Vacuum structure around identity based solutions

Isao Kishimoto

(RIKEN)

Collaboration with Tomohiko Takahashi (Nara Women's Univ.)

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Non-perturbative vacuum in bosonic open string field theory

- Schnabl's solution $\boldsymbol{\Psi}_{\mathbf{Sch}}$

Gauge invariants

(1) Action: D-brane tension

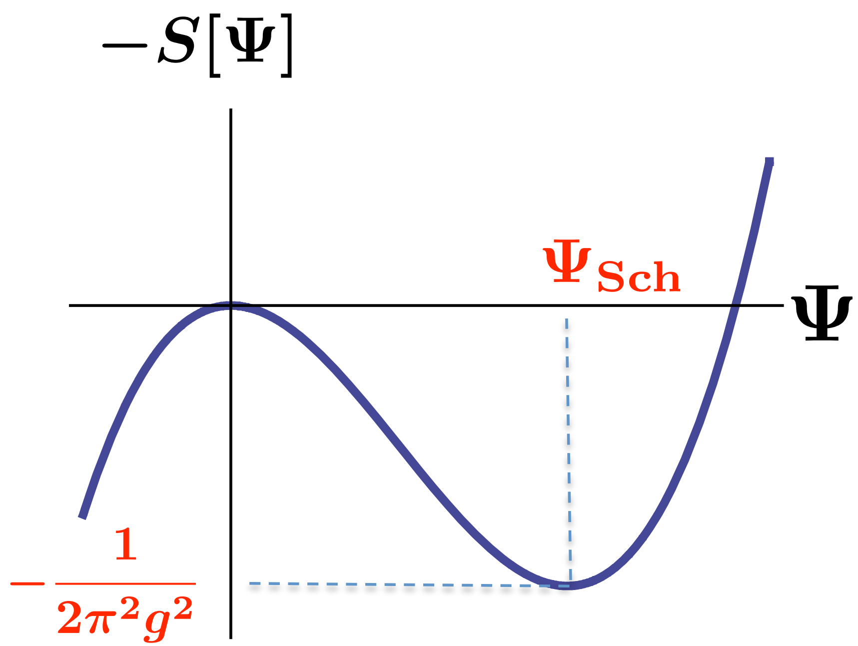
$$S[\boldsymbol{\Psi}_{\mathbf{Sch}}] = \frac{1}{2\pi^2 g^2}$$

[Schnabl(2005),Okawa,Fuchs-Kroyter(2006)]

(2) Gauge invariant overlap:

$$\mathcal{O}_V(\boldsymbol{\Psi}_{\mathbf{Sch}}) = \frac{1}{2\pi}\langle B|c_0^-|\boldsymbol{\Phi}_V\rangle$$

[Ellwood, Kawano-Kishimoto-Takahashi(2008)]



Bosonic cubic open string field theory

Action: $$S[\Psi] = -\frac{1}{g^2}\left(\frac{1}{2}\langle\Psi, Q_{\mathrm{B}}\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right)$$

$$Q_{\mathrm{B}} = \oint \frac{dz}{2\pi i}\left(cT^{(\mathrm{m})} + bc\partial c\right)$$

Equation of motion: $$Q_{\mathrm{B}}\Psi + \Psi * \Psi = 0$$

Gauge transformation: $$\delta_\Lambda \Psi = Q_{\mathrm{B}}\Lambda + \Psi * \Lambda - \Lambda * \Psi$$

$$\rightarrow \quad \delta_\Lambda S[\Psi] = 0$$

Gauge invariant overlap

Gauge invariant for on-shell closed string state

$$\mathcal{O}_V(\Psi) = \langle \mathcal{I} | V(i) | \Psi \rangle = \langle \hat{\gamma}(1_c, 2) | \Phi_V \rangle_{1_c} | \Psi \rangle_2$$

Shapiro-Thorn's open-closed vertex

$$|\Phi_V\rangle = c_1 \bar{c}_1 |V_m\rangle$$

V_m :matter primary with (1,1)-dim.

$$\mathcal{O}_V(Q_B \Lambda) = 0$$

$$\mathcal{O}_V(\Psi * \Lambda) = \mathcal{O}_V(\Lambda * \Psi)$$

$$\Rightarrow \quad \delta_\Lambda \mathcal{O}_V(\Psi) = 0$$

In particular, it vanishes for pure gauge solutions: $\mathcal{O}_V(e^{-\Lambda} Q_B e^{\Lambda}) = 0$

Gauge invariants for Schnabl's solution

Schnabl's solution with one parameter: $(\Psi_{\lambda=1} \equiv \Psi_{\rm Sch})$

$$\Psi_\lambda = \frac{\lambda \partial_r}{\lambda e^{\partial_r} - 1} \psi_r|_{r=0}$$

$$\psi_r \equiv \frac{2}{\pi} U^\dagger_{r+2} U_{r+2} \left[-\frac{1}{\pi}(\mathcal{B}_0 + \mathcal{B}_0^\dagger)\tilde{c}(\frac{\pi r}{4})\tilde{c}(-\frac{\pi r}{4}) + \frac{1}{2}(\tilde{c}(-\frac{\pi r}{4}) + \tilde{c}(\frac{\pi r}{4}))\right]|0\rangle$$

$$U_r \equiv (2/r)^{\mathcal{L}_0}$$

Analytic computation and numerical computation with conventional level truncation:

$$S[\Psi_\lambda] = \begin{cases} 1/(2\pi^2 g^2) & (\lambda = 1) \\ 0 & (|\lambda| < 1) \end{cases}$$

$$\mathcal{O}_V(\Psi_\lambda) = \begin{cases} 1/(2\pi)\langle B|c_0^-|\Phi_V\rangle & (\lambda = 1) \\ 0 & (|\lambda| < 1) \end{cases}$$

↓ *are consistent with*

$\lambda = 1$ → $\Psi_{\rm Sch}$: nontrivial solution

$|\lambda| < 1$ → Ψ_λ : pure gauge solution

Takahashi-Tanimoto's solution

- "Identity based solution" [Takahashi-Tanimoto (2002)]

$$\Psi_0 = Q_L(e^h - 1)\mathcal{I} - C_L((\partial h)^2 e^h)\mathcal{I}$$

$$Q_L(f) \equiv \int_{C_{\mathrm{left}}} \frac{dz}{2\pi i} f(z) j_{\mathrm{B}}(z) \qquad C_L(f) \equiv \int_{C_{\mathrm{left}}} \frac{dz}{2\pi i} f(z) c(z)$$

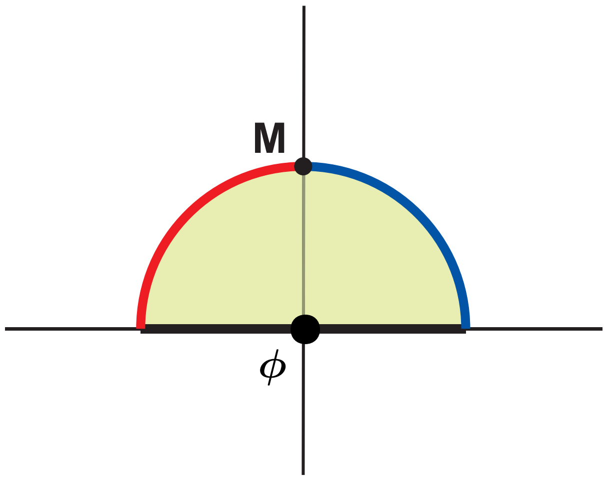
$$h(-1/z) = h(z), \quad h(\pm i) = 0$$

In the following, we take

$$\begin{aligned} h(z) &= \log\left(1 + \frac{a}{2}\left(z + \frac{1}{z}\right)^2\right) \\ &= -\log(1 - Z(a))^2 - \sum_{n=1}^{\infty} \frac{(-1)^n}{n} Z(a)^n (z^{2n} + z^{-2n}) \end{aligned}$$

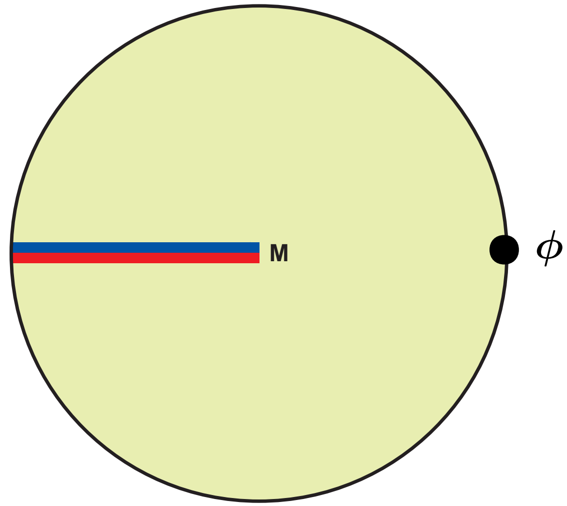
$$Z(a) = (1 + a - \sqrt{1+2a})/a \qquad a \geq -1/2$$

Identity state



$$h(z)^2$$

$$h(z) = \frac{1+iz}{1-iz}$$



$$\langle \mathcal{I} | \phi \rangle = \langle h_{\mathcal{I}}[\phi(0)] \rangle_{\mathrm{UHP}}$$

$$h_{\mathcal{I}}(z) = h^{-1}(h(z)^2) = \frac{2z}{1-z^2}$$

$$\mathcal{L}_0^\dagger = L_0 + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{4k^2-1} L_{-2k}$$

$$\begin{aligned} |\mathcal{I}\rangle &= |r=1\rangle = 2^{\mathcal{L}_0^\dagger}|0\rangle \\ &= \cdots e^{-\frac{1}{2^{k-1}}L_{-2^k}} \cdots e^{-\frac{1}{8}L_{-16}} e^{-\frac{1}{4}L_{-8}} e^{-\frac{1}{2}L_{-4}} e^{L_{-2}} |0\rangle \end{aligned}$$

On the TT solution (1)

- Formal pure gauge form:

$$\Psi_0 = \exp(q_L(h)\mathcal{I})Q_{\mathrm{B}}\exp(-q_L(h)\mathcal{I})$$

Gauge parameter string field:

$$\exp(\pm q_L(h)\mathcal{I}) = \exp(\pm q_L(h))\mathcal{I}$$

$\exp(\pm q_L(h))$: ill-defined for $a = -1/2$

$$q_L(f) \equiv \int_{C_{\mathrm{left}}} \frac{dz}{2\pi i} f(z) :cb:(z)$$

Non-trivial solution (?)

On the TT solution (2)

It is difficult to compute $S[\Psi_0]$, $\mathcal{O}_V(\Psi_0)$ *directly*,
because $\langle\mathcal{I}|(\cdots)|\mathcal{I}\rangle$ is divergent.

→ Identity based solutions may be "singular."

↓

Alternatively, we investigate ***string field theories around*** Ψ_0.

SFT around the TT solution

- Expansion around the TT solution:

$$S_a[\Phi] = S[\Psi_0 + \Phi] - S[\Psi_0]$$
$$= -\frac{1}{g^2}\left[\frac{1}{2}\langle\Phi, Q'\Phi\rangle + \frac{1}{3}\langle\Phi, \Phi * \Phi\rangle\right]$$

$$Q' = (1+a)Q_{\rm B} + \frac{a}{2}(Q_2 + Q_{-2}) + 4aZ(a)c_0 - 2aZ(a)^2(c_2 + c_{-2})$$
$$-2a(1 - Z(a)^2)\sum_{n=2}^{\infty}(-1)^n Z(a)^{n-1}(c_{2n} + c_{-2n})$$

$$j_{\rm B}(z) = cT^{\rm m}(z) + :bc\partial c: + \frac{3}{2}\partial^2 c(z) = \sum_{n=-\infty}^{\infty} Q_n z^{-n-1}$$

$$(Q')^2 = 0$$

$$\delta_\Lambda \Phi = Q'\Lambda + \Phi * \Lambda - \Lambda * \Phi \quad \Rightarrow \quad \delta_\Lambda S_a[\Phi] = 0$$

$$\delta_\Lambda \mathcal{O}_V(\Phi) = 0$$

On the new BRST operator

- cohomology of Q' [I.K.-Takahashi (2002)]

$a > -1/2$ the same as the original Q_{B}

⇒ Ψ_0 : pure gauge

$a = -1/2$ no cohomology at ghost number 1 sector

⇒ no open string

Ψ_0 : tachyon vacuum (!?)

Numerical solution in SFT around the TT solution (1)

- We solve the EOM: $$Q'\Phi + \Phi * \Phi = 0$$

in the Siegel gauge $$b_0\Phi = 0$$

by level truncation with the iterative algorithm:

[cf. Gaiotto-Rastelli(2002)]

$$c_0 b_0 (c_0 L(a)\Phi^{(n+1)} + \Phi^{(n)} * \Phi^{(n+1)} + \Phi^{(n+1)} * \Phi^{(n)} - \Phi^{(n)} * \Phi^{(n)}) = 0$$

$$\begin{aligned} L(a) &= \{b_0, Q'\} \\ &= (1+a)L_0 + \frac{a}{2}(L_2 + L_{-2}) + a(q_2 - q_{-2}) + 4(1 + a - \sqrt{1+2a}) \end{aligned}$$

Using the above we can define $$\Phi^{(n)} \longmapsto \Phi^{(n+1)}$$

Numerical solution in SFT around the TT solution (2)

If the iteration converges, $$c_0 b_0 (Q' \Phi^{(\infty)} + \Phi^{(\infty)} * \Phi^{(\infty)}) = 0$$

Projected part of the equation of motion

We also check the “BRST invariance”: [cf. Hata-Shinohara (2000)]

$$\| b_0 c_0 (Q' \Phi^{(\infty)} + \Phi^{(\infty)} * \Phi^{(\infty)}) \| / \| \Phi^{(\infty)} \| \ll 1$$

⇩

We evaluate the gauge invariants:

(1) potential height: $$f_a(\Phi) = 2\pi^2 \left(\frac{1}{2} \langle \Phi, c_0 L(a) \Phi \rangle + \frac{1}{3} \langle \Phi, \Phi * \Phi \rangle \right)$$

(2) gauge invariant overlap: $$\mathcal{O}_V(\Phi) = 2\pi \langle \hat{\gamma}(1_c, 2) | \Phi_V \rangle_{1_c} | \Phi \rangle_2$$

Construction of stable vacuum solution

- The initial configuration for $a = 0 \ (Q' = Q_{\mathrm{B}})$

$$\Phi^{(0)} = \frac{64}{81\sqrt{3}} c_1 |0\rangle \quad \xrightarrow{\text{iteration}} \quad \Phi_1|_{a=0}$$

the nontrivial solution for (0,0) truncation

conventional tachyon vacuum solution

- The initial configuration for $a = \epsilon \ (0 < |\epsilon| \ll 1)$

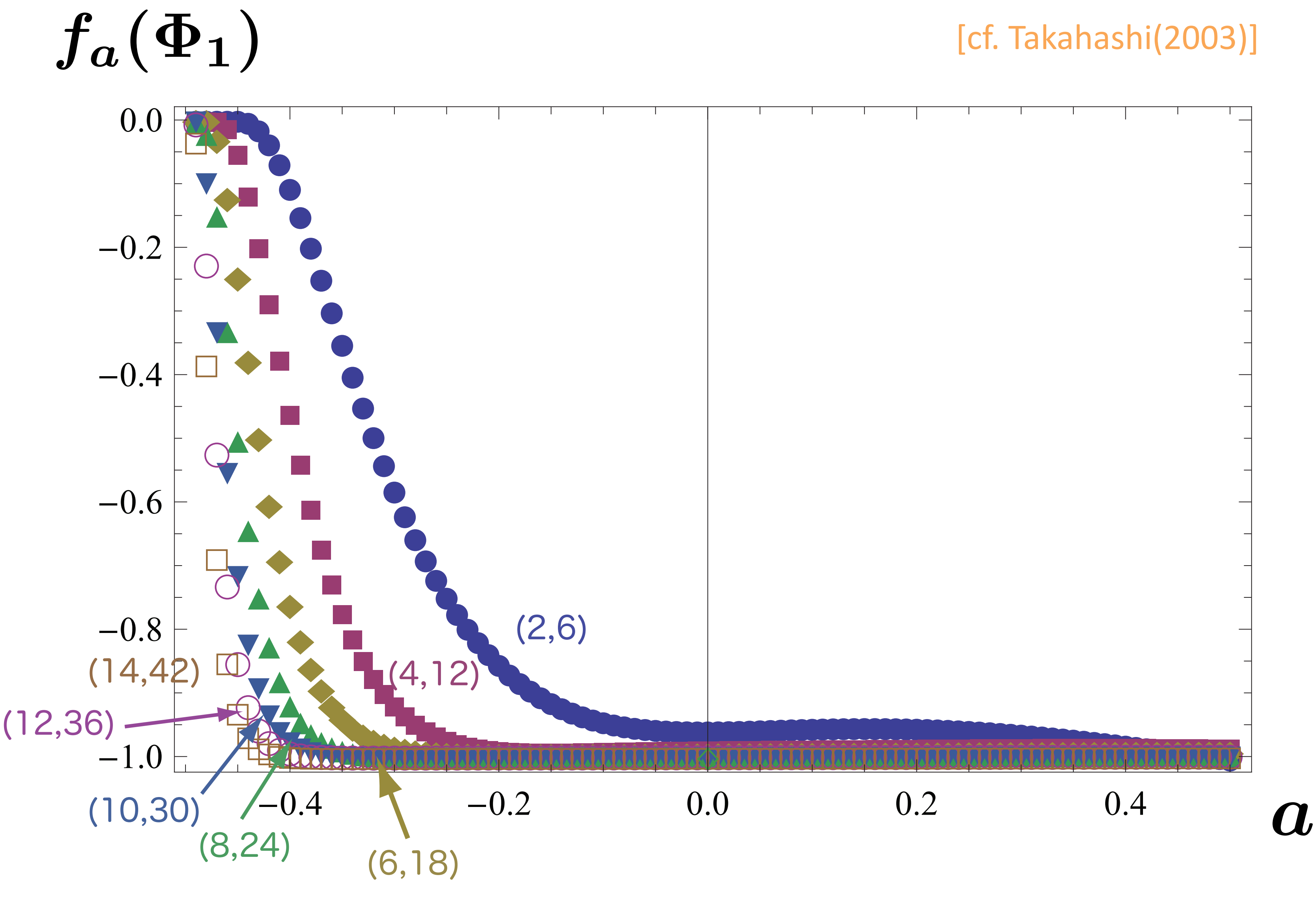
$$\Phi^{(0)} = \Phi_1|_{a=0} \quad \xrightarrow{\text{iteration}} \quad \Phi_1|_{a=\epsilon}$$

- The initial configuration for $a = 2\epsilon$

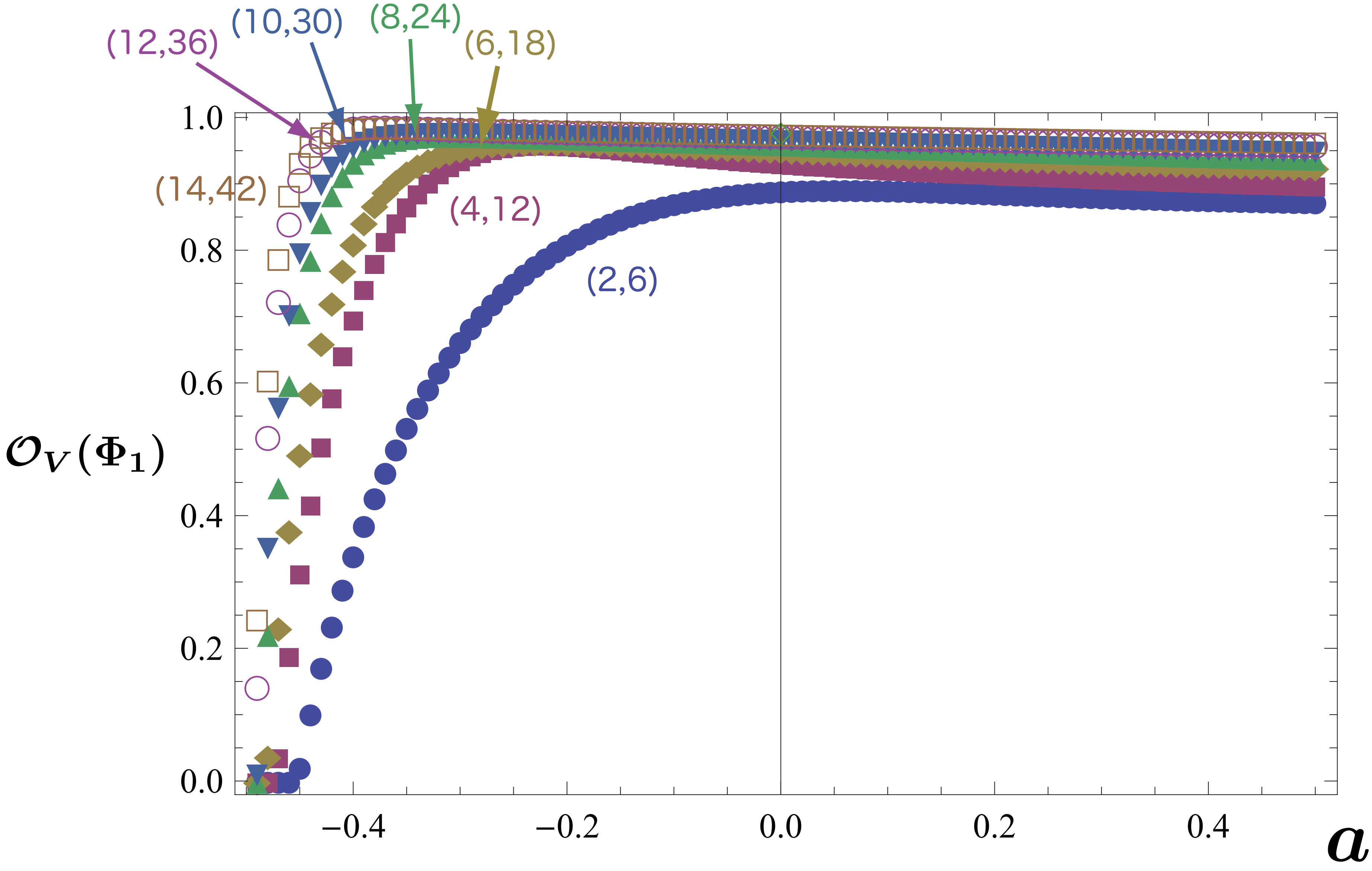
$$\Phi^{(0)} = \Phi_1|_{a=\epsilon} \quad \xrightarrow{\text{iteration}} \quad \Phi_1|_{a=2\epsilon}$$

$$\vdots$$

Potential height for Φ_1

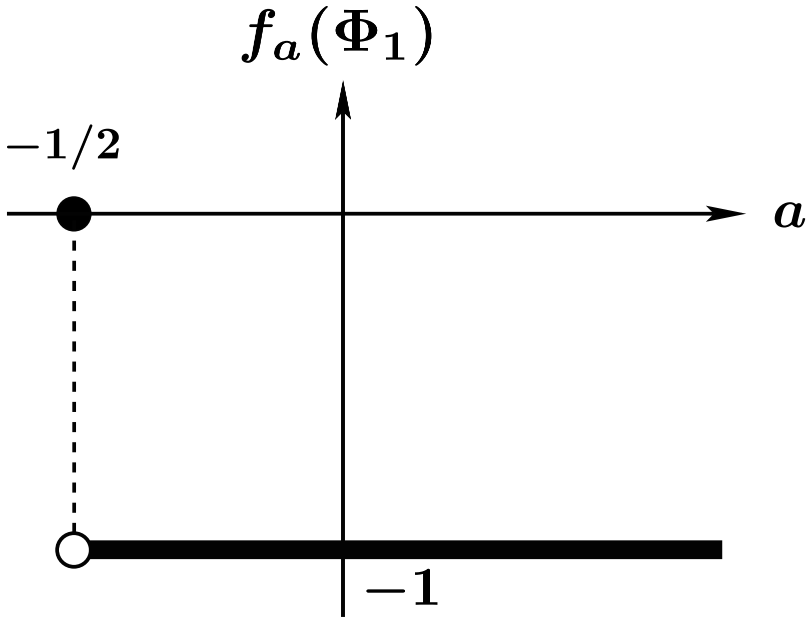


Gauge invariant overlap for Φ_1



Stable vacuum solution

- For $L \to \infty$, numerical results suggest



$a > -1/2$

Φ_1 :nontrivial tachyon vacuum

$a = -1/2$

$\Phi_1 = 0$

Construction of unstable vacuum solution

- The initial configuration for $a = -1/2$

$$\Phi^{(0)} = -\frac{32}{9\sqrt{3}} c_1 |0\rangle \quad \xrightarrow{\text{iteration}} \quad \Phi_2|_{a=-1/2}$$

the nontrivial solution for (0,0) truncation

- The initial configuration for $a = -1/2 + \epsilon \; (0 < \epsilon \ll 1)$

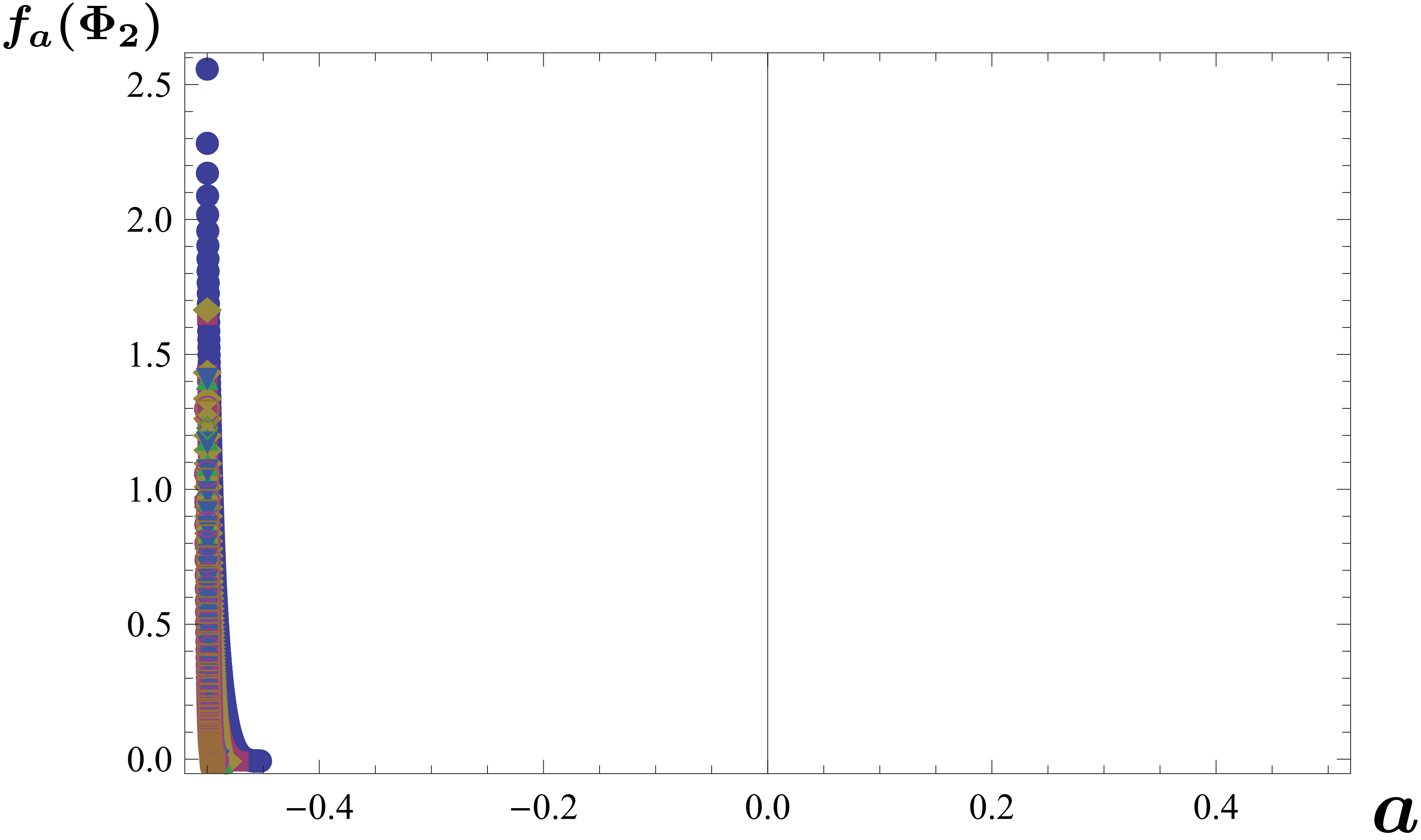
$$\Phi^{(0)} = \Phi_2|_{a=-1/2} \quad \xrightarrow{\text{iteration}} \quad \Phi_2|_{a=-1/2+\epsilon}$$

- The initial configuration for $a = -1/2 + 2\epsilon$

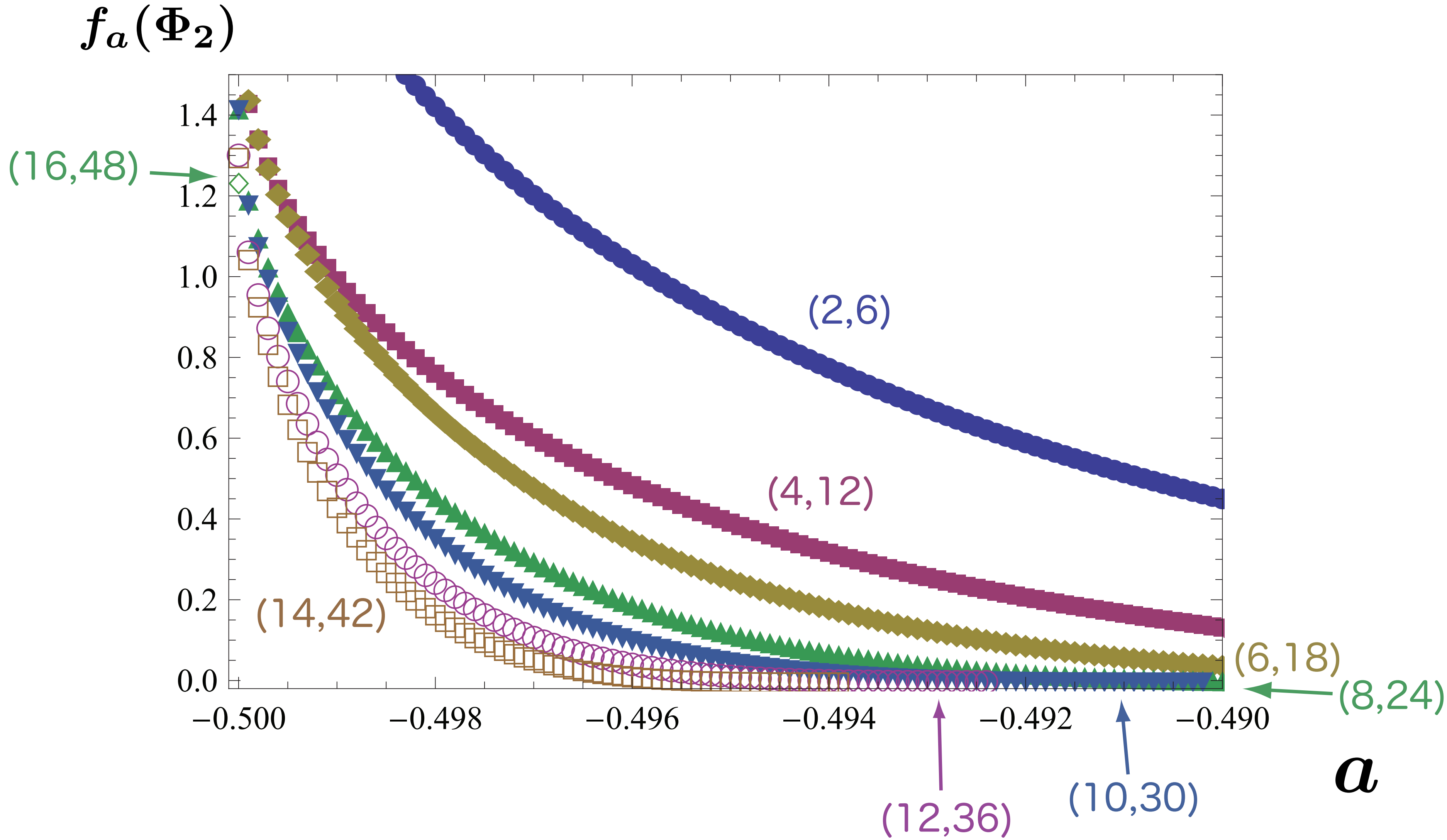
$$\Phi^{(0)} = \Phi_2|_{a=-1/2+\epsilon} \quad \xrightarrow{\text{iteration}} \quad \Phi_2|_{a=-1/2+2\epsilon}$$

$$\vdots$$

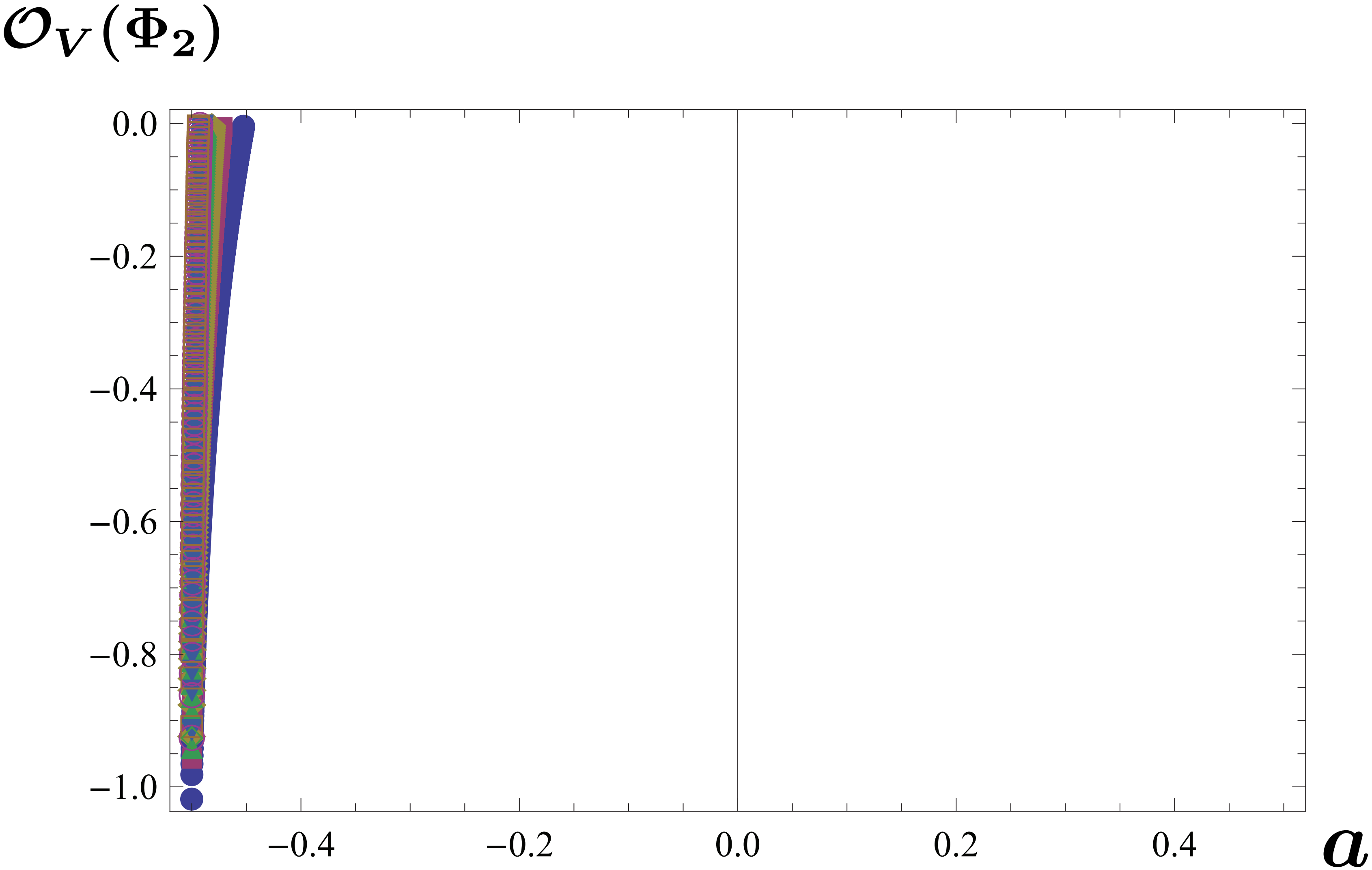
Potential height for Φ_2



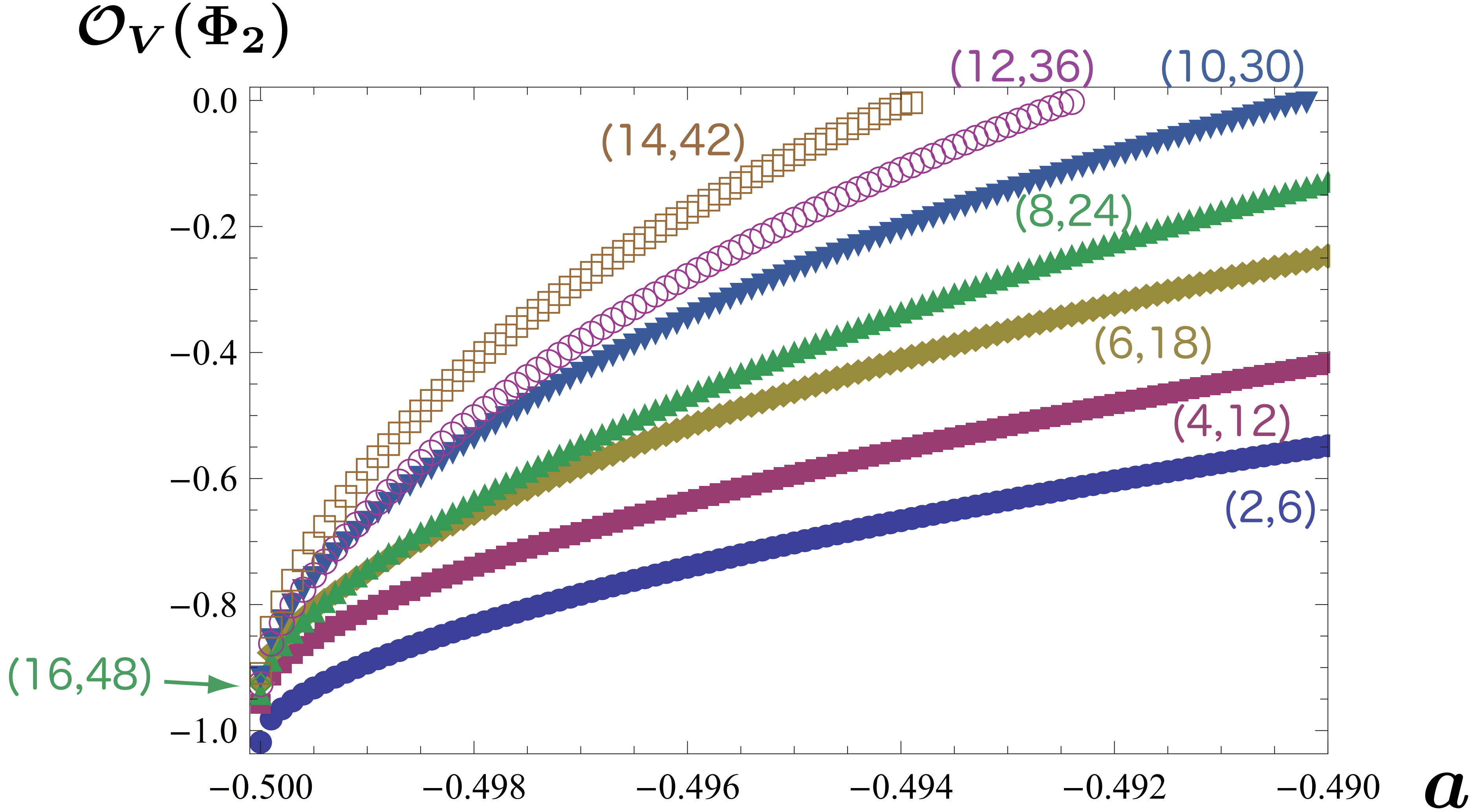
Potential height for Φ_2 (near -1/2)



Gauge invariant overlap for Φ_2



Gauge invariant overlap for Φ_2 (near -1/2)



Gauge invariants for $\Phi_2|_{a=-1/2}$

(L,3L)	$f_a(\Phi_2)$	$\mathcal{O}_V(\Phi_2)$
(0,0)	2.3105796	−1.0748441
(2,6)	2.5641847	−1.0156983
(4,12)	1.6550774	−0.9539832
(6,18)	1.6727496	−0.9207572
(8,24)	1.4193393	−0.9377548
(10,30)	1.4168893	−0.9110994
(12,36)	1.3035715	−0.9237917
(14,42)	1.2986472	−0.9056729
(16,48)	1.2357748	−0.9229035

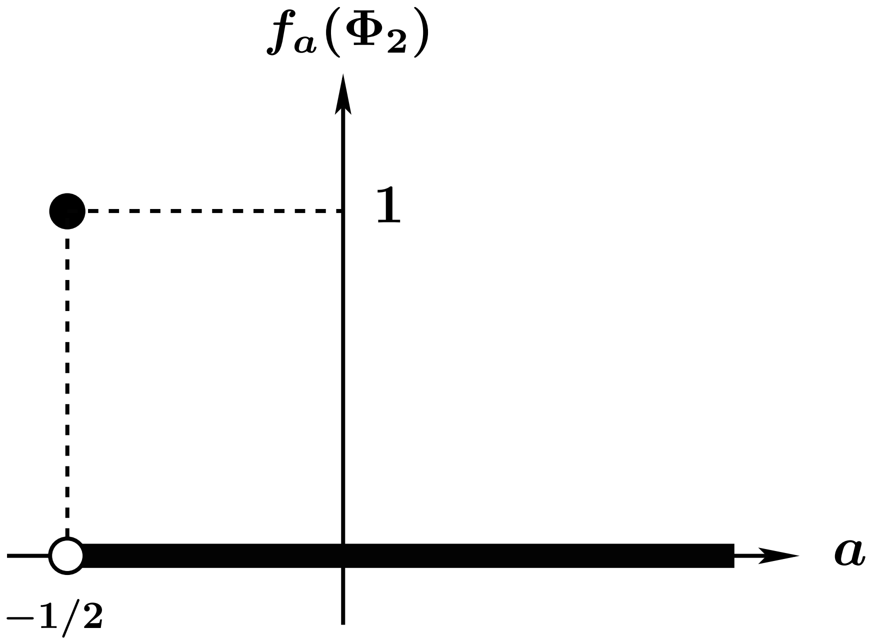
(L,3L)	Extrapolation of $f_a(\Phi_2)$
(4∞,12∞)	0.98107
(4∞+2,12∞+6)	0.98146

Fitting function:

$$F_N(L) = \sum_{n=0}^{N} \frac{a_n}{(L+1)^n}$$

Unstable vacuum solution

- For $L \to \infty$, numerical results suggest



$a > -1/2$

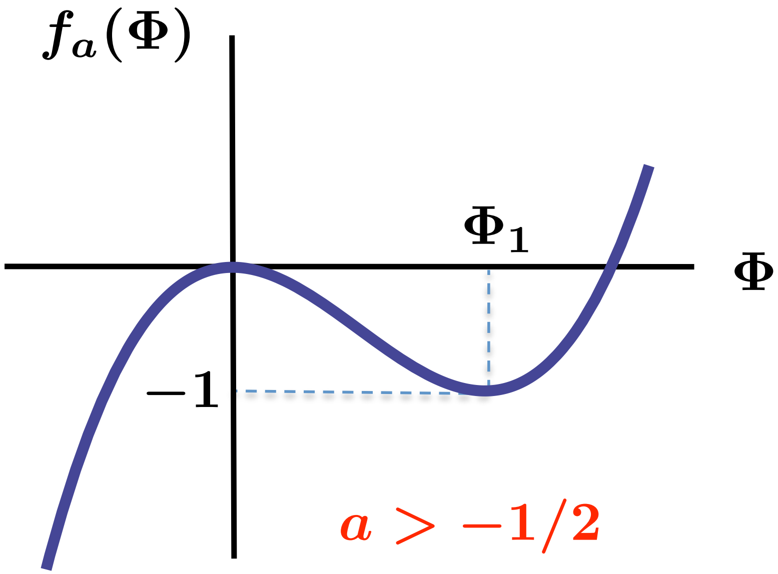
$\Phi_2 = 0$

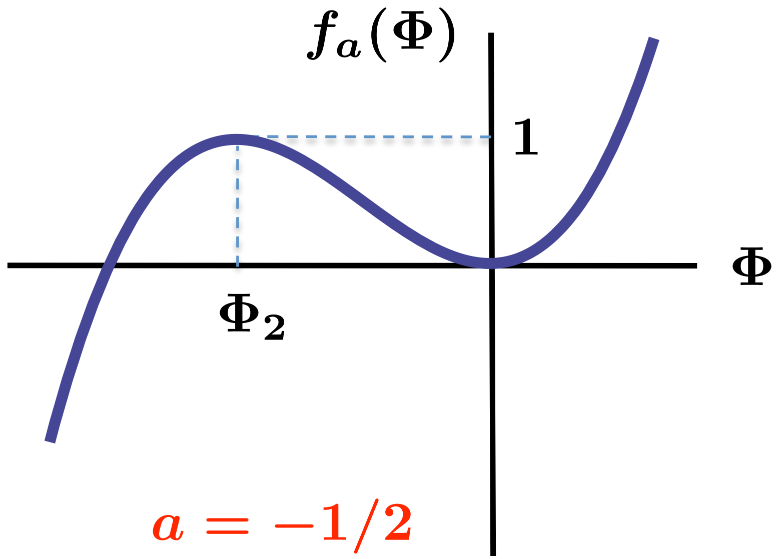
$a = -1/2$

Φ_2 :nontrivial vacuum
(perturbative vacuum!?)

Summary

- We constructed stable solution and unstable solution in the expanded theory around TT's identity based solution.
- We evaluated the gauge invariants for the obtained solutions.
- Numerical results suggest:





- This is consistent with the expectation that

$a > -1/2$ ⇒ Ψ_0 : pure gauge

$a = -1/2$ ⇒ Ψ_0 : tachyon vacuum

Discussion

- Our result on TT's solution suggests that the TT solution ($a=-1/2$) may be "gauge equivalent" to the Schnabl solution ($\lambda=1$) and give an alternative approach to investigating the nonperturbative vacuum.

- *Regular* solutions? *Definition* of the space of string fields?

- Extension to superstring field theory?
  [Erler(2007), Aref'eva-Gorbachev-Medvedev(2008), Fuchs-Kroyter(2008),…]