弦の場の理論における
KBc代数とその拡張について

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References

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- S. Inatomi, I. K., T. Takahashi, to appear

String Field Theory (SFT)

- bosonic SFT [Witten(1986)]

$$S[\Phi] \;=\; -\frac{1}{2}\langle\Phi, Q_{\mathrm{B}}\Phi\rangle - \frac{1}{3}\langle\Phi, \Phi * \Phi\rangle$$

- modified cubic super SFT [Preitschopf-Thorn-Yost, Arefeva-Medvedev-Zubarev (1990)]

$$S[A,\Psi] \;=\; \frac{1}{2}\langle A, Y_{-2}Q_{\mathrm{B}}A\rangle + \frac{1}{3}\langle A, Y_{-2}A * A\rangle + \frac{1}{2}\langle\Psi, YQ_{\mathrm{B}}\Psi\rangle + \langle A, Y\Psi * \Psi\rangle$$

KBc algebra in SFT

- Schnabl(2005): tachyon solution using sliver frame
- Okawa(2006): notation simplified, a class of solutions

$$Bc + cB = 1, \quad B^2 = 0, \quad c^2 = 0$$

$$Q_{\mathrm{B}} B = K, \quad Q_{\mathrm{B}} K = 0, \quad Q_{\mathrm{B}} c = cKc$$

$$BK = KB$$

$$B \equiv \frac{\pi}{2} B_1^L I, \quad K \equiv \frac{\pi}{2} K_1^L I, \quad c \equiv \frac{1}{\pi} c(1) I$$

Erler-Schnabl solution

- "phantomless" tachyon solution (2009)

$$\Phi = \frac{1}{\sqrt{1+K}}(c + cKBc)\frac{1}{\sqrt{1+K}}$$

$$Q_{\mathrm{B}}\Phi + \Phi * \Phi = 0$$

$$E = -S[\Phi] = -\frac{1}{2\pi^2}$$: a D-brane vanishes

Identity-based marginal solution in SSFT

- a solution in super SFT using supercurrent [I.K.-T.Takahashi(2005)] : a super extension of Takahashi-Tanimoto's marginal solution(2001)

$$\Psi_0 = -V_L^a(F_a)I + \frac{1}{8}\Omega^{ab}C_L(F_aF_b)I$$

$$F_a(-1/z) = z^2 F_a(z), \qquad V_L^a(f) \equiv \int_{C_L} \frac{dz}{2\pi i}\frac{1}{\sqrt{2}} f(z)(cJ^a(z) + \gamma\psi^a(z)), \qquad C_L(f) \equiv \int_{C_L} \frac{dz}{2\pi i} f(z)c(z)$$

$$Q_B\Psi_0 + \Psi_0 * \Psi_0 = 0$$:equation of motion in the NS sector

$$\psi^a(y)\psi^b(z) \sim \frac{1}{y-z}\frac{1}{2}\Omega^{ab},$$
$$J^a(y)\psi^b(z) \sim \frac{1}{y-z}f^{ab}{}_c\psi^c(z),$$
$$J^a(y)J^b(z) \sim \frac{1}{(y-z)^2}\frac{1}{2}\Omega^{ab} + \frac{1}{y-z}f^{ab}{}_cJ^c(z)$$

$$\Omega^{ab} = \Omega^{ba}, \quad f^{ab}{}_c\Omega^{cd} + f^{ad}{}_c\Omega^{cb} = 0,$$
$$f^{ab}{}_c = -f^{ba}{}_c, \quad f^{ab}{}_d f^{cd}{}_e + f^{bc}{}_d f^{ad}{}_e + f^{ca}{}_d f^{bd}{}_e = 0$$

structure constants of a Lie algebra

SFT around the solution

- Action around an NS solution:

$$
\begin{aligned}
S'[A,\Psi] &\equiv S[A+A_c,\Psi]-S[A_c,0] \\
&= \frac{1}{2}\langle A, Y_{-2}Q'A\rangle+\frac{1}{3}\langle A, Y_{-2}A*A\rangle+\frac{1}{2}\langle\Psi, YQ'\Psi\rangle+\langle A, Y\Psi*\Psi\rangle
\end{aligned}
$$

$$Q_{\mathrm{B}} \quad\to\quad Q'$$

In the case of our solution:

$$
\begin{aligned}
Q' &= Q_{\mathrm{B}}+[\Psi_0,\ \cdot\ \}_* \\
&= Q_{\mathrm{B}}-V^a(F_a)+\frac{1}{8}\Omega^{ab}C(F_aF_b)
\end{aligned}
$$

⇒

$$L_n\to L'_n,\qquad G_r\to G'_r$$

matter parts of the super Virasoro generators are modified

Algebra of L' and G'

- The same form as the original super Virasoro algebra:

$$[L'_m, L'_n] = (m-n)L'_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}\,,$$
$$\{G'_r, G'_s\} = 2L'_{r+s} + \frac{c}{12}(4r^2 - 1)\delta_{r+s,0}\,,$$
$$[L'_m, G'_r] = \left(\frac{m}{2} - r\right) G'_{m+r}\,,$$

$$L'_n = \{Q', b_n\} = L_n - \frac{1}{\sqrt{2}}\sum_{k\in\mathbb{Z}} F_{a,k}J^a_{n-k} + \frac{1}{8}\Omega^{ab}\sum_{k\in\mathbb{Z}} F_{a,n-k}F_{b,k},$$
$$G'_r = [Q', \beta_r] = G_r - \frac{1}{\sqrt{2}}\sum_{k\in\mathbb{Z}} F_{a,k}\psi^a_{r-k}$$

$$F_a(z) = \sum_{n\in\mathbb{Z}} F_{a,n}z^{-n-1} \qquad F_{a,k} = -(-1)^k F_{a,-k}$$

NS action including GSO(-) sector

- Chan-Paton factor using the Pauli matrix included

$$S'[\Phi] = \frac{1}{2}\langle\langle \Phi \hat{Q}' \Phi \rangle\rangle + \frac{1}{3}\langle\langle \Phi^3 \rangle\rangle, \qquad \langle\langle \Psi \rangle\rangle \equiv \frac{1}{2}\mathrm{Tr}\langle \hat{Y}_{-2} | \Psi \rangle$$

$$\hat{Y}_{-2} = Y_{-2}\sigma_3, \quad \hat{Q}' = Q'\sigma_3,$$

$$\Phi = \Phi_+\sigma_3 + \Phi_-\sigma_2$$

GSO(+) GSO(-)

GSO unprojected:
Non-BPS D-brane

Grassmann parity	worldsheet spinor	CP factor
even	even	1
odd	even	σ_3
even	odd	σ_2
odd	odd	σ_1

K'Bc algebra

- We find the same form as Erler's algebra (2010) using the original *L*, *G*

$$Bc + cB = 1, \quad B^2 = 0, \quad c^2 = 0$$

$$G'^2 = K'$$

$$\hat{Q}'B = K', \quad \hat{Q}'K' = 0, \quad \hat{Q}'G' = 0, \quad \hat{Q}'c = cK'c - \gamma^2$$

$$B\gamma + \gamma B = 0, \quad c\gamma + \gamma c = 0$$

$$BG' - G'B = 0, \quad BK' - K'B = 0, \quad G'K' - K'G' = 0$$

$$B = \frac{\pi}{2} B_1^L I \sigma_3, \qquad c = \frac{1}{\pi} c(1) I \sigma_3 \qquad \gamma = \frac{1}{\sqrt{\pi}} \gamma(1) I \sigma_2$$

$$K' = \frac{\pi}{2} K_1'^L I, \qquad G' = \mathcal{G}_L' I \sigma_1$$

Solutions

- A primed version of Erler's solution:

$$\Phi \;=\; \sqrt{f'}\left(c\frac{K'B}{1-f'}c+B\gamma^2\right)\sqrt{f'}=\sqrt{f'}\,c\frac{K'f'}{1-f'}Bc\sqrt{f'}+\hat{Q}'\left(\sqrt{f'}Bc\sqrt{f'}\right)$$

$$f'=f_+(K')+G'f_-(K')$$

In particular, we investigate a "simple" solution:

$$f'=\frac{1}{1+iG'}=\frac{1}{1+K'}-i\frac{G'}{1+K'}$$

$$\Phi_{\text{simp}}=\frac{1}{\sqrt{1+iG'}}\left(-icG'Bc+\hat{Q}'(Bc)\right)\frac{1}{\sqrt{1+iG'}}$$

Equation of motion

Calculation using the $K'Bc$ algebra:

$$\hat{Q}'\Phi = \sqrt{f'}\Big((cK'c-\gamma^2)\frac{K'f'}{1-f'}Bc - c\frac{K'^2f'}{1-f'}c + c\frac{K'f'}{1-f'}B(cK'c-\gamma^2)\Big)\sqrt{f'},$$

$$\Phi^2 = \sqrt{f'}\left(c\frac{K'B}{1-f'}c + B\gamma^2\right) f' \left(c\frac{K'B}{1-f'}c + B\gamma^2\right)\sqrt{f'}$$

$$= \sqrt{f'}\Big(c\frac{K'B}{1-f'}cf'c\frac{K'B}{1-f'}c + B\gamma^2 f'c\frac{K'B}{1-f'}c + c\frac{K'B}{1-f'}cf'B\gamma^2\Big)\sqrt{f'}$$

$$= \sqrt{f'}\Big(c\frac{K'}{1-f'}(f'-cBf'-f'Bc)\frac{K'}{1-f'}c + \gamma^2\frac{K'f'}{1-f'}Bc + cB\frac{K'f'}{1-f'}\gamma^2\Big)\sqrt{f'}$$

$$\begin{aligned}
\hat{Q}'\Phi + \Phi^2 \;&=\; \sqrt{f'}\left(c\left(-\frac{K'^2f'}{1-f'} + \frac{K'^2f'}{(1-f')^2}\right)c + c\left(K' - \frac{K'}{1-f'}\right)c\frac{K'f'}{1-f'}Bc\right.\\
&\qquad \left. +cB\frac{K'f'}{1-f'}c\left(K' - \frac{K'}{1-f'}\right)c\right)\sqrt{f'}\\
&=\; \sqrt{f'}\left(c\left(\frac{K'f'}{1-f'}\right)^2 c - c\frac{K'f'}{1-f'}(cB+Bc)\frac{K'f'}{1-f'}c\right)\sqrt{f'} = 0
\end{aligned}$$

Evaluation of the action

$$\langle\langle \Phi_{\rm simp}\hat{Q}'\Phi_{\rm simp}\rangle\rangle = -\langle\langle cG'Bc\frac{1}{1+K'}\hat{Q}'(cG'Bc)\frac{1}{1+K'}\rangle\rangle + \langle\langle cG'Bc\frac{G'}{1+K'}\hat{Q}'(cG'Bc)\frac{G'}{1+K'}\rangle\rangle$$

$$\equiv -\mathbf{(1)} + \mathbf{(2)}$$

$$\mathbf{(1)} = (cG'Bc, \hat{Q}'(cG'Bc))' = \frac{3}{\pi^2} - \frac{24}{\pi^4},$$

$$\mathbf{(2)} = (cG'BcG', \hat{Q}'(cG'Bc)G')' = \frac{9}{2\pi^2} - \frac{24}{\pi^4}$$

$$(\Phi, \Psi)' \equiv \langle\langle \Phi\frac{1}{1+K'}\Psi\frac{1}{1+K'}\rangle\rangle$$

$$S'[\Phi_{\rm simp}] = \frac{1}{2}\langle\langle \Phi_{\rm simp}\hat{Q}'\Phi_{\rm simp}\rangle\rangle + \frac{1}{3}\langle\langle \Phi_{\rm simp}^3\rangle\rangle = \frac{1}{6}\langle\langle \Phi_{\rm simp}\hat{Q}'\Phi_{\rm simp}\rangle\rangle = \frac{1}{4\pi^2}$$

a half of D-brane tension!

Details of evaluation

$$\langle\!\langle \Phi\Psi \rangle\!\rangle = (-1)^{(\epsilon(\Phi)+F(\Phi))(\epsilon(\Psi)+F(\Psi))} \langle\!\langle \Psi\Phi \rangle\!\rangle,$$
$$\hat{Q}'(\Phi\Psi) = (\hat{Q}'\Phi)\Psi + (-1)^{\epsilon(\Phi)+F(\Phi)}\Phi(\hat{Q}'\Psi),$$
$$\langle\!\langle \hat{Q}'(\cdots) \rangle\!\rangle = 0,$$

$$\langle\!\langle \cdots \rangle\!\rangle = -\frac{1}{2}\mathrm{Tr}\langle \delta'(\gamma(i))\delta'(\gamma(-i))c(i)c(-i)\cdots\rangle$$

$$\begin{aligned}\mathbf{(1)} \quad = \quad & -(\gamma^2, cK')' + 5(\gamma^2, c\partial cB)' - 4(cB\gamma, \gamma K'c)' \\ & +2(Bc\gamma, \partial c\gamma)' - 2(cB\gamma, \partial\gamma c)' + 2(\gamma, K'\gamma c)',\end{aligned}$$

$$\begin{aligned}\mathbf{(2)} \quad = \quad & (B\gamma^2, K'c\partial c)' + 4(Bc\gamma K', c\gamma K')' + 2(Bc\partial\gamma, c\gamma K')' - 2(B\gamma\partial c, c\gamma K')' + (B\gamma\partial\gamma, c\partial c)' \\ & -(\gamma^2, K'cK')' - (cB\gamma, \partial\gamma\partial c)' - 2(cB\gamma, \partial^2\gamma c)' + (cB\gamma, \gamma\partial^2 c)'.\end{aligned}$$

Inverse of a string field 1+K'

$$\frac{1}{1+K'} = \int_0^\infty dx e^{-x(1+K')}$$: "definition"

$[K' - K, K] \neq 0$

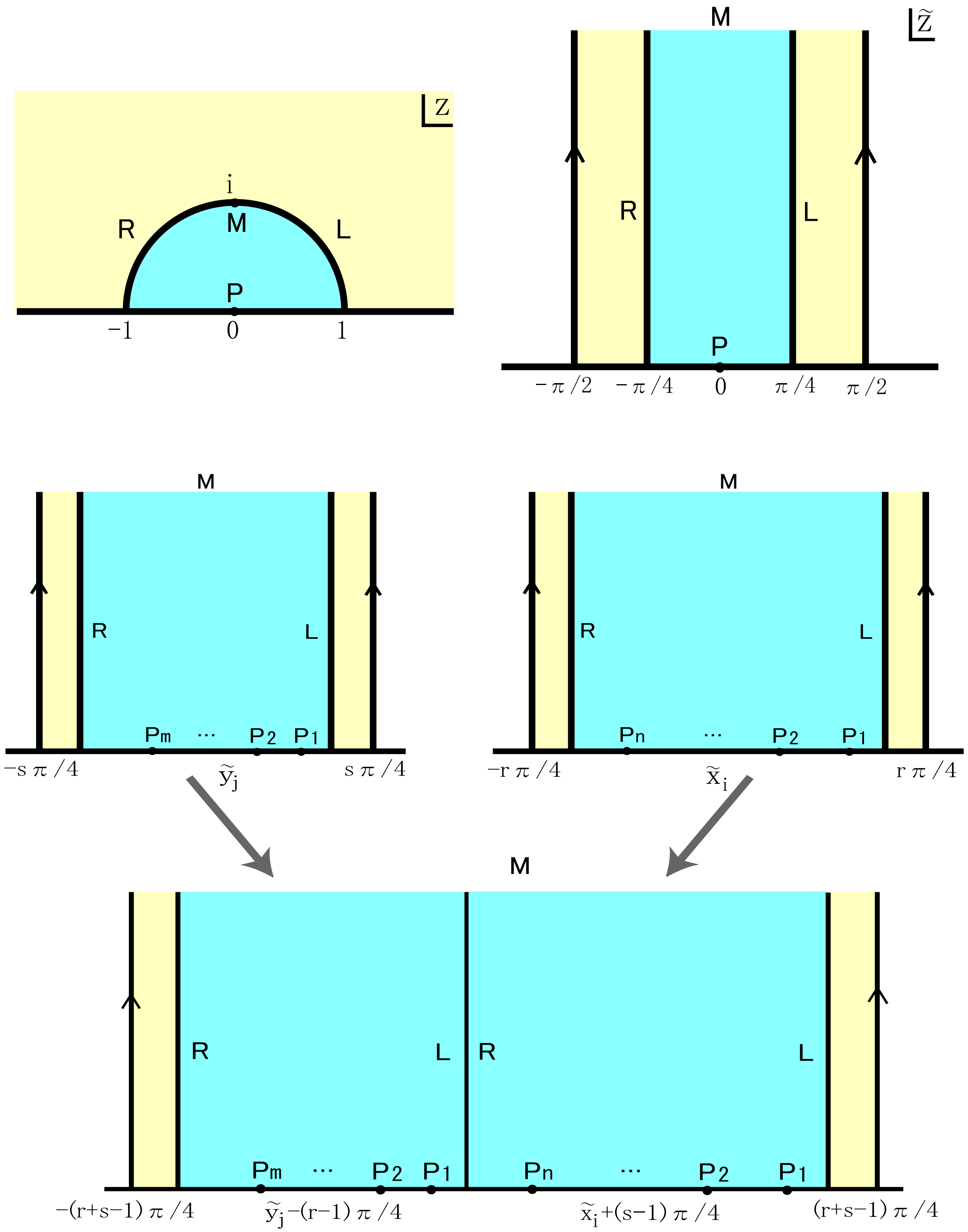
$$e^{-tK'} \;=\; e^{-t\frac{\pi}{2}\mathcal{C}}\hat{U}_{t+1}\mathbf{P}_u \exp\left(\frac{\pi}{4}\int_{-t}^{t} du \int_{-\infty}^{\infty} dt' f_a(t')\tilde{J}^a(it'+\frac{\pi}{4}u)\right)|0\rangle$$

u-ordered exponential

$$f_a(t) \equiv \frac{F_a(\tan(it+\frac{\pi}{4}))}{2\pi\sqrt{2}\cos^2(it+\frac{\pi}{4})}, \qquad F_a(z) = \sum_{n\in\mathbb{Z}} F_{a,n} z^{-n-1},$$

$$\mathcal{C} = \int_{C_{\rm L}} \frac{dz}{2\pi i}(1+z^2)\frac{\Omega^{ab}}{8}F_a(z)F_b(z) = \frac{\pi}{2}\int_{-\infty}^{\infty} dt \Omega^{ab} f_a(t) f_b(t).$$

Sliver frame and star product



sliver frame technique
by Schnabl (2005)

$$\arctan z = \tilde{z}$$

$$\tilde{\phi}(\tilde{z}) = \left(\frac{dz}{d\tilde{z}}\right)^h \phi(z) = (\cos\tilde{z})^{-2h}\phi(\tan\tilde{z})$$

$$e^{-tK} = \hat{U}_{t+1}|0\rangle = U_{t+1}^\dagger U_{t+1}|0\rangle$$:wedge state

$$\begin{aligned}
&U_r^\dagger U_r \tilde{\phi}_1(\tilde{x}_1)\cdots\tilde{\phi}_n(\tilde{x}_n)|0\rangle * U_s^\dagger U_s \tilde{\psi}_1(\tilde{y}_1)\cdots\tilde{\psi}_m(\tilde{y}_m)|0\rangle \\
&= U_{r+s-1}^\dagger U_{r+s-1}\tilde{\phi}_1(\tilde{x}_1 + \frac{\pi}{4}(s-1))\cdots\tilde{\phi}_n(\tilde{x}_n + \frac{\pi}{4}(s-1)) \\
&\qquad \times \tilde{\psi}_1(\tilde{y}_1 - \frac{\pi}{4}(r-1))\cdots\tilde{\psi}_m(\tilde{y}_m - \frac{\pi}{4}(r-1))|0\rangle
\end{aligned}$$

Extended Feynman's formula

$$\delta(e^X) = \int_0^1 d\alpha e^{(1-\alpha)X}(\delta X)e^{\alpha X}$$

$$\Rightarrow \quad e^{X+\delta X} = e^X + \sum_{N=1}^{\infty}(e^{X+\delta X})|_{O((\delta X)^N)}$$

$$(e^{X+\delta X})|_{O((\delta X)^N)}$$
$$= \int_0^1 du_1 \int_0^{1-u_1} du_2 \cdots \int_0^{1-u_1-u_2\cdots-u_{N-1}} du_N \, e^{(1-u_1-u_2\cdots-u_N)X}(\delta X)e^{u_1 X}(\delta X)e^{u_2 X}\cdots(\delta X)e^{u_N X}$$

Note: $$B(p,q) = \int_0^1 dt(1-t)^{p-1}t^{q-1} = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

$$(e^{X+\delta X})|_{O((\delta X)^N)} = \sum_{k_0=0}^{\infty}\sum_{k_1=0}^{\infty}\cdots\sum_{k_N=0}^{\infty}\frac{1}{(k_0+k_1\cdots+k_N+N)!}X^{k_0}(\delta X)X^{k_1}(\delta X)\cdots X^{k_{N-1}}(\delta X)X^{k_N}$$

Evaluation using CFT correlator

$$(B\gamma^2, K'c\partial c)' \;=\; -\int_0^\infty dt \int_0^\infty ds e^{-t-s} \frac{\partial}{\partial t} \langle\!\langle B\gamma^2 e^{-tK'} c\partial c e^{-sK'} \rangle\!\rangle$$

$$\begin{aligned}
&\langle\!\langle B\gamma^2 e^{-tK'} c\partial c e^{-sK'} \rangle\!\rangle \\
&= \frac{4}{(t+s)\pi^2} \langle 0| \tilde{Y}_{-2} \mathcal{B}_0 U_{t+s} \tilde{\gamma}^2(\frac{\pi}{4}(t+s)) \tilde{c}\partial\tilde{c}(\frac{\pi}{4}(s-t)) \\
&\qquad \times e^{-(t+s)\frac{\pi}{2}\mathcal{C}} \mathbf{P}_u \exp\left(\frac{\pi}{4} \int_{-t-s}^{t+s} du \int_{-\infty}^{\infty} dt' f_a(t') \tilde{J}^a(it' + \frac{\pi}{4}u) \right) |0\rangle \\
&= -\frac{t+s}{\pi^2} \lim_{M\to\infty} \langle \delta'(\tilde{\gamma}(iM)) \delta'(\tilde{\gamma}(-iM)) \tilde{\gamma}^2(\frac{\pi}{2}) \rangle_{\beta\gamma} \\
&\qquad \times \left(\frac{\pi(s-t)}{2(s+t)} \langle \tilde{c}(iM)\tilde{c}(-iM)\partial\tilde{c}(\frac{\pi(s-t)}{2(s+t)}) \rangle_{bc} - \langle \tilde{c}(iM)\tilde{c}(-iM)\tilde{c}(\frac{\pi(s-t)}{2(s+t)}) \rangle_{bc} \right) \\
&\qquad \times e^{-(t+s)\frac{\pi}{2}\mathcal{C}} \left\langle \exp\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du \int_{-\infty}^{\infty} dt' f_a(t') \tilde{J}^a(\frac{2it'}{t+s} + u) \right) \right\rangle_{\mathrm{mat}} \\
&= \frac{t+s}{2\pi^2}
\end{aligned}$$

$$\Rightarrow \quad (B\gamma^2, K'c\partial c)' \;=\; -\int_0^\infty dt \int_0^\infty ds e^{-t-s} \frac{1}{2\pi^2} = -\frac{1}{2\pi^2}$$

Conclusion

- リー代数に基づいた(super)currentによるidentity-based marginal解周りの(超)弦理論において$K'Bc$代数を用いて古典解を構成した。
- Erler-Schnabl解（のsuper版）とErlerのhalf-brane解に対応する$K'Bc$代数による解のvacuum energyを計算し、元のKBc代数による解と値が同じことを確かめた。
- 今回の結果は元のidentity-based marginal解のvacuum energyがゼロであるという予想と整合する。

Comments

- bosonic SFTのErler-Schnabl解をレベルトランケーションして数値計算：
  (kinetic term: Erler-Schnabl, cubic term: Arroyo-I.K.)

L	$\tilde{E}_2$	$\tilde{E}_{2,\mathrm{P}}\vert_L^L$	$\tilde{E}_{2,\mathrm{PB}}\vert_L^L$	$\tilde{E}$	$\tilde{E}_{\mathrm{P}}\vert_{3L/2}^{3L/2}$	$\tilde{E}_{\mathrm{PB}}\vert_{3L/2}^{3L/2}$
0	−0.85247	−0.85247	−0.85247	−0.654908	−0.654908	−0.654908
2	−0.914146	−0.85247	−0.85247	−1.33686	−1.38342	−1.38798
4	−1.03467	−0.787834	−0.871988	−0.532599	−0.421667	−0.358173
6	−0.930637	−0.787834	−0.871988	−1.55434	−1.19306	−1.08516
8	−1.06335	−0.992052	−0.983242	−0.167462	−1.14097	−1.00745
10	−0.904984	−0.992052	−0.983242	−1.87271	−0.919443	−1.07258
12	−1.10973	−0.992013	−0.984516	−0.166042	−0.850702	−1.05767*
14	−0.841643	−0.992013	−0.984516	−1.83972	−0.972165	−0.933839**
16	−1.20564	−0.99608	−0.993936	+1.83619	−1.00666	−0.92572*
18	−0.709632	−0.99608	−0.993933	−4.22806	−1.01865	−0.981341**
20	−1.39169	−0.999595	−0.993687	−1.1971	−1.02464	−1.01792*
22	−0.449641	−0.999595	−0.993574	−0.188021	−0.994601	−1.00019**
24	−1.75829	−0.997321	−0.995001	+12.4404	−0.997754	−1.01338**
26	+0.0590993	−0.997321	−0.993171	−24.5744	−0.999148	−1.02392**
28	−2.46306	−0.99769	−0.993253			
30	+1.03342	−0.99769	−0.989787			

$$\tilde{E} = 2\pi^2 E = -2\pi^2 S[\Phi]$$

生の値は「発散」するがレベル毎の寄与に基づく多項式を**パデ近似**すると「厳密値」に近い。