YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

su(2|2) 光円錐型弦の場の理論の代数模型

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reference:
I. Kishimoto and S. Moriyama, JHEP08(2010)013 [arXiv:1005.4719]

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LCSFT on pp-wave

- AdS/CFT対応の観点で応用された：
  - $AdS_5 \times S^5$ のPenrose limit:　pp-wave時空
  - BMN(Berenstein-Maldacena-Nastase)対応

4次元N=4 SU(N) SYMのalmost BPS operator

↕

pp-wave上の超弦理論のstring state

→ 弦の相互作用を含めて調べる

Spradlin-Volovich, Pankiewicz-Stefanski(2002), Pankiewicz(2003)による光円錐ゲージの弦の場の理論(LCSFT)の振動子表示の定式化が使われた

Toward su(2|2) LCSFT

- flat, pp-wave以外のより一般の背景でのSFTの構成を直接、具体的に振動子レベルでやるのは難しい。
- 代わりにflat, pp-waveの例からSFTのbuilding blockを抜き出し代数で形を決めよう。→「模型」の提案
- pp-wave背景の対称性はsu(2|2)を部分代数として含む。
- spin鎖模型でsu(2|2)代数が重要な役割を果たした。

⇩

su(2|2)対称性をもつ背景上のLCSFTの模型

pp-wave上のSFTをより簡潔に再現、
一般化の計算例

Green-Schwarz形式の光円錐ゲージ

- Green-Schwarz形式光円錐ゲージの弦の座標、運動量

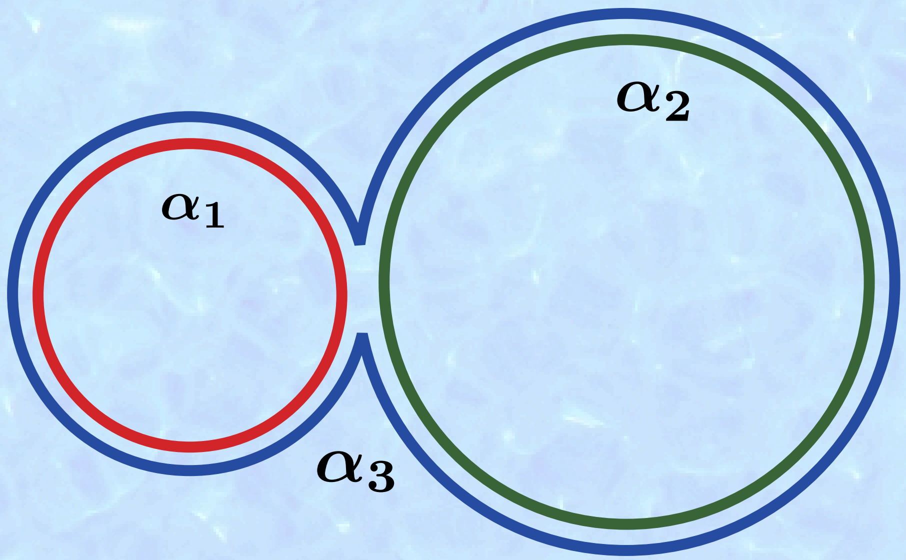
$\boldsymbol{x^i(\sigma), \vartheta^a(\sigma)}$ $\boldsymbol{p^i(\sigma), \lambda^a(\sigma)}$ ：モード展開される

- 超電荷とハミルトニアン（free part）

$$
\begin{aligned}
q^{\dot a} &= \frac{\sqrt{2}}{\alpha} \sum_{n=-\infty}^{\infty} \gamma^i_{a\dot a} Q^a_{-n} \alpha^i_n, \\
\widetilde{q}^{\dot a} &= \frac{\sqrt{2}}{\alpha} \sum_{n=-\infty}^{\infty} \gamma^i_{a\dot a} \widetilde{Q}^a_{-n} \widetilde{\alpha}^i_n, \\
h &= \frac{1}{\alpha} p^i p^i + \frac{1}{\alpha} \sum_{n=1}^{\infty} \left(\alpha^i_{-n}\alpha^i_n + \widetilde{\alpha}^i_{-n}\widetilde{\alpha}^i_n + \frac{n}{\alpha}(Q^a_{-n}Q^a_n + \widetilde{Q}^a_{-n}\widetilde{Q}^a_n) \right).
\end{aligned}
$$

- $\boldsymbol{i, a, \dot a}$ はSO(8)の $\mathbf{8_v, 8_s, 8_c}$

Kinematical overlap



:光円錐型の３弦相互作用

３弦の接続条件を表すδ-汎関数:

$$\delta(\alpha_1+\alpha_2+\alpha_3)\delta^8(x^{i(3)}-\Theta_1 x^{i(1)}-\Theta_2 x^{i(2)})\delta^8(\vartheta^{i(3)}-\Theta_1\vartheta^{i(1)}-\Theta_2\vartheta^{i(2)})$$
$$=\langle\alpha_1,x^{(1)},\vartheta^{(1)}|\langle\alpha_2,x^{(2)},\vartheta^{(2)}|\langle\alpha_3,x^{(3)},\vartheta^{(3)}|V\rangle$$

$|V\rangle$:振動子とノイマン係数で露わに書かれている

Algebra and prefactors

- SUSY代数： $\{Q^{\dot{a}}, Q^{\dot{b}}\} = \{\widetilde{Q}^{\dot{a}}, \widetilde{Q}^{\dot{b}}\} = 2H\delta^{\dot{a}\dot{b}},\ \ \{Q^{\dot{a}}, \widetilde{Q}^{\dot{b}}\} = 0$
- 非自明な最低次（３弦相互作用項）の部分：

$$q^{\dot{a}}|Q^{\dot{b}}\rangle + q^{\dot{b}}|Q^{\dot{a}}\rangle = \tilde{q}^{\dot{a}}|\widetilde{Q}^{\dot{b}}\rangle + \tilde{q}^{\dot{b}}|\widetilde{Q}^{\dot{a}}\rangle = 2\delta^{\dot{a}\dot{b}}|H\rangle,\ \ q^{\dot{a}}|\widetilde{Q}^{\dot{b}}\rangle + \tilde{q}^{\dot{b}}|Q^{\dot{a}}\rangle = 0$$

up to level matching conditionで満たす解

Green-Schwarz-Brinkの公式（の簡潔版 [I.K.-Moriyama(2006)]）

$$|H\rangle = X^i \widetilde{X}^j \left[\cosh \not{Y}\right]^{ij} |V\rangle,$$

$$|Q^{\dot{a}}\rangle = \sqrt{-\alpha_{123}}\,\widetilde{X}^i \left[\sinh \not{Y}\right]^{\dot{a}i} |V\rangle,$$

$$|\widetilde{Q}^{\dot{a}}\rangle = i\sqrt{-\alpha_{123}}\,X^i \left[\sinh \not{Y}\right]^{i\dot{a}} |V\rangle$$

$$\not{Y} = \sqrt{\tfrac{2}{-\alpha_{123}}}\,\eta^* Y^a \hat{\gamma}^a \qquad \hat{\gamma}^a = \begin{pmatrix} 0 & \hat{\gamma}^a_{i\dot{a}} \\ \hat{\gamma}^a_{\dot{a}i} & 0 \end{pmatrix} = \begin{pmatrix} 0 & \gamma^i_{a\dot{a}} \\ \gamma^i_{a\dot{a}} & 0 \end{pmatrix}$$

Building blocks

- 3 弦相互作用項の構成要素:　$|V\rangle$　　$Y^a,\ X^i,\ \widetilde{X}^i$

ここで　$\lambda^{(1)}(\sigma_1)|V\rangle \sim \frac{1}{4\pi|\alpha_{123}|^{1/2}|\sigma_1-\sigma_{\rm int}|^{1/2}} Y^a|V\rangle$

$$\{q^{\dot{a}}, Y^a\} = \frac{1}{\sqrt{2}}\eta\gamma^i_{a\dot{a}}X^i, \qquad \{\tilde{q}^{\dot{a}}, Y^a\} = \frac{1}{\sqrt{2}}\eta^*\gamma^i_{a\dot{a}}\widetilde{X}^i$$

実はハミルトニアンには　$[q, X] \sim \partial Y,\ \ [\tilde{q}, \widetilde{X}] \sim \bar{\partial} Y$

を用いた不定性も入りうる: [Lee-Russo(2004), Dobashi-I.K.-Moriyama(unpublished)]

$$|Q^{\dot{a}}\rangle = \left(\tilde{f}_0\widetilde{X}^i[\sinh \not{Y}]^{i\dot{a}} + X^i\left[f_1\not{Y} + \frac{1}{7!}f_7\not{Y}^7\right]^{i\dot{a}}\right)|V\rangle = \left(\tilde{f}_0\widetilde{X}^i[\sinh \not{Y}]^{i\dot{a}} + q^{\dot{a}}i\sqrt{-\alpha_{123}}(f_1 + f_7 y_0^8\delta^8(Y))\right)|V\rangle$$

$$|\widetilde{Q}^{\dot{a}}\rangle = \left(g_0[\sinh \not{Y}]_{\dot{a}i}X^i + \left[\tilde{g}_1\not{Y} + \frac{1}{7!}\tilde{g}_7\not{Y}^7\right]^{\dot{a}i}\widetilde{X}^i\right)|V\rangle = \left(g_0[\sinh \not{Y}]^{\dot{a}i}X^i + \tilde{q}^{\dot{a}}\sqrt{-\alpha_{123}}(\tilde{g}_1 + \tilde{g}_7 y_0^8\delta^8(Y))\right)|V\rangle$$

$$|H\rangle = \left(\frac{1}{\sqrt{-\alpha_{123}}}g_0\tilde{X}^iX^j[\cosh \not{Y}]^{ij} + h\sqrt{-\alpha_{123}}(\tilde{g}_1 + y_0^8\tilde{g}_7\delta^8(Y))\right)|V\rangle$$

Algebraic model

- ここで扱う超代数： $\mathfrak{psu}(2|2) \ltimes \mathbb{R}^3$

$$
\begin{aligned}
\{\mathcal{Q}^{\alpha}{}_{a}, \mathcal{Q}^{\beta}{}_{b}\} &= \epsilon^{\alpha\beta}\epsilon_{ab}\mathcal{H}, \\
\{\widetilde{\mathcal{Q}}^{\alpha}{}_{a}, \widetilde{\mathcal{Q}}^{\beta}{}_{b}\} &= \epsilon^{\alpha\beta}\epsilon_{ab}\widetilde{\mathcal{H}}, \\
\{\mathcal{Q}^{\alpha}{}_{a}, \widetilde{\mathcal{Q}}^{\beta}{}_{b}\} &= \epsilon^{\alpha\beta}\epsilon_{ac}\mathcal{R}^{c}{}_{b} + \epsilon_{ab}\mathcal{L}^{\alpha}{}_{\gamma}\epsilon^{\gamma\beta} + \epsilon^{\alpha\beta}\epsilon_{ab}\mathcal{N}.
\end{aligned}
$$

$\mathcal{J} = j + g_s|J\rangle + \cdots$ のように展開しfree部分は知っているとし、

$$
\begin{aligned}
q^{\alpha}{}_{a}|Q^{\beta}{}_{b}\rangle + q^{\beta}{}_{b}|Q^{\alpha}{}_{a}\rangle &= \epsilon^{\alpha\beta}\epsilon_{ab}|H\rangle, \\
\widetilde{q}^{\alpha}{}_{a}|\widetilde{Q}^{\beta}{}_{b}\rangle + \widetilde{q}^{\beta}{}_{b}|\widetilde{Q}^{\alpha}{}_{a}\rangle &= \epsilon^{\alpha\beta}\epsilon_{ab}|\widetilde{H}\rangle, \\
q^{\alpha}{}_{a}|\widetilde{Q}^{\beta}{}_{b}\rangle + \widetilde{q}^{\beta}{}_{b}|Q^{\alpha}{}_{a}\rangle &= \epsilon^{\alpha\beta}\epsilon_{ab}|N\rangle.
\end{aligned}
$$

を満たす「３弦相互作用項」を構成しよう！

$\mathcal{L}^{\alpha}{}_{\beta} = l^{\alpha}{}_{\beta}, \quad \mathcal{R}^{a}{}_{b} = r^{a}{}_{b}$ はkinematicalだと仮定している。

Building blocks

- 3弦相互作用項 $|H\rangle$, $|\widetilde{H}\rangle$, $|N\rangle$, $|Q^{\alpha}{}_{a}\rangle$, $|\widetilde{Q}^{\alpha}{}_{a}\rangle$
  を構成するもの（仮定）

$|V\rangle$: kinematical overlap ～ 弦の接続条件を表すδ-汎函数

$Y^{\alpha}{}_{\dot{b}}, Y'^{a}{}_{\dot{\beta}}, X^{a}{}_{\dot{b}}, \widetilde{X}^{a}{}_{\dot{b}}, X'^{\alpha}{}_{\dot{\beta}}, \widetilde{X}'^{\alpha}{}_{\dot{\beta}}, W^{\alpha}{}_{\dot{b}}, \widetilde{W}^{\alpha}{}_{\dot{b}}, W'^{a}{}_{\dot{\beta}}, \widetilde{W}'^{a}{}_{\dot{\beta}}$

: prefactorの構成要素 $(\alpha, \dot{\alpha}, a, \dot{a}, \cdots = 1, 2)$

$Y^{\alpha}{}_{\dot{b}}, Y'^{a}{}_{\dot{\beta}}$ ～fermion運動量を $|V\rangle$ にかけ相互作用点に近づけ発散を取り除いた部分

交換関係:

$\{q^{\alpha}{}_{a}, Y^{\beta}{}_{\dot{b}}\} = -\epsilon^{\beta\alpha}(\epsilon X)_{a\dot{b}}$, $\{q^{\alpha}{}_{a}, Y'^{b}{}_{\dot{\beta}}\} = \delta^{b}_{a} X'^{\alpha}{}_{\dot{\beta}}$, ～X,X'の定義

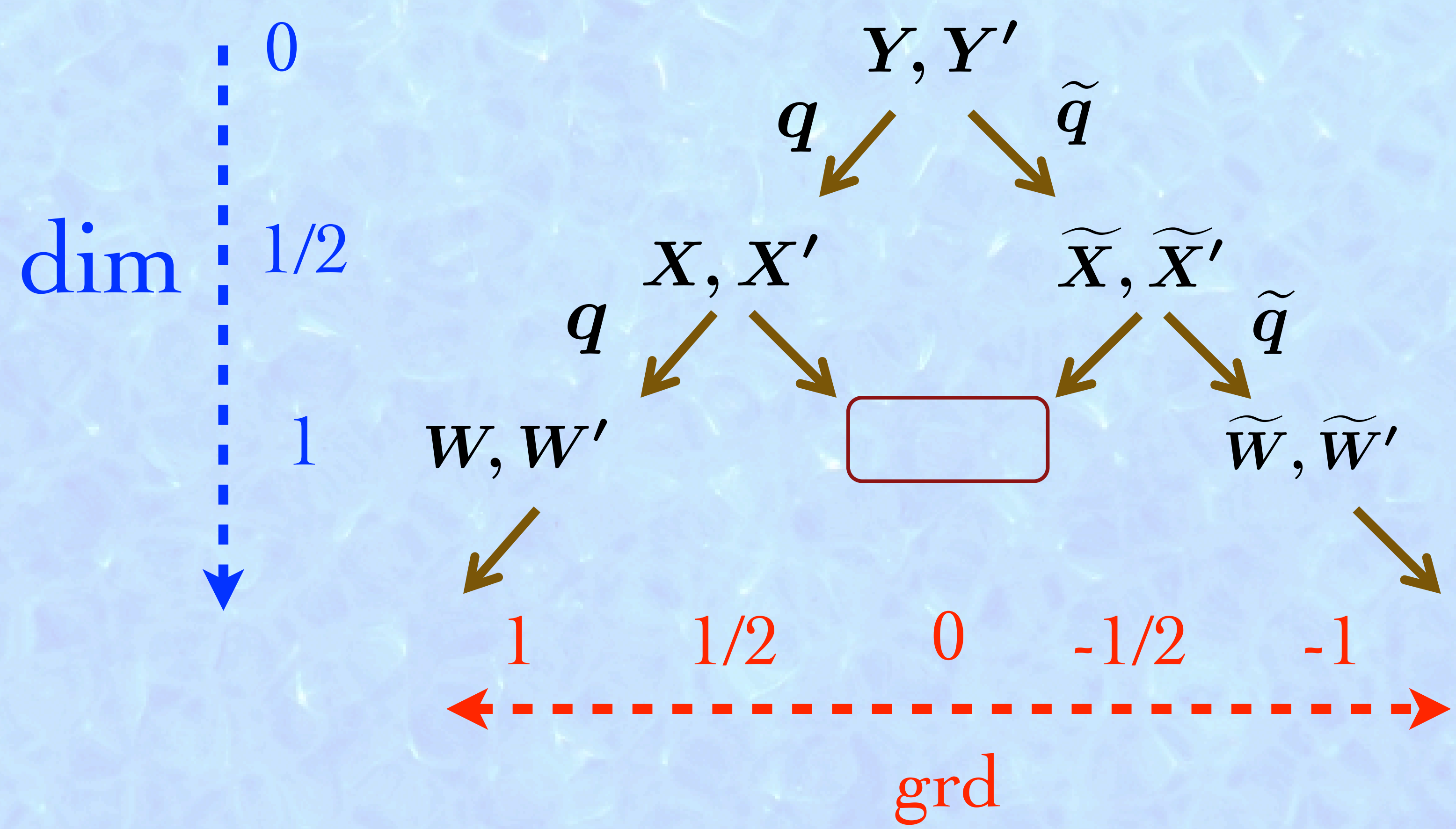
$[q^{\alpha}{}_{a}, X^{b}{}_{\dot{b}}] = \delta^{b}_{a} W^{\alpha}{}_{\dot{b}}$, $[q^{\alpha}{}_{a}, X'^{\beta}{}_{\dot{\beta}}] = -\epsilon^{\beta\alpha}(\epsilon W')_{a\dot{\beta}}$. ～W,W'の定義

$\tilde{q}^{\alpha}{}_{a}$ との交換関係も同様

※さらに $q^{\alpha}{}_{a}|V\rangle$, $[q^{\alpha}{}_{a}, \widetilde{X}^{b}{}_{\dot{b}}]$, $[q^{\alpha}{}_{a}, \widetilde{X}'^{\beta}{}_{\dot{\beta}}]$ の形を決めておく必要がある！

Dimension and grade

- dim とgrdをassignする:



- consistency

$$\{[q^\alpha{}_a, \widetilde{X}^c{}_{\dot{c}}], q^\beta{}_b\} + \{q^\alpha{}_a, [q^\beta{}_b, \widetilde{X}^c{}_{\dot{c}}]\} = \epsilon^{\alpha\beta}\epsilon_{ab}[h, \widetilde{X}^c{}_{\dot{c}}],$$

$\{q^\alpha{}_a, q^\beta{}_b\}|V\rangle = \epsilon^{\alpha\beta}\epsilon_{ab}h|V\rangle, \quad \{q^\alpha{}_a, \tilde{q}^\beta{}_b\}|V\rangle = \epsilon^{\alpha\beta}\epsilon_{ab}n|V\rangle,$ 等は満たすように決める

$r^a{}_b|V\rangle = l^\alpha{}_\beta|V\rangle = 0$ は仮定

Ansatz

$$\boldsymbol{q^{\alpha}{}_{a}|V\rangle \ = \ \sum_{n,m}\Big\{v_{nm}(Y^{n}XY'^{m})^{\alpha}{}_{a} + v'_{mn}(Y^{m}X'Y'^{n})^{\alpha}{}_{a}\Big\}|V\rangle}$$

up to dim 1/2,
grd=1/2 (12項)

$$\begin{aligned}
&[q^{\alpha}{}_{a},\tilde{X}^{b}{}_{\dot b}] = v_1\delta^b_aY^{\alpha}{}_{\dot b} + v_2\delta^b_a(Y^3)^{\alpha}{}_{\dot b} + v_3Y^{\alpha}{}_{\dot b}(Y'^2)^b{}_a + v_4\delta^b_aY^{\alpha}{}_{\dot b}(Y'^4)^c{}_c + v_5(Y^3)^{\alpha}{}_{\dot b}(Y'^2)^b{}_a + v_6\delta^b_a(Y^3)^{\alpha}{}_{\dot b}(Y'^4)^c{}_c + v_7Y^{\alpha}{}_{\dot b}(\tilde XX)^b{}_a \\
&+v_8Y^{\alpha}{}_{\dot b}(X\tilde X)^b{}_a + v_9(YX\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b} + v_{10}(YX)^{\alpha}{}_a\tilde X^b{}_{\dot b} + v_{11}(Y\tilde X\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{12}(Y\tilde X)^{\alpha}{}_aX^b{}_{\dot b} + v_{13}(Y^3)^{\alpha}{}_{\dot b}(\tilde XX)^b{}_a \\
&+v_{14}(Y^3)^{\alpha}{}_{\dot b}(X\tilde X)^b{}_a + v_{15}(Y^3X\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b} + v_{16}(Y^3X)^{\alpha}{}_a\tilde X^b{}_{\dot b} + v_{17}(Y^3\tilde X\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{18}(Y^3\tilde X)^{\alpha}{}_aX^b{}_{\dot b} \\
&+v_{19}Y^{\alpha}{}_{\dot b}(Y'^2)^b{}_a(X\tilde X)^c{}_c + v_{20}Y^{\alpha}{}_{\dot b}(\tilde XXY'^2)^b{}_a + v_{21}Y^{\alpha}{}_{\dot b}(Y'^2X\tilde X)^b{}_a + v_{22}Y^{\alpha}{}_{\dot b}(X\tilde XY'^2)^b{}_a + v_{23}Y^{\alpha}{}_{\dot b}(Y'^2\tilde XX)^b{}_a \\
&+v_{24}(YX\tilde X)^{\alpha}{}_{\dot b}(Y'^2)^b{}_a + v_{25}(YXY'^2)^{\alpha}{}_a\tilde X^b{}_{\dot b} + v_{26}(YXY'^2\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b} + v_{27}(Y\tilde XX)^{\alpha}{}_{\dot b}(Y'^2)^b{}_a + v_{28}(Y\tilde XY'^2)^{\alpha}{}_aX^b{}_{\dot b} \\
&+v_{29}(Y\tilde XY'^2\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{30}(YX)^{\alpha}{}_a(Y'^2\tilde X)^b{}_{\dot b} + v_{31}(YX\epsilon)^{\alpha b}(\epsilon Y'^2\tilde X)_{a\dot b} + v_{32}(Y\tilde X)^{\alpha}{}_a(Y'^2X)^b{}_{\dot b} + v_{33}(Y\tilde X\epsilon)^{\alpha b}(\epsilon Y'^2X)_{a\dot b} \\
&+v_{34}Y^{\alpha}{}_{\dot b}(\tilde XX)^b{}_a(Y'^4)^c{}_c + v_{35}Y^{\alpha}{}_{\dot b}(X\tilde X)^b{}_a(Y'^4)^c{}_c + v_{36}(YX\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b}(Y'^4)^c{}_c + v_{37}(YX)^{\alpha}{}_a\tilde X^b{}_{\dot b}(Y'^4)^c{}_c \\
&+v_{38}(Y\tilde X\epsilon)^{\alpha b}(\epsilon X)_{a\dot b}(Y'^4)^c{}_c + v_{39}(Y\tilde X)^{\alpha}{}_aX^b{}_{\dot b}(Y'^4)^c{}_c + v_{40}(Y^3)^{\alpha}{}_{\dot b}(Y'^2)^b{}_a(X\tilde X)^c{}_c + v_{41}(Y^3)^{\alpha}{}_{\dot b}(\tilde XXY'^2)^b{}_a \\
&+v_{42}(Y^3)^{\alpha}{}_{\dot b}(Y'^2X\tilde X)^b{}_a + v_{43}(Y^3)^{\alpha}{}_{\dot b}(X\tilde XY'^2)^b{}_a + v_{44}(Y^3)^{\alpha}{}_{\dot b}(Y'^2\tilde XX)^b{}_a + v_{45}(Y^3X\tilde X)^{\alpha}{}_{\dot b}(Y'^2)^b{}_a + v_{46}(Y^3XY'^2)^{\alpha}{}_a\tilde X^b{}_{\dot b} \\
&+v_{47}(Y^3XY'^2\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b} + v_{48}(Y^3\tilde XX)^{\alpha}{}_{\dot b}(Y'^2)^b{}_a + v_{49}(Y^3\tilde XY'^2)^{\alpha}{}_aX^b{}_{\dot b} + v_{50}(Y^3\tilde XY'^2\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{51}(Y^3X)^{\alpha}{}_a(Y'^2\tilde X)^b{}_{\dot b} \\
&+v_{52}(Y^3X\epsilon)^{\alpha b}(\epsilon Y'^2\tilde X)_{a\dot b} + v_{53}(Y^3\tilde X)^{\alpha}{}_a(Y'^2X)^b{}_{\dot b} + v_{54}(Y^3\tilde X\epsilon)^{\alpha b}(\epsilon Y'^2X)_{a\dot b} + v_{55}(Y^3)^{\alpha}{}_{\dot b}(\tilde XX)^b{}_a(Y'^4)^c{}_c \\
&+v_{56}(Y^3)^{\alpha}{}_{\dot b}(X\tilde X)^b{}_a(Y'^4)^c{}_c + v_{57}(Y^3X\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b}(Y'^4)^c{}_c + v_{58}(Y^3X)^{\alpha}{}_a\tilde X^b{}_{\dot b}(Y'^4)^c{}_c + v_{59}(Y^3\tilde X\epsilon)^{\alpha b}(\epsilon X)_{a\dot b}(Y'^4)^c{}_c \\
&+v_{60}(Y^3\tilde X)^{\alpha}{}_aX^b{}_{\dot b}(Y'^4)^c{}_c + v_{61}Y^{\alpha}{}_{\dot b}\delta^b_a(X'\tilde X')^{\beta}{}_{\beta} + v_{62}(X'\tilde X'Y)^{\alpha}{}_{\dot b}\delta^b_a + v_{63}(\tilde X'X'Y)^{\alpha}{}_{\dot b}\delta^b_a + v_{64}(Y^3)^{\alpha}{}_{\dot b}\delta^b_a(X'\tilde X')^{\beta}{}_{\beta} \\
&+v_{65}(X'\tilde X'Y^3)^{\alpha}{}_{\dot b}\delta^b_a + v_{66}(\tilde X'X'Y^3)^{\alpha}{}_{\dot b}\delta^b_a + v_{67}Y^{\alpha}{}_{\dot b}(Y'^2)^b{}_a(X'\tilde X')^{\beta}{}_{\beta} + v_{68}Y^{\alpha}{}_{\dot b}\delta^b_a(X'\tilde X')^{\beta}{}_{\beta}(Y'^4)^c{}_c + v_{69}(X'\tilde X'Y)^{\alpha}{}_{\dot b}\delta^b_a(Y'^4)^c{}_c \\
&+v_{70}(\tilde X'X'Y)^{\alpha}{}_{\dot b}\delta^b_a(Y'^4)^c{}_c + v_{71}(Y^3)^{\alpha}{}_{\dot b}(Y'^2)^b{}_a(X'\tilde X')^{\beta}{}_{\beta} + v_{72}(Y^3)^{\alpha}{}_{\dot b}\delta^b_a(X'\tilde X')^{\beta}{}_{\beta}(Y'^4)^c{}_c + v_{73}(X'\tilde X'Y^3)^{\alpha}{}_{\dot b}\delta^b_a(Y'^4)^c{}_c \\
&+v_{74}(\tilde X'X'Y^3)^{\alpha}{}_{\dot b}\delta^b_a(Y'^4)^c{}_c + v_{75}(\tilde X'Y'\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{76}(\tilde X'Y')^{\alpha}{}_aX^b{}_{\dot b} + v_{77}(X'Y'\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b} + v_{78}(X'Y')^{\alpha}{}_a\tilde X^b{}_{\dot b} \\
&+v_{79}(\tilde X'Y'^3\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{80}(\tilde X'Y'^3)^{\alpha}{}_aX^b{}_{\dot b} + v_{81}(X'Y'^3\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b} + v_{82}(X'Y'^3)^{\alpha}{}_a\tilde X^b{}_{\dot b} + v_{83}(\tilde X'Y'\epsilon)^{\alpha b}(\epsilon X)_{a\dot b}(Y^4)^{\beta}{}_{\beta} \\
&+v_{84}(\tilde X'Y')^{\alpha}{}_aX^b{}_{\dot b}(Y^4)^{\beta}{}_{\beta} + v_{85}(X'Y'\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b}(Y^4)^{\beta}{}_{\beta} + v_{86}(X'Y')^{\alpha}{}_a\tilde X^b{}_{\dot b}(Y^4)^{\beta}{}_{\beta} + v_{87}(\tilde X'Y'^3\epsilon)^{\alpha b}(\epsilon X)_{a\dot b}(Y^4)^{\beta}{}_{\beta} \\
&+v_{88}(\tilde X'Y'^3)^{\alpha}{}_aX^b{}_{\dot b}(Y^4)^{\beta}{}_{\beta} + v_{89}(X'Y'^3\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b}(Y^4)^{\beta}{}_{\beta} + v_{90}(X'Y'^3)^{\alpha}{}_a\tilde X^b{}_{\dot b}(Y^4)^{\beta}{}_{\beta} + v_{91}(Y^2\tilde X'Y'\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} \\
&+v_{92}(Y^2\tilde X'Y')^{\alpha}{}_aX^b{}_{\dot b} + v_{93}(\tilde X'Y'\epsilon)^{\alpha b}(\epsilon XY^2)_{a\dot b} + v_{94}(\tilde X'Y')^{\alpha}{}_a(XY^2)^b{}_{\dot b} + v_{95}(Y^2X'Y'\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b} + v_{96}(Y^2X'Y')^{\alpha}{}_a\tilde X^b{}_{\dot b} \\
&+v_{97}(X'Y'\epsilon)^{\alpha b}(\epsilon\tilde XY^2)_{a\dot b} + v_{98}(X'Y')^{\alpha}{}_a(\tilde XY^2)^b{}_{\dot b} + v_{99}(Y^2\tilde X'Y'^3\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{100}(Y^2\tilde X'Y'^3)^{\alpha}{}_aX^b{}_{\dot b} \\
&+v_{101}(\tilde X'Y'^3\epsilon)^{\alpha b}(\epsilon XY^2)_{a\dot b} + v_{102}(\tilde X'Y'^3)^{\alpha}{}_a(XY^2)^b{}_{\dot b} + v_{103}(Y^2X'Y'^3\epsilon)^{\alpha b}(\epsilon\tilde X)_{a\dot b} + v_{104}(Y^2X'Y'^3)^{\alpha}{}_a\tilde X^b{}_{\dot b} \\
&+v_{105}(X'Y'^3\epsilon)^{\alpha b}(\epsilon\tilde XY^2)_{a\dot b} + v_{106}(X'Y'^3)^{\alpha}{}_a(\tilde XY^2)^b{}_{\dot b}
\end{aligned}$$

up to dim 1,
grd=0 (106項)

$\boldsymbol{[q^{\alpha}{}_{a},\widetilde{X}'^{\beta}{}_{\dot\beta}]}$ も同様に106項

※consistencyから係数に関係がつく

Linearized version

- Y,Y'について線形な場合： $q^{\alpha}{}_a|V\rangle = \frac{i}{2}[(YX)^{\alpha}{}_a + (X'Y')^{\alpha}{}_a]|V\rangle$

$$[q^{\alpha}{}_a, \widetilde{X}^b{}_{\dot{b}}] = \frac{i}{2}\delta^b_a Y^{\alpha}{}_{\dot{b}}, \quad [q^{\alpha}{}_a, \widetilde{X}'^{\beta}{}_{\dot{\beta}}] = \frac{i}{2}\epsilon^{\beta\alpha}(\epsilon Y')_{a\dot{\beta}}$$

- このときsu(2|2)代数を満たすもの：

$$\begin{aligned}
|Q^{\alpha}{}_a\rangle &= \frac{1}{2}q_1\Big\{\eta^*\big[(\sinh\overline{Y})\widetilde{X}(\cosh\overline{Y}')\big]^{\alpha}{}_a + \eta\big[(\cosh\overline{Y})\widetilde{X}'(\sinh\overline{Y}')\big]^{\alpha}{}_a\Big\}|V\rangle \\
&\quad + q^{\alpha}{}_a|W\rangle \\
|H\rangle &= \frac{1}{2}q_1\Big\{\frac{1}{12}\big[\mathrm{Tr}Y^4 - \mathrm{Tr}Y'^4\big] \\
&\quad + \mathrm{Tr}X\cosh\overline{Y}\widetilde{X}\cosh\overline{Y}' - i\mathrm{Tr}\sinh\overline{Y}\widetilde{X}\sinh\overline{Y}'X' \\
&\quad + \mathrm{Tr}\cosh\overline{Y}\widetilde{X}'\cosh\overline{Y}'X' + i\mathrm{Tr}X\sinh\overline{Y}\widetilde{X}'\sinh\overline{Y}'\Big\}|V\rangle \\
&\quad + h|W\rangle
\end{aligned}$$

$$\overline{Y} = Y\eta, \ \overline{Y}' = Y'\eta^*, \quad (\eta \equiv e^{i\pi/4})$$

pp-waveの場合のLCSFTの３弦相互作用項

[Pankiewicz(2003)] + [Di Vecchia-Petersen-Petrini-Russo-Tanzini(2003)]
を再現している！

Toy model Ⅰ

仮定： $q^{\alpha}{}_a|V\rangle = \frac{i}{2}[(YX)^{\alpha}{}_a + (X'Y')^{\alpha}{}_a]|V\rangle + (v_{30}Y^3X + v_{34}Y^3XY'^4 + v'_{03}X'Y'^3 - v_{34}Y^4X'Y'^3)^{\alpha}{}_a|V\rangle$

$[q^{\alpha}{}_a, \widetilde{X}^b{}_{\dot{b}}] = \frac{i}{2}\delta^b_a Y^{\alpha}{}_{\dot{b}}, \quad [q^{\alpha}{}_a, \widetilde{X}'^{\beta}{}_{\dot{\beta}}] = \frac{i}{2}\epsilon^{\beta\alpha}(\epsilon Y')_{a\dot{\beta}}$ ⟵ pp-waveのときと同じ

このときsu(2|2)代数を満たすもの：

$$|Q^{\alpha}{}_a\rangle = \frac{1}{2}q_1\Big\{\eta^*\big[(\sinh\overline{Y})\widetilde{X}\big(\cosh\overline{Y}' - (v'_{03}/2)Y'^4\big)\big]^{\alpha}{}_a + \eta\big[\big(\cosh\overline{Y} + (v_{30}/2)Y^4\big)\widetilde{X}'\sinh\overline{Y}'\big]^{\alpha}{}_a\Big\}|V\rangle$$
$$+q^{\alpha}{}_a|W\rangle$$

$$|H\rangle = \frac{1}{2}q_1\Big\{\frac{1}{12}\big[\mathrm{Tr}\,Y^4(1-(v'_{03}/4)\mathrm{Tr}\,Y'^4) - \mathrm{Tr}\,Y'^4(1+(v_{30}/4)\mathrm{Tr}\,Y^4)\big]$$
$$+\mathrm{Tr}\,X(\cosh\overline{Y} - (v_{30}/2)Y^4)\widetilde{X}(\cosh\overline{Y}' - (v'_{03}/2)Y'^4) - i\mathrm{Tr}\,\sinh\overline{Y}\widetilde{X}\sinh\overline{Y}'X'$$
$$+\mathrm{Tr}\,(\cosh\overline{Y} + (v_{30}/2)Y^4)\widetilde{X}'(\cosh\overline{Y}' + (v'_{03}/2)Y'^4)X' + i\mathrm{Tr}\,X\sinh\overline{Y}\widetilde{X}'\sinh\overline{Y}'$$
$$-(v_{34}/4)(\mathrm{Tr}XY^4\widetilde{X}Y'^4 + \mathrm{Tr}\,Y^4\widetilde{X}'Y'^4X')\Big\}|V\rangle$$
$$+h|W\rangle$$

Toy model II

仮定： $q^{\alpha}{}_{a}|V\rangle = \frac{i}{2}[(YX)^{\alpha}{}_{a} + (X'Y')^{\alpha}{}_{a}]|V\rangle$ ⟵ pp-waveのときと同じ

$$[q^{\alpha}{}_{a}, \widetilde{X}^{\dot{b}}{}_{b}] = -\frac{i}{2}(Y\epsilon)^{\dot{b}\alpha}\epsilon_{ab} + y_{03}\big[(Y^{3}\epsilon)^{\dot{b}\alpha}\epsilon_{ab} + 2i(Y\epsilon)^{\dot{b}\alpha}(\epsilon X\widetilde{X})_{ab} + 4i(YX)^{\alpha}{}_{a}\widetilde{X}^{\dot{b}}{}_{b}\big],$$

$$[q^{\alpha}{}_{a}, \widetilde{X}'^{\beta}{}_{\dot{\beta}}] = \frac{i}{2}\epsilon^{\beta\alpha}(\epsilon Y')_{a\dot{\beta}} - y'_{30}\big[\epsilon^{\beta\alpha}(\epsilon Y'^{3})_{a\dot{\beta}} + 2i(\widetilde{X}'X'\epsilon)^{\beta\alpha}(\epsilon Y')_{a\dot{\beta}} + 4i(X'Y')^{\alpha}{}_{a}\widetilde{X}'^{\beta}{}_{\dot{\beta}}\big]$$

up to dim 1でconsistentに変形

このときup to dim 1でsu(2|2)代数を満たすものを求めた。

pp-waveからのハミルトニアンのズレ：

$$\frac{1}{2}q_{1}\Big[y_{03}\mathrm{Tr}Y^{4}\Big(1 - \frac{1}{48}\mathrm{Tr}Y'^{4}\Big) + y'_{30}\mathrm{Tr}Y'^{4}\Big(1 - \frac{1}{48}\mathrm{Tr}Y^{4}\Big)\Big]|V\rangle.$$

超電荷はpp-waveのときと同じ。

Summary and discussion

- su(2|2)対称性をもつ背景でのLCSFTの構成に向けた模型を提案。
- 従来知られているpp-waveの場合の形を簡潔に再現した。
- “SUGRA極限”： $\widetilde{X}^{a}{}_{\dot{a}} = X^{a}{}_{\dot{a}},\ \widetilde{X}'^{\alpha}{}_{\dot{\alpha}} = X'^{\alpha}{}_{\dot{\alpha}} \qquad W^{\alpha}{}_{\dot{b}} = \widetilde{W}^{\alpha}{}_{\dot{b}} = W'^{a}{}_{\dot{\beta}} = \widetilde{W}'^{a}{}_{\dot{\beta}} = 0$
- LCSFTの相互作用項をdim.に関する展開で系統的に構成できる。
- pp-waveから少し一般化した具体例(toy model I,II)を計算した。
- pp-waveより一般の背景の場合の正当化？それに対応する背景？(bubbling geometry?) 高次の相互作用項は？
- AdS/CFT (BMN) 対応の一般化への応用？