On gauge invariant observables for identity-based marginal solutions in bosonic and super string field theory

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References

This talk is based on

I. K. and Tomohiko Takahashi,
"Comments on observables for identity-based marginal solutions in Berkovits' superstring field theory,"
JHEP07(2014)031 [arXiv:1404.4427]

and references therein:

I. K. and T. T. (2005, 2013); Shoko Inatomi, I. K. and T. T. (2012).

cf. Erler, Ishibashi and Maccaferri's talks

Contents

Introduction and summary

In my previous talk@SFT2012, we have constructed [IKT(2012)] nontrivial solutions around identity-based marginal solutions in modified cubic SSFT using the "$G'K'Bc\gamma$" algebra.

Some time ago in [KT(2005)], we constructed identity-based marginal solution Φ_J in Berkovits' WZW-like SSFT.
Recntly, Erler has constructed the tachyon vacuum solution $\Phi_T^{\rm E}$ in Berkovits' WZW-like SSFT.

$\Downarrow$

◯ We will construct the tachyon vacuum solution Φ_T around the identity-based marignal solution Φ_J, using a version of the extended KBc algebra in the framework of Berkovits' WZW-like SSFT.

It is *difficult* to evaluate vacuum energy and gauge invariant overlaps for "identity-based" solutions because of singular property of the identity state: $\langle I|(\cdots)|I\rangle$, corresponding to *zero width* in the sliver frame, is indefinite at least naively.

Instead of straightforward calculations, there have been various studies about the gauge invariants indirectly for identity-based solutions in bosonic SFT. (2001∼, cf. Takahashi's talks)

$$\updownarrow$$

On the other hand, gauge invariants for "wedge-based" solutions using KBc algebra and its extention have been evaluated exactly. ([Schnabl(2005)]∼)

In our previous paper [KT(2013)], we have evaluated the gauge invariant overlaps (GIO), $\langle\Psi\rangle_{\mathcal{V}}$, for *identity-based* marginal solution Ψ_J in *bosonic* SFT:

$$\langle\Psi_J\rangle_{\mathcal{V}} = \langle\Psi_T^{\mathrm{ES}}\rangle_{\mathcal{V}} - \langle\Psi_T\rangle_{\mathcal{V}}$$

using the relationship to (wedge-based) tachyon vacuum solutions:

$$\boxed{\Psi_J = \Psi_T^{\mathrm{ES}} - \Psi_T + \int_0^1 Q_{\Psi_T^t}\Lambda_t dt.}$$

In fact, we can show that Ψ_T^{ES} [Erler-Schnabl(2009)] and the sum of Ψ_J [Takahashi-Tanimoto(2001)] and Ψ_T [IKT(2012)] are gauge equivalent:

$$\Psi_J + \Psi_T = g^{-1}\Psi_T^{\mathrm{ES}} g + g^{-1} Q_{\mathrm{B}} g, \qquad g = \mathrm{P}\exp\left(\int_0^1 \Lambda_t dt\right)$$

$$\rightarrow \qquad S[\Psi_J; Q_{\mathrm{B}}] = S[\Psi_T^{\mathrm{ES}}; Q_{\mathrm{B}}] - S[\Psi_T; Q_{\Psi_J}] = 0$$

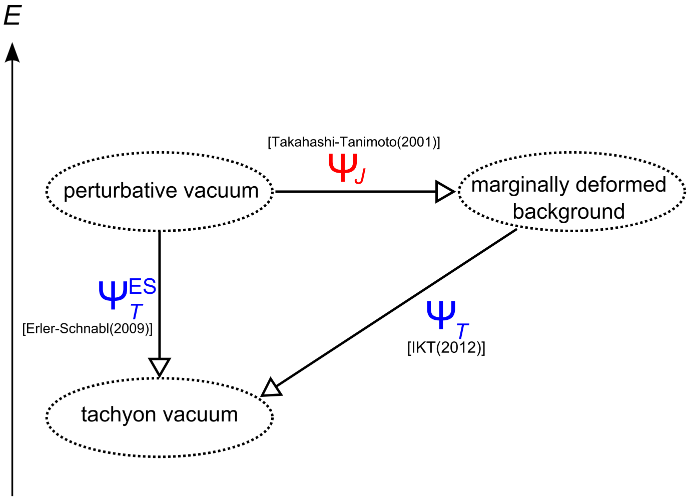

Figure : Identity/wedge-based solutions in bosonic SFT

$\Downarrow$ extention to SSFT

$$e^{-\Phi_J}\hat{Q}e^{\Phi_J} = e^{-\Phi_T^{\mathrm{E}}}\hat{Q}e^{\Phi_T^{\mathrm{E}}} - e^{-\Phi_T}\hat{Q}_{\Phi_J}e^{\Phi_T} + \int_0^1 \hat{Q}_{\tilde{\Phi}_T(t)}\Lambda_t\, dt$$

Using this relation among identity-based marginal solution Φ_J and wedge-based tachyon vacuum solutions, Φ_T^{E} and Φ_T, the GIO for identity-based marginal solution can be evaluated:

$$\langle \Phi_J \rangle_{\mathcal{V}} = \langle \Phi_T^{\mathrm{E}} \rangle_{\mathcal{V}} - \langle \Phi_T \rangle_{\mathcal{V}}.$$

Actually, Φ_T^{E} and $\log(e^{\Phi_J}e^{\Phi_T})$ are *gauge equivalent* and we have

$$S[\Phi_J;\hat{Q}] = S[\Phi_T^{\mathrm{E}};\hat{Q}] - S[\Phi_T;\hat{Q}_{\Phi_J}] = 1/(2\pi^2) - 1/(2\pi^2) = 0.$$

The energy for the identity-based marginal solution Φ_J is zero, which agrees with the previous result as a consequence of ξ zeromode counting [KT(2005)].

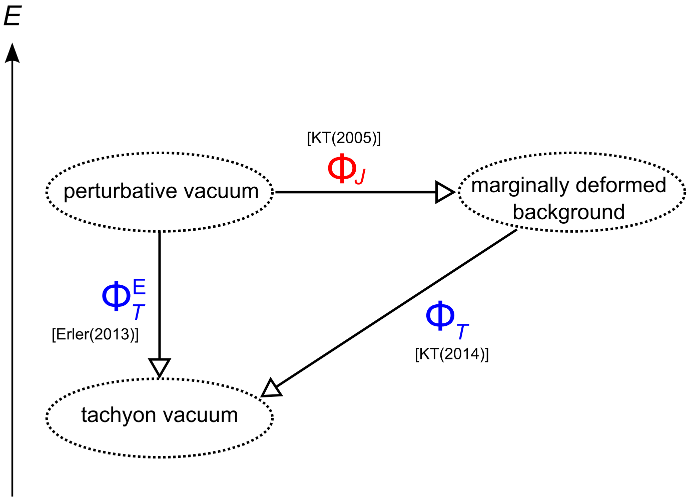

Figure : Identity/wedge-based solutions in Berkovits' WZW-like SSFT including GSO($-$) sector

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Identity-based marginal solutions

Identity-based marginal solution in Berkovits' WZW-like SSFT:

$$\Phi_J = \tilde{V}_L^a(F_a)I,$$

$$\tilde{V}_L^a(f) \equiv \int_{C_{\rm L}} \frac{dz}{2\pi i} f(z) \frac{1}{\sqrt{2}} c\gamma^{-1}\psi^a(z), \quad \gamma^{-1}(z) = e^{-\phi}\xi(z).$$

$F_a(-1/z) = z^2 F_a(z)$, $C_{\rm L}$: a half unit circle: $|z| = 1$, ${\rm Re}\, z \geq 0$

I: the identity state, ψ^a: matter worldsheet fermion.

$\psi^a(y)\psi^b(z) \sim \frac{1}{y-z}\frac{1}{2}\Omega^{ab}$, $J^a(y)\psi^b(z) \sim \frac{1}{y-z} f^{ab}{}_c \psi^c(z)$,

$J^a(y)J^b(z) \sim \frac{1}{(y-z)^2}\frac{1}{2}\Omega^{ab} + \frac{1}{y-z} f^{ab}{}_c J^c(z)$,

$\Omega^{ab} = \Omega^{ba}$, $f^{ab}{}_c\Omega^{cd} + f^{ad}{}_c\Omega^{cb} = 0$, $f^{ab}{}_c = -f^{ba}{}_c$, $f^{ab}{}_d f^{cd}{}_e + f^{bc}{}_d f^{ad}{}_e + f^{ca}{}_d f^{bd}{}_e = 0$.

EOM in the NS sector is satisfied:

$$\eta_0(e^{-\Phi_J} Q_{\rm B} e^{\Phi_J}) = 0.$$

By expanding the NS action $S[\Phi;Q_{\rm B}]$ of Berkovits' WZW-like SSFT around Φ_J as

$$e^{\Phi}=e^{\Phi_J}e^{\Phi'},$$

we have

$$S[\Phi;Q_{\rm B}]=S[\Phi_J;Q_{\rm B}]+S[\Phi';Q_{\Phi_J}],$$

where $S[\Phi';Q_{\Phi_J}]$ is given by a deformed BRST operator:

$$Q_{\Phi_J}=Q_{\rm B}-V^a(F_a)+\frac{1}{8}\Omega^{ab}C(F_aF_b).$$

$V^a(F_a)$ and $C(F_aF_b)$ are given by integrations along the whole unit circle:

$$V^a(f)\equiv\oint\frac{dz}{2\pi i}\frac{1}{\sqrt{2}}f(z)(cJ^a(z)+\gamma\psi^a(z)),\quad C(f)\equiv\oint\frac{dz}{2\pi i}f(z)c(z).$$

Deformed algebra

A version of the extended KBc algebra with

$Q_{\rm B} \to Q' \equiv Q_{\Phi_J}$

$L_n \to L'_n = \{Q', b_n\} = L_n - \frac{1}{\sqrt{2}}\sum_{k\in\mathbb{Z}} F_{a,k} J^a_{n-k} + \frac{1}{8}\Omega^{ab}\sum_{k\in\mathbb{Z}} F_{a,n-k}F_{b,k}$

$F_{a,n} \equiv \oint \frac{d\sigma}{2\pi} e^{i(n+1)\sigma} F_a(e^{i\sigma}), \quad F_{a,n} = -(-1)^n F_{a,-n}$

Relations among string fields K', B, c, γ:

$$B^2 = 0,\ c^2 = 0,\ Bc + cB = 1, BK' = K'B,$$

$$K'c - cK' = Kc - cK \equiv \partial c,\ \gamma B + B\gamma = 0,\ c\gamma + \gamma c = 0,$$

$$K'\gamma - \gamma K' = K\gamma - \gamma K \equiv \partial\gamma,\ \hat{Q}'B = K',\ \hat{Q}'K' = 0,$$

$$\hat{Q}'c = cK'c - \gamma^2 = cKc - \gamma^2 = c\partial c - \gamma^2,\ \hat{Q}'\gamma = \hat{Q}\gamma = c\partial\gamma - \frac{1}{2}(\partial c)\gamma$$

where $\hat{Q}' \equiv Q'\sigma_3, \hat{Q} \equiv Q_{\rm B}\sigma_3$ and σ_i are Pauli matrices (CP factor) for the GSO$(-)$ sector

$B = \frac{\pi}{2}B_1^L I\sigma_3, \quad c = \frac{2}{\pi}\hat{U}_1\tilde{c}(0)|0\rangle\sigma_3, \quad \gamma = \sqrt{\frac{2}{\pi}}\hat{U}_1\tilde{\gamma}(0)|0\rangle\sigma_2 \quad K' = \frac{\pi}{2}K_1'^L I, \quad K = \frac{\pi}{2}K_1^L I$

For the string fields γ^{-1}, ζ, V, we have

$$\gamma^{-1}\gamma = \gamma\gamma^{-1} = 1, \quad \gamma^{-1}B + B\gamma^{-1} = 0, \quad \gamma^{-1}c + c\gamma^{-1} = 0,$$
$$K'\gamma^{-1} - \gamma^{-1}K' = K\gamma^{-1} - \gamma^{-1}K \equiv \partial\gamma^{-1},$$
$$\hat{Q}'\gamma^{-1} = \hat{Q}\gamma^{-1} = c\partial\gamma^{-1} + \frac{1}{2}(\partial c)\gamma^{-1}, \qquad \hat{Q}'\zeta = \hat{Q}\zeta = cV + \gamma$$

where

$\gamma^{-1} = \sqrt{\frac{\pi}{2}}\hat{U}_1\tilde{\gamma}^{-1}(0)|0\rangle\sigma_2, \quad \zeta = \gamma^{-1}c = \sqrt{\frac{2}{\pi}}\hat{U}_1\tilde{\gamma}^{-1}\tilde{c}(0)|0\rangle i\sigma_1,$
$V = \frac{1}{2}\gamma^{-1}\partial c = \sqrt{\frac{\pi}{2}}\hat{U}_1\frac{1}{2}\tilde{\gamma}^{-1}\tilde{\partial}\tilde{c}(0)|0\rangle i\sigma_1.$

K', B, c, γ, γ^{-1}, ζ, V and $\hat{Q}'$ have the same algebraic structure as that of the extended KBc algebra with $\hat{Q}$ [Erler(2013)].

From the result in [Erler(2013)] and the above algebra, we can immediately construct a solution Φ_T in the theory with $\hat{Q}'$:

$$e^{\Phi_T} = 1 - c\frac{B}{1+K'} + q\left(\zeta + (\hat{Q}'\zeta)\frac{B}{1+K'}\right)$$

(q is a nonzero constant.) Actually, Φ_T satisfies

$$e^{-\Phi_T}\hat{Q}'e^{\Phi_T} = c - (\hat{Q}'c)\frac{B}{1+K'} = (c + \hat{Q}'(Bc))\frac{1}{1+K'}$$

which is in the *small* Hilbert space, and therefore the EOM in the NS sector holds:

$$\hat{\eta}(e^{-\Phi_T}\hat{Q}'e^{\Phi_T}) = 0$$

($\hat{\eta} \equiv \eta_0\sigma_3$)

Expanding the action $S[\Phi';\hat{Q}']$ around the solution Φ_T as $e^{\Phi'} = e^{\Phi_T} e^{\Phi''}$, we have a new BRST operator $\hat{Q}'_{\Phi_T}$:

$$\hat{Q}'_{\Phi_T}\,\Xi = \hat{Q}'\,\Xi + (e^{-\Phi_T}\hat{Q}'e^{\Phi_T})\Xi - (-1)^{|\Xi|}\Xi(e^{-\Phi_T}\hat{Q}'e^{\Phi_T}).$$

Note that $e^{-\Phi_T}\hat{Q}'e^{\Phi_T}$ is the tachyon vacuum solution on the marginally deformed background in the modified cubic SSFT.

$\rightarrow$ We can find a homotopy operator $\hat{A}'$ for $\hat{Q}'_{\Phi_T}$:
$\hat{A}'\,\Xi = \frac{1}{2}(A'\,\Xi + (-1)^{|\Xi|}\Xi\,A')$, such as $\{\hat{Q}'_{\Phi_T}, \hat{A}'\} = 1,\ (\hat{A}')^2 = 0$, where $A' \equiv \dfrac{B}{1+K'}$ is a homotopy state in the *small* Hilbert space.

$\rightarrow$ No physical open string state around the solution Φ_T
$\sim$ the tachyon vacuum solution in the marginally deformed background.

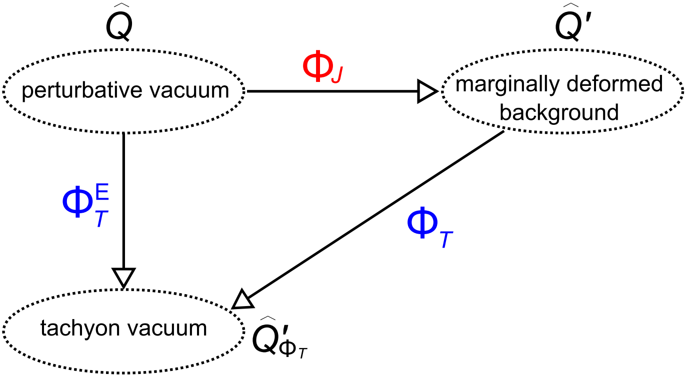

Figure : Identity/wedge-based solutions and BRST operators around them. $\hat{Q}'_{\Phi_T}$ has no cohomology in the small Hilbert space.

Energy and gauge invariant overlaps

NS action $S[\Phi';\hat{Q}_{\Phi_J}]$ around Φ_J:

$$S[\Phi';\hat{Q}_{\Phi_J}] = -\int_0^1 dt\,\mathrm{Tr}\left[\left(\hat{\eta}(g(t)^{-1}\partial_t g(t))\right)\left(g(t)^{-1}\hat{Q}_{\Phi_J}g(t)\right)\right].$$

$g(t)$: an interpolating string field s.t. $g(0)=1$ and $g(1)=e^{\Phi'}$.
$\mathrm{Tr}\,A \equiv \frac{1}{2}\mathrm{tr}\langle I|A\rangle$; tr : trace for the Chan-Paton factor for GSO($\pm$) sector

We take an interpolating string field as $g_T(t)=1+t(e^{\Phi_T}-1)$ and the integrand in the action for the solution: $S[\Phi_T;\hat{Q}_{\Phi_J}]$, can be manipulated in the same way as the Erler solution, with $Q_{\mathrm{B}}\to Q'\equiv Q_{\Phi_J},\ L_n\to L'_n$. As a result, we have

$$\begin{aligned}&\mathrm{Tr}\left[\left(\hat{\eta}(g_T(t)^{-1}\partial_t g_T(t))\right)\left(g_T(t)^{-1}\hat{Q}_{\Phi_J}g_T(t)\right)\right]\\ &=\frac{-2q^2t^2(1-t)(2q^2t-1)}{(1-t+q^2t^2)^3}\int_0^1 d\theta(1-\theta)\mathcal{X}(\theta)+\frac{q^2t(1-t)}{(1-t+q^2t^2)^2}\int_0^1 d\theta\mathcal{X}(\theta)\end{aligned}$$

where $\quad \mathcal{X}(\theta)=\mathrm{Tr}\Big[B(\hat{\eta}(cV))e^{-\theta K'}cVe^{-(1-\theta)K'}\Big].$

We use the result for the modified cubic SSFT in the marginally deformed background [IKT(2012)]:

$$e^{-\alpha K'} = e^{-\alpha\frac{\pi}{2}\mathcal{C}}\hat{U}_{\alpha+1}\mathbf{T}\exp\left(\frac{\pi}{4}\int_{-\alpha}^{\alpha}du\int_{-\infty}^{\infty}dv f_a(v)\tilde{J}^a(iv+\frac{\pi}{4}u)\right)|0\rangle,$$

$$f_a(v) \equiv \frac{F_a(\tan(iv+\frac{\pi}{4}))}{2\pi\sqrt{2}\cos^2(it+\frac{\pi}{4})}, \qquad \mathcal{C} \equiv \frac{\pi}{2}\int_{-\infty}^{\infty}dv\,\Omega^{ab}f_a(v)f_b(v),$$

where $\mathbf{T}$ is an ordering symbol with respect to the real part of the argument of $\tilde{J}^a$.

Finally, the trace can be evaluated as

$$\begin{aligned}
&\mathrm{Tr}\Big[B(\hat{\eta}(cV))e^{-\theta K'}cVe^{-(1-\theta)K'}\Big]\\
&= -\frac{1}{4}\left\langle(\eta_0\gamma^{-1}(\frac{\pi}{2}))\gamma^{-1}(\frac{\pi}{2}(1-2\theta))\right\rangle_{\xi\eta\phi}\left\langle B_1^L c\partial c(\frac{\pi}{2})c\partial c(\frac{\pi}{2}(1-2\theta))\right\rangle_{bc}\\
&\qquad\times e^{-\frac{\pi}{2}\mathcal{C}}\left\langle\exp\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}du\int_{-\infty}^{\infty}dv f_a(v)J^a(2iv+u)\right)\right\rangle_{\mathrm{mat}}
\end{aligned}$$

The last factor in the trace, which comes from the matter sector, is 1:

$$e^{-\frac{\pi}{2}\mathcal{C}}\left\langle \exp\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}du\int_{-\infty}^{\infty}dv f_a(v)J^a(2iv+u)\right)\right\rangle_{\mathrm{mat}}=1$$

as proved in [IKT(2012)], and so the trace becomes the same result as the case of the Erler solution.

Consequently, the vacuum energy is unchanged from the case in the original background without marginal deformations, namely,

$$E=-S[\Phi_T;\hat{Q}_{\Phi_J}]=-1/(2\pi^2).$$

Next, we will evaluate the gauge invariant overlap (GIO) in Berkovits' WZW-like SSFT. We define the GIO $\langle\Phi\rangle_{\mathcal{V}}$ as

$$\langle\Phi\rangle_{\mathcal{V}} \equiv \mathrm{Tr}[\mathcal{V}(i)\Phi].$$

$\mathcal{V}(i)$: a midpoint insertion of a primary closed string vertex operator with picture #: -1, ghost #: 2, conformal dim.: $(0,0)$, BRST invariant in the small Hilbert space: $[Q_{\mathrm{B}},\mathcal{V}(i)]=0$, $[\eta_0,\mathcal{V}(i)]=0$. Then, $\forall\Lambda,\Xi$

$$\langle\hat{Q}\Lambda\rangle_{\mathcal{V}}=0,\qquad \langle\hat{\eta}\Lambda\rangle_{\mathcal{V}}=0,\qquad \langle\Lambda\Xi\rangle_{\mathcal{V}}=(-)^{|\Lambda||\Xi|}\langle\Xi\Lambda\rangle_{\mathcal{V}}.$$

Infinitesimal gauge transformation: $\delta_\Lambda e^{\Phi}=(\hat{Q}\Lambda_0)e^{\Phi}+e^{\Phi}\hat{\eta}\Lambda_1$
(Λ_0, Λ_1: gauge parameter string field with the picture # $0,1$.) Namely,

$$\delta_\Lambda\Phi=\frac{\mathrm{ad}_\Phi}{e^{\mathrm{ad}_\Phi}-1}\hat{Q}\Lambda_0+\frac{-\mathrm{ad}_\Phi}{e^{-\mathrm{ad}_\Phi}-1}\hat{\eta}\Lambda_1=\sum_{n=0}^{\infty}\frac{B_n}{n!}(\mathrm{ad}_\Phi)^n(\hat{Q}\Lambda_0+(-1)^n\hat{\eta}\Lambda_1)$$

$(\mathrm{ad}_B(A)\equiv[B,A]=BA-AB)$

Thanks to the above properties, the GIO is invariant under this gauge transformation: $\langle \delta_\Lambda \Phi \rangle_{\mathcal{V}} = 0$.

Inserting $1 = \{Q_{\rm B}, \xi Y(i)\}$, ($Y(z) = c\partial\xi e^{-2\phi}(z)$ the inverse picture changing operator) the GIO can be rewritten as

$$\begin{aligned}\langle \Phi \rangle_{\mathcal{V}} &= \mathrm{Tr}[\mathcal{V}(i)\{Q_{\rm B}, \xi Y(i)\}\Phi] = \mathrm{Tr}[\xi Y \mathcal{V}(i)\sigma_3 \hat{Q}\Phi] = \mathrm{Tr}[\xi Y \mathcal{V}(i)\sigma_3 \hat{Q}'\Phi] \\ &= \mathrm{Tr}[\xi Y \mathcal{V}(i)\sigma_3\, e^{-\Phi}\hat{Q}' e^{\Phi}].\end{aligned}$$

GIO for the tachyon vacuum Φ_T: $\langle \Phi_T \rangle_{\mathcal{V}} = \mathrm{Tr}\left[\xi Y \mathcal{V}(i)\sigma_3\, c\dfrac{1}{1+K'}\right]$

In a similar way to the calculation of the vacuum energy, we obtain an expression of the GIO in the marginally deformed background:

$$\langle \Phi_T \rangle_{\mathcal{V}} = \frac{e^{-\pi\mathcal{C}}}{\pi}\left\langle \xi Y \mathcal{V}(i\infty) c(\frac{\pi}{2}) \exp\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du\, \mathcal{J}(u)\right)\right\rangle_{C_\pi}$$

$$\mathcal{J}(u) = \int_{-\infty}^{\infty} dv\, f_a(v) J^a(iv+u)$$

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Interpolation

Let us calculate two observables, the vacuum energy and the GIO, for the *identity-based* marginal solutions.

An interpolation: $\Phi_J(t) = t\Phi_J$ s.t. $\Phi_J(0) = 0,\ \Phi_J(1) = \Phi_J$.
$\Phi_J(t)$: a replacement of weighting function: $F_a(z) \to tF_a(z)$ in Φ_J.
$\Rightarrow$ EOM is satisfied: $\hat{\eta}(e^{-\Phi_J(t)}\hat{Q}e^{\Phi_J(t)}) = 0$

A new BRST operator $Q_{\Phi_J(t)}$ for the theory around $\Phi_J(t)$:

$$Q_{\Phi_J(t)} = Q_{\mathrm{B}} - tV^a(F_a) + \frac{t^2}{8}\Omega^{ab}C(F_aF_b)$$

Following the same procedure as before, we define a string field $K'(t) \equiv \hat{Q}_{\Phi_J(t)}B$ ($\hat{Q}_{\Phi_J(t)} \equiv Q_{\Phi_J(t)}\sigma_3$). Then, we can construct a tachyon vacuum solution $\Phi_T(t)$ as

$$e^{\Phi_T(t)} = 1 - c\frac{B}{1+K'(t)} + q\left(\zeta + (\hat{Q}_{\Phi_J(t)}\zeta)\frac{B}{1+K'(t)}\right)$$

It satisfies the EOM around the identity-based solution $\Phi_J(t)$:

$$\hat{\eta}(e^{-\Phi_T(t)}\hat{Q}_{\Phi_J(t)}e^{\Phi_T(t)}) = 0.$$

In particular, $\Phi_T(t)$ satisfies $\Phi_T(1) = \Phi_T$ and $\Phi_T(0) = \Phi_T^{\mathrm{E}}$ (the Erler solution (2013)) because $Q_{\Phi_J(1)} = Q_{\Phi_J}$ and $Q_{\Phi_J(0)} = Q_{\mathrm{B}}$.

Using the above string fields, we define a string field $\tilde{\Phi}_T(t)$ with the parameter t as $e^{\tilde{\Phi}_T(t)} \equiv e^{\Phi_J(t)}e^{\Phi_T(t)}$ and then we find a relation:

$$e^{-\tilde{\Phi}_T(t)}\hat{Q}e^{\tilde{\Phi}_T(t)} = e^{-\Phi_J(t)}\hat{Q}e^{\Phi_J(t)} + e^{-\Phi_T(t)}\hat{Q}_{\Phi_J(t)}e^{\Phi_T(t)}.$$

Hence, $\tilde{\Phi}_T(t)$ satisfies the EOM of the *original* theory:

$$\hat{\eta}(e^{-\tilde{\Phi}_T(t)}\hat{Q}e^{\tilde{\Phi}_T(t)}) = 0.$$

Expanding around the solution $\tilde{\Phi}_T(t)$ in the theory with $\hat{Q}$, we have the theory with the deformed BRST operator $\hat{Q}_{\tilde{\Phi}_T(t)}$:

$$\begin{aligned}\hat{Q}_{\tilde{\Phi}_T(t)}\Xi &= \hat{Q}\Xi + (e^{-\tilde{\Phi}_T(t)}\hat{Q}e^{\tilde{\Phi}_T(t)})\Xi - (-1)^{|\Xi|}\Xi(e^{-\tilde{\Phi}_T(t)}\hat{Q}e^{\tilde{\Phi}_T(t)}) \\ &= \hat{Q}_{\Phi_J(t)}\Xi + (e^{-\Phi_T(t)}\hat{Q}_{\Phi_J(t)}e^{\Phi_T(t)})\Xi - (-1)^{|\Xi|}\Xi(e^{-\Phi_T(t)}\hat{Q}_{\Phi_J(t)}e^{\Phi_T(t)}).\end{aligned}$$

The last expression implies that $\hat{Q}_{\tilde{\Phi}_T(t)}$ is the same as the BRST operator $\hat{Q}'_{\Phi_T(t)}$ in the theory around $\Phi_T(t)$, which is a tachyon vacuum solution in the theory around $\Phi_J(t)$.

Following the previous results with appropriate replacement, we find that there exists a homotopy state: $A'(t) \equiv \frac{B}{1+K'(t)}$ such as $\hat{Q}_{\tilde{\Phi}_T(t)}A'(t) = 1$, which implies that

there is no cohomology for $\hat{Q}_{\tilde{\Phi}_T(t)}$ in the small Hilbert space.

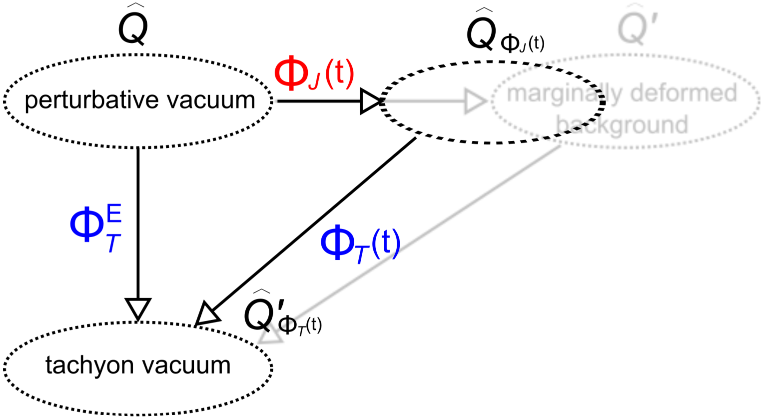

Figure : Interporating identity/wedge-based solutions and BRST operators around them. With $e^{\tilde{\Phi}_T(t)} \equiv e^{\Phi_J(t)} e^{\Phi_T(t)}$, a BRST operator around $\tilde{\Phi}_T(t)$ $\hat{Q}'_{\Phi_T(t)} = \hat{Q}_{\tilde{\Phi}_T(t)}$ has no cohomology in the small Hilbert space.

Differentiating an identity: $\hat{Q}(e^{-\tilde{\Phi}_T(t)}\hat{Q}e^{\tilde{\Phi}_T(t)}) + (e^{-\tilde{\Phi}_T(t)}\hat{Q}e^{\tilde{\Phi}_T(t)})^2 = 0$ with respect to t, we have

$$\hat{Q}_{\tilde{\Phi}_T(t)}\frac{d}{dt}(e^{-\tilde{\Phi}_T(t)}\hat{Q}e^{\tilde{\Phi}_T(t)}) = 0.$$

Therefore, there exists a state Λ_t in the small Hilbert space such as

$$\frac{d}{dt}(e^{-\tilde{\Phi}_T(t)}\hat{Q}e^{\tilde{\Phi}_T(t)}) = \hat{Q}_{\tilde{\Phi}_T(t)}\Lambda_t.$$

Integrating the above, we have

$$e^{-\tilde{\Phi}_T(1)}\hat{Q}e^{\tilde{\Phi}_T(1)} = e^{-\Phi_T^{\mathrm{E}}}\hat{Q}e^{\Phi_T^{\mathrm{E}}} + \int_0^1 \hat{Q}_{\tilde{\Phi}_T(t)}\Lambda_t\, dt.$$

This relation implies that $\tilde{\Phi}_T(1) = \log(e^{\Phi_J}e^{\Phi_T})$ is *gauge equivalent* to the Erler solution Φ_T^{E}.

On gauge equivalence relations

Here, we discuss some gauge equivalence relations in terms of the NS sector of Berkovits' WZW-like SSFT.

A gauge transformation of the superstring field Φ by group elements $h(t)$ and $g(t)$ with one parameter t s.t. $g(0) = h(0) = 1$ is

$$e^{\Phi(t)} = h(t)\, e^{\Phi}\, g(t), \qquad Q_{\rm B} h(t) = \eta_0 g(t) = 0. \tag{1}$$

For the string fields, Φ and $\Phi(t)$, "one-form" string fields are defined by $\Psi \equiv e^{-\Phi}\, Q_{\rm B} e^{\Phi}$ and $\Psi(t) \equiv e^{-\Phi(t)}\, Q_{\rm B} e^{\Phi(t)}$. From (1), these turn out to be related by a transformation:

$$\Psi(t) = g(t)^{-1}\, Q_{\rm B}\, g(t) + g(t)^{-1} \Psi\, g(t), \qquad \eta_0 g(t) = 0. \tag{2}$$

It is the same form as gauge transformations in the modified cubic SSFT.

Conversely, given the relation (2), we find that the relation (1) holds for $h(t) = e^{\Phi(t)} g(t)^{-1} e^{-\Phi}$. In fact, $Q_{\rm B}(e^{\Phi(t)} g(t)^{-1} e^{-\Phi}) = 0$ holds from (2)

Differentiating (2) with respect to t and integrating it again, we find another relation between Ψ and $\Psi(t)$:

$$\Psi(t) = \Psi + \int_0^t Q_{\Phi(t')}\Lambda(t')\,dt', \qquad \eta_0\Lambda(t) = 0, \tag{3}$$

where $\Lambda(t) = g(t)^{-1}\frac{d}{dt}g(t)$ and Q_ϕ is a modified BRST operator associated with $\psi \equiv e^{-\phi}Q_{\rm B}e^{\phi}$: $Q_\phi\lambda = Q_{\rm B}\lambda + \psi\lambda - (-1)^{|\lambda|}\psi\lambda$.

Conversely, supposing that the equations (3) for a given $\Lambda(t)$ hold, we find the relations (2) hold for the group element $g(t)$ such as $g(0) = 1$:

$$g(t) = \mathrm{P}\exp\left(\int_0^t \Lambda(t')dt'\right),$$

where $\mathrm{P}\exp$ means a t-ordered exponent.

Consequently, the above relations (1), (2) and (3) are all equivalent. (We can include internal Chan-Paton factors in these relations.)

Energy and gauge invariant overlaps

From the gauge equivalence:

$$\log(e^{\Phi_J} e^{\Phi_T}) \sim \Phi_T^{\mathrm{E}}$$

we can analytically evaluate gauge invariants for the *identity-based* marginal solutions.

The value of the action: $S[\Phi_J; \hat{Q}] + S[\Phi_T; \hat{Q}_{\Phi_J}] = S[\Phi_T^{\mathrm{E}}; \hat{Q}]$

$$\therefore \; E = -S[\Phi_J; \hat{Q}] = S[\Phi_T; \hat{Q}_{\Phi_J}] - S[\Phi_T^{\mathrm{E}}; \hat{Q}] = 0$$

The result agrees with the previous one derived from ξ zeromode counting:

$$S[\Phi_J, Q_{\mathrm{B}}] = -\int_0^1 dt\, \mathrm{Tr}[(\eta_0 \Phi_J)\,(e^{-t\Phi_J} Q_{\mathrm{B}} e^{t\Phi_J})] = 0$$

Evaluation of the GIO for the identity-based marginal solution $\langle \Phi_J \rangle_{\mathcal{V}}$:

Using an identity,

$$e^{-\tilde{\Phi}_T(1)} \hat{Q} e^{\tilde{\Phi}_T(1)} = e^{-\Phi_J} \hat{Q} e^{\Phi_J} + e^{-\Phi_T} \hat{Q}_{\Phi_J} e^{\Phi_T},$$

we have obtained

$$e^{-\Phi_J} \hat{Q} e^{\Phi_J} = e^{-\Phi_T^{\mathrm{E}}} \hat{Q} e^{\Phi_T^{\mathrm{E}}} - e^{-\Phi_T} \hat{Q}_{\Phi_J} e^{\Phi_T} + \int_0^1 \hat{Q}_{\tilde{\Phi}_T(t)} \Lambda_t \, dt.$$

It leads to a relation for the GIOs:

$$\begin{aligned} \langle \Phi_J \rangle_{\mathcal{V}} &= \langle \Phi_T^{\mathrm{E}} \rangle_{\mathcal{V}} - \langle \Phi_T \rangle_{\mathcal{V}} \\ &= \frac{1}{\pi} \left\langle \xi Y \mathcal{V}(i\infty) c(\frac{\pi}{2}) \left\{ 1 - e^{-\pi \mathcal{C}} \exp\Big(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du \, \mathcal{J}(u) \Big) \right\} \right\rangle_{C_\pi} . \end{aligned}$$

In the same way as the case of bosonic SFT [KT(2013)], it can be rewritten as a difference between two disk amplitudes with the boundary deformation by taking

$$F_a(z;s) = \frac{2\lambda_a s(1-s^2)}{\arctan\frac{2s}{1-s^2}} \frac{1+z^{-2}}{1-s^2(z^2+z^{-2})+s^4}$$

for the function $F_a(z)$ in Φ_J, which satisfies

$$\int_{C_{\mathrm{L}}} \frac{dz}{2\pi i} F_a(z;s) = \frac{2\lambda_a}{\pi},$$
$$F_a(z;s) \to 4\lambda_a\{\delta(\theta)+\delta(\pi-\theta)\}, \qquad (s\to 1, \ \ z=e^{i\theta}).$$

Because the form of weighting function F_a except the half-integration mode can be changed by a kind of gauge transformation [KT(2005)], we have the same value of the GIO for a fixed value of λ_a, which corresponds to a marginal deformation parameter).

Namely, for the limit $s \to 1$, the GIO is expressed as

$$\langle \Phi_J \rangle_{\mathcal{V}} = \frac{1}{\pi} \left\langle \xi Y \mathcal{V}(i\infty) c(\frac{\pi}{2}) \left\{ 1 - e^{-\pi \mathcal{C}} \exp\left(\frac{\sqrt{2}}{\pi} \lambda_a \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du\, J^a(u) \right) \right\} \right\rangle_{C_\pi} .$$

In this expression, $\mathcal{C}$ becomes divergent for the function $\lim_{s \to 1} F_a(z; s)$ and then it cancels the contact term divergence due to singular OPE among the currents.

This expression of the GIO corresponds to the result in [Ellwood(2008)] for a *wedge-based* marginal solution.

Conclusions

We have applied the method in [IKT(2012)] for cubic (S)SFT to the Erler solution Φ_T^{E} for Berkovits' WZW-like SSFT.

- We have constructed a tachyon vacuum solution Φ_T around the identity-based marginal solution Φ_J [KT(2005)] in SSFT with an extended KBc algebra in the marginally deformed background.
- Around Φ_T, we have obtained a homotopy operator and evaluated vacuum energy and gauge invariant overlap (GIO) for it.
  The energy is the same value as that on the original background, but the GIO is deformed by the marginal operators.

Using the above, we have extended our computation for bosonic SFT [KT(2013)] to superstring: We have evaluated the energy and the GIO for the identity-based marginal solution Φ_J in the framework of Berkovits' WZW-like SSFT.

- The gauge equivalence relation between Φ_T^{E} and $\log(e^{\Phi_J}e^{\Phi_T})$ is essential. It is derived from the vanishing cohomology in the small Hilbert space around the interpolating tachyon vacuum solutions.
- The energy for Φ_J vanishes and it is consistent with our previous result using ξ zeromode counting.
- The GIO for Φ_J is expressed by a difference of those of the tachyon vacuum solutions on the undeformed and deformed backgrounds.

We hope that this approach to identity-based solutions will be useful to deeply understand bosonic and super SFT.