Gauge invariants for identity-based tachyon vacuum solutions

Isao Kishimoto (Niigata University)

Reference: Isao Kishimoto, Toru Masuda and Tomohiko Takahashi, PTEP (2014) 103B02 [arXiv:1408.6318]

(See also: Nobuyuki Ishibashi, arXiv:1408.6319)

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Summary

In my previous talk@Niigata-Yamgata School (Nov. 2, 2013), we have computed gauge invariant overlaps for identity-based *marginal* solution [KT(2013)] using "$K'Bc$ algebra" in cubic bosonic open string field theory (SFT).

⇓ Extension to the Takahashi-Tanimoto (TT) identity-based scalar solution

Here, we compute gauge invariants, vacuum energy and gauge invariant overlap (GIO), for the TT identity-based *scalar* solution, using a deformed $K'Bc$ algebra. The result is consistent with the previous one obtained with other (indirect) methods.

String field theory

The gauge invariant action for cubic bosonic open SFT

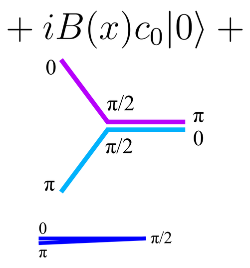
$$S[\Psi;Q_{\rm B}]=-\int\left(\frac{1}{2}\Psi*Q_{\rm B}\Psi+\frac{1}{3}\Psi*\Psi*\Psi\right)$$

string field: $|\Psi\rangle=t(x)c_1|0\rangle+A_\mu(x)\alpha^\mu_{-1}c_1|0\rangle+iB(x)c_0|0\rangle+\cdots$

$Q_{\rm B}$: Kato-Ogawa's BRST operator

$*$: interaction of open strings (star product)

$\int$: contraction with the identity state $\langle I|$



The equation of motion

$$Q_{\rm B}\Psi+\Psi*\Psi=0$$

Gauge transformation and gauge invariant overlap

The action is invariant under the following transformation:

Gauge transformation

$$\Psi'=e^{-\Lambda}Q_{\rm B}e^{\Lambda}+e^{-\Lambda}\Psi e^{\Lambda}$$

Λ: gauge parameter string field, the symbol "$*$" is omitted.

Gauge invariants defined by an onshell closed string vertex V at the string midpoint:

GIO

$$O_V(\Psi)=\langle I|V(i)|\Psi\rangle$$

The TT identity-based scalar solution

Takahashi-Tanimoto (2002)

$$\Psi_0=Q_L(e^h-1)I-C_L((\partial h)^2e^h)I$$

$Q_L(f)=\int_{C_{\rm left}}\frac{dz}{2\pi i}f(z)j_{\rm B}(z)$, $C_L(f)=\int_{C_{\rm left}}\frac{dz}{2\pi i}f(z)c(z)$: half integration

A simple choice of a function with one parameter a:

$$h_a(z)=\log\left(1+\frac{a}{2}(z+z^{-1})^2\right),\quad (a\geq -1/2)$$

Interpretation of the solution

$$\Psi_0(a)=\begin{cases}a>-1/2 & : \text{trivial pure gauge solution}\\ a=-1/2 & : \text{tachyon vacuum solution}\end{cases}\qquad\diamondsuit$$

There are several evidences for this expectation.

KBc algebra and the ES simple solution

KBc algebra [Schnabl(2005), Okawa(2006)]

$K=Q_{\rm B}B,\ Q_{\rm B}K=0,\ Q_{\rm B}c=cKc,\ B^2=0,\ c^2=0,\ Bc+cB=1$

$B=\frac{\pi}{2}(B_1)_LI,\quad c=\frac{1}{\pi}c(1)I,\quad (B_1)_L=\int_{C_{\rm left}}\frac{dz}{2\pi i}(1+z^2)b(z)$

The simple tachyon vacuum solution [Erler-Schnabl(2009)]

$$\Psi_0(K,B,c)=\frac{1}{\sqrt{1+K}}(c+cKBc)\frac{1}{\sqrt{1+K}}$$

Gauge invariants for the ES simple solution can be evaluated:

$$-S[\Psi_0(K,B,c),Q_{\rm B}]=-\frac{1}{2\pi^2},\quad O_V(\Psi_0(K,B,c))=\frac{1}{\pi}\left\langle V(i\infty)c(\tfrac{\pi}{2})\right\rangle$$

The theory around the TT identity-based solution

Action expanding around the TT solution: $\Psi=\Psi_0(a)+\Phi$

$$S[\Psi;Q_{\rm B}]=S[\Psi_0(a);Q_{\rm B}]+S[\Phi;Q']$$

where $Q'=\oint\frac{dz}{2\pi i}\left(e^{h_a(z)}j_{\rm B}(z)-(\partial h_a(z))^2\,e^{h_a(z)}c(z)\right)$

The BRST operator Q' around the solution $\Psi_0(a)$

$a>-1/2$: $Q'=e^{\tilde q(h_a)}Q_{\rm B}e^{-\tilde q(h_a)}$, $\tilde q(h_a)=\oint\frac{dz}{2\pi i}h_a(z)\left(j_{\rm gh}(z)-\frac{3}{2z}\right)$

$a=-1/2$: Q' has no cohomology [KT(2002), Inatomi-KT(2011)]

EOM to the theory around $\Psi_0(a)$

$$Q'\Phi+\Phi*\Phi=0\qquad\circledcirc$$

$K'Bc$ algebra and the ES-like solution $(a>-1/2)$

$K'Bc$ algebra for $\boldsymbol{a>-1/2}$

$K'=Q'B,\ Q'K'=0,\ Q'c=cK'c,\ B^2=0,\ c^2=0,\ Bc+cB=1$

The ES-like wedge-based solution to the EOM $\circledcirc$

$$\Phi_0(K',B,c)=\frac{1}{\sqrt{1+K'}}(c+cK'Bc)\frac{1}{\sqrt{1+K'}}$$

In order to evaluate the gauge invariants, we find a relation:

$$\Phi_0(K',B,c)=e^{\tilde q(h_a)}\,U_f^{-1}\Psi_0(K,B,c)$$

where $U_f=\exp\left(\sum_n v_n(L_n-(-1)^nL_{-n})\right)$ is given by a conformal transformation determined by $h_a(z)$.

Using the above relation, we have

$$S[\Phi_0(K',B,c);\,Q']=S[\Psi_0(K,B,c);\,U_fQ_{\rm B}U_f^{-1}]=S[\Psi_0(K,B,c);Q_{\rm B}].$$

We also note

$$\langle I|V(i)\,\tilde q(h_a)=0,\quad \langle I|V(i)\,(L_n-(-1)^nL_{-n})=0.$$

Gauge invariants for the ES-like solution:

$$-S[\Phi_0(K',B,c),Q']=-\frac{1}{2\pi^2},\quad O_V(\Phi_0(K',B,c))=\frac{1}{\pi}\left\langle V(i\infty)c(\tfrac{\pi}{2})\right\rangle$$

These are the same value for the ES simple solution for tachyon condensation in the original theory.

$K'Bc$ algebra and the ES-like solution $(a=-1/2)$

$K'Bc$ algebra for $\boldsymbol{a=-1/2}$

$K'=Q'B,\ Q'K'=0,\ Q'c=0\ \ B^2=0,\ c^2=0,\ Bc+cB=1$

In particular, we have $K'c=cK'$ in this case.

The ES-like solution to the EOM $\circledcirc$ is reduced to a Q'-closed form

$$\Phi_0(K',B,c)=\frac{1}{\sqrt{1+K'}}(c+cK'Bc)\frac{1}{\sqrt{1+K'}}=c$$

Actually, this should also be exact because Q' has no cohomology.

Gauge invariants for the above solution become trivial:

$$-S[\Phi_0(K',B,c),Q']=0,\quad O_V(\Phi_0(K',B,c))=0$$

An interpolating string field Ψ_a

$\Psi_a\equiv\Psi_0(a)+\Phi_0(K',B,c)$ satisfies EOM: $Q_{\rm B}\Psi_a+\Psi_a*\Psi_a=0$.

Note: $\Psi_{a=0}=\Psi_0(K,B,c)$ (the ES simple solution)

Differentiating the EOM with respect to a, we have

$$Q_{\Psi_a}\frac{d}{da}\Psi_a=0;\quad Q_{\Psi_a}A=Q'A+\Phi_0(K',B,c)*A-(-1)^{|A|}A*\Phi_0(K',B,c).$$

Q_{Ψ_a} has a homotopy operator, $\frac{B}{1+K'}$, and it has no cohomology.

$\Rightarrow\quad \exists\Lambda_a$ s.t. $\quad\frac{d}{da}\Psi_a=Q_{\Psi_a}\Lambda_a$

Integrating this expression, we obtain a gauge equivalence relation.

Gauge equivalence relation

$\Psi_a\sim\Psi_0(K,B,c)$

$$\Psi_0(a)+\Phi_0(K',B,c)=\Psi_0(K,B,c)+\int_0^aQ_{\Psi_a}\Lambda_ada$$

Or equivaletly, this can be rewritten as

$$\Psi_0(a)+\Phi_0(K',B,c)=g^{-1}Q_{\rm B}g+g^{-1}\,\Psi_0(K,B,c)\,g,$$

where $g={\rm P}\exp\left(\int_0^a\Lambda_ada\right)$.

The above implies following relations among gauge invariants:

$$S[\Psi_0(a);Q_{\rm B}]+S[\Phi_0(K',B,c);Q']=S[\Psi_0(K,B,c);Q_{\rm B}]$$

$$O_V(\Psi_0(a))+O_V(\Phi_0(K',B,c))=O_V(\Psi_0(K,B,c))$$

Gauge invariants for the identity-based solution

Then, we immediately obtain:

Energy for the *identity-based* solution

$$-S[\Psi_0(a);Q_{\rm B}]=\begin{cases}0 & (a>-1/2)\\ -\frac{1}{2\pi^2} & (a=-1/2)\end{cases}$$

GIO for the *identity-based* solution

$$O_V(\Psi_0(a))=\begin{cases}0 & (a>-1/2)\\ \frac{1}{\pi}\left\langle V(i\infty)c(\tfrac{\pi}{2})\right\rangle & (a=-1/2)\end{cases}$$

These results strongly support our previous expectation $\diamondsuit$.