超弦の場の理論における単位弦場に基づく解とホモトピー演算子

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2012年3月25日 [25pKA-10]

日本物理学会第67回年次大会@関西学院大学

Introduction

- 前回の学会講演では：

  S.Inatomi, I.Kishimoto, T.Takahashi, PTP126(2011)1077[arXiv:1106.5314]

  - タキオン凝縮を表すidentity-based解（TT解）周りの理論のBRST演算子 Q' に対するhomotopy演算子 $\hat{A}$ を構成した:

$$\{Q', \hat{A}\} = 1$$

- 今回はこれ（つまりTT解および対応するhomotopy演算子）を超弦の場の理論の場合に拡張する。

  S.Inatomi, I.Kishimoto, T.Takahashi, JHEP1110(2011)114[arXiv:1109.2406]

Comment

- ここで考える「解」は、（bosonicの場合もsuperの場合も）近年、弦の場の理論の業界でよく用いられるようになったいわゆる「KBc代数」（とその超弦への拡張）[Schnabl(2005), Okawa(2006), Erler(2006,2007),...]を用いた解とは違う。

- KBc代数を用いた解に対しては対応するhomotopy演算子は構成されている。[Ellwood-Schnabl(2006), Erler(2007),...]

OPE and an identity-based solution in bosonic SFT

- The following OPEs were essential to prove the equation of motion $Q_{\rm B}\Psi_h + \Psi_h * \Psi_h = 0$ for the TT-solution:

$$\Psi_h = Q_L(e^h - 1)I - C_L((\partial h)^2 e^h)I$$

$$j_{\rm B}(y)j_{\rm B}(z) \sim \frac{-4}{(y-z)^3}c\partial^3 c(z) + \frac{-2}{(y-z)^2}c\partial^2 c(z)$$

$$j_{\rm B}(y)c(z) \sim \frac{1}{y-z}c\partial c(z)$$

$j_{\rm B}, c$ form a closed algebra.

- The identity state I is an identity element of the star product.

OPE in RNS superstring

- BRST current and c-ghost and...

$$j_{\mathrm{B}} = cT^{\mathrm{m}} + \gamma G^{\mathrm{m}} + bc\partial c + \frac{1}{4}c\partial\beta\gamma - \frac{3}{4}c\beta\partial\gamma + \frac{3}{4}\partial c\beta\gamma - b\gamma^2 + \frac{3}{4}\partial^2 c$$

:primary, dim. 1, s.t., $\{Q_{\mathrm{B}}, b(z)\} = T(z)$

$\theta \equiv c\beta\gamma - \partial c$:primary, dim. 0

$$j_{\mathrm{B}}(y)\, j_{\mathrm{B}}(z) \sim \frac{1}{(y-z)^3}\left(-\frac{17}{8}c\partial c(z) + 3\gamma^2(z)\right) + \frac{1}{(y-z)^2}\frac{1}{2}\partial\left(-\frac{17}{8}c\partial c(z) + 3\gamma^2(z)\right) + \frac{1}{y-z}\partial\left(\frac{1}{4}c\gamma G^{\mathrm{m}}(z) + \frac{1}{2}bc\gamma^2(z) + \frac{1}{4}\beta\gamma^3(z)\right)$$

$$j_{\mathrm{B}}(y)\, \theta(z) \sim \frac{1}{(y-z)^2}\left(\frac{1}{4}c\partial c(z) - \gamma^2(z)\right) + \frac{1}{y-z}\left(-c\gamma G^{\mathrm{m}}(z) - 2bc\gamma^2(z) - \beta\gamma^3(z)\right)$$

$$j_{\mathrm{B}}(y)\, c(z) \sim \frac{1}{y-z}(c\partial c(z) - \gamma^2(z)) \qquad \theta(y)\, \theta(z) \sim \frac{1}{y-z}c\partial c(z)$$

Ansatz for an identity-based solution in super SFT

- A super extension of the TT-solution (ghost#=1, picture#=0):

$$A_c = Q_L(f)I + C_L(g)I + \Theta_L(h)I$$

$$Q_L(f) = \int_{C_{\mathrm{L}}} \frac{dz}{2\pi i} f(z) j_{\mathrm{B}}(z), \quad C_L(g) = \int_{C_{\mathrm{L}}} \frac{dz}{2\pi i} g(z) c(z), \quad \Theta_L(h) = \int_{C_{\mathrm{L}}} \frac{dz}{2\pi i} h(z) \theta(z)$$

$$C_{\mathrm{L}} = \{ z \in \mathbb{C} \mid |z| = 1, \ \mathrm{Re}\, z > 0 \}$$

$$f(-1/z) = f(z), \quad g(-1/z) = z^4 g(z), \quad h(-1/z) = z^2 h(z), \quad f(\pm i) = 0$$

- Calculation using OPEs and properties of the identity state:

$$\begin{aligned} &Q_{\mathrm{B}} A_c + A_c * A_c \\ &= \Big[\Big\{ Q_{\mathrm{B}}, C_L\Big((1+f)g + \frac{3}{4}(\partial f)^2 + h\partial f \Big) \Big\} + \Big\{ Q_{\mathrm{B}}, \Theta_L\Big((1+f)\Big(h + \frac{1}{4}\partial f\Big) \Big) \Big\} \\ &\quad - \frac{7}{32}\{\Theta_L(\partial f), \Theta_L(\partial f)\} + \frac{1}{2}\{\Theta_L(h), \Theta_L(h)\} - \frac{3}{4}\{\Theta_L(\partial f), \Theta_L(h)\} \Big] I. \end{aligned}$$

An identity-based solution in modified cubic super SFT

- Equations of motion of modified cubic SSFT:

$$Y_{-2}(Q_{\rm B}A + A * A) + Y\Psi * \Psi = 0,$$
$$Y(Q_{\rm B}\Psi + A * \Psi + \Psi * A) = 0.$$

- A class of identity-based solution in the NS sector (as an extension of Takahashi-Tanimoto's scalar solution to SSFT):

$$A_c = Q_L(e^{\lambda} - 1)I + C_L\Big(-\frac{1}{2}(\partial\lambda)^2 e^{\lambda}\Big)I + \Theta_L\Big(-\frac{1}{4}\partial e^{\lambda}\Big)I$$

$$\lambda(-1/z) = \lambda(z),\ \lambda(\pm i) = 0.$$

$$\Rightarrow \quad Q_{\rm B}A_c + A_c * A_c = 0$$

BRST operator at the solution

- Re-expansion of the action of SSFT around the solution:

$$S'[A,\Psi] \equiv S[A+A_c,\Psi] - S[A_c,0]$$
$$= \frac{1}{2}\langle A, Y_{-2}Q'A\rangle + \frac{1}{3}\langle A, Y_{-2}A*A\rangle + \frac{1}{2}\langle \Psi, YQ'\Psi\rangle + \langle A, Y\Psi*\Psi\rangle$$

- BRST operator at the solution can be expressed as:

$$Q' = Q_{\rm B} + [A_c,\ \cdot\ \}_*$$
$$= Q_{\rm B} + (Q_L(f) + C_L(g) + \Theta_L(h)) + (Q_R(f) + C_R(g) + \Theta_R(h))$$
$$= Q(e^{\lambda}) + C\left(-\frac{1}{2}(\partial\lambda)^2 e^{\lambda}\right) + \Theta\left(-\frac{1}{4}\partial e^{\lambda}\right)$$

$$Q(f) = \oint \frac{dz}{2\pi i} f(z) j_{\rm B}(z),\quad C(g) = \oint \frac{dz}{2\pi i} g(z) c(z),\qquad \Theta(h) = \oint \frac{dz}{2\pi i} h(z)\theta(z)$$

Homotopy operator for Q'

- Anti-commutation relation from OPE:

$$\{Q', b(z)\} = \frac{1}{2}(\partial^2\lambda(z))e^{\lambda(z)} + (\partial e^{\lambda(z)})j_{\rm gh}(z) + e^{\lambda(z)}T(z).$$

It becomes a c-number at a <u>second order zero</u> $z = z_0$ of $e^{\lambda(z)}$

- Example of the function $\lambda(z) = h^l_a(z)$ as in the bosonic case:

$$h^l_a(z) = \log\left(1 - \frac{a}{2}(-1)^l(z^l - (-1)^l z^{-l})^2\right), \quad (a \geq -1/2;\ l = 1, 2, 3, \cdots).$$

$e^{h^l_a(z)}$ has second order zeros z_k $(z_k^{2l} = -(-1)^l)$ only for $a = -\frac{1}{2}$

- Homotopy operator $\hat{A}$ for $\lambda(z) = h^l_{a=-1/2}(z)$

$$\hat{A} = \sum_{k=1}^{2l} a_k l^{-2} z_k^2\, b(z_k), \qquad \sum_{k=1}^{2l} a_k = 1.$$

:the same form with the bosonic case

$$\Rightarrow \quad \{Q', \hat{A}\} = 1, \quad \hat{A}^2 = 0.$$

Similarity transform of BRST operator

- It turns out that *Q'* can be rewritten as a similarity transform using the ghost number current: $j_{\rm gh} = -bc - \beta\gamma$

$$Q' = e^{q(\lambda)} Q_{\rm B} e^{-q(\lambda)}$$

$$q(\lambda) = \oint \frac{dz}{2\pi i} \lambda(z) j_{\rm gh}(z)$$

$$= Q(e^{\lambda}) + C\left(-\frac{1}{2}(\partial\lambda)^2 e^{\lambda}\right) + \Theta\left(-\frac{1}{4}\partial e^{\lambda}\right).$$

- Unlike the bosonic case, $e^{\pm q(\lambda)}$ is <u>not</u> singular even for $\lambda = h^l_{a=-\frac{1}{2}}$

$j_{\rm gh}(y) j_{\rm gh}(z) \quad (y \to z)$:regular for superstring

- Nevertheless, there exists a homotopy operator for $\lambda = h^l_{a=-\frac{1}{2}}$
It implies:

$$Q'\psi = 0 \quad \Leftrightarrow \quad \psi = Q'(\hat{A}\psi)$$

Vanishing cohomology for all ghost number sectors!

On cohomology of the BRST operator

- At least formally, we have

$$Q_{\mathrm{B}}\phi = 0 \quad \Leftrightarrow \quad Q'(e^{q(h^l_{-1/2})}\phi) = 0$$
$$\Leftrightarrow \quad e^{q(h^l_{-1/2})}\phi = Q'(\hat{A}e^{q(h^l_{-1/2})}\phi)$$

- Using the explicit form of the nontrivial part φ of Q_{B}-cohomology in the NS and R sector, we find:

$$\phi = \varphi + Q_{\mathrm{B}}\chi$$

$$\Rightarrow \quad e^{q(h^l_{-1/2})}\phi = U2^{-2g}\varphi + Q'(e^{q(h^l_{-1/2})}\chi)$$

g: ghost number

$$U = \exp\left(-\sum_{n=1}^{\infty}\frac{(-1)^{n(l+1)}}{n}q_{-2nl}\right) \qquad j_{\mathrm{gh}}(z) = \sum_n q_n z^{-n-1}$$

Zero in the Fock space but nonzero in a larger space (?)

- In a similar way to the bosonic case,

$$[q_n, b_m] = -b_{n+m} \qquad U^{-1} b(z) U = e^{-\sum_{n=1}^{\infty} \frac{(-1)^{n(l+1)}}{n} z^{-2nl}} b(z)$$

$$\Rightarrow \quad \hat{A} U 2^{-2g} |\varphi\rangle = \exp\left(-\sum_{n=1}^{\infty} \frac{1}{n}\right) U \hat{A} 2^{-2g} |\varphi\rangle = 0$$

- It implies that all coefficients of $\hat{A} U 2^{-2g} \varphi$ vanish in the Fock space.

- However, we should have $e^{q(h^l_{-1/2})} \varphi = Q'(\hat{A} U 2^{-2g} \varphi)$

$$\neq 0$$

- Nontrivial part of Q_{B}-cohomology becomes *Q'*-exact <u>outside</u> the Fock space by $e^{q(h^l_{-1/2})}$ *as far as we respect the homotopy relation:*

$$\{Q', \hat{A}\} = 1$$

Summary

- bosonic SFTのTT解をSSFTのidentity-based解に拡張した。
- このSSFTの解周りの Q'のhomotopy演算子を構成した。
- 実はこのQ' は（bosonicの場合と異なりhomotopy演算子が存在しても）通常の Q_Bからのsimilarity変換として書き直せる。
- Q_B cohomologyの非自明な部分はこのsimilarity変換により(通常のFock spaceを超えた意味で) Q'-exactになる。

Future problem

- ∃ homotopy演算子:cohomologyが自明→BPS D-braneが消えた？

  [cf. Erler(2007)]

- vacuum energy, gauge invariant overlapの直接計算。（あるいはレベル切断による数値的かつ間接的評価。）

  [cf. Kishimoto-Takahashi(2009), Kishimoto(2010)]

- bosonicの場合と異なりhomotopy演算子が存在する場合にも解はpure gaugeの形に書き直せる？

- $\hat{A}U2^{-2g}\varphi \simeq 0$ かつ $e^{q(h^l_{-1/2})}\varphi = Q'(\hat{A}U2^{-2g}\varphi)$ という意味の精密化。
  $\neq 0$

- 結合則の破れ（？）　$(Q'\hat{A} + \hat{A}Q')U|\cdot\rangle \neq Q'(\hat{A}U|\cdot\rangle) + \hat{A}(Q'U|\cdot\rangle)$