LINEAR GAUGES AND BV FORMALISM IN OPEN STRING FIELD THEORY

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INTRODUCTION AND MOTIVATION

- Classical open string field theory

$$S_0[A] = -\frac{1}{g^2}\left(\frac{1}{2}\langle A, QA\rangle + \frac{1}{3}\langle A, A * A\rangle\right)$$

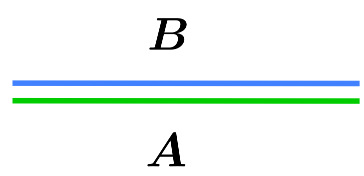
Q : BRST operator of worldsheet theory

$A = |A\rangle = \sum_{\alpha} \phi_\alpha |\alpha\rangle_1$: string field

Space-time field

Basis of worldsheet ghost number 1 state

$$\langle A, B\rangle = \sum_{\alpha,\beta} \phi_\alpha \langle |\alpha\rangle, |\beta\rangle\rangle \varphi_\beta$$

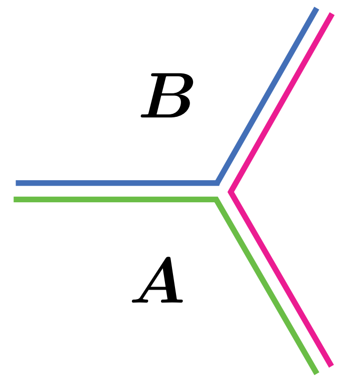


BPZ inner product

$$\langle |\alpha\rangle, |\beta\rangle\rangle = \langle I[\mathcal{O}_\alpha]\mathcal{O}_\beta\rangle, \quad I(z) = -1/z$$

$$|\alpha\rangle = \mathcal{O}_\alpha|0\rangle, \cdots$$

$$|A * B\rangle = \sum_{\alpha,\beta} \phi_\alpha \, |\alpha\rangle * |\beta\rangle \, \varphi_\beta$$



Star product

$$|\alpha\rangle * |\beta\rangle = \sum_\gamma |\gamma\rangle\langle|\tilde{\gamma}\rangle, |\alpha\rangle * |\beta\rangle\rangle = \sum_\gamma |\gamma\rangle\langle f_{(1)}[\mathcal{O}_{\tilde{\gamma}}] f_{(2)}[\mathcal{O}_\alpha] f_{(3)}[\mathcal{O}_\beta]\rangle$$

$$f_{(r)}(z) = h^{-1}(e^{(1-r)\frac{2\pi}{3}i} h(z)^{\frac{2}{3}}) \qquad h(z) = \frac{1+iz}{1-iz}$$

Gauge symmetry

$$\delta_{\Lambda_0} A = Q\Lambda_0 + A * \Lambda_0 - \Lambda_0 * A$$

$$\Lambda_0 = \sum_\alpha \lambda_0^\alpha |\alpha\rangle_0$$

Basis of worldsheet ghost number 0 state

Space-time gauge transformation parameter

$$\longrightarrow \quad \delta_{\Lambda_0} S_0[A] = 0$$

Note:

Gauge transformation parameter is degenerated.

If we ignore the interaction (star product), gauge transformation:

$\delta_{\Lambda_0} A = Q\Lambda_0$ is invariant under $\delta_{\Lambda_{-1}} \Lambda_0 = Q\Lambda_{-1}$

Similarly,

$\delta_{\Lambda_{-1}} \Lambda_0 = Q\Lambda_{-1}$ is invariant under $\delta_{\Lambda_{-2}} \Lambda_{-1} = Q\Lambda_{-2}$

$\vdots \qquad \qquad \vdots$

Gauge fixing? The most familiar condition is the Siegel gauge: $\boldsymbol{b_0 A = 0}$

To quantize string field, we should introduce

ghosts (for ghosts for …) corresponding to gauge transformation parameters.

It is said that

the Batalin-Vilkovisky (BV) formalism can be applied to string field theory.

New consistent gauge conditions were proposed by Asano-Kato(2006).

"a-gauge" $$\boldsymbol{(b_0 M + a b_0 c_0 \tilde{Q}) A = 0}$$

$$\boldsymbol{Q = \tilde{Q} + c_0 L_0 + b_0 M}$$

How to interpret a-gauge condition in terms of the BV formalism?

CONTENTS

A SKETCH OF THE BV FORMALISM

- Gauge invariant action $S_0[A]$
- Introduce fields and anti-fields corresponding to the gauge structure: $\Phi = (A, \mathcal{C}),\ \Phi^* = (A^*, \mathcal{C}^*)$
- Add some terms to the action so that it satisfies the "master equation": $(S, S)_{\mathrm{a.b.}} = 0$

$$S[\Phi, \Phi^*] = S_0[A] + \cdots$$

$$(X, Y)_{\mathrm{a.b.}} \equiv \frac{\partial_r X}{\partial \Phi^A} \frac{\partial_l Y}{\partial \Phi^*_A} - \frac{\partial_r X}{\partial \Phi^*_A} \frac{\partial_l Y}{\partial \Phi^A}$$

- Give a "gauge fixing fermion": $\Upsilon[\Phi]$

⇒ gauge fixed action: $S[\Phi, \frac{\partial \Upsilon}{\partial \Phi}]$

- BRST transformation and BRST invariance

$$\delta_{\mathrm{B}} X = (X, S)_{\mathrm{a.b.}}$$: (off-shell) nilpotent

$$\delta_{\mathrm{B}} S(\Phi, \Phi^*) = 0$$

$$\delta_{\mathrm{B}_\Upsilon} X = (X, S)_{\mathrm{a.b.}}\big|_{\Phi^* = \partial\Upsilon/\partial\Phi}$$: on-shell nilpotent

$$\delta_{\mathrm{B}_\Upsilon} S(\Phi, \Phi^* = \tfrac{\partial\Upsilon}{\partial\Phi}) = 0$$

- Path integral

$$\int [D\Phi][D\Phi^*]\delta\left(\Phi^* - \frac{\partial\Upsilon}{\partial\Phi}\right)\exp\left(\frac{i}{\hbar}W(\Phi, \Phi^*)\right)(\cdots)$$

- Quantum master equation

$$\frac{1}{2}(W, W)_{\mathrm{a.b.}} = i\hbar\Delta W$$

$$\Delta \equiv (-1)^{\epsilon_A+1}\frac{\partial_r}{\partial\Phi^A}\frac{\partial_r}{\partial\Phi^*_A}$$

$$W = S + \hbar M_1 + \hbar^2 M_2 + \cdots$$

$\longrightarrow$ Gauge (Υ) independence of the path integral

BV formalism in open string field theory

- Gauge invariant action

$$S_0[A] = -\frac{1}{g^2}\left(\frac{1}{2}\langle A, QA\rangle + \frac{1}{3}\langle A, A * A\rangle\right)$$

$$A = \sum_{\alpha} \mathcal{C}^{\alpha}_{-1}|\alpha\rangle_1$$: "classical" string field

Grassmann even

Grassmann odd

worldsheet ghost number: $g(A) = 1$

space-time ghost number: $\mathrm{gh}(A) = 0$

Grassmannality (mod 2): $s(A) = 1$

- Gauge transformation s.t. $\delta_{\Lambda_0} S_0[A] = 0$

$$\begin{aligned} \delta_{\Lambda_0} A &= Q\Lambda_0 + A * \Lambda_0 - \Lambda_0 * A, \\ & g(\Lambda_0) = 0, \quad s(\Lambda_0) = 0. \end{aligned}$$

This is on-shell invariant under

$$\begin{aligned} \delta_{\Lambda_{-1}} \Lambda_0 &= Q\Lambda_{-1} + A * \Lambda_{-1} + \Lambda_{-1} * A, \\ & g(\Lambda_{-1}) = -1, \quad s(\Lambda_{-1}) = 1. \end{aligned}$$

Namely,

$$\delta_{\Lambda_{-1}}(\delta_{\Lambda_0} A) = (QA + A * A) * \Lambda_{-1} - \Lambda_{-1} * (QA + A * A)$$

vanishes if the equation of motion is satisfied: $QA + A * A = 0$

- Similarly, under the following "gauge transformations:"

$$\delta_{\Lambda_{-n-1}}\Lambda_{-n} = Q\Lambda_{-n-1} + A * \Lambda_{-n-1} + (-1)^n \Lambda_{-n-1} * A,$$
$$g(\Lambda_{-n}) = -n, \quad s(\Lambda_{-n}) \equiv -n \,(\mathrm{mod}\ 2),$$
$$(n = 0, 1, 2, 3, \cdots)$$

we have

$$\delta_{\Lambda_{-n-2}}(\delta_{\Lambda_{-n-1}}\Lambda_{-n}) = (QA + A * A) * \Lambda_{-n-2} - \Lambda_{-n-2} * (QA + A * A).$$

equation of motion

Namely, they are on-shell invariant.

Gauge transformations are not independent on-shell.

Gauge transformations are (∞-stage) reducible.

- Space-time ghost fields are introduced corresponding to the gauge transformation parameters:

n-th stage gauge tr. $(n=0,1,2,\cdots)$

$$\Lambda_{-n}=\sum_{\alpha}\lambda^{\alpha}_{-n}|\alpha\rangle_{-n}$$

$$g(\Lambda_{-n})=-n,\quad s(\Lambda_{-n})\equiv -n\ (\mathrm{mod}\ 2)$$

$$\Downarrow$$

$$\mathcal{C}_n=\sum_{\alpha}\mathcal{C}^{\alpha}_{n}|\alpha\rangle_{-n}$$

$$\mathrm{gh}(\mathcal{C}_n)=1+n,\quad \epsilon(\mathcal{C}_n)\equiv 1+n\ (\mathrm{mod}\ 2),\quad g(\mathcal{C}_n)=-n,\quad s(\mathcal{C}_n)=1\,.$$

Grassmannality of space-time field

"total" Grassmannality

Ghost number of space-time field

For $n=-1$, we denote the original field as $A\equiv\mathcal{C}_{-1}=\sum_{\alpha}\mathcal{C}^{\alpha}_{-1}|\alpha\rangle_{1}$

- Anti-fields are introduced corresponding to (ghost) fields

$$\mathcal{C}_n = \sum_\alpha \mathcal{C}_n^\alpha |\alpha\rangle_{-n} \qquad n = -1, 0, 1, 2, \cdots$$

$$\mathrm{gh}(\mathcal{C}_n) = 1 + n, \quad \epsilon(\mathcal{C}_n) \equiv 1 + n \pmod 2, \quad g(\mathcal{C}_n) = -n, \quad s(\mathcal{C}_n) = 1\,.$$

$$\Downarrow$$

$$\mathcal{C}_n^* = \sum_\alpha \mathcal{C}_{n,\alpha}^* |\alpha\rangle_{-n} \qquad n = -1, 0, 1, 2, \cdots$$

$$\mathrm{gh}(\mathcal{C}_n^*) = -2 - n, \quad \epsilon(\mathcal{C}_n^*) \equiv n \pmod 2, \quad g(\mathcal{C}_n^*) = -n, \quad s(\mathcal{C}_n) = 0\,.$$

Note:

space-time ghost number and Grassmannality are assigned s.t.

$$\mathrm{gh}(\mathcal{C}_n^\alpha) + \mathrm{gh}(\mathcal{C}_{n,\alpha}^*) = -1 \qquad \epsilon(\mathcal{C}_n^\alpha) + \epsilon(\mathcal{C}_{n,\alpha}^*) = 1 \pmod 2$$

Summary of fields and anti-fields introduced

"original" field

	$\cdots,$	$\mathcal{C}_2^*,$	$\mathcal{C}_1^*,$	$\mathcal{C}_0^*,$	$\mathcal{C}_{-1}^*,$	$\boxed{\mathcal{C}_{-1},}$	$\mathcal{C}_0,$	$\mathcal{C}_1,$	$\mathcal{C}_2,$	$\cdots$
gh#	… ,	-4,	-3,	-2,	-1,	0,	1,	2,	3,	…
Grassmannality of spacetime field	$\cdots,$	$+,$	$-,$	$+,$	$-,$	$+,$	$-,$	$+,$	$-,$	$\cdots$
w.s. g#	… ,	-2,	-1,	0,	1,	1,	0,	-1,	-2,	…
"total" Grassmannality	$\cdots,$	$+,$	$+,$	$+,$	$+,$	$-,$	$-,$	$-,$	$-,$	$\cdots$

ANTI-BRACKET AND MASTER EQUATION

- Anti-bracket is defined by

$$(X, Y)_{\mathrm{a.b.}} \equiv \frac{\partial_r X}{\partial \Phi^A}\frac{\partial_l Y}{\partial \Phi^*_A} - \frac{\partial_r X}{\partial \Phi^*_A}\frac{\partial_l Y}{\partial \Phi^A}$$

right derivative　　left derivative

In this case,

$$\Phi^A = (\mathcal{C}^\alpha_n),\ \Phi^*_A = (\mathcal{C}^*_{n,\alpha})$$

- Classical master equation for the action

$$(S, S)_{\mathrm{a.b.}} = 2\frac{\partial_r S}{\partial \Phi^A}\frac{\partial_l S}{\partial \Phi^*_A} = 0$$

- Boundary condition for a "proper solution"

$$S[\Phi, \Phi^* = 0] = S_0[A]$$

original gauge invariant action

PROPER SOLUTION FOR OPEN STRING FIELD THEORY

- String field including spacetime fields and antifields

$$\boldsymbol{\Psi} = \sum_{n=-1}^{\infty} \mathcal{C}_n + \sum_{n=-1}^{\infty} {}^*\mathcal{C}_n^* \qquad \text{: Gassmann odd}$$

$$= \sum_{n\geq -1,\alpha} \mathcal{C}_n^{\alpha} |\alpha\rangle_{-n} + \sum_{n\geq -1,\alpha} \mathcal{C}^*_{n,\alpha} |\tilde{\alpha}\rangle_{n+3} (-1)^n$$

${}^*\mathcal{C}_n^*$: a "string Hodge dual" of $\mathcal{C}_n^*$

$$\mathrm{gh}({}^*\mathcal{C}_n^*) = -2 - n, \quad \epsilon({}^*\mathcal{C}_n^*) \equiv n \,(\mathrm{mod}\, 2), \quad g({}^*\mathcal{C}_n^*) = 3 + n, \quad s({}^*\mathcal{C}_n^*) = 1\,.$$

$${}^*\boldsymbol{\Phi} \equiv \sum_{g\leq 1,\alpha} (|\alpha\rangle_g \, {}_g\langle\alpha| + |\tilde{\alpha}\rangle_{3-g} \, {}_{3-g}\langle\tilde{\alpha}|)\boldsymbol{\Phi}$$

$\{|\alpha\rangle_g, |\tilde{\alpha}\rangle_{3-g}\}_{g\leq 1}$:basis of worldsheet state s.t.

$$\langle |\tilde{\alpha}\rangle_{3-g}, |\alpha'\rangle_g \rangle = \delta_{\alpha,\alpha'}$$

- Summary of fields and anti-fields included in Ψ

“original” field

	$\cdots$	${}^*\mathcal{C}_2^*$	${}^*\mathcal{C}_1^*$	${}^*\mathcal{C}_0^*$	${}^*\mathcal{C}_{-1}^*$	$\boxed{\mathcal{C}_{-1}}$	$\mathcal{C}_0$	$\mathcal{C}_1$	$\mathcal{C}_2$	$\cdots$
gh#	… ,	-4,	-3,	-2,	-1,	0,	1,	2,	3,	…
Grassmannality of spacetime field	$\cdots,$	$+,$	$-,$	$+,$	$-,$	$+,$	$-,$	$+,$	$-,$	$\cdots$
w.s. g#	… ,	5,	4,	3,	2,	1,	0,	-1,	-2,	…
“total” Grassmannality	$\cdots,$	$-,$	$-,$	$-,$	$-,$	$-,$	$-,$	$-,$	$-,$	$\cdots$

- Proper solution: the same "form" as the original action!

$$S[\Phi,\Phi^*] = S[\Psi] = -\frac{1}{g^2}\left(\frac{1}{2}\langle\Psi, Q\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi*\Psi\rangle\right)$$

Note:

$$\frac{\partial_r S}{\partial \mathcal{C}_n^\alpha} = -\frac{1}{g^2}\langle|\alpha\rangle_{-n}, (Q\Psi + \Psi*\Psi)\rangle,$$

$$\frac{\partial_l S}{\partial \mathcal{C}_{n,\alpha}^*} = -\frac{1}{g^2}\langle|\tilde{\alpha}\rangle_{n+3}, (Q\Psi + \Psi*\Psi)\rangle$$

$$(S,S)_{\text{a.b.}} = 2\sum_{n\geq -1,\alpha}\frac{\partial_r S}{\partial \mathcal{C}_n^\alpha}\frac{\partial_l S}{\partial \mathcal{C}_{n,\alpha}^*} = \frac{1}{g^4}\langle Q\Psi + \Psi*\Psi, Q\Psi + \Psi*\Psi\rangle = 0$$

$$\sum_{g\leq 1,\alpha}\left(|\alpha\rangle_{g\ 3-g}\langle\tilde{\alpha}| + |\tilde{\alpha}\rangle_{3-g\ g}\langle\alpha|\right) = 1$$

$$S[\Psi]|_{\Phi_A^*=0} = S[\textstyle\sum_{n\geq -1}\mathcal{C}_n] = S[\mathcal{C}_{-1}] = S_0[A]$$

due to worldsheet ghost number

GAUGE FIXING FERMION

- Gauge fixing condition in the BV formalism

$$\Phi^*_A = \frac{\partial \Upsilon[\Phi]}{\partial \Phi^A}$$

Gauge fixing fermion $\Upsilon[\Phi]$ should satisfy

$$\mathrm{gh}(\Upsilon) = -1, \quad \epsilon(\Upsilon) = 1.$$

$$\mathrm{gh}(\Phi^*_A) = -1 - \mathrm{gh}(\Phi^A)$$

$$\epsilon(\Phi^*_A) = 1 - \epsilon(\Phi^A) \pmod 2$$

Fields (not anti-fields) with **negative** ghost number are needed.

Trivial pairs

We further introduce fields

$$(\bar{\mathcal{C}}_s^k, \bar{\pi}_s^k), \quad (s = 0, 1, 2, \cdots; k : \text{even}, 0 \le k \le s),$$
$$(\mathcal{C}_s^k, \pi_s^k), \quad (s = 1, 2, 3, \cdots; k : \text{odd}, 1 \le k \le s),$$

and their anti-fields.

$$\text{gh}(\bar{\mathcal{C}}_s^k) = -1 + \text{gh}(\bar{\pi}_s^k) \qquad \text{gh}(\mathcal{C}_s^k) = -1 + \text{gh}(\pi_s^k)$$

$$\epsilon(\mathcal{C}_s) = \epsilon(\bar{\mathcal{C}}_s^k) = 1 + \epsilon(\bar{\pi}_s^k) = \epsilon(\mathcal{C}_s^k) = 1 + \epsilon(\pi_s^k) \pmod 2$$

The label s corresponds to the s-th stage gauge transformation.

	$\bar{\mathcal{C}}_s^k$	$\bar{\pi}_s^k$	$\mathcal{C}_s^k$	π_s^k	$\bar{\mathcal{C}}_s^{k*}$	$\mathcal{C}_s^{k*}$
gh( )	$k-s-1$	$k-s$	$s-k$	$s-k+1$	$s-k$	$k-s-1$
$\epsilon(\ )\pmod 2$	$s+1$	s	$s+1$	s	s	s
$g(\ )$	$1+k-s$	$1+k-s$	$1+k-s$	$1+k-s$	$1+k-s$	$1+k-s$
$s(\ )$	0	1	1	0	1	0

Adding extra terms to the action:

$$\begin{aligned} S_{\text{aux}} &= \sum_{\substack{k=0 \\ k:\text{even}}}^{\infty} \sum_{s=k}^{\infty} \langle {}^{*}\bar{\pi}_s^k, \bar{\mathcal{C}}_s^{k*} \rangle + \sum_{\substack{k=1 \\ k:\text{odd}}}^{\infty} \sum_{s=k}^{\infty} \langle {}^{*}\pi_s^k, \mathcal{C}_s^{k*} \rangle \\ &= \sum_{\substack{k=0 \\ k:\text{even}}}^{\infty} \sum_{s=k}^{\infty} \sum_{\alpha} \bar{\pi}_{s,\alpha}^k \bar{\mathcal{C}}_s^{k*,\alpha} + \sum_{\substack{k=1 \\ k:\text{odd}}}^{\infty} \sum_{s=k}^{\infty} \sum_{\alpha} \mathcal{C}_{s,\alpha}^{k*} \pi_s^{k,\alpha} \end{aligned}$$

$\Downarrow$

$S[\Psi] + S_{\text{aux}}$ is also a proper solution to the master equation.

$$(S[\Psi] + S_{\text{aux}}, S[\Psi] + S_{\text{aux}})_{\text{a.b.}} = 0$$

In this case, $\Phi^A = (\mathcal{C}_s^\alpha, \bar{\mathcal{C}}_{s,\alpha}^k, \bar{\pi}_{s,\alpha}^k, \mathcal{C}_s^{k,\alpha}, \pi_s^{k,\alpha}),\ \Phi_A^* = (\mathcal{C}_{n,\alpha}^*, \bar{\mathcal{C}}_s^{k*,\alpha}, \bar{\pi}_s^{k*,\alpha}, \mathcal{C}_{s,\alpha}^{k*}, \pi_{s,\alpha}^{k*})$

Boundary condition:

$$(S[\Psi] + S_{\text{aux}})|_{\Phi_A^*=0} = S_0[A]$$

Summary of introduced fields

$$\boldsymbol{A}$$

$$(\bar{\mathcal{C}}_0^0, \bar{\pi}_0^0) \quad \boldsymbol{\mathcal{C}_0}$$

$$(\mathcal{C}_1^1, \pi_1^1) \quad (\bar{\mathcal{C}}_1^0, \bar{\pi}_1^0) \quad \boldsymbol{\mathcal{C}_1}$$

+anti-fields

$$(\bar{\mathcal{C}}_2^2, \bar{\pi}_2^2) \quad (\mathcal{C}_2^1, \pi_2^1) \quad (\bar{\mathcal{C}}_2^0, \bar{\pi}_2^0) \quad \boldsymbol{\mathcal{C}_2}$$

$$(\mathcal{C}_3^3, \pi_3^3) \quad (\bar{\mathcal{C}}_3^2, \bar{\pi}_3^2) \quad (\mathcal{C}_3^1, \pi_3^1) \quad (\bar{\mathcal{C}}_3^0, \bar{\pi}_3^0) \quad \boldsymbol{\mathcal{C}_3}$$

$$\iddots \qquad \vdots \qquad \ddots$$

GAUGE FIXING FERMION FOR LINEAR GAUGES

$$\Upsilon[\Phi] \;=\; \sum_{\substack{k=0\\ k:\text{even}}}^{\infty}\sum_{s=k}^{\infty}\langle {}^*\bar{\mathcal{C}}_s^k, \mathcal{P}_{1+k-s}\mathcal{C}_{s-1}^{k-1}\rangle + \sum_{\substack{k=1\\ k:\text{odd}}}^{\infty}\sum_{s=k}^{\infty}\langle {}^*\mathcal{C}_s^k, \tilde{\mathcal{P}}_{1+k-s}\bar{\mathcal{C}}_{s-1}^{k-1}\rangle$$

$\mathcal{C}_n^{-1} \equiv \mathcal{C}_n$: original fields included in Ψ

$\mathcal{P}_n, \tilde{\mathcal{P}}_n$: projections on the worldsheet ghost number n states s.t.

$$\text{bpz}(\mathcal{P}_n) = \tilde{\mathcal{P}}_{3-n}, \quad \mathcal{P}_n + \tilde{\mathcal{P}}_n = 1, \quad (\mathcal{P}_n)^2 = \mathcal{P}_n, \quad (\tilde{\mathcal{P}}_n)^2 = \tilde{\mathcal{P}}_n$$

$$\Phi_A^* = \frac{\partial \Upsilon[\Phi]}{\partial \Phi^A}$$

$$\begin{aligned}
\bar{\mathcal{C}}_s^{k*} &= \mathcal{P}_{1+k-s}\mathcal{C}_{s-1}^{k-1} + (-1)^{s+1}\tilde{\mathcal{P}}_{1+k-s}\mathcal{C}_{s+1}^{k+1}\,, \quad (k = 0, 2, 4, \cdots; s \geq k) \\
\mathcal{C}_s^{k*} &= (-1)^s\tilde{\mathcal{P}}_{1+k-s}\bar{\mathcal{C}}_{s-1}^{k-1} + \mathcal{P}_{1+k-s}\bar{\mathcal{C}}_{s+1}^{k+1}\,, \quad (k = 1, 3, 5, \cdots; s \geq k) \\
\mathcal{C}_s^* &= \mathcal{P}_{-s}\bar{\mathcal{C}}_{s+1}^{0}\,, \quad (s \geq -1)
\end{aligned}$$

Gauge fixed action

$$\left(S[\Psi] + S_{\mathrm{aux}}\right)\Big|_{\Phi^*_A = \frac{\partial \Upsilon}{\partial \Phi_A}}$$

integrate out auxiliary fields $\bar{\pi}^k_s, \pi^k_s$

$$\mathcal{C}^k_s = 0\,, \qquad (k = 1, 3, 5, \cdots; s \geq k),$$

$$\bar{\mathcal{C}}^k_s = 0\,, \qquad (k = 2, 4, 6 \cdots; s \geq k),$$

$$\mathcal{C}^*_s = \bar{\mathcal{C}}^0_{s+1}, \quad \mathcal{P}_{-s}\mathcal{C}_s = 0, \quad \tilde{\mathcal{P}}_{-s}\bar{\mathcal{C}}^0_{s+1} = 0, \quad (s \geq -1)$$

$$S_{\mathrm{fix}} = -\frac{1}{g^2}\left(\frac{1}{2}\langle \Psi_{\mathrm{f}}, Q\Psi_{\mathrm{f}}\rangle + \frac{1}{3}\langle \Psi_{\mathrm{f}}, \Psi_{\mathrm{f}} * \Psi_{\mathrm{f}}\rangle\right),$$

$$\Psi_{\mathrm{f}} = \sum_{n=-1}^{\infty} \mathcal{C}_n + \sum_{n=-1}^{\infty} {}^*\bar{\mathcal{C}}^0_{n+1},$$

includes all worldsheet ghost number sectors

$$\mathcal{P}_{-n}\mathcal{C}_n = 0, \quad \mathcal{P}_{3+n}{}^*\bar{\mathcal{C}}^0_{n+1} = 0, \quad (n \geq -1)$$

linear gauge condition

Summary of remained fields

$$
\begin{array}{ccccc}
 & & & & \boldsymbol{A} \\
 & & & (\bar{\mathcal{C}}^0_0, \bar{\pi}^0_0) & \mathcal{C}_0 \\
 & & (\mathcal{C}^1_1, \pi^1_1) & (\bar{\mathcal{C}}^0_1, \bar{\pi}^0_1) & \mathcal{C}_1 \\
 & (\bar{\mathcal{C}}^2_2, \bar{\pi}^2_2) & (\mathcal{C}^1_2, \pi^1_2) & (\bar{\mathcal{C}}^0_2, \bar{\pi}^0_2) & \mathcal{C}_2 \\
(\mathcal{C}^3_3, \pi^3_3) & (\bar{\mathcal{C}}^2_3, \bar{\pi}^2_3) & (\mathcal{C}^1_3, \pi^1_3) & (\bar{\mathcal{C}}^0_3, \bar{\pi}^0_3) & \mathcal{C}_3 \\
\iddots & & \vdots & \ddots & \ddots
\end{array}
$$

Some explicit examples

- Siegel gauge

$$\mathcal{P}_{-n} = c_0 b_0, \quad n \in \mathbb{Z}$$

[Gomis-Paris-Samuel(1994)]

$$\longleftrightarrow \qquad b_0 \Psi_{\mathrm{f}} = 0$$

- Linear b-gauge [Kiermaier-Sen-Zwiebach(2007)]

$$\mathcal{P}_{2k+1} = \frac{1}{v_0} c_0 \Big(\sum_m v_m^{(2k+1)} b_m\Big), \quad \mathcal{P}_{2-2k} = \frac{1}{v_0} c_0 \,\mathrm{bpz}\Big(\sum_m v_m^{(2k+1)} b_m\Big)$$

Asano-Kato's *a*-gauge

$$\mathrm{bpz}(\mathcal{O}_a^{\langle -n+4\rangle})\Phi_n = 0\,,$$

for worldsheet ghost number n sector

$$\mathrm{bpz}(\mathcal{O}_a^{\langle n+4\rangle}) = \frac{1}{1-a}(b_0 + ab_0c_0W_{n+2}M^{n+1}\tilde{Q}), \quad (n \geq -1)\,,$$

$$\mathrm{bpz}(\mathcal{O}_a^{\langle 1-n\rangle}) = b_0(1-P_{n+1}) + \frac{1}{1-a}(b_0P_{n+1} + ab_0c_0\tilde{Q}M^{n+1}W_{n+2}), \quad (n \geq -1)$$

$$Q = \tilde{Q} + c_0L_0 + b_0M \qquad W_n = \sum_{i=0}^{\infty}\frac{(-1)^i(n+i-1)!}{i!(n-1)!((n+i)!)^2}M^i(M^-)^{n+i}$$

$$P_n = -\frac{1}{L_0}\tilde{Q}M^nW_{n+1}\tilde{Q} \qquad M^- = -\sum_{n=1}^{\infty}\frac{1}{2n}b_{-n}b_n$$

$$\mathcal{P}_n = e^{\frac{\tilde{Q}}{L_0}b_0}e^{\chi_n}c_0b_0\,e^{-\chi_n}e^{-\frac{\tilde{Q}}{L_0}b_0}$$

$$\chi_n = \begin{cases} \frac{a}{a-1}c_0W_{-n+2}M^{-n+1}\tilde{Q} & (n \leq 1) \\ \frac{a}{a-1}c_0\tilde{Q}M^{n-2}W_{n-1} & (n \geq 2) \end{cases}$$

COMMENT ON ANOTHER CONVENTION OF THE BV FORMALISM IN OPEN STRING FIELD THEORY

- For given projections $\mathcal{P}_n$ for a linear gauge condition

$$\Psi = \sum_n \Phi_n, \quad \mathcal{P}_n \Phi_n = 0$$

we define a basis such as

$$\{|\beta\rangle_g, |\gamma\rangle_g, |\tilde{\beta}\rangle_{3-g}, |\tilde{\gamma}\rangle_{3-g}\}, \quad (g \le 1),$$

$$\langle|\tilde{\beta}\rangle_{3-g}, |\beta'\rangle_g\rangle = \delta_{\beta,\beta'}, \quad \langle|\tilde{\gamma}\rangle_{3-g}, |\gamma'\rangle_g\rangle = \delta_{\gamma,\gamma'},$$

$$\mathcal{P}_g|\beta\rangle_g = |\beta\rangle_g,\ \tilde{\mathcal{P}}_g|\gamma\rangle_g = |\gamma\rangle_g,\ \tilde{\mathcal{P}}_{3-g}|\tilde{\beta}\rangle_{3-g} = |\tilde{\beta}\rangle_{3-g},\ \mathcal{P}_{3-g}|\tilde{\gamma}\rangle_{3-g} = |\tilde{\gamma}\rangle_{3-g}.$$

Then, we define component fields as

$$\begin{aligned}\Psi \ &= \ \sum_{m\ge -1}\Big(\sum_{\gamma}(\mathcal{C}'^{\gamma}_m|\gamma\rangle_{-m} + (-1)^m \mathcal{C}'^{*}_{m,\gamma}|\tilde{\gamma}\rangle_{3+m})\Big)\\ &\quad + \sum_{m\ge -1}\Big(\sum_{\beta}(\mathcal{C}'^{\beta}_m|\tilde{\beta}\rangle_{3+m} + (-1)^{m+1}\mathcal{C}'^{*}_{m,\beta}|\beta\rangle_{-m})\Big).\end{aligned}$$

- Compared to the previous definition, component fields are linearly transformed:

$$
\begin{aligned}
\mathcal{C}'^{\gamma}_{m} &= \sum_{\alpha} {}_{3+m}\langle\tilde{\gamma}|\alpha\rangle_{-m}\mathcal{C}^{\alpha}_{m}, \\
\mathcal{C}'^{*}_{m,\gamma} &= \sum_{\alpha} {}_{-m}\langle\gamma|\alpha\rangle_{3+m}\mathcal{C}^{*}_{m,\alpha}, \\
\mathcal{C}'^{\beta}_{m} &= (-1)^{m}\sum_{\alpha} {}_{-m}\langle\beta|\tilde{\alpha}\rangle_{3+m}\mathcal{C}^{*}_{m,\alpha}, \\
\mathcal{C}'^{*}_{m,\beta} &= (-1)^{m+1}\sum_{\alpha} {}_{3+m}\langle\tilde{\beta}|\alpha\rangle_{-m}\mathcal{C}^{\alpha}_{m}
\end{aligned}
$$

fields and anti-fields are mixed

Note: $\mathrm{gh}(\mathcal{C}'^{\gamma}_{m}) + \mathrm{gh}(\mathcal{C}'^{*}_{m,\gamma}) = -1, \quad \mathrm{gh}(\mathcal{C}'^{\beta}_{m}) + \mathrm{gh}(\mathcal{C}'^{*}_{m,\beta}) = -1,$

$\epsilon(\mathcal{C}'^{\gamma}_{m}) + \epsilon(\mathcal{C}'^{*}_{m,\gamma}) \equiv 1 \pmod 2, \quad \epsilon(\mathcal{C}'^{\beta}_{m}) + \epsilon(\mathcal{C}'^{*}_{m,\beta}) \equiv 1 \pmod 2$

Under the above transformation, anti-bracket is invariant

$$
\begin{aligned}
(X,Y)_{\mathrm{a.b.}} &\equiv \sum_{n\geq-1,\alpha}\left(\frac{\partial_r X}{\partial\mathcal{C}^{\alpha}_{n}}\frac{\partial_l Y}{\partial\mathcal{C}^{*}_{n,\alpha}} - \frac{\partial_r X}{\partial\mathcal{C}^{*}_{n,\alpha}}\frac{\partial_l Y}{\partial\mathcal{C}^{\alpha}_{n}}\right) \\
&= \sum_{n\geq-1,\gamma}\left(\frac{\partial_r X}{\partial\mathcal{C}'^{\gamma}_{n}}\frac{\partial_l Y}{\partial\mathcal{C}'^{*}_{n,\gamma}} - \frac{\partial_r X}{\partial\mathcal{C}'^{*}_{n,\gamma}}\frac{\partial_l Y}{\partial\mathcal{C}'^{\gamma}_{n}}\right) + \sum_{n\geq-1,\beta}\left(\frac{\partial_r X}{\partial\mathcal{C}'^{\beta}_{n}}\frac{\partial_l Y}{\partial\mathcal{C}'^{*}_{n,\beta}} - \frac{\partial_r X}{\partial\mathcal{C}'^{*}_{n,\beta}}\frac{\partial_l Y}{\partial\mathcal{C}'^{\beta}_{n}}\right) \\
&\equiv (X,Y)'_{\mathrm{a.b.}}
\end{aligned}
$$

→ The above is a canonical transformation w.r.t. anti-bracket!

- If we take the trivial gauge fixing fermion: $\boldsymbol{\Upsilon' \equiv 0}$

  the gauge fixing condition of the BV formalism is

$$\mathcal{C}'^{*}_{m,\gamma} = 0, \mathcal{C}'^{*}_{m,\beta} = 0 \quad \longleftrightarrow \quad \Psi = \sum_n \Phi_n, \quad \mathcal{P}_n \Phi_n = 0$$

and the gauge fixed action is the same as previous one:

$$S[\Psi]|_{\mathcal{C}'^{*}_{m,\gamma}=0, \mathcal{C}'^{*}_{m,\beta}=0} = S_{\mathbf{fix}}$$

Different "boundary condition"

In the case of Siegel gauge: $\mathcal{P}_{-n} = c_0 b_0, \quad n \in \mathbb{Z}$

this convention is adopted by Hata et. al.

Summary and discussion

- We reviewed the BV formalism in open bosonic string field theory.
- Linear gauge conditions for string field can be realized by choosing projections for gauge fixing fermion appropriately.
- Another convention of the BV formalism with the trivial gauge fixing fermion can be obtained by a canonical transformation with respect to the anti-bracket.

- What about superstring field theory?

- Cubic super SFT ← [Kohriki-Kugo-Kunitomo-Murata]
- WZW-type super SFT ← [Michishita, Okawa-Torii]

The original actions are gauge invariant.

→ In principle, there should exist proper solutions of the BV master equation.

($\because$ existence theorem)

For open **bosonic** string field theory, a proper solution is the **same** form as the original action with the replacement of string fields

$$A \to \Psi = \sum_{n=-1}^{\infty} \mathcal{C}_n + \sum_{n=-1}^{\infty} {}^*\mathcal{C}_n^*$$

However, in the case of superstring field theory, the form of a proper solution might be changed…(?).