Nontrivial solutions around identity-based marginal solutions in cubic superstring field theory

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October 28 (2012)

SFT2012@IAS, The Hebrew University of Jerusalem

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Introduction

- In our previous work, we have investigated the tachyon vacuum solution in marginally deformed background using ***K'Bc*** algebra. [T. Takahashi's talk]
- Here, we will extend it to the case of SSFT.
- Identity-based marginal solution in SSFT [I.K.-Takahashi (2005)]
- $KBc \rightarrow GKBc\gamma$ [Erler (2010)]
- **$K'Bc \rightarrow G'K'Bc\gamma$ [This talk]**

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Identity-based marginal solution in Berkovits' WZW-like SSFT

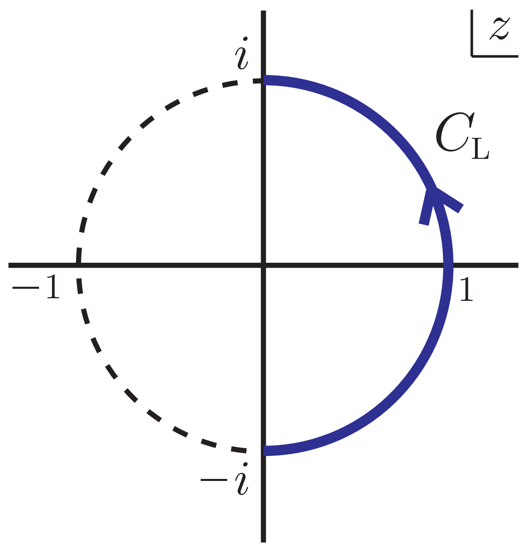
- In [I.K.-T.Takahashi(2005)], we have constructed a type of marginal solution in WZW-like SSFT:

$$\tilde{\Phi}_0 = -\tilde{V}_L^a(F_a) I$$

$$\tilde{V}_L^a(F_a) = \int_{C_{\rm L}} \frac{dz}{2\pi i} F_a(z) \tilde{v}^a(z)$$

$$F_a(-1/z) = z^2 F_a(z)$$

$$\tilde{v}^a(z) = \frac{1}{\sqrt{2}} c\xi e^{-\phi}\psi^a(z)$$



$$\Rightarrow \quad \eta_0 \left(e^{-\tilde{\Phi}_0} Q_{\rm B} e^{\tilde{\Phi}_0} \right) = 0$$:EOM is satisfied.

Identity-based marginal solution in modified cubic SSFT

- A solution is obtained from $\tilde{\Phi}_0 = -\tilde{V}_L^a(F_a)I$:

$$\Psi_J = e^{-\tilde{\Phi}_0} Q_{\rm B} e^{\tilde{\Phi}_0} \quad \Rightarrow \quad Q_{\rm B}\Psi_J + \Psi_J * \Psi_J = 0$$

$$\Psi_J = -V_L^a(F_a)I + \frac{1}{8}\Omega^{ab} C_L(F_a F_b) I$$

$$V_L^a(f) \equiv \int_{C_{\rm L}} \frac{dz}{2\pi i} \frac{1}{\sqrt{2}} f(z)(cJ^a(z) + \gamma\psi^a(z))$$

$$C_L(f) \equiv \int_{C_{\rm L}} \frac{dz}{2\pi i} f(z) c(z)$$

$$J^a(z,\theta) = \psi^a(z) + \theta J^a(z)$$

:supercurrent

OPEs of supercurrent

- Component fields of supercurrent satisfy

$$\psi^a(y)\psi^b(z) \sim \frac{1}{y-z}\frac{1}{2}\Omega^{ab}, \qquad J^a(y)\psi^b(z) \sim \frac{1}{y-z}f^{ab}{}_c\psi^c(z),$$

$$J^a(y)J^b(z) \sim \frac{1}{(y-z)^2}\frac{1}{2}\Omega^{ab} + \frac{1}{y-z}f^{ab}{}_cJ^c(z),$$

$$\Omega^{ab} = \Omega^{ba}, \qquad f^{ab}{}_c\Omega^{cd} + f^{ad}{}_c\Omega^{cb} = 0,$$
$$f^{ab}{}_c = -f^{ba}{}_c, \qquad f^{ab}{}_df^{cd}{}_e + f^{bc}{}_df^{ad}{}_e + f^{ca}{}_df^{bd}{}_e = 0.$$

$\psi^a(z),\ J^a(z)$ are primary fields and have dimension 1/2 and 1, respectively.

$$\begin{aligned} T(z) &= \Omega_{ab}:(J^aJ^b+\partial\psi^a\psi^b):(z) + \frac{2}{3}\Omega_{ad}\Omega_{be}f^{de}{}_c:(J^a:\psi^b\psi^c: + \psi^a:(\psi^bJ^c - J^b\psi^c):):(z), \\ G(z) &= 2\Omega_{ab}:J^a\psi^b:(z) + \frac{4}{3}\Omega_{ad}\Omega_{be}f^{de}{}_c:\psi^a:\psi^b\psi^c::(z), \end{aligned}$$

BRST operator around the solution

- Expanding around the solution in the action of the modified cubic SSFT, the BRST operator becomes:

$$Q' = Q_{\mathrm{B}} + [\Psi_J, \cdot\}_* = Q_{\mathrm{B}} - V^a(F_a) + \frac{1}{8}\Omega^{ab}C(F_aF_b)$$

$$V^a(f) \equiv \oint \frac{dz}{2\pi i}\frac{1}{\sqrt{2}}f(z)(cJ^a(z) + \gamma\psi^a(z)), \quad C(f) \equiv \oint \frac{dz}{2\pi i}f(z)c(z)$$

L, G are deformed in the matter sector.

$$L'_n \equiv \{Q', b_n\} = L_n - \frac{1}{\sqrt{2}}\sum_{k\in\mathbb{Z}} F_{a,k}J^a_{n-k} + \frac{1}{8}\Omega^{ab}\sum_{k\in\mathbb{Z}} F_{a,n-k}F_{b,k},$$

$$G'_r \equiv [Q', \beta_r] = G_r - \frac{1}{\sqrt{2}}\sum_{k\in\mathbb{Z}} F_{a,k}\psi^a_{r-k},$$

$$F_{a,n} \equiv \oint \frac{d\sigma}{2\pi}e^{i(n+1)\sigma}F_a(e^{i\sigma})$$

NS action without GSO projection in modified cubic SSFT

- String fields have Chan-Paton factor as in the table:

Grassmann parity(ε)	worldsheet spinor(F)	CP factor
even	even	1
odd	even	σ_3
even	odd	σ_2
odd	odd	σ_1

- NS action:

$$S[\Psi] = \frac{1}{2}\langle\langle \Psi\, \hat{Q}\Psi \rangle\rangle + \frac{1}{3}\langle\langle \Psi^3 \rangle\rangle$$

$$\Psi = \Psi_+ \sigma_3 + \Psi_- \sigma_2$$

GSO(+) GSO(-)

$$\hat{Q} \equiv Q_{\rm B}\sigma_3$$

$$\langle\langle A \rangle\rangle \equiv \frac{1}{2}\mathrm{Tr}\left(\sigma_3 \langle I | Y_{-2} A\rangle\right)$$

$$Y_{-2} = Y(i)Y(-i), \quad Y(z) \equiv c(z)\delta'(\gamma(z)).$$

Picture changing operator with picture number (-2)

$$\hat{Q}(\Phi\Psi) = (\hat{Q}\Phi)\Psi + (-1)^{\epsilon(\Phi)+F(\Phi)}\Phi(\hat{Q}\Psi),$$

$$\langle\langle \Phi\Psi \rangle\rangle = (-1)^{(\epsilon(\Phi)+F(\Phi))(\epsilon(\Psi)+F(\Psi))}\langle\langle \Psi\Phi \rangle\rangle,$$

$$\langle\langle \hat{Q}(\cdots) \rangle\rangle = 0.$$

GKBcγ string fields in SSFT

- KBc with CP factor

$$K=\frac{\pi}{2}K_1^L I,\quad B=\frac{\pi}{2}B_1^L I\sigma_3,\quad c=\frac{1}{\pi}c(1)I\sigma_3$$

- G,γ : $\quad G=\mathcal{G}_L I\sigma_1,\quad \gamma=\frac{1}{\sqrt{\pi}}\gamma(1)I\sigma_2$

[Erler(2010)]

$$\mathcal{G}_L=\frac{1}{2}(\mathcal{G}+\mathcal{G}^\star),$$

$$\mathcal{G}=\oint\frac{dz}{2\pi i}\sqrt{\frac{\pi}{2}}\sqrt{1+z^2}G(z)=\sqrt{\frac{\pi}{2}}\sum_{n=0}^{\infty}\binom{1/2}{n}G_{2n-\frac{1}{2}},$$

$$\mathcal{G}^\star=\oint\frac{dz}{2\pi i}\sqrt{\frac{\pi}{2}}z\sqrt{1+z^{-2}}G(z)=\sqrt{\frac{\pi}{2}}\sum_{n=0}^{\infty}\binom{1/2}{n}G_{\frac{1}{2}-2n}.$$

GKBcγ algebra

- *KBc* algebra in SSFT

$$Bc + cB = 1, \quad BK = KB, \quad B^2 = 0, \quad c^2 = 0,$$
$$\hat{Q}B = K, \quad \hat{Q}K = 0, \quad \hat{Q}c = cKc - \gamma^2$$

[Erler(2010)]

- *G*,*γ* : $G^2 = K, \quad \hat{Q}G = 0, \quad \hat{Q}\gamma = c\partial\gamma - \frac{1}{2}(\partial c)\gamma,$

$$B\gamma + \gamma B = 0, \quad c\gamma + \gamma c = 0$$

- *δ* : $\delta c = 2i\gamma, \quad \delta\gamma = -\frac{i}{2}\partial c, \quad \delta G = 2K, \quad \delta K = 0, \quad \delta B = 0$

$$\delta\Phi \equiv G\Phi - (-)^{F(\Phi)}\Phi G = (\mathcal{G}\sigma_1)\Phi, \quad \partial\Phi \equiv K\Phi - \Phi K = \frac{\pi}{2}K_1\Phi$$

String fields: G', K'

- Around the solution Ψ_J : $\quad Q_{\rm B} \to Q'$

$$\Rightarrow \quad G_r \to G'_r, \quad L_n \to L'_n$$

- Replacement: $\hat{Q} = Q_{\rm B}\sigma_3 \to \hat{Q}' = Q'\sigma_3$

$$G, K \to G' = \mathcal{G}'_L I\sigma_1,\ K' = \frac{\pi}{2}K_1'^L I$$

$$\delta \to \delta'\Phi \equiv G'\Phi - (-)^{F(\Phi)}\Phi G' = (\mathcal{G}'\sigma_1)\Phi,$$

$$\partial \to \partial'\Phi \equiv K'\Phi - \Phi K' = \frac{\pi}{2}K'_1\Phi$$

G′K′Bcγ algebra

- *K′Bc* algebra in SSFT

$$Bc + cB = 1, \quad BK' = K'B, \quad B^2 = 0, \quad c^2 = 0,$$

$$\hat{Q}'B = K', \qquad \hat{Q}'K' = 0, \qquad \hat{Q}'c = cK'c - \gamma^2 = cKc - \gamma^2$$

- *G′*,*γ*: $G'^2 = K'$, $\hat{Q}'G' = 0, \quad \hat{Q}'\gamma = c\partial'\gamma - \frac{1}{2}(\partial' c)\gamma = c\partial\gamma - \frac{1}{2}(\partial c)\gamma$

$$B\gamma + \gamma B = 0, \quad c\gamma + \gamma c = 0$$

- *δ′*:

$$\delta' c = 2i\gamma, \quad \delta'\gamma = -\frac{i}{2}\partial' c = -\frac{i}{2}\partial c, \quad \delta' G' = 2K', \quad \delta' K' = 0, \quad \delta' B = 0$$

Solutions using G′K′Bcγ

- NS action without GSO projection around the solution $\Psi_J \sigma_3$:

$$S'[\Phi] \equiv S[\Phi + \Psi_J \sigma_3] - S[\Psi_J \sigma_3] = \frac{1}{2}\langle\langle \Phi\, \hat{Q}' \Phi \rangle\rangle + \frac{1}{3}\langle\langle \Phi^3 \rangle\rangle$$

- Equation of motion:

$$\hat{Q}'\Phi + \Phi^2 = 0$$

- A class of solutions to EOM (a version of [Erler(2010)])

$$\Phi_{f'} = \sqrt{f'}\left(c\frac{K'B}{1-f'}c + B\gamma^2\right)\sqrt{f'} = \sqrt{f'}\left(c\frac{K'f'}{1-f'}Bc + \hat{Q}'(Bc)\right)\sqrt{f'}$$

$f'(G') = f_0(K') + f_1(K')G'$:a function of G′

Note: $G'^2 = K'$

Tachyon vacuum and half-brane solution in the marginally deformed background

- We consider two examples: $$f' = \frac{1}{1+K'}, \quad \frac{1}{1+iG'} = \frac{1-iG'}{1+K'}$$

- Tachyon vacuum solution: (a version of [Erler-Schnabl(2009), Gorbachev(2010)])

$$\Phi_T = \frac{1}{\sqrt{1+K'}}\left(c + \hat{Q}'(Bc)\right)\frac{1}{\sqrt{1+K'}}$$

- Half-brane solution: (a version of [Erler(2010)])

$$\Phi_H = \frac{1}{\sqrt{1+iG'}}\left(-icG'Bc + \hat{Q}'(Bc)\right)\frac{1}{\sqrt{1+iG'}}$$

Vacuum energy for tachyon vacuum solution

- The action is evaluated as: $$S'[\Phi_T] = -\frac{1}{6}\langle\langle \gamma^2 \frac{1}{1+K'} c \frac{1}{1+K'} \rangle\rangle$$
- We use an expression: $$\frac{1}{1+K'} = \int_0^\infty dt\, e^{-t(1+K')}$$
- K' is given by

$\tilde{J}^a(\tilde{z}) = (\cos\tilde{z})^{-2} J^a(\tan\tilde{z})$ in the sliver frame

$$K' = K - J + \frac{\pi}{2}\mathcal{C} I,$$

$$J = \frac{\pi}{2}\int_{-\infty}^{\infty} dt f_a(t) \hat{U}_1 \tilde{J}^a(it)|0\rangle, \qquad f_a(t) \equiv \frac{F_a(\tan(it+\frac{\pi}{4}))}{2\pi\sqrt{2}\cos^2(it+\frac{\pi}{4})},$$

$$\mathcal{C} = \int_{C_L} \frac{dz}{2\pi i}(1+z^2)\frac{\Omega^{ab}}{8} F_a(z) F_b(z) = \frac{\pi}{2}\int_{-\infty}^{\infty} dt \Omega^{ab} f_a(t) f_b(t).$$

Expansion of the exponential

- The N-th order of J in the exponential can be computed as

$$
\begin{aligned}
&e^{-tK+tJ}|_{O(J^N)} \\
&= \int_0^1 du_1 \int_0^{1-u_1} du_2 \cdots \int_0^{1-u_1-u_2\cdots-u_{N-1}} du_N t^N e^{-t(1-u_1-u_2\cdots-u_N)K} J e^{-tu_1K} J e^{-tu_2K} \cdots J e^{-tu_NK} \\
&= t^N \int_0^1 du_1 \int_0^{1-u_1} du_2 \cdots \int_0^{1-u_1-u_2\cdots-u_{N-1}} du_N \int_{-\infty}^{\infty} dt_1 f_{a_1}(t_1) \int_{-\infty}^{\infty} dt_2 f_{a_2}(t_2) \cdots \int_{-\infty}^{\infty} dt_N f_{a_N}(t_N) \\
&\quad \times \frac{\pi^N}{2^N} \hat{U}_{t+1} \tilde{J}^{a_1}(it_1 + \frac{\pi}{4} y_1) \tilde{J}^{a_2}(it_2 + \frac{\pi}{4} y_2) \cdots \tilde{J}^{a_N}(it_N + \frac{\pi}{4} y_N)|0\rangle
\end{aligned}
$$

$$
(y_1 = 2t \sum_{k=1}^{N} u_k - t, \ \cdots, \ y_i = 2t \sum_{k=i}^{N} u_k - t, \ \cdots, \ y_N = 2tu_N - t.)
$$

$$
\begin{aligned}
&= \frac{\pi^N}{4^N} \int_{-t}^{t} dy_1 \int_{-t}^{y_1} dy_2 \cdots \int_{-t}^{y_{N-1}} dy_N \int_{-\infty}^{\infty} dt_1 f_{a_1}(t_1) \int_{-\infty}^{\infty} dt_2 f_{a_2}(t_2) \cdots \int_{-\infty}^{\infty} dt_N f_{a_N}(t_N) \\
&\quad \times \hat{U}_{t+1} \tilde{J}^{a_1}(it_1 + \frac{\pi}{4} y_1) \tilde{J}^{a_2}(it_2 + \frac{\pi}{4} y_2) \cdots \tilde{J}^{a_N}(it_N + \frac{\pi}{4} y_N)|0\rangle.
\end{aligned}
$$

Extended Feynman's formula

$$\delta(e^X) = \int_0^1 d\alpha e^{(1-\alpha)X}(\delta X)e^{\alpha X}$$

$$\Rightarrow \quad e^{X+\delta X} = e^X + \sum_{N=1}^{\infty}(e^{X+\delta X})|_{O((\delta X)^N)}$$

$$(e^{X+\delta X})|_{O((\delta X)^N)}$$
$$= \int_0^1 du_1 \int_0^{1-u_1} du_2 \cdots \int_0^{1-u_1-u_2\cdots-u_{N-1}} du_N\, e^{(1-u_1-u_2\cdots-u_N)X}(\delta X)e^{u_1 X}(\delta X)e^{u_2 X}\cdots(\delta X)e^{u_N X}$$

Note: $$B(p,q) = \int_0^1 dt(1-t)^{p-1}t^{q-1} = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

$$(e^{X+\delta X})|_{O((\delta X)^N)} = \sum_{k_0=0}^{\infty}\sum_{k_1=0}^{\infty}\cdots\sum_{k_N=0}^{\infty}\frac{1}{(k_0+k_1\cdots+k_N+N)!}X^{k_0}(\delta X)X^{k_1}(\delta X)\cdots X^{k_{N-1}}(\delta X)X^{k_N}$$

Evaluation of the action

- u-ordered exponential:

$$e^{-tK'} = e^{-t\frac{\pi}{2}\mathcal{C}}\hat{U}_{t+1}\mathrm{T}\exp\left(\frac{\pi}{4}\int_{-t}^{t}du\int_{-\infty}^{\infty}dt' f_a(t')\tilde{J}^a(it'+\frac{\pi}{4}u)\right)|0\rangle$$

- Computation of the CFT correlator:

$$\begin{aligned}
&\langle\!\langle \gamma^2 e^{-tK'} c e^{-sK'} \rangle\!\rangle \\
&= \frac{(t+s)^2}{\pi^2}\langle 0|\tilde{Y}(i\infty)\tilde{Y}(-i\infty)\tilde{\gamma}^2(\frac{\pi}{2})\tilde{c}(\frac{\pi(s-t)}{2(t+s)}) \\
&\qquad \times e^{-(t+s)\frac{\pi}{2}\mathcal{C}}\,\mathrm{T}\exp\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}du\int_{-\infty}^{\infty}dt' f_a(t')\tilde{J}^a(\frac{2it'}{t+s}+u)\right)|0\rangle \\
&= -\frac{(t+s)^2}{\pi^2}\lim_{M\to\infty}\langle\delta'(\tilde{\gamma}(iM))\delta'(\tilde{\gamma}(-iM))\tilde{\gamma}^2(\frac{\pi}{2})\rangle_{\beta\gamma}\langle\tilde{c}(iM)\tilde{c}(-iM)\tilde{c}(\frac{\pi(s-t)}{2(s+t)})\rangle_{bc} \\
&\qquad \times e^{-(t+s)\frac{\pi}{2}\mathcal{C}}\left\langle\exp\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}du\int_{-\infty}^{\infty}dt' f_a(t')\tilde{J}^a(\frac{2it'}{t+s}+u)\right)\right\rangle_{\mathrm{mat}} \\
&= -\frac{(t+s)^2}{\pi^2}\lim_{M\to\infty}(-4ie^{-4M})\frac{i}{8}e^{4M}\cdot 1 = -\frac{(t+s)^2}{2\pi^2}
\end{aligned}$$

Result for tachyon vacuum solution

- Finally, we obtain

$$S'[\Phi_T] = \int_0^\infty dt \int_0^\infty ds\, e^{-t-s} \frac{(t+s)^2}{12\pi^2} = \frac{1}{2\pi^2}$$

- This value is the same as a D-brane tension. Namely, the vacuum energy of Φ_T is the same as that of the tachyon vacuum in the original theory. [Erler(2007), Gorbachev(2010)]

- A key relation is the same as the bosonic case: [T.Takahashi's talk]

$$e^{-T\frac{\pi}{2}c} \left\langle \exp\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du \int_{-\infty}^{\infty} dt' f_a(t') \tilde{J}^a(\frac{2it'}{T} + u) \right) \right\rangle_{\text{mat}} = 1$$

Vacuum energy for half-brane solution

- Noting the structure of the CP factor, we have

$$S'[\Phi_H] = \frac{1}{6}(-A_1 + A_2),$$

$$A_1 = (cG'Bc, \hat{Q}'(cG'Bc))', \quad A_2 = (cG'BcG', \hat{Q}'(cG'Bc)G')'$$

$$(\Phi, \Psi)' \equiv \langle\langle \Phi \frac{1}{1+K'} \Psi \frac{1}{1+K'} \rangle\rangle = \int_0^\infty dt \int_0^\infty ds\, e^{-t-s} \langle\langle \Phi e^{-tK'} \Psi e^{-sK'} \rangle\rangle$$

- Thanks to the *J*-independence of the matter sector, computations are almost the same as [Erler(2010)] using ***G′K′Bcγ*** algebra.

- The result is the same as that of the undeformed background:

$$A_1 = \frac{3}{\pi^2} - \frac{24}{\pi^4}, \quad A_2 = \frac{9}{2\pi^2} - \frac{24}{\pi^4} \quad \Rightarrow \quad S'[\Phi_H] = \frac{1}{4\pi^2}$$

a half of a D brane tension

Gauge invariant overlaps for the solutions

- Definition: $\langle\!\langle \Phi \rangle\!\rangle_{\mathcal{V}} = \frac{1}{2}\mathrm{Tr}(\sigma_3 \langle I | \mathcal{V}(i) | \Phi \rangle)$

  $$\mathcal{V}(i) = c(i)c(-i)\delta(\gamma(i))\delta(\gamma(-i))V_{\mathrm{m}}(i,-i)$$

  $V_{\mathrm{m}}(z,\bar{z})$: a matter primary field with dim (1/2,1/2)

- Note: $\langle\!\langle \Phi\Psi \rangle\!\rangle_{\mathcal{V}} = (-1)^{(\epsilon(\Phi)+F(\Phi))(\epsilon(\Psi)+F(\Psi))}\langle\!\langle \Psi\Phi \rangle\!\rangle_{\mathcal{V}},$

  $$\langle\!\langle \hat{Q}'(\cdots) \rangle\!\rangle_{\mathcal{V}} = 0.$$

- In the same way as [Erler(2010)], we can evaluate the gauge invariant overlaps for the solutions using $G'K'Bc\gamma$ algebra:

$$\langle\!\langle \Phi_T \rangle\!\rangle_{\mathcal{V}} = \langle\!\langle c\, e^{-K'} \rangle\!\rangle_{\mathcal{V}} \qquad \langle\!\langle \Phi_H \rangle\!\rangle_{\mathcal{V}} = \frac{1}{2}\langle\!\langle c\, e^{-K'} \rangle\!\rangle_{\mathcal{V}}$$

Evaluation of the gauge invariant overlap

- Performing calculation in the ghost sector, we have more explicit expression in the sliver frame:

$$\langle\!\langle \Phi_T \rangle\!\rangle_{\mathcal{V}} = 2\langle\!\langle \Phi_H \rangle\!\rangle_{\mathcal{V}}$$
$$= \frac{e^{-\frac{\pi}{2}\mathcal{C}}}{\pi} \lim_{M\to\infty} \frac{e^{2M}}{4} \left\langle \exp\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du \int_{-\infty}^{\infty} dt' f_a(t') \tilde{J}^a(2it'+u) \right) \tilde{V}_{\mathrm{m}}(iM, -iM) \right\rangle_{\mathrm{mat}}$$

- If there is no correlation between J and V_{m} , the value is the same as the original theory thanks to the J-independence formula.

Ex. $J \sim \partial X^9$ $\quad V_{\mathrm{m}} \sim \psi^0 \bar{\psi}^0$

- However, generally, the above gauge invariant overlap may depend on the current J.

Phase shift for the solutions in the gauge invariant overlap

- Let us consider the case:

$$\boldsymbol{J = \frac{i}{\sqrt{2\alpha'}}\partial X^9} \qquad \boldsymbol{V_{\mathrm{m}}(i,-i) = e^{\frac{i}{2}k_9 X^9(i)} e^{-\frac{i}{2}k_9 X^9(-i)}}$$

$$\boldsymbol{(k_9)^2 = 2/\alpha'}$$: on-shell condition

- The result of explicit computation is

$$\boldsymbol{\langle\!\langle \Phi_T \rangle\!\rangle_{\mathcal{V}} = 2\langle\!\langle \Phi_H \rangle\!\rangle_{\mathcal{V}} = \frac{1}{2\pi i}\exp\left(i\pi k_9\sqrt{\alpha'}\int_{C_{\mathrm{L}}}\frac{dz}{2\pi i}F(z)\right)}$$

Phase factor due to the deformed background

Field redefinition induced by an identity-based marginal solution

- Let us consider the identity-based marginal solution Ψ_J corresponding to $J(z,\theta) = \psi^9(z) + \theta \frac{i}{\sqrt{2\alpha'}} \partial X^9(z)$
- The BRST operator around it can be rewritten as [I.K.-Takahashi(2005)]

$$Q' = e^{\frac{i}{2\sqrt{\alpha'}} X(F)} Q_{\mathrm{B}} e^{-\frac{i}{2\sqrt{\alpha'}} X(F)} \qquad X(F) = \oint \frac{dz}{2\pi i} F(z) X^9(z)$$

- It induces a field redefinition:

$$\Phi = e^{\frac{i}{2\sqrt{\alpha'}} X(F)} \Phi' = e^{\frac{i}{2\sqrt{\alpha'}} X_L(F) I} * \Phi' * e^{-\frac{i}{2\sqrt{\alpha'}} X_L(F) I}$$

$$X_L(F) = \int_{C_{\mathrm{L}}} \frac{dz}{2\pi i} F(z) X^9(z)$$

- The action can be rewritten as

$$S'[\Phi] = \frac{1}{2} \langle\langle \Phi \, \hat{Q}' \Phi \rangle\rangle + \frac{1}{3} \langle\langle \Phi^3 \rangle\rangle = \frac{1}{2} \langle\langle \Phi' \, \hat{Q} \Phi' \rangle\rangle + \frac{1}{3} \langle\langle \Phi'^3 \rangle\rangle = S[\Phi']$$

Gauge invariant overlap and field redefinition

- We consider a gauge invariant overlap with

$$V_{\mathrm{m}}(i,-i)=e^{\frac{i}{2}k_9X^9(i)}e^{-\frac{i}{2}k_9X^9(-i)}$$

- In this case, we find

$$\langle I|\mathcal{V}(i)\propto\langle 0|\exp\left(-\sum_{n=1}^{\infty}(-1)^n\frac{1}{2n}(\alpha_n^9)^2-\sum_{n=1}^{\infty}\frac{2i\sqrt{2\alpha'}(-1)^n}{2n-1}k_9\alpha_{2n-1}^9\right)$$

- Using the above, the gauge invariant overlap is evaluated as

$$\langle\langle\Phi\rangle\rangle_{\mathcal{V}}=\frac{1}{2}\mathrm{Tr}(\sigma_3\langle I|\mathcal{V}(i)\,e^{\frac{i}{2\sqrt{\alpha'}}X(F)}|\Phi'\rangle)=\exp\left(i\pi\sqrt{\alpha'}k_9\int_{C_{\mathrm{L}}}\frac{dz}{2\pi i}F(z)\right)\langle\langle\Phi'\rangle\rangle_{\mathcal{V}}$$

This is the same phase factor as that of two solutions around the deformed background corresponding to the bosonic case in [Katsumata-Takahashi-Zeze(2004)].

Concluding remarks I

- We have constructed some solutions of the theory around an identity-based marginal solution Ψ_J in cubic SSFT using $G'K'Bc\gamma$ algebra.
- Around Ψ_J , the BRST operator is deformed: $Q_{\mathrm{B}} \to Q'$
- Correspondingly, string fields are deformed: $G, K \to G', K'$
- $GKBc\gamma$ and $G'K'Bc\gamma$ have the same algebraic structure.
- We have explicitly computed vacuum energies for **tachyon vacuum** Φ_T and **half-brane** Φ_H solutions in $G'K'Bc\gamma$ algebra and it turned out that they are the same as those of the original theory.
- It implies **vanishing vacuum energy for the identity-based marginal solution** Ψ_J as in the bosonic case although it seems to be difficult to evaluate it directly.

Concluding remarks II

- We also evaluated the gauge invariant overlap for those solutions and found the relation: $\langle\langle \boldsymbol{\Phi_T} \rangle\rangle_{\mathcal{V}} = \mathbf{2}\langle\langle \boldsymbol{\Phi_H} \rangle\rangle_{\mathcal{V}}$ which is the same as that of the undeformed background.
- If we take a closed tachyon vertex of Dirichlet type for the gauge invariant overlap for Φ_T and Φ_H, the **phase factor appears** according to the marginal deformation.
- The phase shift is consistent with the value due to a field redefinition induced by the identity-based marginal solution Ψ_J.
- These results just correspond to the bosonic ones [T.Takahashi's talk].
- It may be interesting to investigate the algebra in the theory around the identity-based universal solution in SSFT, which has a homotopy operator, constructed in [Inatomi-I.K.-Takahashi(2011)] in SSFT.

Comment on level truncation

- Level truncation of the Erler-Schnabl solution in bosonic SFT（evaluation of kinetic term: Erler-Schnabl, including cubic term: Arroyo-I.K.(2011-2012))

L	$\tilde{E}_2$	$\tilde{E}_{2,\mathrm{P}}\vert_L^L$	$\tilde{E}_{2,\mathrm{PB}}\vert_L^L$	$\tilde{E}$	$\tilde{E}_{\mathrm{P}}\vert_{3L/2}^{3L/2}$	$\tilde{E}_{\mathrm{PB}}\vert_{3L/2}^{3L/2}$
0	−0.85247	−0.85247	−0.85247	−0.654908	−0.654908	−0.654908
2	−0.914146	−0.85247	−0.85247	−1.33686	−1.38342	−1.38798
4	−1.03467	−0.787834	−0.871988	−0.532599	−0.421667	−0.358173
6	−0.930637	−0.787834	−0.871988	−1.55434	−1.19306	−1.08516
8	−1.06335	−0.992052	−0.983242	−0.167462	−1.14097	−1.00745
10	−0.904984	−0.992052	−0.983242	−1.87271	−0.919443	−1.07258
12	−1.10973	−0.992013	−0.984516	−0.166042	−0.850702	−1.05767*
14	−0.841643	−0.992013	−0.984516	−1.83972	−0.972165	−0.933839**
16	−1.20564	−0.99608	−0.993936	+1.83619	−1.00666	−0.92572*
18	−0.709632	−0.99608	−0.993933	−4.22806	−1.01865	−0.981341**
20	−1.39169	−0.999595	−0.993687	−1.1971	−1.02464	−1.01792*
22	−0.449641	−0.999595	−0.993574	−0.188021	−0.994601	−1.00019**
24	−1.75829	−0.997321	−0.995001	+12.4404	−0.997754	−1.01338**
26	+0.0590993	−0.997321	−0.993171	−24.5744	−0.999148	−1.02392**
28	−2.46306	−0.99769	−0.993253			
30	+1.03342	−0.99769	−0.989787			

(L,3L) truncation

$$\tilde{E} = 2\pi^2 E = -2\pi^2 S[\Phi]$$

P: Pade approximation, PB: Pade-Borel approximation