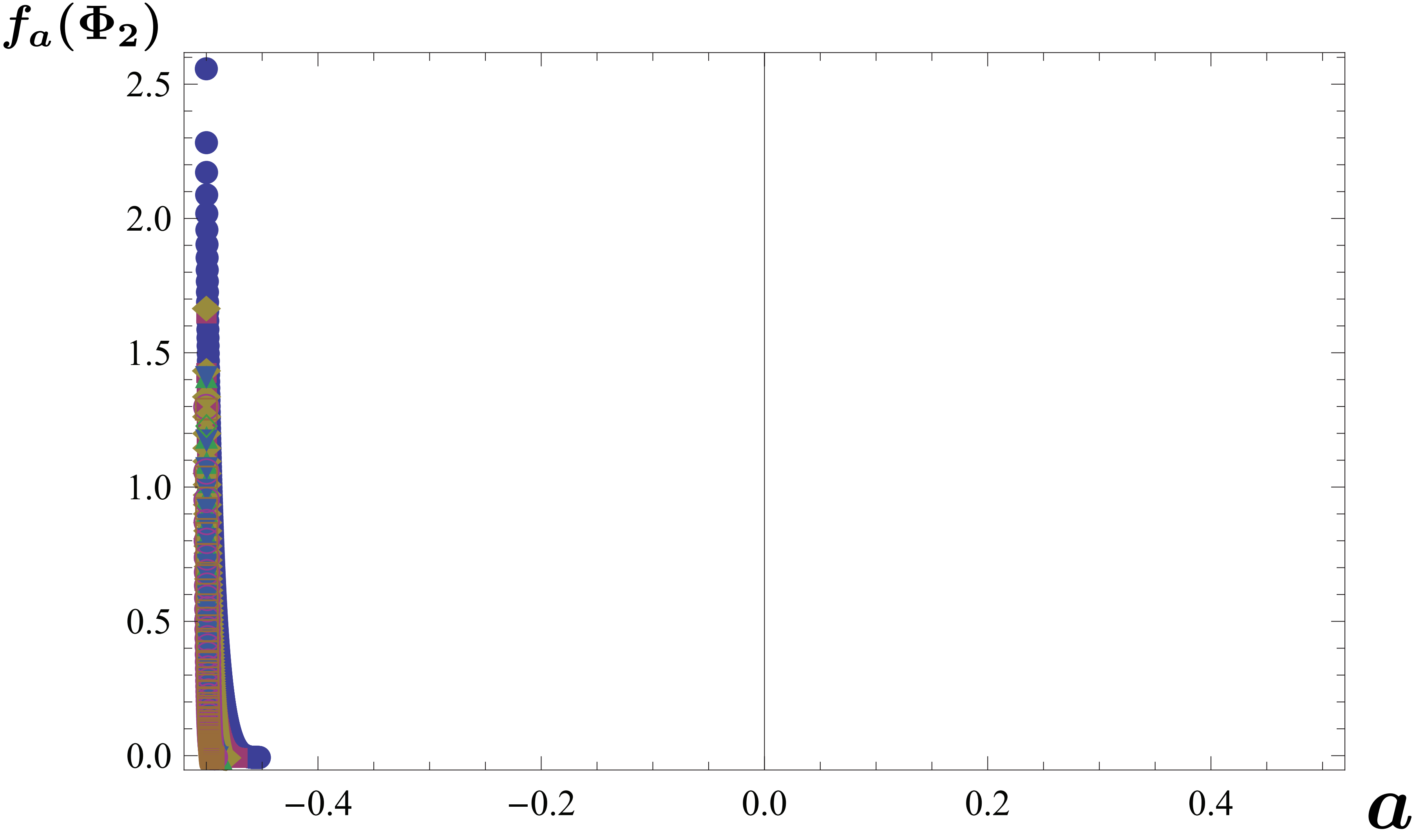
$$\Phi^{(0)} = \Phi_2|_{a=-1/2} \xrightarrow{\text{iteration}} \Phi_2|_{a=-1/2+\epsilon}$$

- The initial configuration for $a = -1/2 + 2\epsilon$

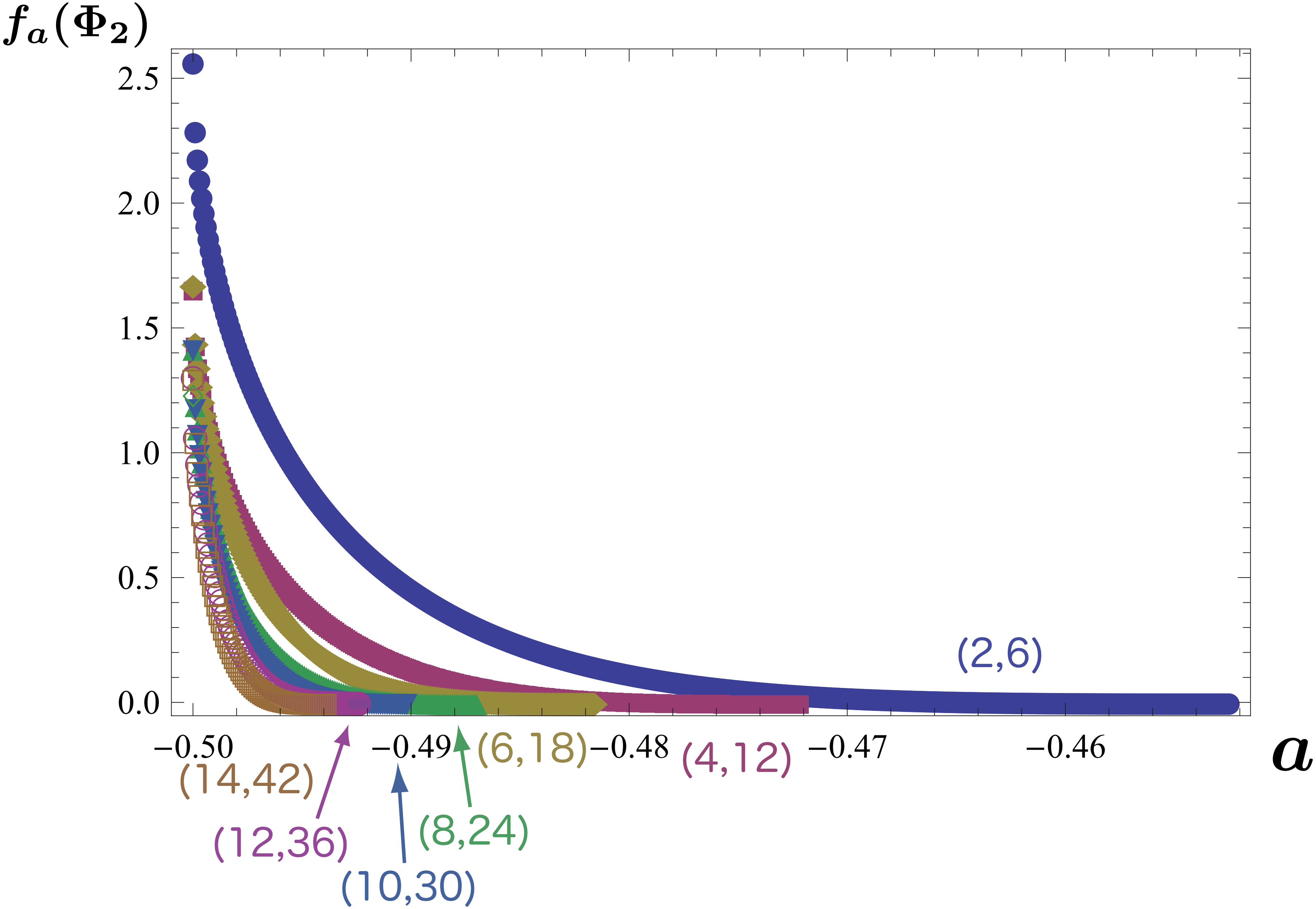
$$\Phi^{(0)} = \Phi_2|_{a=-1/2+\epsilon} \xrightarrow{\text{iteration}} \Phi_2|_{a=-1/2+2\epsilon}$$

$$\vdots$$

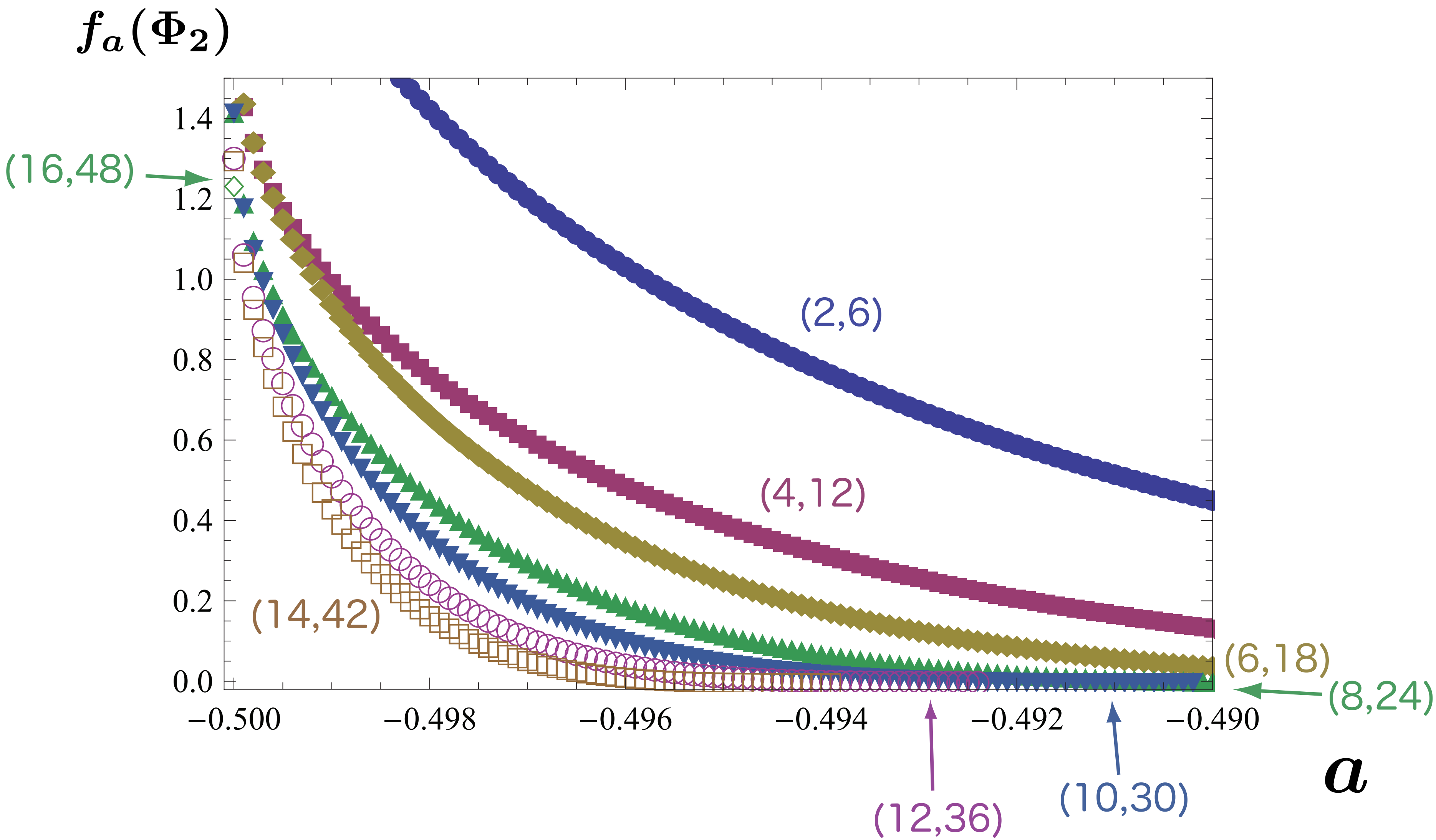
Potential height for Φ_2



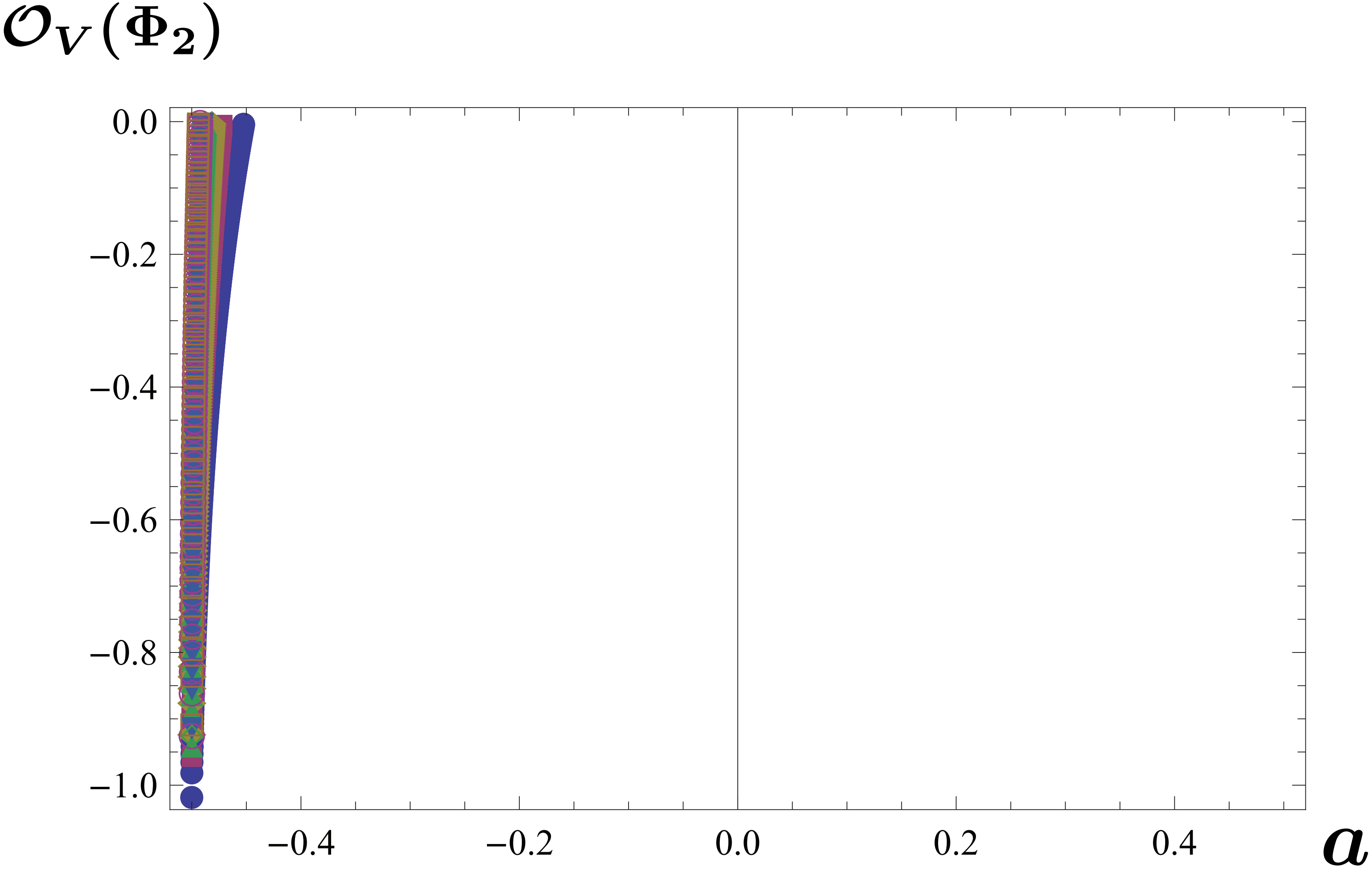
Potential height for Φ_2



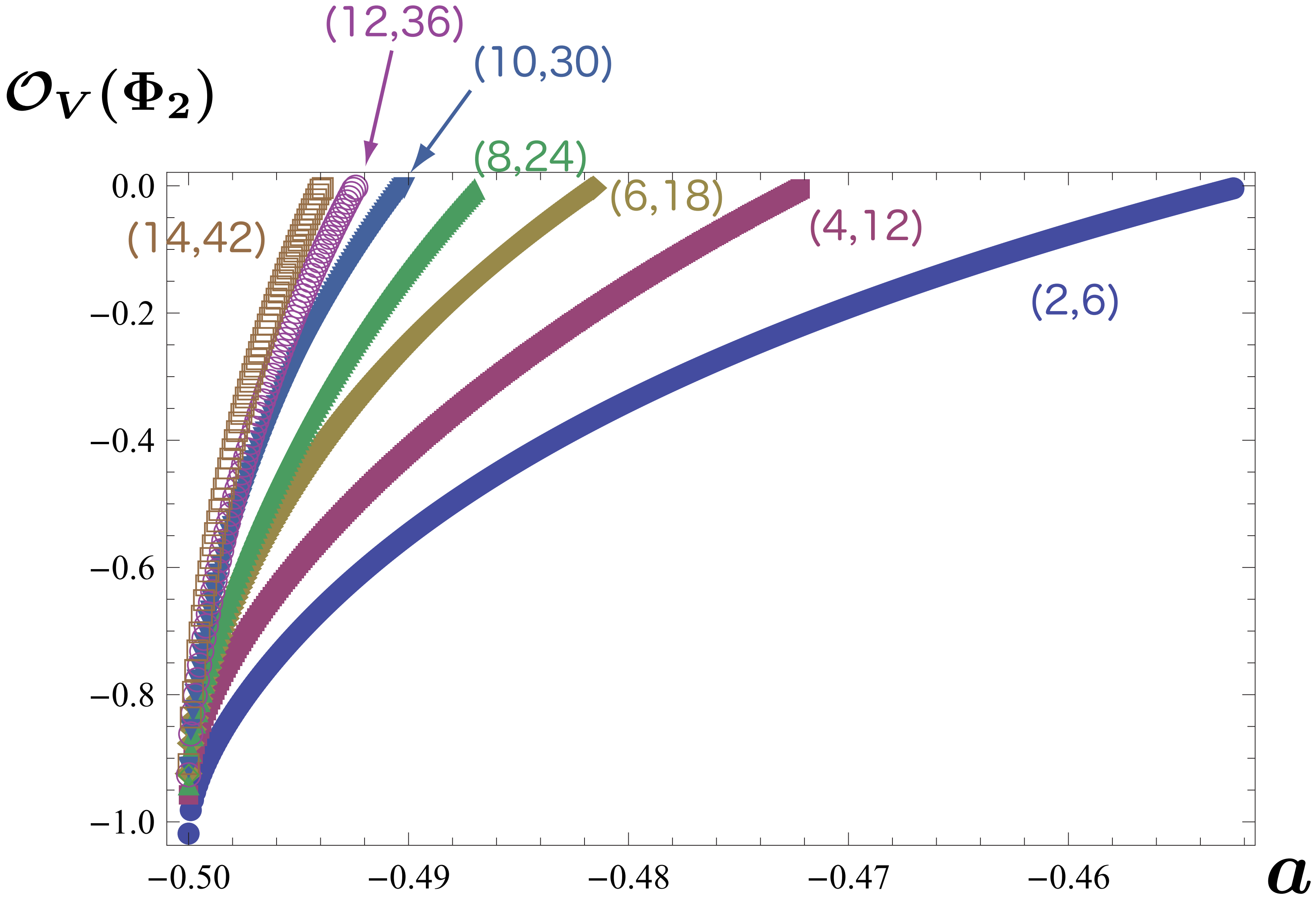
Potential height for Φ_2 (near -1/2)



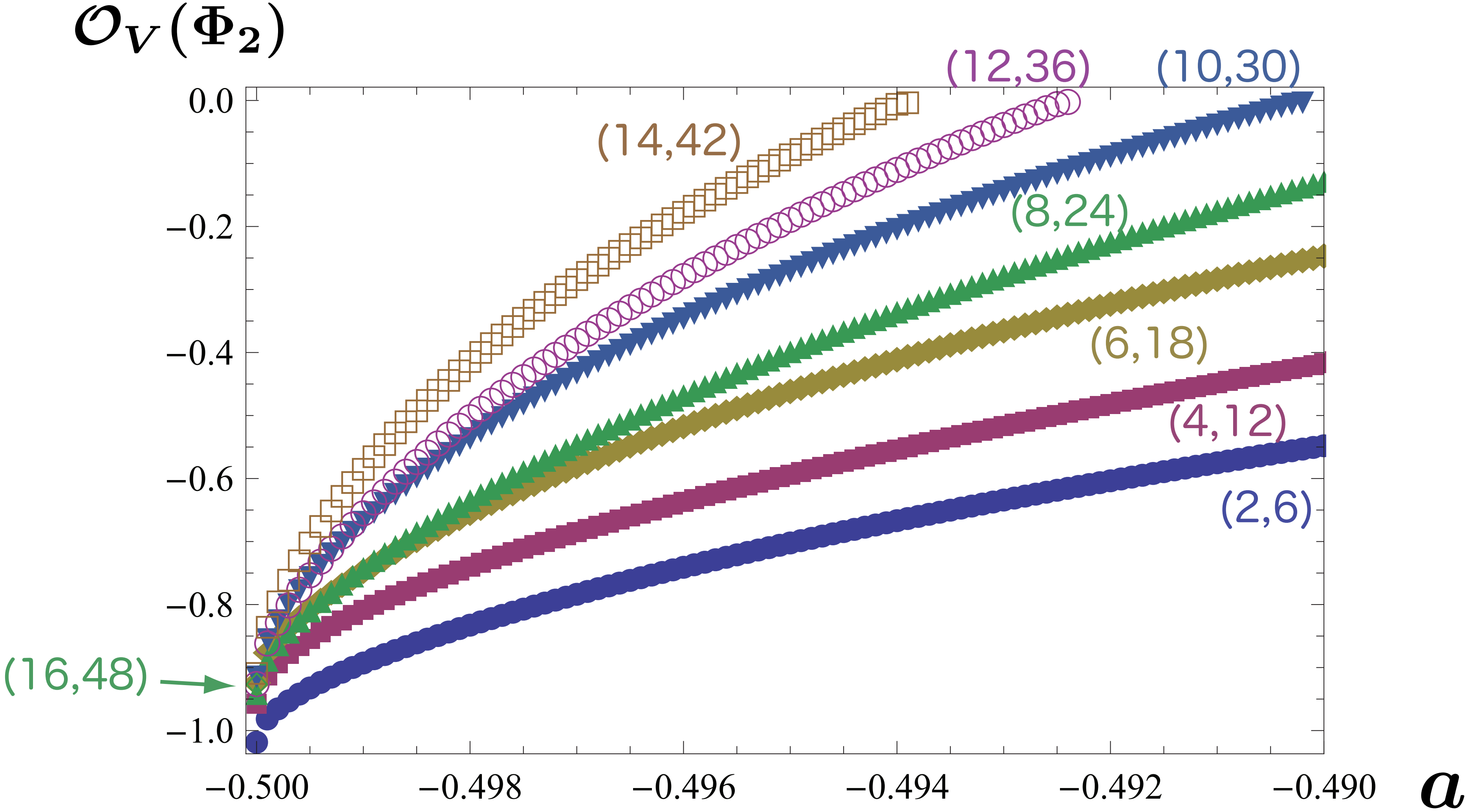
Gauge invariant overlap for Φ_2



Gauge invariant overlap for Φ_2



Gauge invariant overlap for Φ_2 (near -1/2)



Gauge invariants for $\Phi_2|_{a=-1/2}$

(L,3L)	$f_a(\Phi_2)$	$\mathcal{O}_V(\Phi_2)$
(0,0)	2.3105796	−1.0748441
(2,6)	2.5641847	−1.0156983
(4,12)	1.6550774	−0.9539832
(6,18)	1.6727496	−0.9207572
(8,24)	1.4193393	−0.9377548
(10,30)	1.4168893	−0.9110994
(12,36)	1.3035715	−0.9237917
(14,42)	1.2986472	−0.9056729
(16,48)	1.2357748	−0.9229035

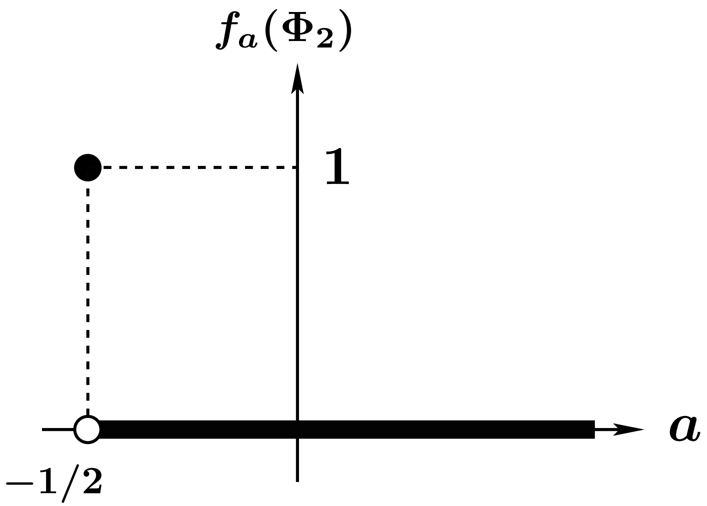
(L,3L)	**Extrapolation of** $f_a(\Phi_2)$
(4∞,12∞)	0.98107
(4∞+2,12∞+6)	0.98146

Fitting function:

$$F_N(L) = \sum_{n=0}^{N} \frac{a_n}{(L+1)^n}$$

Unstable vacuum solution

- For $L \to \infty$, numerical results suggest



$a > -1/2$

$\Phi_2 = 0$

$a = -1/2$

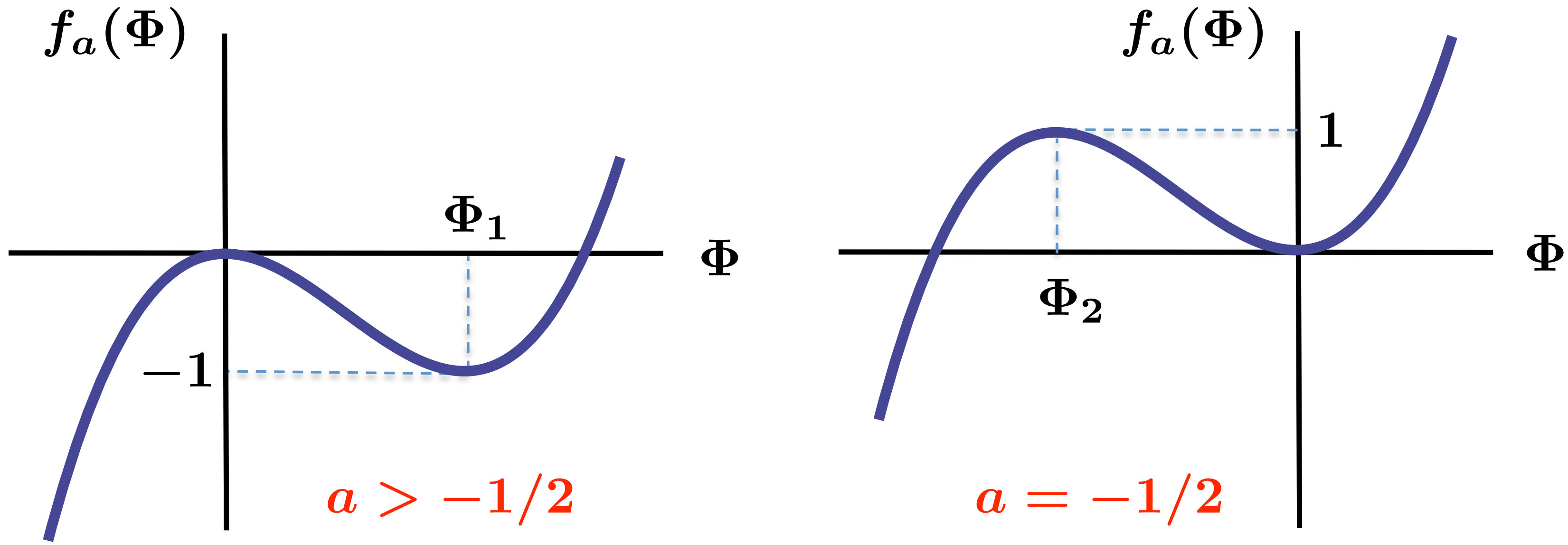
Φ_2 :nontrivial vacuum
(perturbative vacuum!?)

$a > -1/2$ ⇒ Ψ_0 : pure gauge

$a = -1/2$ ⇒ Ψ_0 : tachyon vacuum (!?)

Summary (2)

- We constructed stable solution and unstable solution in the expanded theory around TT's identity based solution.
- We evaluated the gauge invariants for the obtained solutions.
- Numerical results suggest the vacuum structure is like this:



- This is consistent with the expectation that

$a > -1/2$ ⇒ Ψ_0 : pure gauge

$a = -1/2$ ⇒ Ψ_0 : tachyon vacuum

Discussion

- Our result on TT's solution suggests that the TT solution (a=−1/2) may be "gauge equivalent" to the Schnabl solution (λ=1) and give an alternative approach to investigating the nonperturbative vacuum.

- *Regular* solutions? *Definition* of the space of string fields?

- Extension to superstring field theory?
  [Erler(2007), Aref'eva-Gorbachev-Medvedev(2008), Fuchs-Kroyter(2008),…]