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On Numerical Solutions in Open String Field Theory

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Based on collaboration with
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Refs.
I.K.-T.Takahashi, Theor.Math.Phys.163:710-716,2010;
Theor.Math.Phys.163:717-724,2010

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Non-perturbative vacuum in open bosonic string field theory (1)

- Schnabl's solution $\mathbf{\Psi_{Sch}}$

Gauge invariants

(1) Action: D-brane tension

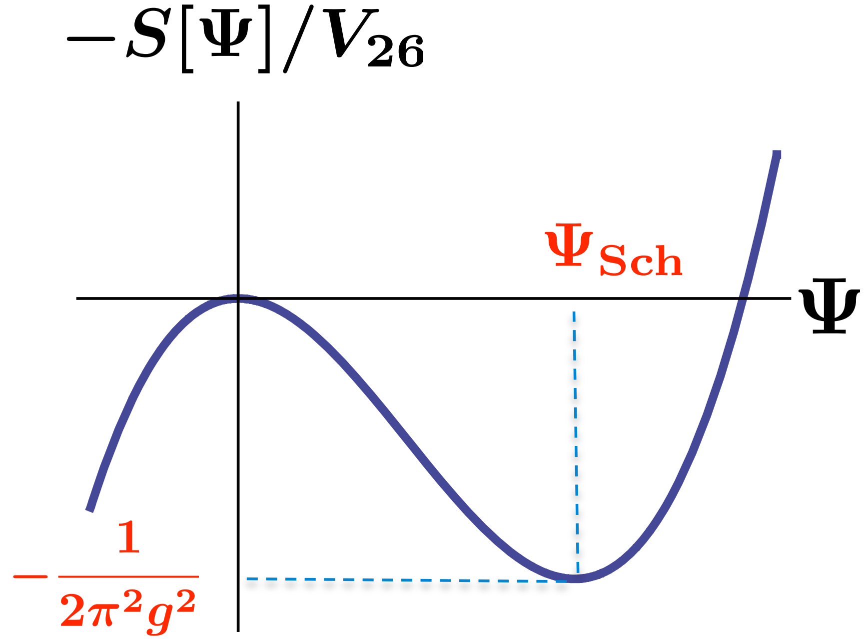
$$S[\Psi_{\rm Sch}]/V_{26} = \frac{1}{2\pi^2 g^2}$$

[Schnabl(2005),Okawa,Fuchs-Kroyter(2006)]

(2) Gauge invariant overlap:

$$\mathcal{O}_V(\Psi_{\rm Sch}) = \langle \hat{\gamma}(1_c, 2)|\phi_V\rangle_{1_c}|\Psi_{\rm Sch}\rangle_2 = \frac{1}{2\pi}\langle B|c_0^-|\phi_V\rangle$$

[Ellwood, Kawano-Kishimoto-Takahashi(2008)]



Non-perturbative vacuum in open bosonic string field theory (2)

$$Q' \equiv Q_{\mathrm{B}} + [\Psi_{\mathrm{Sch}},\ \cdot\ \}_*$$

no cohomology (in all ghost number sectors) [Ellwood-Schnabl(2006)]

On the other hand,…

in 2002, Takahashi and Tanimoto constructed a class of analytic solutions ("identity based solutions") and conjectured that it represents a non-perturbative vacuum for a particular value of the parameter. $(a=-1/2)$

Actually, around the solution $\Psi_{l,a=-1/2}$ there is no cohomology in the ghost number 1 sector. [Kishimoto-Takahashi (2002)]

We have obtained further quantitative evidences. [Kishimoto-Takahashi (2009)]

↑

Numerical solutions in Siegel gauge

NUMERICAL SOLUTION IN SIEGEL GAUGE

- Sen-Zwiebach(1999), Moeller-Taylor(2000), Gaiotto-Rastelli(2002)

  Level (4,8) Level (10,20) Level (18,54)

  constructed a numerical solution to $Q\Psi + \Psi * \Psi = 0$ $b_0\Psi = 0$

  by level truncation approximation.

  Note: any explicit form of the exact solution in Siegel gauge is not yet known.

On the other hand, solutions to $Q'\Phi + \Phi * \Phi = 0$ $b_0\Phi = 0$

- Takahashi(2003) in the theory around $\Psi_{l=1,2,a>-1/2}$

  Level (6,18)

- Zeze(2003), Drukker-Okawa(2005)

  Level 6

  ⇨ Kishimoto-Takahashi (in SFT2009 proceedings)

  up to Level (20,60)

  in the theory around $\Psi_{l=1,a=-1/2}$

SUMMARY OF NUMERICAL RESULTS (1)

- Numerical solution to $Q\Psi + \Psi * \Psi = 0$ in Siegel gauge

Level	Potential height	Gauge inv. overlap
(0,0)	-0.6846162	0.7165627
(2,6)	-0.9593766	0.8898618
(4,12)	-0.9878218	0.9319524
(6,18)	-0.9951771	0.9510789
(8,24)	-0.9979301	0.9611748
(10,30)	-0.9991825	0.9681148
(12,24)	-0.9998223	0.9725595
(14,42)	-1.0001737	0.9761715
(16,48)	-1.0003755	0.9786768
(18,54)	-1.0004937	0.9809045
(20,60)	-1.0005630	0.9825168
(22,66)	-1.0006023	0.9840334
(24,72)	-1.0006227	0.9851603
(26,78)	-1.0006312	0.9862619

Normalization

Potential height:

$$V(\Psi_{\rm Sch}) = -\frac{2\pi^2 g^2 S[\Psi_{\rm Sch}]}{V_{26}} = -1$$

Gauge inv. overlap:

$$\mathcal{O}_V(\Psi_{\rm Sch}) = 1$$

Summary of numerical results (2)

- Numerical solution to $Q'\Phi + \Phi * \Phi = 0$ in Siegel gauge

Level	Potential height	Gauge inv. overlap
(0,0)	2.3105795	-1.0748441
(2,6)	2.5641847	-1.0156983
(4,12)	1.6550774	-0.9539832
(6,18)	1.6727496	-0.9207572
(8,24)	1.4193393	-0.9377548
(10,30)	1.4168893	-0.9110994
(12,24)	1.3035715	-0.9237917
(14,42)	1.2986472	-0.9056729
(16,48)	1.2357748	-0.9229035
(18,54)	1.2310583	-0.9086563
(20,60)	1.1915648	-0.9212376
(22,66)	1.1874828	-0.9103838
(24,72)	1.1605884	-0.9231608
(26,78)	1.1571287	-0.9142181

in the theory around

$\Psi_{l=1,a=-1/2}$

Contents

- Introduction and summary ✔
- A brief review of TT-solution
- Level truncation in Siegel gauge
- Evaluation of gauge invariants
- Concluding remarks

Takahashi-Tanimoto (TT) solution

- TT-solution is a type of identity based solution.

$$\Psi_{l,a} = Q_L(e^{h_a^l} - 1)\mathcal{I} - C_L((\partial h_a^l)^2 e^{h_a^l})\mathcal{I}$$

$\mathcal{I}$: identity state

$$Q_L(f) \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i} f(z) j_{\rm B}(z) \qquad C_L(f) \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i} f(z) c(z)$$

$$\begin{aligned} h_a^l(z) &= \log\left(1 - \frac{a}{2}(-1)^l \left(z^l - (-1)^l \frac{1}{z^l}\right)^2\right) \\ &= -\log((1 - Z(a))^2) - \sum_{n=1}^{\infty} \frac{(-1)^{ln}}{n} Z(a)^n (z^{2ln} + z^{-2ln}) \end{aligned}$$

$$l = 1, 2, 3, \cdots \qquad Z(a) = \frac{1 + a - \sqrt{1 + 2a}}{a} \qquad a \geq -1/2$$

ACTION AROUND THE TT-SOLUTION

- TT-solution satisfies EOM: $Q\Psi_{l,a} + \Psi_{l,a} * \Psi_{l,a} = 0$

- Around it, the action can be re-expanded as

Direct evaluation is difficult for identity-based sols.

$$S_{l,a}[\Phi] \equiv S[\Psi_{l,a} + \Phi] - S[\Psi_{l,a}]$$
$$= -\frac{1}{g^2}\left(\frac{1}{2}\langle\Phi, Q'\Phi\rangle + \frac{1}{3}\langle\Phi, \Phi * \Phi\rangle\right)$$

BRST operator around the TT solution:

$$Q' = Q + [\Psi_{l,a}, \cdot\}_* = Q(e^{h^l_a}) - C((\partial h^l_a)^2 e^{h^l_a})$$

$$Q(f) \equiv \oint \frac{dz}{2\pi i} f(z) j_{\mathrm{B}}(z), \quad C(f) \equiv \oint \frac{dz}{2\pi i} f(z) c(z)$$

Equation of motion in Siegel gauge

- Let us consider EOM of SFT around the TT solution:

$$Q'\Phi + \Phi * \Phi = 0$$

Note: $Q'|_{a=0} = Q$

- Although this can be solved by $\Phi = -\Psi_{l,a}$
  we construct numerical solution in Siegel gauge *in order to evaluate gauge invariants*.
- EOM in the Siegel gauge: $b_0\Phi = 0$

$$L_{l,a}\Phi + b_0(\Phi * \Phi) = 0$$

$$L_{l,a} = \{b_0, Q'\}$$

$$= (1+a)(L'_0 - 1) - \frac{(-1)^l}{2}a(L'_{2l} + L'_{-2l}) + 4l^2 a Z(a)$$

$$L'_n = L^{\text{mat}}_n + L^{\text{gh}\prime}_n$$

$$L^{\text{gh}\prime}_n \equiv L^{\text{gh}}_n + n q_n + \delta_{n,0} \quad \longleftarrow \quad j_{\text{gh}}(z) = cb(z) = \sum_n q_n z^{-n-1}$$

Constructing solution

- Iterative approach (Newton's method)

$$L_{l,a}\Phi^{(n)} + b_0(\Phi^{(n)} * \Phi^{(n)})$$
$$+L_{l,a}(\Phi^{(n+1)} - \Phi^{(n)}) + b_0(\Phi^{(n)} * (\Phi^{(n+1)} - \Phi^{(n)}) + (\Phi^{(n+1)} - \Phi^{(n)}) * \Phi^{(n)}) = 0$$

$$\downarrow$$

$$L_{l,a}\Phi^{(n+1)} + b_0(\Phi^{(n)} * \Phi^{(n+1)} + \Phi^{(n+1)} * \Phi^{(n)}) = b_0(\Phi^{(n)} * \Phi^{(n)})$$

Linear equation with respect to $\Phi^{(n+1)}$

$n \longrightarrow \infty$

$$\Rightarrow \quad L_{l,a}\Phi^{(\infty)} + b_0(\Phi^{(\infty)} * \Phi^{(\infty)}) = 0$$

if iteration converges

We should choose an appropriate initial configuration $\Phi^{(0)}$

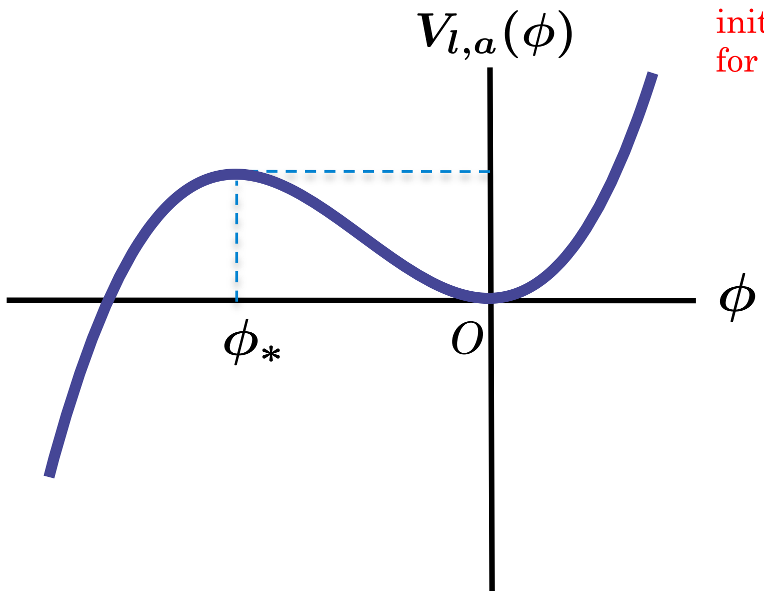
Lowest level analysis

- Ansatz: $\Phi_{L=0} = \phi\, c_1 |0\rangle$

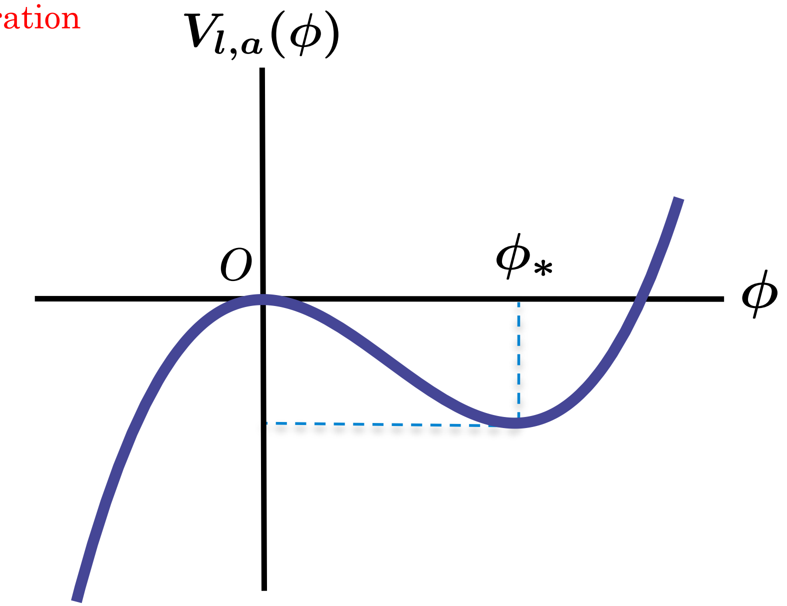
$$
\begin{aligned}
V_{l,a}(\phi) &= -2\pi^2 g^2 S_{l,a}[\Phi_{L=0}]/V_{26} \\
&= 2\pi^2 \left(\frac{1}{2}\left((4l^2-1)(a+1) - 4l^2\sqrt{1+2a}\right)\phi^2 + \frac{27\sqrt{3}}{64}\phi^3 \right)
\end{aligned}
$$

$$
\Rightarrow \quad \phi_* = \frac{-64}{81\sqrt{3}}\left((4l^2-1)(a+1) - 4l^2\sqrt{1+2a}\right)
$$

initial configuration for a= −1/2, 0



unstable solution $(a \gtrsim -1/2)$



stable solution $(a \sim 0)$

TRUNCATION OF STRING FIELD

- Level (L,3L) truncation $L = L_0 + 1$

  Φ : up to level L

  $$|A * B\rangle = \sum_i |\phi^i\rangle \langle V_3(1,2,3)|\phi_i\rangle_1 |A\rangle_2 |B\rangle_3$$

  up to total level 3L

- Further consistent truncation

  $(-1)^{L_0+1}\Phi = \Phi$: twist even

  $\Phi \sim L^{\mathrm{mat}}_{-n_1} L^{\mathrm{mat}}_{-n_2} \cdots L^{\mathrm{gh}\prime}_{-n_1} L^{\mathrm{gh}\prime}_{-n_2} \cdots c_1|0\rangle$: universal and su(1,1) singlet

  $$\mathcal{G}\Phi = X\Phi = Y\Phi = 0$$

  $$\mathcal{G} = \sum_{n=1}^{\infty}(c_{-n}b_n - b_{-n}c_n) \qquad X = -\sum_{n=1}^{\infty} n c_{-n} c_n \qquad Y = \sum_{n=1}^{\infty} \frac{1}{n} b_{-n} b_n$$

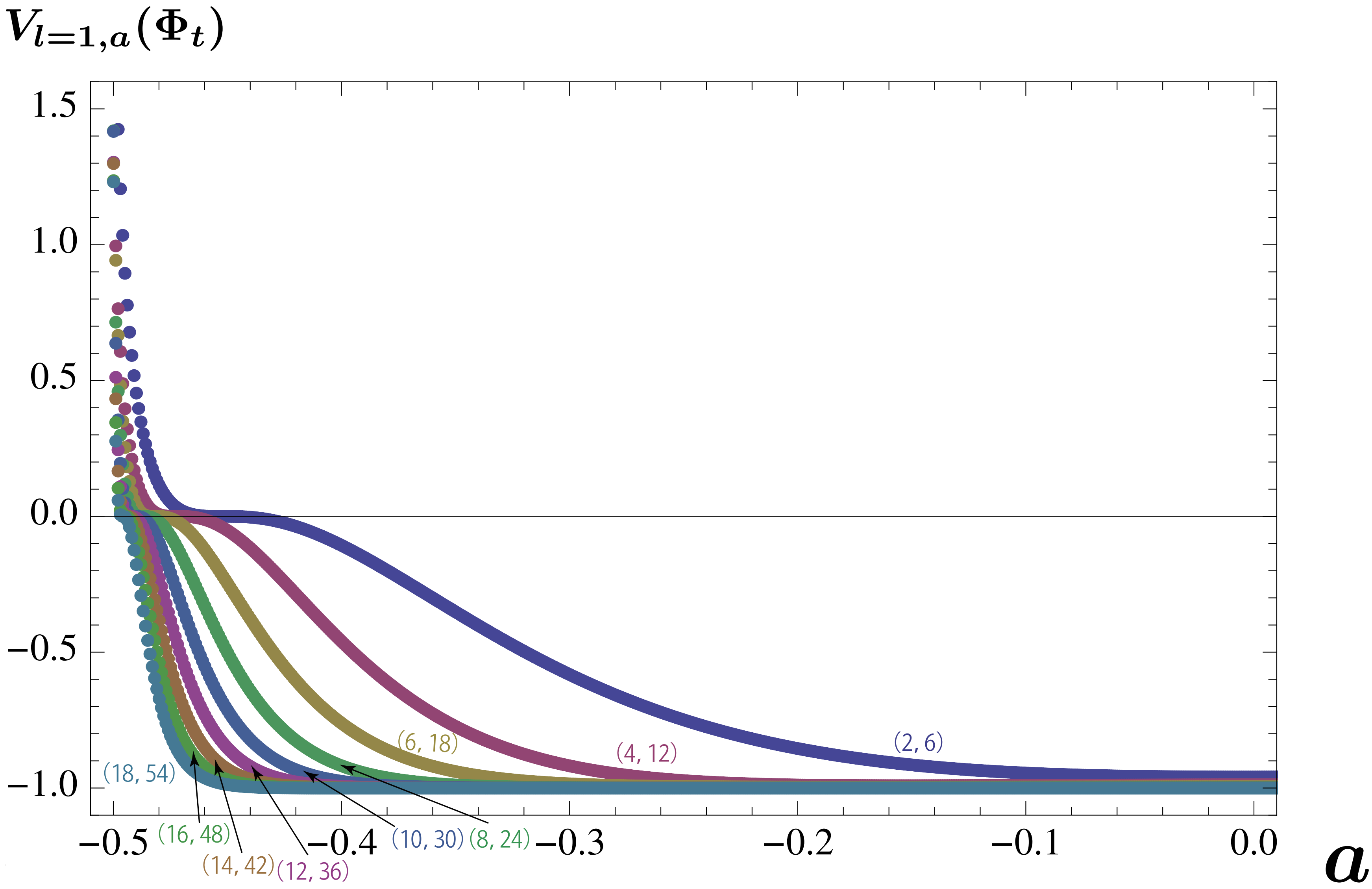
COMPUTERS FOR LEVEL (26,78)

PC cluster @ Nara Women's Univ.

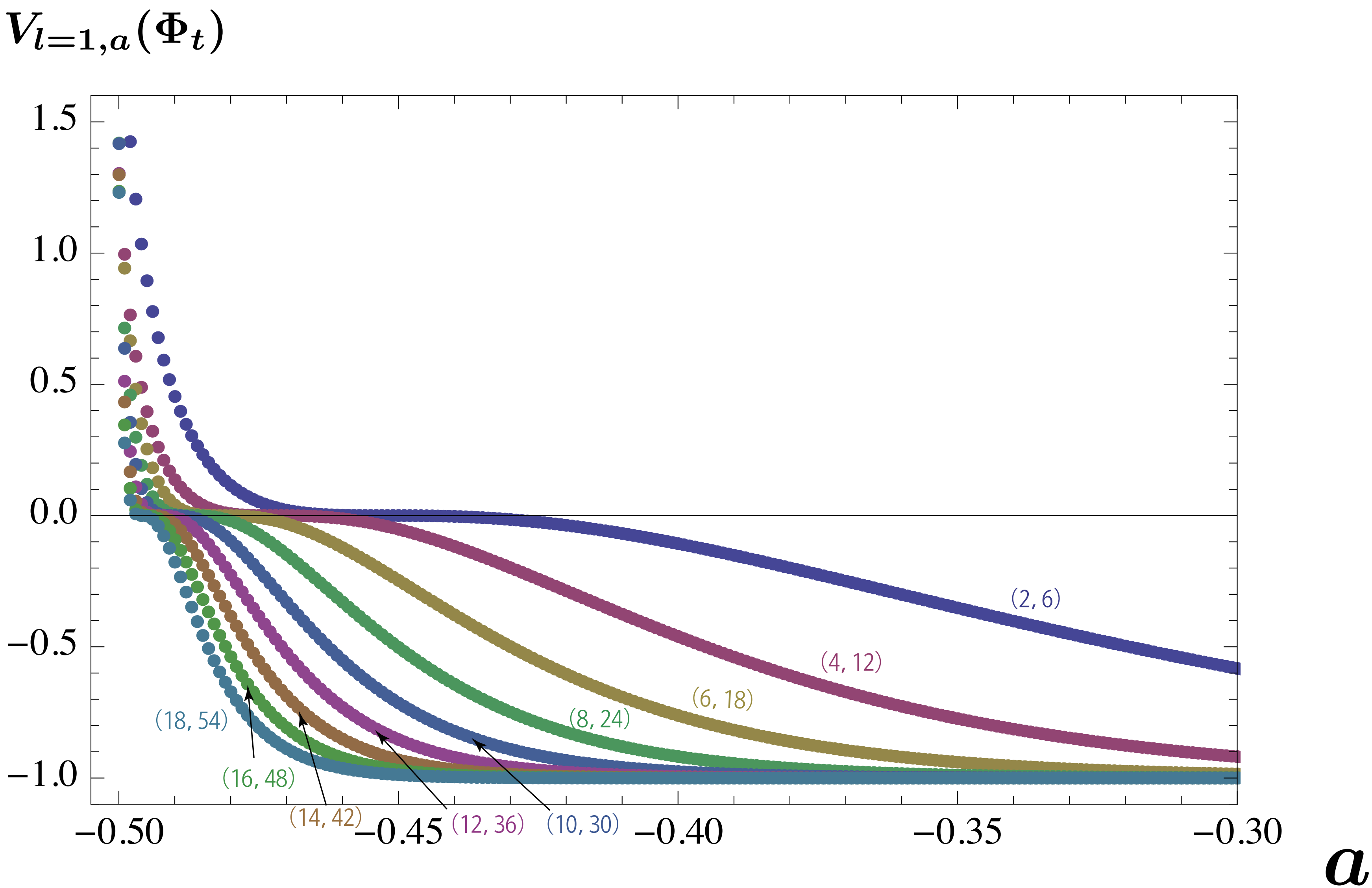
CPU: Intel Xeon($8\times3+12+8$cores,…)
+ Intel compiler (C++, fortran)
Memory: 64GB×3+48GB+120GB,…

RIKEN Integrated Cluster of Clusters (RICC)

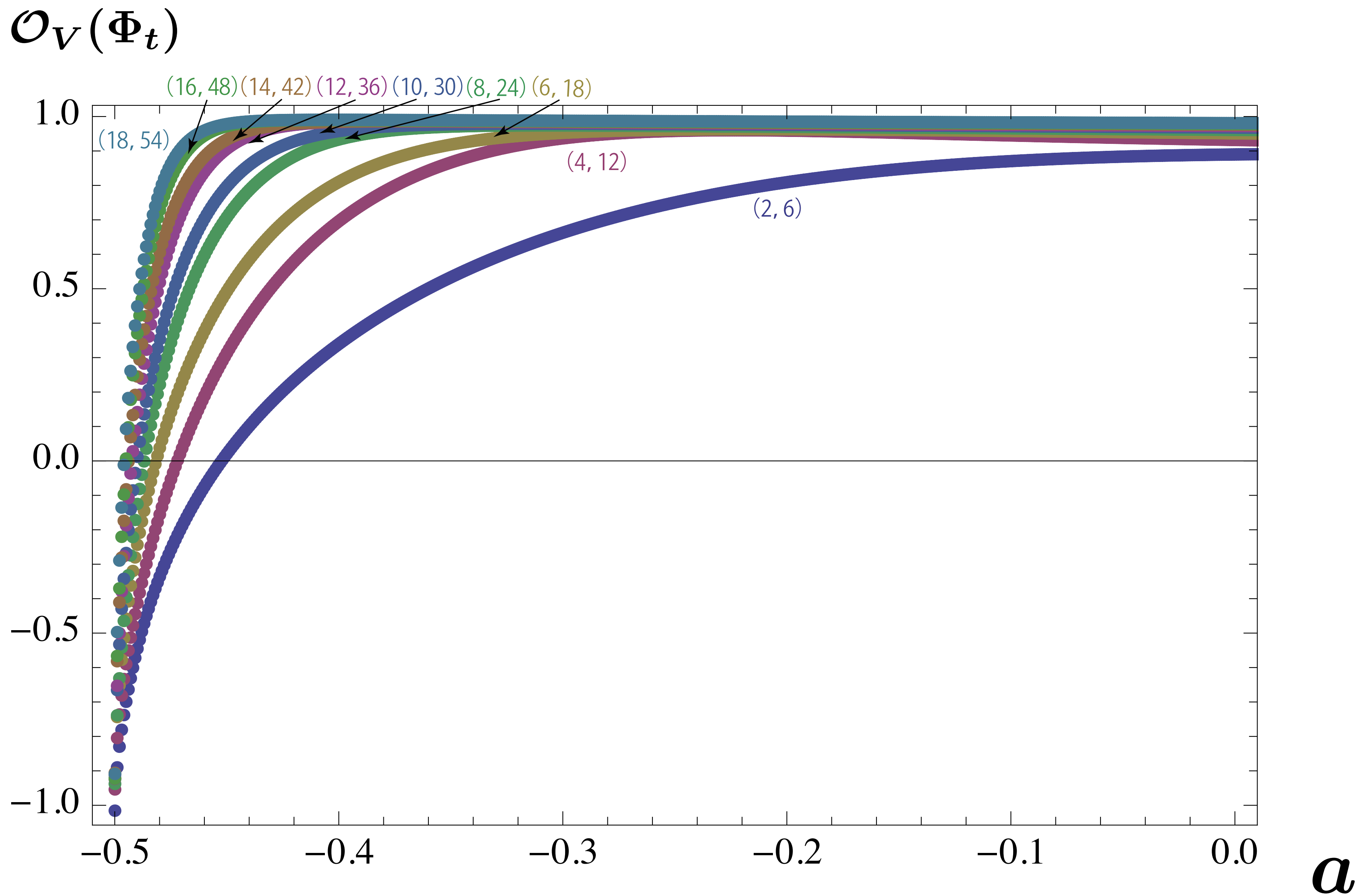
POTENTIAL HEIGHT AT Φ_t IN SFT AROUND $\Psi_{l=1,a}$



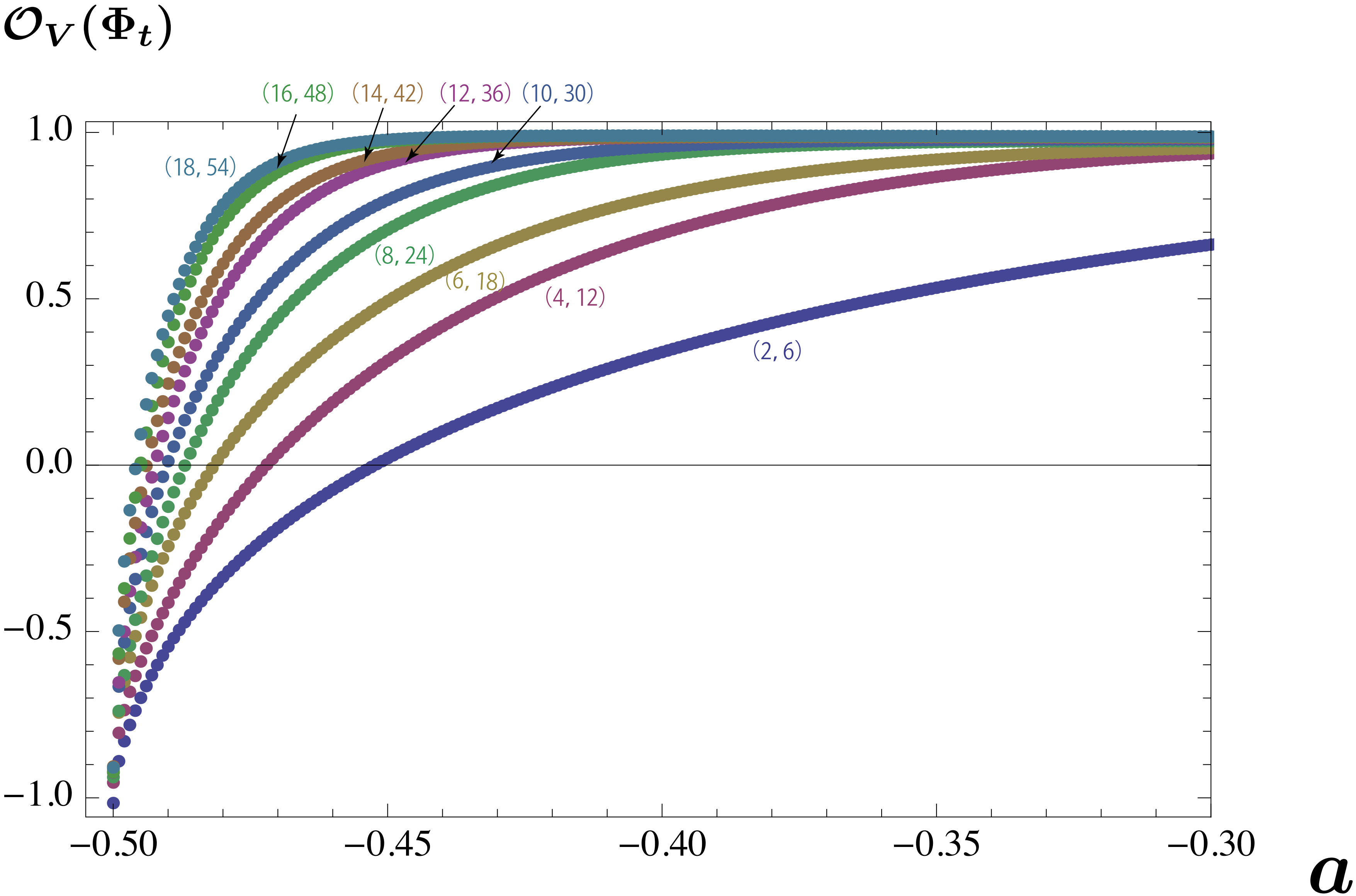
Potential height at Φ_t in SFT around $\Psi_{l=1,a}$



GAUGE INV OVERLAP AT Φ_t IN SFT AROUND $\Psi_{l=1,a}$



Gauge inv overlap at Φ_t in SFT around $\Psi_{l=1,a}$



Comments on solution for $a = 0$ $(Q' = Q)$

- Straightforward extrapolation of potential height

fitting function: $F_N(L) = \sum_{n=0}^{N} \frac{a_n}{(L+1)^n}$ [Gaiotto-Rastelli (2002)]

Using data for L=0,2,4,6,8,10,12,14,16 and N=9, we have

$$F_{N=9}(L=18) = -1.0004937$$

$$F_{N=9}(L=20) = -1.0005630$$

$$F_{N=9}(L=22) = -1.0006023$$

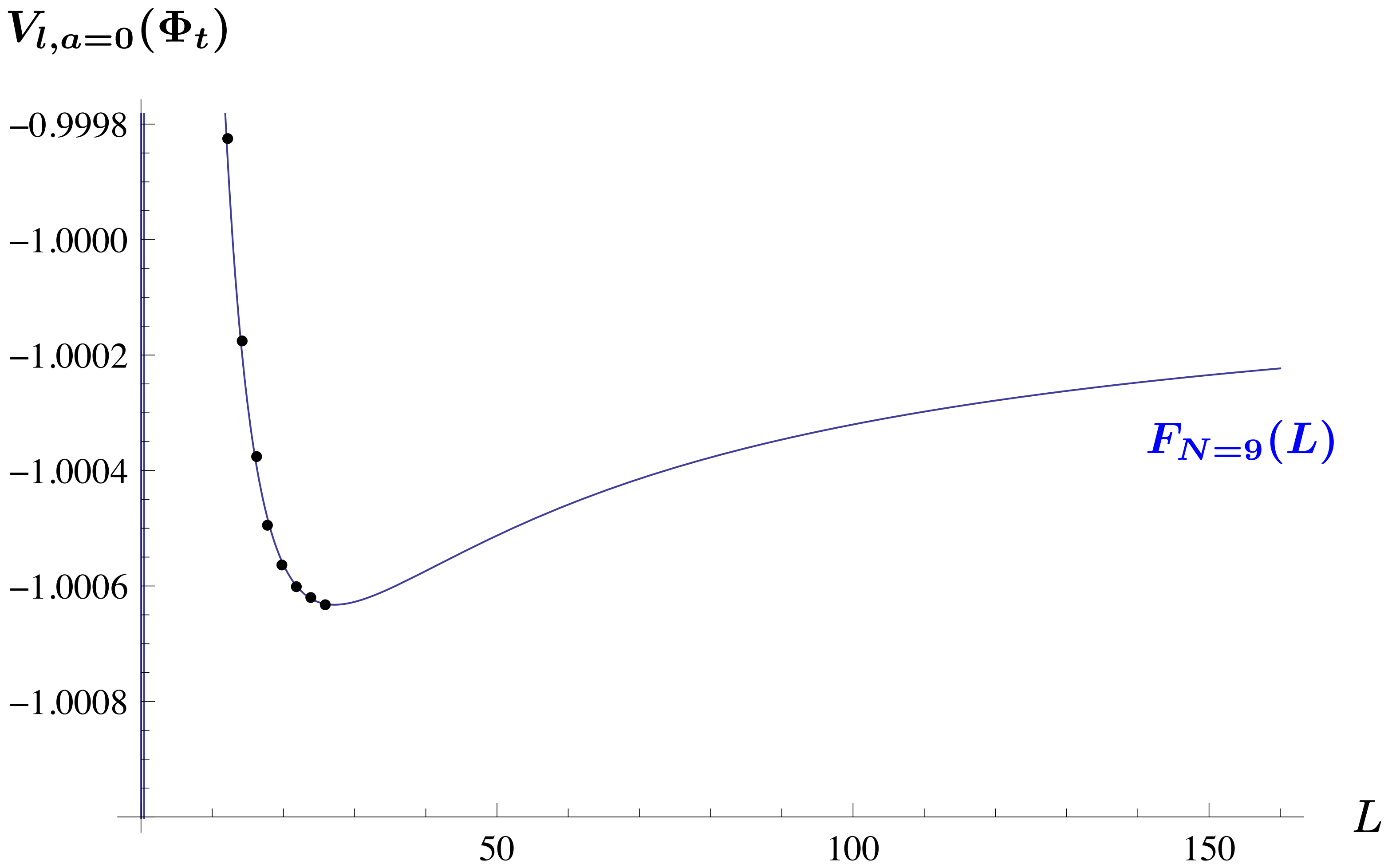
Good coincidence with our direct computation!

$$F_{N=9}(L=24) = -1.0006229 \quad \leftarrow \quad -1.0006227$$

$$F_{N=9}(L=26) = -1.0006313 \quad \leftarrow \quad -1.0006312$$

$$F_{N=9}(L=\infty) = -1.0000293$$

EXTRAPOLATION OF POTENTIAL HEIGHT AT $a = 0$ $(Q' = Q)$



EXTRAPOLATION OF POTENTIAL HEIGHT FOR $l=1, a=-\frac{1}{2}$

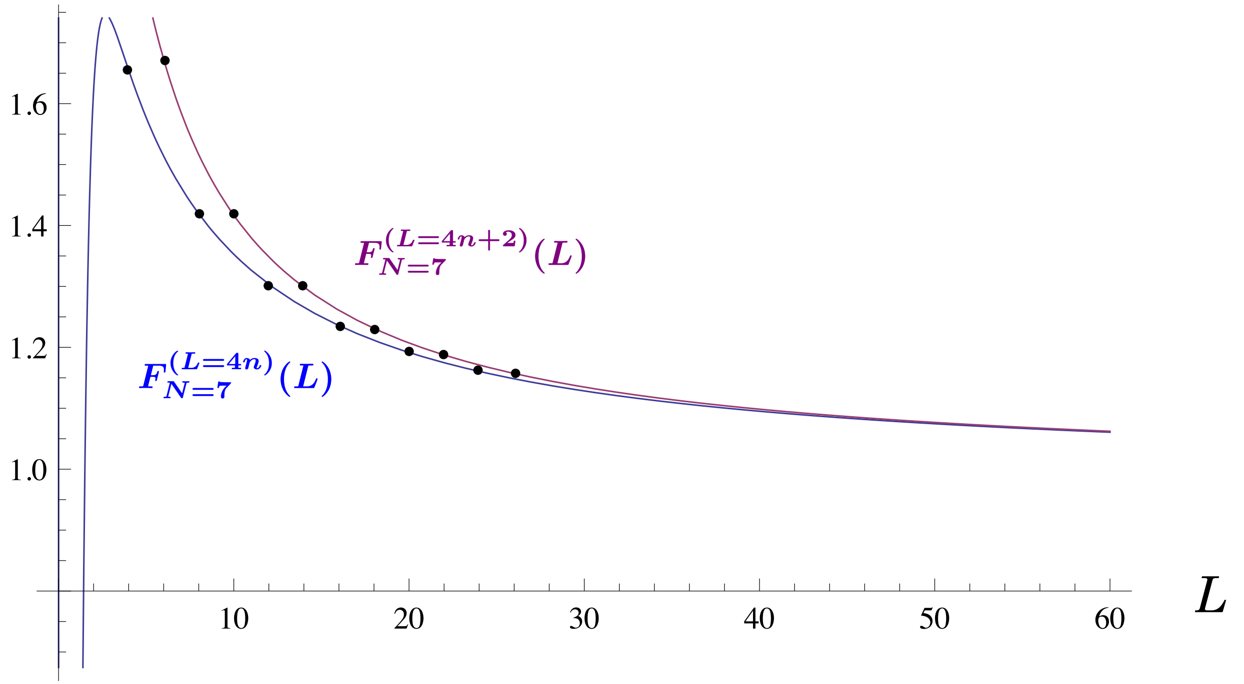
L	Extrapolation of $V_{l=1,a=-1/2}(\Phi_t)$	
4∞	**0.9893181**	← $(L=0,4,8,12,16,20,24;N=7)$
$4\infty+2$	**0.9909238**	← $(L=2,6,10,14,18,22,26;N=7)$

99% of +1 (!?)

fitting function:

$$F_N(L) = \sum_{n=0}^{N} \frac{a_n}{(L+1)^n}$$

$V_{l=1,a=-1/2}(\Phi_t)$

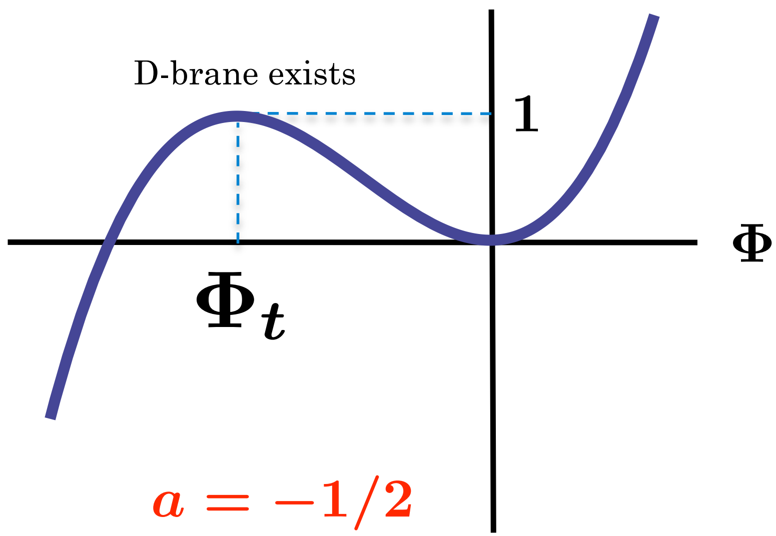


Confirmed vacuum structure of SFT around $\Psi_{l=1,a}$

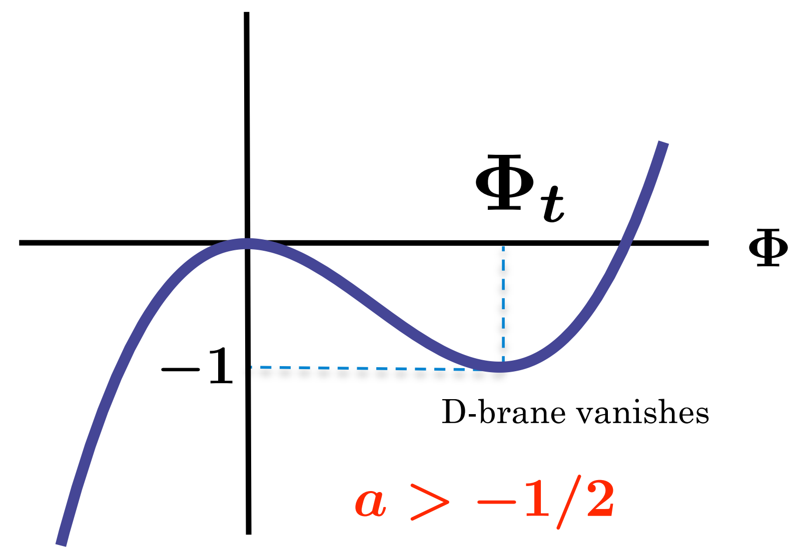
Numerical results for Φ_t s.t. $Q'\Phi_t + \Phi_t * \Phi_t = 0$ are consistent with

[Takahashi-Tanimoto(2002), Kishimoto-Takahashi (2002, 2009), Takahashi(2003)].

$V_{l=1,a}(\Phi)$



$V_{l=1,a}(\Phi)$



$\Psi_{l=1,a=-1/2}$: tachyon vacuum

$\Psi_{l=1,a>-1/2}$: pure gauge

CONCLUDING REMARKS

- We have constructed numerical solution in Siegel gauge (up to level (26,78)) in the theory around $\Psi_{l=1,a\geq -1/2}$
  and evaluated gauge invariants (action and gauge inv overlap).
- The results are consistent with the expectation that:

  $\Psi_{l=1,a>-1/2}$:pure gauge

  $\Psi_{l=1,a=-1/2}$:tachyon vacuum
- We have checked quadratic identities for obtained solutions

  $$R_m^{\rm mat} \to +1 \qquad R_m^{\rm gh} \to +1 \qquad (l=1; a=0,-1/2; \quad L\to\infty)$$
- How to extrapolate the value of gauge invariant overlap?
- What about the cases of $l=2,3,\cdots$

  We have performed similar computation for $l=2,3$ up to level (24,72).
  Behavior of values of gauge invariants for $a=-1/2$ is unclear.
- Exact solution in Siegel gauge?