su(2|2) 光円錐型弦の場の理論の代数模型

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References:

I. Kishimoto and S. Moriyama,
"An Algebraic Model for the su(2|2) Light-Cone String Field Theory,"
[arXiv:1005.4719];
"On LCSFT/MST Correspondence,"
Adv.Theor.Math.Phys.13(2009)111[hep-th/0611113v2]

String field theory

- 弦の場の理論(SFT)：弦理論の非摂動的定式化の候補
  - ボゾニックな<u>開</u>弦の場の理論

    特にタキオン凝縮の問題に関して発展があった。

    シュナブル解(2005年11月）以降、技術的にも再び進展しつつある。 [… ,I.K.-Michishita,I.K.-Kawano-Takahashi,I.K.,I.K.-Takahashi,…]

↓

超対称化

Modified cubic SFT

WZW-type SFT

への拡張

Closed SFT

- 閉弦の場の理論は？
  - ボゾニックな場合、

    Non-polynomial closed SFTは具体的な計算は複雑過ぎる

    cubic closed SFT：HIKKO版 冪等方程式への応用

    [I.K.-Matsuo-Watanabe, I.K.-Matsuo]

    OSp covariantized版 …
  - 超弦の場合は？

    共変なものはまだよくわかっていない。

    光円錐ゲージなら

    Green-Schwarz(-Brink) SFT (1983)

    → Matrix string theoryとの対応

    [… ,I.K.-Moriyama-Teraguchi, I.K.-Moriyama]

LCSFT and BMN

- pp-wave上の光円錐ゲージSFT

  [Spradlin-Volovich, Pankiewicz-Stefanski (2002),… , Pankiewicz(2003),…]

  - AdS/CFT対応の観点で応用された：
    - $AdS_5 \times S^5$ のPenrose limit: pp-wave時空
    - BMN(Berenstein-Maldacena-Nastase)対応

4次元N=4 SU(N) SYMのalmost BPS operator

↕

pp-wave上の超弦理論のstring state

弦の相互作用を含めて調べる

pp-wave

- pp-wave時空：10D IIB SUGRAのmax. SUSY解

$$
\begin{aligned}
ds^2 &= -2dx^+dx^- - \mu^2\sum_{I=1}^{8}(x^I)^2(dx^+)^2 + \sum_{I=1}^{8}(dx^I)^2, \\
F &= \mu dx^+ \wedge (dx^1\wedge dx^2\wedge dx^3\wedge dx^4 + dx^5\wedge dx^6\wedge dx^7\wedge dx^8)
\end{aligned}
$$

←RR-flux

- この背景上のGreen-Schwarz作用を光円錐ゲージで量子化

[Metzaev,Metzaev-Tseytlin(2003)]

  - free levelでの対称性のgeneratorの構成

$$P^+, P^I, J^{+I}, J^{ij}, J^{i'j'}, Q^+, \overline{Q}^+$$: kinematical

$$P^-, Q^-, \overline{Q}^-$$: dynamical

$$I = 1,\cdots,8;\ i,j = 1,2,3,4;\ i',j' = 5,6,7,8$$

bosonic 30個、fermionic 32個

LCSFT on pp-wave

- 代数を尊重するように３弦相互作用項を構成

$$P^-, Q^-, \overline{Q}^- \longrightarrow |P^-\rangle, |Q^-\rangle, |\overline{Q}^-\rangle \sim (\cdots)|V\rangle$$

prefactor

kinematical overlap

Spradlin-Volovichのとは違う別の解（不定性）もある:

$$|P^-\rangle = p^-|V\rangle, |Q^-\rangle = q^-|V\rangle, |\overline{Q}^-\rangle = \overline{q}^-|V\rangle$$

[Di Vecchia-Petersen-Petrini-Russo-Tanzini(2003)]

（BMN対応などを考えると）線形結合が正しい？

$$|P^-\rangle = \frac{1}{2}(|P^-\rangle_{\mathrm{SV}} + |P^-\rangle_{\mathrm{D}}), \ldots$$

[Dobashi-Yoneya, Lee-Russo(2004)]

Strategy

- flat, pp-wave以外のより一般の背景でのSFTの構成を直接、具体的に振動子レベルでやるのは難しい。
- flat, pp-waveの例からSFTのbuilding blockを抜き出して、代数で形を決めていこう。　→　「模型」の提案
- spin chain模型でsu(2|2)代数が重要な役割を果たした。

[…,Beisert,…]

su(2|2)対称性をもつ背景上の光円錐ゲージSFTの模型

pp-wave上のSFTをより簡潔に再現、
一般化の計算例

Contents

- Introduction ✔
- Review of GSB's LCSFT (flat space)
- Algebraic model for su(2|2) LCSFT
  - ansatz for pp-wave and solutions
  - generalization, toy model I,II
- Summary and Discussion

Review of GSB's LCSFT

- Green-Schwarz形式光円錐ゲージの弦の座標、運動量

$$x^i(\sigma), \vartheta^a(\sigma) \qquad p^i(\sigma), \lambda^a(\sigma)$$

- i, a はそれぞれSO(8)の $\mathbf{8_v}, \mathbf{8_s}$

- モード展開：

$$[\alpha_n^i, \alpha_m^j] = n\delta^{ij}\delta_{n+m},\ [\tilde{\alpha}_n^i, \tilde{\alpha}_m^j] = n\delta^{ij}\delta_{n+m},$$
$$\{Q_n^a, Q_m^b\} = \alpha\delta^{ab}\delta_{n+m},\ \{\tilde{Q}_n^a, \tilde{Q}_m^b\} = \alpha\delta^{ab}\delta_{n+m}$$

$$x^i(\sigma) = x^i + i\sum_{n\neq 0}\frac{1}{n}(\alpha_n^i e^{in\sigma/|\alpha|} + \widetilde{\alpha}_n^i e^{-in\sigma/|\alpha|}),$$

$$\vartheta^a(\sigma) = \vartheta^a + \sum_{n\neq 0}\frac{1}{\alpha}(\eta^* Q_n^a e^{in\sigma/|\alpha|} + \eta\widetilde{Q}_n^a e^{-in\sigma/|\alpha|})$$

$$p^i(\sigma) = \frac{1}{2\pi|\alpha|}\left(p^i + \frac{1}{2}\sum_{n\neq 0}(\alpha_n^i e^{in\sigma/|\alpha|} + \tilde{\alpha}_n^i e^{-in\sigma/|\alpha|})\right),$$

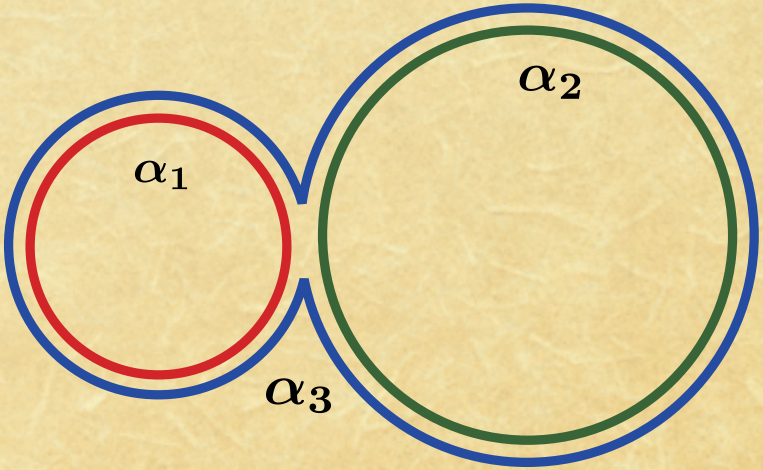
$$\lambda^a(\sigma) = \frac{1}{2\pi|\alpha|}\left(\lambda^a + \frac{1}{2}\sum_{n\neq 0}(\eta Q_n^a e^{in\sigma/|\alpha|} + \eta^*\widetilde{Q}_n^a e^{-in\sigma/|\alpha|})\right)$$

$$[x^i, p^j] = i\delta^{ij},$$
$$\{\vartheta^a, \lambda^b\} = \delta^{ab}$$

$$\eta = e^{i\pi/4}, \eta^* = e^{-i\pi/4}$$

Overlapping

- ３つの閉弦の接続条件：



デルタ汎函数

$$\delta(\alpha_1+\alpha_2+\alpha_3)\delta^8(x^{i(3)}-\Theta_1 x^{i(1)}-\Theta_2 x^{i(2)})\delta^8(\vartheta^{i(3)}-\Theta_1\vartheta^{i(1)}-\Theta_2\vartheta^{i(2)})$$
$$=\langle\alpha_1,x^{(1)},\vartheta^{(1)}|\langle\alpha_2,x^{(2)},\vartheta^{(2)}|\langle\alpha_3,x^{(3)},\vartheta^{(3)}|V\rangle$$

⇩

3-string interaction vertex
（kinematicalなoverlapの部分）

On 3-string vertex

- 振動子によるあらわな表示

$$|V\rangle = (2\pi)^9\delta(\alpha_1+\alpha_2+\alpha_3)\delta^8(p_1^i+p_2^i+p_3^i)\delta^8(\lambda_1^a+\lambda_2^a+\lambda_3^a)$$
$$\times e^{\frac{1}{2}\sum \bar{N}^{rs}_{nm}(\alpha^{(r)}_{-n}\alpha^{(s)}_{-m}+\widetilde{\alpha}^{(r)}_{-n}\widetilde{\alpha}^{(s)}_{-m})+\sum \bar{N}^r_n(\alpha^{(r)}_{-n}+\widetilde{\alpha}^{(r)}_{-n})P-\frac{\tau_0}{\alpha_{123}}P^2}$$
$$\times e^{\sum Q^{\mathrm{II}(r)}_{-n}\alpha_r^{-1}n\bar{N}^{rs}_{nm}Q^{\mathrm{I}(s)}_{-m}-\sqrt{2}\Lambda\sum\alpha_r^{-1}n\bar{N}^r_nQ^{\mathrm{II}(r)}_{-n}}|0\rangle$$

$$P^i=\alpha_1p_2^i-\alpha_2p_1^i,\ \Lambda^a=\alpha_1\lambda_2^a-\alpha_2\lambda^a,\ Q^{\mathrm{I/II}a}_{-n}=\frac{1}{\sqrt{2}}(\eta^{\pm1}Q^a_{-n}+\eta^{\mp1}\widetilde{Q}^a_{-n}) \qquad \alpha_{123}\equiv\alpha_1\alpha_2\alpha_3$$

$$\bar{N}^{rs}_{nm}=-\frac{\alpha_{123}}{\alpha_r/n+\alpha_s/m}\bar{N}^r_n\bar{N}^s_m,\quad \bar{N}^r_n=\frac{\Gamma(-n\alpha_{r+1}/\alpha_r)e^{n\tau_0/\alpha_r}}{\alpha_rn!\Gamma(1-n(1+\alpha_{r+1}/\alpha_r))},\ (\alpha_4\equiv\alpha_1),\quad \tau_0=\sum_{r=1}^3\alpha_r\log|\alpha_r|$$

これを用いて $\sigma_1\sim\sigma_{\mathrm{int}}$ で具体的に評価すると...

$$\lambda^{(1)}(\sigma_1)|V\rangle \sim \frac{1}{4\pi|\alpha_{123}|^{1/2}|\sigma_1-\sigma_{\mathrm{int}}|^{1/2}}Y^a|V\rangle$$

↑

$$Y^a=\Lambda^a-\frac{\alpha_{123}}{2}\sum_{r=1}^3\sum_{n=1}^\infty\alpha_r^{-1}n\bar{N}^r_n(\eta Q^{(r)a}_{-n}+\eta^*\widetilde{Q}^{(r)a}_{-n})$$

Algebra and prefactors

- SUSY代数：　$\{Q^{\dot a}, Q^{\dot b}\} = \{\widetilde{Q}^{\dot a}, \widetilde{Q}^{\dot b}\} = 2H\delta^{\dot a \dot b},\ \ \{Q^{\dot a}, \widetilde{Q}^{\dot b}\} = 0$

を尊重するように３弦相互作用項を決める：

$$q^{\dot a}|Q^{\dot b}\rangle + q^{\dot b}|Q^{\dot a}\rangle = \tilde{q}^{\dot a}|\widetilde{Q}^{\dot b}\rangle + \tilde{q}^{\dot b}|\widetilde{Q}^{\dot a}\rangle = 2\delta^{\dot a \dot b}|H\rangle,\ \ q^{\dot a}|\widetilde{Q}^{\dot b}\rangle + \tilde{q}^{\dot b}|Q^{\dot a}\rangle = 0$$

Green-Schwarz-Brinkの公式（の簡潔版）[I.K.-Moriyama(2006)]）：

$$|H\rangle = X^i \widetilde{X}^j \big[\cosh \not{Y}\big]^{ij}|V\rangle,$$

$$|Q^{\dot a}\rangle = \sqrt{-\alpha_{123}}\widetilde{X}^i \big[\sinh \not{Y}\big]^{\dot a i}|V\rangle,$$

$$|\widetilde{Q}^{\dot a}\rangle = i\sqrt{-\alpha_{123}}X^i \big[\sinh \not{Y}\big]^{i\dot a}|V\rangle$$

$$\not{Y} = \sqrt{\tfrac{2}{-\alpha_{123}}}\eta^* Y^a \hat\gamma^a \qquad \hat\gamma^a = \begin{pmatrix} 0 & \hat\gamma^a_{i\dot a} \\ \hat\gamma^a_{\dot a i} & 0 \end{pmatrix} = \begin{pmatrix} 0 & \gamma^i_{a\dot a} \\ \gamma^i_{a\dot a} & 0 \end{pmatrix}$$

$$\{q^{\dot a}, Y^a\} = \tfrac{1}{\sqrt{2}}\eta\gamma^i_{a\dot a}X^i, \qquad \{\tilde{q}^{\dot a}, Y^a\} = \tfrac{1}{\sqrt{2}}\eta^*\gamma^i_{a\dot a}\widetilde{X}^i$$

On the prefactors

- ３弦相互作用項の構成要素： $|V\rangle \qquad Y^a,\ X^i,\ \widetilde{X}^i$
- Ansatz : $$|Q^{\dot{a}}\rangle = (X^i[f(\not{Y})]^{i\dot{a}} + \widetilde{X}^i[\widetilde{f}(\not{Y})]^{i\dot{a}})|V\rangle,$$ $$|\widetilde{Q}^{\dot{a}}\rangle = ([g(\not{Y})]^{\dot{a}i}X^i + [\widetilde{g}(\not{Y})]^{\dot{a}i}\widetilde{X}^i)|V\rangle$$

SUSY代数をup to level matching conditionで課す

$$|Q^{\dot{a}}\rangle = \left(\tilde{f}_0\widetilde{X}^i[\sinh\not{Y}]^{i\dot{a}} + X^i\left[f_1\not{Y} + \frac{1}{7!}f_7\not{Y}^7\right]^{i\dot{a}}\right)|V\rangle = \left(\tilde{f}_0\widetilde{X}^i[\sinh\not{Y}]^{i\dot{a}} + q^{\dot{a}}i\sqrt{-\alpha_{123}}(f_1 + f_7 y_0^8\delta^8(Y))\right)|V\rangle$$

$$|\widetilde{Q}^{\dot{a}}\rangle = \left(g_0[\sinh\not{Y}]_{\dot{a}i}X^i + \left[\tilde{g}_1\not{Y} + \frac{1}{7!}\tilde{g}_7\not{Y}^7\right]^{\dot{a}i}\widetilde{X}^i\right)|V\rangle = \left(g_0[\sinh\not{Y}]^{\dot{a}i}X^i + \tilde{q}^{\dot{a}}\sqrt{-\alpha_{123}}(\tilde{g}_1 + \tilde{g}_7 y_0^8\delta^8(Y))\right)|V\rangle$$

$$|H\rangle = \left(\frac{1}{\sqrt{-\alpha_{123}}}g_0\tilde{X}^iX^j[\cosh\not{Y}]^{ij} + h\sqrt{-\alpha_{123}}(\tilde{g}_1 + y_0^8\tilde{g}_7\delta^8(Y))\right)|V\rangle$$

[Lee-Russo(2004), Dobashi-I.K.-Moriyama (unpublished)]

つまり GSB part + “trivial” part の形になる。

$[q, X] \sim \partial Y,\quad [\tilde{q}, \widetilde{X}] \sim \overline{\partial} Y$ も含まれる

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Algebraic model (1)

- superalgebra $\mathfrak{psu}(2|2)\ltimes\mathbb{R}^3$

$$
\begin{aligned}
\{\mathcal{Q}^{\alpha}{}_{a},\mathcal{Q}^{\beta}{}_{b}\} &= \epsilon^{\alpha\beta}\epsilon_{ab}\mathcal{H},\\
\{\widetilde{\mathcal{Q}}^{\alpha}{}_{a},\widetilde{\mathcal{Q}}^{\beta}{}_{b}\} &= \epsilon^{\alpha\beta}\epsilon_{ab}\widetilde{\mathcal{H}},\\
\{\mathcal{Q}^{\alpha}{}_{a},\widetilde{\mathcal{Q}}^{\beta}{}_{b}\} &= \epsilon^{\alpha\beta}\epsilon_{ac}\mathcal{R}^{c}{}_{b}+\epsilon_{ab}\mathcal{L}^{\alpha}{}_{\gamma}\epsilon^{\gamma\beta}+\epsilon^{\alpha\beta}\epsilon_{ab}\mathcal{N}.
\end{aligned}
$$

$$
\begin{aligned}
[\mathcal{L}^{\alpha}{}_{\beta},\mathcal{L}^{\gamma}{}_{\delta}] &= i(\delta^{\gamma}_{\beta}\mathcal{L}^{\alpha}{}_{\delta}-\delta^{\alpha}_{\delta}\mathcal{L}^{\gamma}{}_{\beta}),\\
[\mathcal{R}^{a}{}_{b},\mathcal{R}^{c}{}_{d}] &= -i(\delta^{c}_{b}\mathcal{R}^{a}{}_{d}-\delta^{a}_{d}\mathcal{R}^{c}{}_{b}).
\end{aligned}
$$

$\alpha,\beta,\cdots=1,2;\quad a,b,\cdots=1,2$

$(\mathcal{L}\epsilon)^{\alpha\beta}=(\mathcal{L}\epsilon)^{\beta\alpha}\quad\Leftrightarrow\quad \mathrm{Tr}\mathcal{L}=0$

$(\epsilon R)_{ab}=(\epsilon R)_{ba}\quad\Leftrightarrow\quad \mathrm{Tr}\mathcal{R}=0$

$$[\mathcal{L}^{\alpha}{}_{\beta},Q^{\gamma}{}_{c}]=i(\delta^{\gamma}_{\beta}Q^{\alpha}{}_{c}-\frac{1}{2}\delta^{\alpha}_{\beta}Q^{\gamma}{}_{c}),\;\;[\mathcal{L}^{\alpha}{}_{\beta},\widetilde{Q}^{\gamma}{}_{c}]=i(\delta^{\gamma}_{\beta}\widetilde{Q}^{\alpha}{}_{c}-\frac{1}{2}\delta^{\alpha}_{\beta}\widetilde{Q}^{\gamma}{}_{c}),$$

$$[\mathcal{R}^{a}{}_{b},Q^{\gamma}{}_{c}]=-i(\delta^{a}_{c}Q^{\gamma}{}_{b}-\frac{1}{2}\delta^{a}_{b}Q^{\gamma}{}_{c}),\;\;[\mathcal{R}^{a}{}_{b},\widetilde{Q}^{\gamma}{}_{c}]=-i(\delta^{a}_{c}\widetilde{Q}^{\gamma}{}_{b}-\frac{1}{2}\delta^{a}_{b}\widetilde{Q}^{\gamma}{}_{c}).$$

Algebraic model (2)

- Expansion of generators

$$\mathcal{J} = j + g_s J + \cdots$$

$J \rightarrow |J\rangle$ ：3-string interaction vertexで表される

$$q^{\alpha}{}_{a}|Q^{\beta}{}_{b}\rangle + q^{\beta}{}_{b}|Q^{\alpha}{}_{a}\rangle = \epsilon^{\alpha\beta}\epsilon_{ab}|H\rangle,$$
$$\widetilde{q}^{\alpha}{}_{a}|\widetilde{Q}^{\beta}{}_{b}\rangle + \widetilde{q}^{\beta}{}_{b}|\widetilde{Q}^{\alpha}{}_{a}\rangle = \epsilon^{\alpha\beta}\epsilon_{ab}|\widetilde{H}\rangle,$$
$$q^{\alpha}{}_{a}|\widetilde{Q}^{\beta}{}_{b}\rangle + \widetilde{q}^{\beta}{}_{b}|Q^{\alpha}{}_{a}\rangle = \epsilon^{\alpha\beta}\epsilon_{ab}|N\rangle.$$

$\mathcal{L}^{\alpha}{}_{\beta} = l^{\alpha}{}_{\beta}, \quad \mathcal{R}^{a}{}_{b} = r^{a}{}_{b}$ ：kinematicalであると仮定。

Ansatz (1)

- Building blocks for $|H\rangle,\ |\widetilde{H}\rangle,\ |N\rangle,\ |Q^{\alpha}{}_{a}\rangle,\ |\widetilde{Q}^{\alpha}{}_{a}\rangle$

  $|V\rangle$: kinematical overlap ～ 弦の接続条件を表すデルタ汎函数

$$Y^{\alpha}{}_{\dot b}, Y'^{a}{}_{\dot\beta}, X^{a}{}_{\dot b}, \widetilde{X}^{a}{}_{\dot b}, X'^{\alpha}{}_{\dot\beta}, \widetilde{X}'^{\alpha}{}_{\dot\beta}, W^{\alpha}{}_{\dot b}, \widetilde{W}^{\alpha}{}_{\dot b}, W'^{a}{}_{\dot\beta}, \widetilde{W}'^{a}{}_{\dot\beta}$$

：**prefactor**の構成要素 $(\alpha, \dot\alpha, a, \dot a, \cdots = 1, 2)$

- Commutation relations (assumptions) :

$$\{q^{\alpha}{}_{a}, Y^{\beta}{}_{\dot b}\} = -\epsilon^{\beta\alpha}(\epsilon X)_{a\dot b}, \qquad \{q^{\alpha}{}_{a}, Y'^{b}{}_{\dot\beta}\} = \delta^{b}_{a} X'^{\alpha}{}_{\dot\beta},$$

$$[q^{\alpha}{}_{a}, X^{b}{}_{\dot b}] = \delta^{b}_{a} W^{\alpha}{}_{\dot b}, \qquad [q^{\alpha}{}_{a}, X'^{\beta}{}_{\dot\beta}] = -\epsilon^{\beta\alpha}(\epsilon W')_{a\dot\beta},$$

$$[q^{\alpha}{}_{a}, \widetilde{X}^{b}{}_{\dot b}] = \frac{i}{2}\delta^{b}_{a} Y^{\alpha}{}_{\dot b}, \qquad [q^{\alpha}{}_{a}, \widetilde{X}'^{\beta}{}_{\dot\beta}] = \frac{i}{2}\epsilon^{\beta\alpha}(\epsilon Y')_{a\dot\beta}.$$

$\widetilde{q}^{\alpha}{}_{a}$ との交換関係も同様

Note on indices

$$SO(8) \;\rightarrow\; SO(4)\times SO(4)$$

GSB's SFT(flat)

$$SU(2)\times SU(2)\times SU(2)\times SU(2)$$

$$\alpha \qquad \dot\alpha \qquad a \qquad \dot a$$

$$\mathbf{8_v} \;\rightarrow\; (1,1,2,2)+(2,2,1,1)$$

$$X^i \;\rightarrow\; X^a{}_{\dot a},\, X'^{\alpha}{}_{\dot\alpha}$$

$$\widetilde{X}^i \;\rightarrow\; \widetilde{X}^a{}_{\dot a},\, \widetilde{X}'^{\alpha}{}_{\dot\alpha}$$

$$\mathbf{8_s} \;\rightarrow\; (2,1,1,2)+(1,2,2,1)$$

$$Y^a \;\rightarrow\; Y^{\alpha}{}_{\dot a},\, Y'^{a}{}_{\dot\alpha}$$

$$\partial Y^a \;\rightarrow\; W^{\alpha}{}_{\dot a},\, W'^{a}{}_{\dot\alpha}$$

$$\overline{\partial} Y^a \;\rightarrow\; \widetilde{W}^{\alpha}{}_{\dot a},\, \widetilde{W}'^{a}{}_{\dot\alpha}$$

$$\mathbf{8_c} \;\rightarrow\; (2,1,2,1)+(1,2,1,2)$$

$$q^{\dot a} \;\rightarrow\; q^{\alpha}{}_{a}\ (,\ q^{\dot\alpha}{}_{\dot a})$$

$$\widetilde{q}^{\dot a} \;\rightarrow\; \widetilde{q}^{\alpha}{}_{a}\ (,\ \widetilde{q}^{\dot\alpha}{}_{\dot a})$$

Ansatz (2)

- assumptions:

$$q^{\alpha}{}_a|V\rangle = \frac{i}{2}[(YX)^{\alpha}{}_a + (X'Y')^{\alpha}{}_a]|V\rangle, \quad \tilde{q}^{\alpha}{}_a|V\rangle = -\frac{i}{2}[(Y\widetilde{X})^{\alpha}{}_a + (\widetilde{X}'Y')^{\alpha}{}_a]|V\rangle.$$

- anti-chiral ansatz:

$$|Q^{\alpha}{}_a\rangle = \sum_{n,m}\left\{q_{nm}(Y^n\widetilde{X}Y'^m)^{\alpha}{}_a + q'_{mn}(Y^m\widetilde{X}'Y'^n)^{\alpha}{}_a\right\}|V\rangle,$$

$$|\widetilde{Q}^{\alpha}{}_a\rangle = \sum_{n,m}\left\{\tilde{q}_{nm}(Y^nXY'^m)^{\alpha}{}_a + \tilde{q}'_{mn}(Y^mX'Y'^n)^{\alpha}{}_a\right\}|V\rangle.$$

- chiral ansatz:

$$|Q^{\alpha}{}_a\rangle = \sum_{n,m}\left\{p_{nm}(Y^nXY'^m)^{\alpha}{}_a + p'_{mn}(Y^mX'Y'^n)^{\alpha}{}_a\right\}|V\rangle,$$

$$|\widetilde{Q}^{\alpha}{}_a\rangle = \sum_{n,m}\left\{\tilde{p}_{nm}(Y^n\widetilde{X}Y'^m)^{\alpha}{}_a + \tilde{p}'_{mn}(Y^m\widetilde{X}'Y'^n)^{\alpha}{}_a\right\}|V\rangle.$$

ここで和は $n = 1, 3; \quad m = 0, 2, 4.$

Solution (anti-chiral)

- $q^{\alpha}{}_a|Q^{\beta}{}_b\rangle + q^{\beta}{}_b|Q^{\alpha}{}_a\rangle = \epsilon^{\alpha\beta}\epsilon_{ab}|H\rangle$ を解くと

$$|Q^{\alpha}{}_a\rangle = \frac{1}{2}q_1\Big\{\eta^*\big[(\sinh\overline{Y})\widetilde{X}(\cosh\overline{Y}')\big]^{\alpha}{}_a + \eta\big[(\cosh\overline{Y})\widetilde{X}'(\sinh\overline{Y}')\big]^{\alpha}{}_a\Big\}|V\rangle,$$

$$\begin{aligned}|H\rangle = \frac{1}{2}q_1\Big\{&\frac{1}{12}\big[\mathrm{Tr}Y^4 - \mathrm{Tr}Y'^4\big]\\ &+ \mathrm{Tr}X\cosh\overline{Y}\widetilde{X}\cosh\overline{Y}' - i\mathrm{Tr}\sinh\overline{Y}\widetilde{X}\sinh\overline{Y}'X'\\ &+ \mathrm{Tr}\cosh\overline{Y}\widetilde{X}'\cosh\overline{Y}'X' + i\mathrm{Tr}X\sinh\overline{Y}\widetilde{X}'\sinh\overline{Y}'\Big\}|V\rangle.\end{aligned}$$

ここで $\overline{Y} = Y\eta,\ \overline{Y}' = Y'\eta^*,\quad (\eta \equiv e^{i\pi/4})$

$\widetilde{q}^{\alpha}{}_a|\widetilde{Q}^{\beta}{}_b\rangle + \widetilde{q}^{\beta}{}_b|\widetilde{Q}^{\alpha}{}_a\rangle = \epsilon^{\alpha\beta}\epsilon_{ab}|\widetilde{H}\rangle$ の解も同様に求まる。

[Pankiewicz(2003)]によるpp-wave上のSFTの相互作用項の形を再現！

On consistency

- $q^{\alpha}{}_a|\widetilde{Q}^{\beta}{}_b\rangle + \widetilde{q}^{\beta}{}_b|Q^{\alpha}{}_a\rangle = \epsilon^{\alpha\beta}\epsilon_{ab}|N\rangle$ の左辺を計算すると

$$q^{\alpha}{}_a|\widetilde{Q}^{\beta}{}_b\rangle + \widetilde{q}^{\beta}{}_b|Q^{\alpha}{}_a\rangle = \frac{i}{2}\left[\widetilde{q}_1 h - q_1 \widetilde{h}\right](\cosh\overline{Y}\epsilon)^{\alpha\beta}(\epsilon\cosh\overline{Y}')_{ba}|V\rangle$$

⇩

係数を $\widetilde{q}_1 = q_1$ とすればup to level matching condition:

$h - \widetilde{h} = 0$ で代数は成立。このとき $|N\rangle = 0$

ここで $h, \widetilde{h}$ はHamiltonianのfree part:

$$h = \frac{1}{4}\epsilon^{ab}\epsilon_{\alpha\beta}\{q^{\alpha}{}_a, q^{\beta}{}_b\},\ \widetilde{h} = \frac{1}{4}\epsilon^{ab}\epsilon_{\alpha\beta}\{\widetilde{q}^{\alpha}{}_a, \widetilde{q}^{\beta}{}_b\}$$

Solution (chiral)

- $$q^{\alpha}{}_a|Q^{\beta}{}_b\rangle + q^{\beta}{}_b|Q^{\alpha}{}_a\rangle = \epsilon^{\alpha\beta}\epsilon_{ab}|H\rangle$$ を解くと

$$|Q^{\alpha}{}_a\rangle = q^{\alpha}{}_a|W\rangle, \quad |H\rangle = h|W\rangle$$

ここで $|W\rangle = \left(p_1 + \frac{p_>}{2}\mathrm{Tr}Y^4 + \frac{p_<}{2}\mathrm{Tr}Y'^4 + \frac{p_7}{4}\mathrm{Tr}Y^4\mathrm{Tr}Y'^4\right)|V\rangle$

$$\tilde{q}^{\alpha}{}_a|\tilde{Q}^{\beta}{}_b\rangle + \tilde{q}^{\beta}{}_b|\tilde{Q}^{\alpha}{}_a\rangle = \epsilon^{\alpha\beta}\epsilon_{ab}|\widetilde{H}\rangle$$ の解も同様に求まる。

$$q^{\alpha}{}_a|\tilde{Q}^{\beta}{}_b\rangle + \tilde{q}^{\beta}{}_b|Q^{\alpha}{}_a\rangle = \epsilon^{\alpha\beta}\epsilon_{ab}|N\rangle$$ については

$|W\rangle = |\widetilde{W}\rangle$ とすることにより、up to level matching projection:

$h - \tilde{h} = 0$ で代数は成立。このとき $|N\rangle = 0$

[Di Vecchia et al.(2003)]によるpp-wave上のSFTの相互作用項の形を含む。

On “SUGRA” limit

- 今の模型での超重力極限は…

$$\widetilde{X}^{a}{}_{\dot{a}} = X^{a}{}_{\dot{a}},\ \widetilde{X'}^{\alpha}{}_{\dot{\alpha}} = X'^{\alpha}{}_{\dot{\alpha}}$$
: left moving = right moving

$$W^{\alpha}{}_{\dot{b}} = \widetilde{W}^{\alpha}{}_{\dot{b}} = W'^{a}{}_{\dot{\beta}} = \widetilde{W}'^{a}{}_{\dot{\beta}} = 0$$
: zero modeを含まない

このとき $|H\rangle = |H\rangle_{\mathrm{A.C.}} + |H\rangle_{\mathrm{C.}} \sim O((Y \text{ or } Y')^4)|V\rangle$

を要請すると係数に制限がつく：

$$p_1 = iq_1, \quad p_7 = \frac{-iq_1}{(4!)^2}$$

flatのときのLC SUGRA
[Green-Schwarz]

$u = 2 - \frac{1}{2}\lambda^a \vartheta^a$ -chargeがゼロ：

$$|H\rangle \sim O(Y^4)|V\rangle$$

Contents

- Introduction ✔
- Review of GSB's LCSFT (flat space) ✔
- Algebraic model for su(2|2) LCSFT
  - ansatz for pp-wave and solutions ✔
  - generalization, toy model I,II
- Summary and Discussion

On generalization (1)

- pp-waveよりも一般のsu(2|2)対称性をもつ背景の場合に向けて

$$q^{\alpha}{}_{a}|V\rangle, \; \tilde{q}^{\alpha}{}_{a}|V\rangle,$$

$$[q^{\alpha}{}_{a}, \widetilde{X}^{b}{}_{\dot{b}}], \; [q^{\alpha}{}_{a}, \widetilde{X}'^{\beta}{}_{\dot{\beta}}], \quad [\tilde{q}^{\alpha}{}_{a}, X^{b}{}_{\dot{b}}], \; [\tilde{q}^{\alpha}{}_{a}, X'^{\beta}{}_{\dot{\beta}}]$$

ansatzを一般化しよう。

$$\mathrm{grd}\boldsymbol{Y} = 0, \quad \mathrm{grd}\boldsymbol{X} = -\mathrm{grd}\widetilde{\boldsymbol{X}} = 1/2, \quad \mathrm{grd}\boldsymbol{W} = -\mathrm{grd}\widetilde{\boldsymbol{W}} = 1,$$

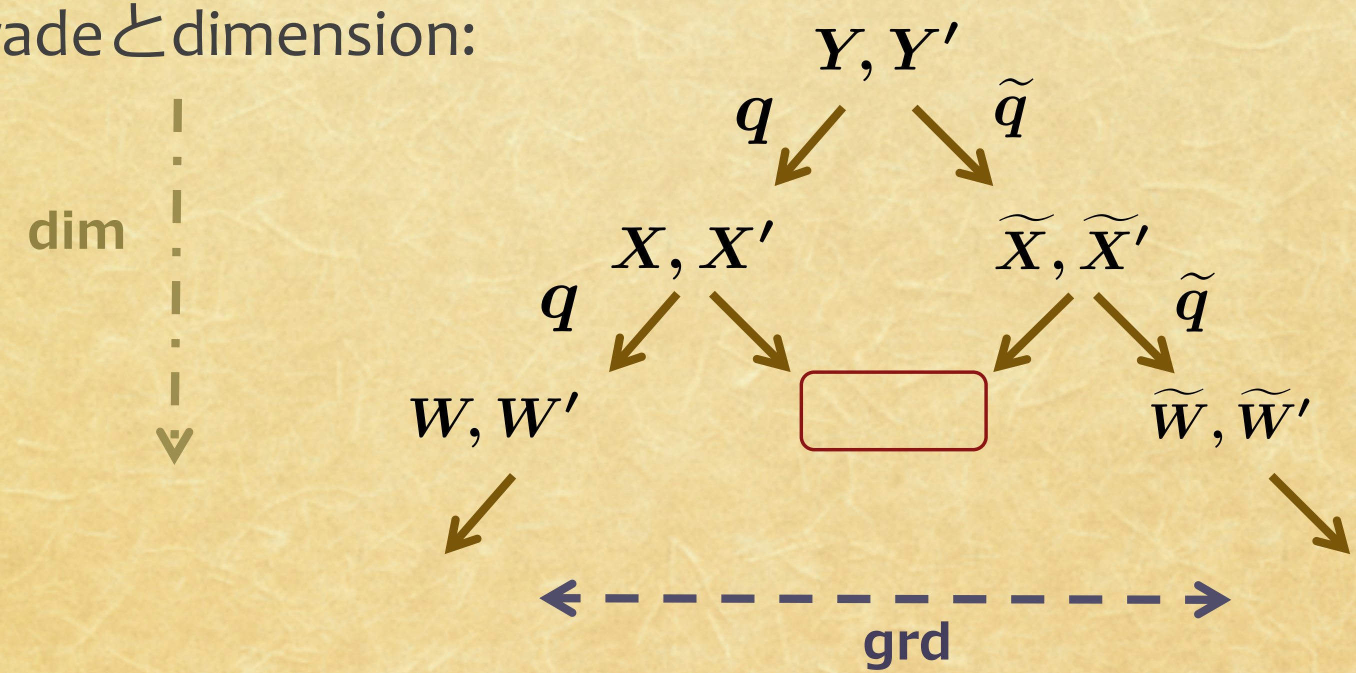
$$\mathrm{dim}\boldsymbol{Y} = 0, \quad \mathrm{dim}\boldsymbol{X} = \mathrm{dim}\widetilde{\boldsymbol{X}} = 1/2, \quad \mathrm{dim}\boldsymbol{W} = \mathrm{dim}\widetilde{\boldsymbol{W}} = 1.$$

とし、gradeを保ちながらdimensionに関して展開

⇒ 代数のconsistencyから係数を決めていく

On generalization (2)

- Gradeとdimension:



- consistency:

$$\{[q^\alpha{}_a, \widetilde{X}^c{}_{\dot c}], q^\beta{}_b\} + \{q^\alpha{}_a, [q^\beta{}_b, \widetilde{X}^c{}_{\dot c}]\} = \epsilon^{\alpha\beta}\epsilon_{ab}[h, \widetilde{X}^c{}_{\dot c}],$$

$$\{q^\alpha{}_a, q^\beta{}_b\}|V\rangle = \epsilon^{\alpha\beta}\epsilon_{ab}h|V\rangle, \quad \{q^\alpha{}_a, \widetilde{q}^\beta{}_b\}|V\rangle = \epsilon^{\alpha\beta}\epsilon_{ab}n|V\rangle,$$

$r^a{}_b|V\rangle = l^\alpha{}_\beta|V\rangle = 0$ を仮定

Generalized ansatz (1)

$$q^{\alpha}{}_a|V\rangle = \frac{i}{2}[(YX)^{\alpha}{}_a + (X'Y')^{\alpha}{}_a]|V\rangle, \quad \widetilde{q}^{\alpha}{}_a|V\rangle = -\frac{i}{2}[(Y\widetilde{X})^{\alpha}{}_a + (\widetilde{X}'Y')^{\alpha}{}_a]|V\rangle.$$

の一般化 (up to dim=1/2)

$$q^{\alpha}{}_a|V\rangle = \sum_{n,m}\Big\{v_{nm}(Y^nXY'^m)^{\alpha}{}_a + v'_{mn}(Y^mX'Y'^n)^{\alpha}{}_a\Big\}|V\rangle$$

$$\widetilde{q}^{\alpha}{}_a|V\rangle = \sum_{n,m}\Big\{\widetilde{v}_{nm}(Y^n\widetilde{X}Y'^m)^{\alpha}{}_a + \widetilde{v}'_{mn}(Y^m\widetilde{X}'Y'^n)^{\alpha}{}_a\Big\}|V\rangle$$

ここで和は $n = 1, 3; \quad m = 0, 2, 4.$

Generalized ansatz (2)

- 交換関係　$[q^{\alpha}{}_{a}, \widetilde{X}^{b}{}_{\dot{b}}] = \frac{i}{2}\delta^{b}_{a} Y^{\alpha}{}_{\dot{b}}$, $[q^{\alpha}{}_{a}, \widetilde{X}'^{\beta}{}_{\dot{\beta}}] = \frac{i}{2}\epsilon^{\beta\alpha}(\epsilon Y')_{a\dot{\beta}}$　の一般化
  (up to dim 1, grd=0)

$$
\begin{aligned}
&[q^{\alpha}{}_{a}, \tilde{X}^{b}{}_{\dot{b}}] = v_1\delta^b_a Y^{\alpha}{}_{\dot b} + v_2\delta^b_a (Y^3)^{\alpha}{}_{\dot b} + v_3 Y^{\alpha}{}_{\dot b}(Y'^2)^b{}_a + v_4\delta^b_a Y^{\alpha}{}_{\dot b}(Y'^4)^c{}_c + v_5 (Y^3)^{\alpha}{}_{\dot b}(Y'^2)^b{}_a + v_6\delta^b_a (Y^3)^{\alpha}{}_{\dot b}(Y'^4)^c{}_c + v_7 Y^{\alpha}{}_{\dot b}(\tilde X X)^b{}_a \\
&+v_8 Y^{\alpha}{}_{\dot b}(X\tilde X)^b{}_a + v_9 (YX\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b} + v_{10}(YX)^{\alpha}{}_a\tilde X^b{}_{\dot b} + v_{11}(Y\tilde X\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{12}(Y\tilde X)^{\alpha}{}_a X^b{}_{\dot b} + v_{13}(Y^3)^{\alpha}{}_{\dot b}(\tilde X X)^b{}_a \\
&+v_{14}(Y^3)^{\alpha}{}_{\dot b}(X\tilde X)^b{}_a + v_{15}(Y^3X\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b} + v_{16}(Y^3X)^{\alpha}{}_a\tilde X^b{}_{\dot b} + v_{17}(Y^3\tilde X\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{18}(Y^3\tilde X)^{\alpha}{}_a X^b{}_{\dot b} \\
&+v_{19} Y^{\alpha}{}_{\dot b}(Y'^2)^b{}_a(X\tilde X)^c{}_c + v_{20} Y^{\alpha}{}_{\dot b}(\tilde X X Y'^2)^b{}_a + v_{21} Y^{\alpha}{}_{\dot b}(Y'^2 X\tilde X)^b{}_a + v_{22} Y^{\alpha}{}_{\dot b}(X\tilde X Y'^2)^b{}_a + v_{23} Y^{\alpha}{}_{\dot b}(Y'^2\tilde X X)^b{}_a \\
&+v_{24}(YX\tilde X)^{\alpha}{}_{\dot b}(Y'^2)^b{}_a + v_{25}(YXY'^2)^{\alpha}{}_a\tilde X^b{}_{\dot b} + v_{26}(YXY'^2\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b} + v_{27}(Y\tilde X X)^{\alpha}{}_{\dot b}(Y'^2)^b{}_a + v_{28}(Y\tilde X Y'^2)^{\alpha}{}_a X^b{}_{\dot b} \\
&+v_{29}(Y\tilde X Y'^2\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{30}(YX)^{\alpha}{}_a(Y'^2\tilde X)^b{}_{\dot b} + v_{31}(YX\epsilon)^{\alpha b}(\epsilon Y'^2\tilde X)_{a\dot b} + v_{32}(Y\tilde X)^{\alpha}{}_a(Y'^2X)^b{}_{\dot b} + v_{33}(Y\tilde X\epsilon)^{\alpha b}(\epsilon Y'^2 X)_{a\dot b} \\
&+v_{34} Y^{\alpha}{}_{\dot b}(\tilde X X)^b{}_a(Y'^4)^c{}_c + v_{35} Y^{\alpha}{}_{\dot b}(X\tilde X)^b{}_a(Y'^4)^c{}_c + v_{36}(YX\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b}(Y'^4)^c{}_c + v_{37}(YX)^{\alpha}{}_a\tilde X^b{}_{\dot b}(Y'^4)^c{}_c \\
&+v_{38}(Y\tilde X\epsilon)^{\alpha b}(\epsilon X)_{a\dot b}(Y'^4)^c{}_c + v_{39}(Y\tilde X)^{\alpha}{}_a X^b{}_{\dot b}(Y'^4)^c{}_c + v_{40}(Y^3)^{\alpha}{}_{\dot b}(Y'^2)^b{}_a(X\tilde X)^c{}_c + v_{41}(Y^3)^{\alpha}{}_{\dot b}(\tilde X X Y'^2)^b{}_a \\
&+v_{42}(Y^3)^{\alpha}{}_{\dot b}(Y'^2X\tilde X)^b{}_a + v_{43}(Y^3)^{\alpha}{}_{\dot b}(X\tilde X Y'^2)^b{}_a + v_{44}(Y^3)^{\alpha}{}_{\dot b}(Y'^2\tilde X X)^b{}_a + v_{45}(Y^3X\tilde X)^{\alpha}{}_{\dot b}(Y'^2)^b{}_a + v_{46}(Y^3XY'^2)^{\alpha}{}_a\tilde X^b{}_{\dot b} \\
&+v_{47}(Y^3XY'^2\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b} + v_{48}(Y^3\tilde X X)^{\alpha}{}_{\dot b}(Y'^2)^b{}_a + v_{49}(Y^3\tilde X Y'^2)^{\alpha}{}_a X^b{}_{\dot b} + v_{50}(Y^3\tilde X Y'^2\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{51}(Y^3X)^{\alpha}{}_a(Y'^2\tilde X)^b{}_{\dot b} \\
&+v_{52}(Y^3X\epsilon)^{\alpha b}(\epsilon Y'^2\tilde X)_{a\dot b} + v_{53}(Y^3\tilde X)^{\alpha}{}_a(Y'^2X)^b{}_{\dot b} + v_{54}(Y^3\tilde X\epsilon)^{\alpha b}(\epsilon Y'^2X)_{a\dot b} + v_{55}(Y^3)^{\alpha}{}_{\dot b}(\tilde X X)^b{}_a(Y'^4)^c{}_c \\
&+v_{56}(Y^3)^{\alpha}{}_{\dot b}(X\tilde X)^b{}_a(Y'^4)^c{}_c + v_{57}(Y^3X\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b}(Y'^4)^c{}_c + v_{58}(Y^3X)^{\alpha}{}_a\tilde X^b{}_{\dot b}(Y'^4)^c{}_c + v_{59}(Y^3\tilde X\epsilon)^{\alpha b}(\epsilon X)_{a\dot b}(Y'^4)^c{}_c \\
&+v_{60}(Y^3\tilde X)^{\alpha}{}_a X^b{}_{\dot b}(Y'^4)^c{}_c + v_{61} Y^{\alpha}{}_{\dot b}\delta^b_a(X'\tilde X')^{\beta}{}_{\beta} + v_{62}(X'\tilde X'Y)^{\alpha}{}_{\dot b}\delta^b_a + v_{63}(\tilde X'X'Y)^{\alpha}{}_{\dot b}\delta^b_a + v_{64}(Y^3)^{\alpha}{}_{\dot b}\delta^b_a(X'\tilde X')^{\beta}{}_{\beta} \\
&+v_{65}(X'\tilde X'Y^3)^{\alpha}{}_{\dot b}\delta^b_a + v_{66}(\tilde X'X'Y^3)^{\alpha}{}_{\dot b}\delta^b_a + v_{67} Y^{\alpha}{}_{\dot b}(Y'^2)^b{}_a(X'\tilde X')^{\beta}{}_{\beta} + v_{68} Y^{\alpha}{}_{\dot b}\delta^b_a(X'\tilde X')^{\beta}{}_{\beta}(Y'^4)^c{}_c + v_{69}(X'\tilde X'Y)^{\alpha}{}_{\dot b}\delta^b_a(Y'^4)^c{}_c \\
&+v_{70}(\tilde X'X'Y)^{\alpha}{}_{\dot b}\delta^b_a(Y'^4)^c{}_c + v_{71}(Y^3)^{\alpha}{}_{\dot b}(Y'^2)^b{}_a(X'\tilde X')^{\beta}{}_{\beta} + v_{72}(Y^3)^{\alpha}{}_{\dot b}\delta^b_a(X'\tilde X')^{\beta}{}_{\beta}(Y'^4)^c{}_c + v_{73}(X'\tilde X'Y^3)^{\alpha}{}_{\dot b}\delta^b_a(Y'^4)^c{}_c \\
&+v_{74}(\tilde X'X'Y^3)^{\alpha}{}_{\dot b}\delta^b_a(Y'^4)^c{}_c + v_{75}(\tilde X'Y'\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{76}(\tilde X'Y')^{\alpha}{}_a X^b{}_{\dot b} + v_{77}(X'Y'\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b} + v_{78}(X'Y')^{\alpha}{}_a\tilde X^b{}_{\dot b} \\
&+v_{79}(\tilde X'Y'^3\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{80}(\tilde X'Y'^3)^{\alpha}{}_a X^b{}_{\dot b} + v_{81}(X'Y'^3\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b} + v_{82}(X'Y'^3)^{\alpha}{}_a\tilde X^b{}_{\dot b} + v_{83}(\tilde X'Y'\epsilon)^{\alpha b}(\epsilon X)_{a\dot b}(Y^4)^{\beta}{}_{\beta} \\
&+v_{84}(\tilde X'Y')^{\alpha}{}_a X^b{}_{\dot b}(Y^4)^{\beta}{}_{\beta} + v_{85}(X'Y'\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b}(Y^4)^{\beta}{}_{\beta} + v_{86}(X'Y')^{\alpha}{}_a\tilde X^b{}_{\dot b}(Y^4)^{\beta}{}_{\beta} + v_{87}(\tilde X'Y'^3\epsilon)^{\alpha b}(\epsilon X)_{a\dot b}(Y^4)^{\beta}{}_{\beta} \\
&+v_{88}(\tilde X'Y'^3)^{\alpha}{}_a X^b{}_{\dot b}(Y^4)^{\beta}{}_{\beta} + v_{89}(X'Y'^3\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b}(Y^4)^{\beta}{}_{\beta} + v_{90}(X'Y'^3)^{\alpha}{}_a\tilde X^b{}_{\dot b}(Y^4)^{\beta}{}_{\beta} + v_{91}(Y^2\tilde X'Y'\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} \\
&+v_{92}(Y^2\tilde X'Y')^{\alpha}{}_a X^b{}_{\dot b} + v_{93}(\tilde X'Y'\epsilon)^{\alpha b}(\epsilon XY^2)_{a\dot b} + v_{94}(\tilde X'Y')^{\alpha}{}_a(XY^2)^b{}_{\dot b} + v_{95}(Y^2X'Y'\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b} + v_{96}(Y^2X'Y')^{\alpha}{}_a\tilde X^b{}_{\dot b} \\
&+v_{97}(X'Y'\epsilon)^{\alpha b}(\epsilon\tilde XY^2)_{a\dot b} + v_{98}(X'Y')^{\alpha}{}_a(\tilde XY^2)^b{}_{\dot b} + v_{99}(Y^2\tilde X'Y'^3\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{100}(Y^2\tilde X'Y'^3)^{\alpha}{}_a X^b{}_{\dot b} \\
&+v_{101}(\tilde X'Y'^3\epsilon)^{\alpha b}(\epsilon XY^2)_{a\dot b} + v_{102}(\tilde X'Y'^3)^{\alpha}{}_a(XY^2)^b{}_{\dot b} + v_{103}(Y^2X'Y'^3\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b} + v_{104}(Y^2X'Y'^3)^{\alpha}{}_a\tilde X^b{}_{\dot b} \\
&+v_{105}(X'Y'^3\epsilon)^{\alpha b}(\epsilon\tilde XY^2)_{a\dot b} + v_{106}(X'Y'^3)^{\alpha}{}_a(\tilde XY^2)^b{}_{\dot b}
\end{aligned}
$$

←右辺は106項

$[q^{\alpha}{}_{a}, \widetilde{X}'^{\beta}{}_{\dot{\beta}}]$

についても同様に106項。

Mathematicaを用いてconsistency
を課していく…
しかし、一般には煩雑。

Toy model I

- 交換関係は元のまま

$$q^{\alpha}{}_a|V\rangle = \sum_{n,m}\Big\{v_{nm}(Y^n X Y'^m)^{\alpha}{}_a + v'_{mn}(Y^m X' Y'^n)^{\alpha}{}_a\Big\}|V\rangle$$
$$\widetilde{q}^{\alpha}{}_a|V\rangle = \sum_{n,m}\Big\{\widetilde{v}_{nm}(Y^n \widetilde{X} Y'^m)^{\alpha}{}_a + \widetilde{v}'_{mn}(Y^m \widetilde{X}' Y'^n)^{\alpha}{}_a\Big\}|V\rangle$$

とした場合、consistencyから

$$v_{12} = v_{32} = v_{14} = v'_{21} = v'_{23} = v'_{41} = \widetilde{v}_{12} = \widetilde{v}_{32} = \widetilde{v}_{34} = \widetilde{v}'_{21} = \widetilde{v}'_{23} = \widetilde{v}'_{43} = 0,$$
$$v_{30} = \widetilde{v}_{30}, \quad v_{34} = \widetilde{v}_{34}, \quad v'_{03} = \widetilde{v}'_{03}, \quad v'_{43} = \widetilde{v}'_{43}, \quad v_{34} + v'_{43} = \widetilde{v}_{34} + \widetilde{v}'_{43} = 0,$$

となり「anti-chiral」な解は少し変形を受けて

$$|Q^{\alpha}{}_a\rangle = \frac{1}{2}q_1\Big\{\eta^*\big[(\sinh\overline{Y})\widetilde{X}\big(\cosh\overline{Y}' - \widehat{v}'Y'^4\big)\big]^{\alpha}{}_a + \eta\big[\big(\cosh\overline{Y} + \widehat{v}Y^4\big)\widetilde{X}'\sinh\overline{Y}'\big]^{\alpha}{}_a\Big\}|V\rangle,$$
$$\begin{aligned}|H\rangle = \frac{1}{2}q_1\Big\{&\frac{1}{12}\big[\mathrm{Tr}\,Y^4(1-(\widehat{v}'/2)\mathrm{Tr}\,Y'^4) - \mathrm{Tr}\,Y'^4(1+(\widehat{v}/2)\mathrm{Tr}\,Y^4)\big]\\ &+\mathrm{Tr}\,X(\cosh\overline{Y} - \widehat{v}Y^4)\widetilde{X}(\cosh\overline{Y}' - \widehat{v}'Y'^4) - i\mathrm{Tr}\,\sinh\overline{Y}\widetilde{X}\sinh\overline{Y}'X'\\ &+\mathrm{Tr}\,(\cosh\overline{Y} + \widehat{v}Y^4)\widetilde{X}'(\cosh\overline{Y}' + \widehat{v}'Y'^4)X' + i\mathrm{Tr}\,X\sinh\overline{Y}\widetilde{X}'\sinh\overline{Y}'\\ &-\widehat{b}(\mathrm{Tr}XY^4\widetilde{X}Y'^4 + \mathrm{Tr}\,Y^4\widetilde{X}'Y'^4X')\Big\}|V\rangle\end{aligned}$$

$$(\widehat{v} = v_{30}/2,\ \widehat{v}' = v'_{03}/2,\ \widehat{b} = v_{34}/4 = -v'_{43}/4)$$

「chiral」な解は元と同じ形の解

Toy model II

- $q^\alpha{}_a|V\rangle,\ \widetilde{q}^\alpha{}_a|V\rangle$: 元と同じで交換関係をup to dim 1でconsistentに変形

$$[q^\alpha{}_a, \widetilde{X}^{\dot{b}}{}_b] = -\frac{i}{2}(Y\epsilon)^{\dot{b}\alpha}\epsilon_{ab} + y_{03}\big[(Y^3\epsilon)^{\dot{b}\alpha}\epsilon_{ab} + 2i(Y\epsilon)^{\dot{b}\alpha}(\epsilon X\widetilde{X})_{ab} + 4i(YX)^\alpha{}_a\widetilde{X}^{\dot{b}}{}_b\big],$$

$$[q^\alpha{}_a, \widetilde{X}'^\beta{}_{\dot{\beta}}] = \frac{i}{2}\epsilon^{\beta\alpha}(\epsilon Y')_{a\dot{\beta}} - y'_{30}\big[\epsilon^{\beta\alpha}(\epsilon Y'^3)_{a\dot{\beta}} + 2i(\widetilde{X}'X'\epsilon)^{\beta\alpha}(\epsilon Y')_{a\dot{\beta}} + 4i(X'Y')^\alpha{}_a\widetilde{X}'^\beta{}_{\dot{\beta}}\big]$$

$[\widetilde{q}^\alpha{}_a, X^{\dot{b}}{}_b],\ [\widetilde{q}^\alpha{}_a, X'^\beta{}_{\dot{\beta}}]$ も同様、ただし $y_{03} = \widetilde{y}_{03},\quad y'_{30} = \widetilde{y}'_{30}.$

このときup to dim 1で代数を満たす $|Q^\alpha{}_a\rangle,\ |\widetilde{Q}^\alpha{}_a\rangle$ は変形前と**同じ形**で

ハミルトニアンは

$$\frac{1}{2}q_1\Big[y_{03}\mathrm{Tr}Y^4\Big(1-\frac{1}{48}\mathrm{Tr}Y'^4\Big) + y'_{30}\mathrm{Tr}Y'^4\Big(1-\frac{1}{48}\mathrm{Tr}Y^4\Big)\Big]|V\rangle.$$

だけ変形される。

Summary

- su(2|2)対称性をもつ背景でのLCSFTの構成に向けた模型を提案。
- pp-waveの場合の形を簡潔に再現した。
  - anti-chiral部分はPankiewiczの3弦相互作用項の形。
  - chiral部分はDi Vecchia et al.の3弦相互作用項の一般化に対応。
- “SUGRA”極限でanti-chiral部分とchiral部分の関係がつく。
- (ansatzが正しければ)より一般の背景でのLCSFTの相互作用項をdim.に関する展開により系統的に構成できる。
- 一般化の具体例(toy model I,II)の計算をした。

Discussion

- pp-wave以外でのansatzの正当化？
  - Building blockの具体形
  - それらの代数関係
- Toy model I, IIに対応する背景は？
- 別の背景に対応する $q^{\alpha}{}_{a}|V\rangle,\ [q^{\alpha}{}_{a}, \widetilde{X}^{b}{}_{\dot{b}}],\ [q^{\alpha}{}_{a}, \widetilde{X}'^{\beta}{}_{\dot{\beta}}]\cdots$

  と代数を満たす解？
- AdS/CFT (BMN) 対応の一般化への応用？
- LCSFTのより高次の相互作用項については？