Analysis of tree level 5-point amplitudes in open superstring field theory

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- Introduction to the superstring field theory
- gauge fixing procedure
- Computation of 5-point amplitudes

1. 5 bosons
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4. 2 fermions and 3 bosons (order FBFBB)

Berkovits' open superstring field theory

- based on NSR formalism
- Star products: the same ones as Witten's
- String fields have ghost numbers and picture numbers
- state space: large Hilbert space
- 2 gauge symmetries
- action: Q and η_0 insertions in interaction terms
- infinitely many higher order terms

$$-g^2 S = \sum_{M,N=0}^{\infty} \frac{1}{(M+N+2)!} (-1)^N \binom{M+N}{N} (Q\Phi)\Phi^M (\eta_0 \Phi)\Phi^N$$

Gauge fixing

To compute amplitudes, we have to fix the gauge symmetry.

- Witten's bosonic SFT

gauge fixing condition: Siegel gauge $b_0\Phi = 0$

(Other conditions: Asano-Kato (2006),Schnabl(2005), Nakayama-Fuji-Suzuki 2006)

Application of Batalin-Vilkovisky (BV) formalism

"fields" $\cdots, \Phi_{-1}, \Phi_0, \Phi_1, \Phi_2, \Phi_3, \cdots$ "antifields"

original field (Φ_1)

spectrum: as in the 1st quantized formulation

Physics must be independent of gauge fixing condition.
⇒ (classical) master equation

$$\sum_s (-1)^s \frac{\partial}{\partial \phi_s} S \frac{\partial}{\partial \bar{\phi}_s} S = 0$$

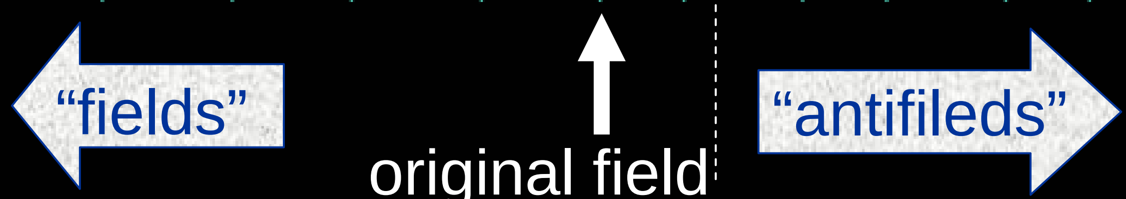
action: in the same form as the original one with ghost number restriction removed

- Superstring case

gauge fixing condition: $b_0 \Phi = 0 \quad \xi_0 \Phi = 0$

Analogy to the bosonic case:

$$\cdots \Phi_{(-2,0)}, \Phi_{(-1,0)}, \Phi_{(0,0)}; \Phi_{(1,0)}, \Phi_{(2,0)} \cdots$$



picture number fixed ⇒1st quantized spectrum reproduced

action: original one with ghost number restriction removed?
(denoted by S_1)

seems quite natural, for example:

in loop amplitudes contributions with any ghost should contribute with the same weight.

However,

cubic contribution to the classical master equation

$$\left[\sum_s (-1)^s \frac{\partial}{\partial \phi_s} S_1 \frac{\partial}{\partial \bar{\phi}_s} S_1\right]_{\Phi^3} \sim \Phi Q(Q\Phi\eta_0\Phi + \eta_0\Phi Q\Phi) \neq 0$$

⇒BV formalism does not work for S_1.

This means that S_1 is different from the one (S_0) given by solving master equation (possible only order by order).

$$\begin{aligned} -g^2 S_0 \;=\; & \frac{1}{2} Q \left(\sum_N \Phi_N\right) \eta_0 \left(\sum_M \Phi_M\right) - \frac{1}{6} \Phi_0 (Q\Phi_0\eta_0\Phi_0 + \eta_0\Phi_0 Q\Phi_0) \\ & - \frac{1}{2} \Phi_1 \eta_0 [\Phi_0, (1+\eta_0\xi_0) Q\Phi_{-1}] + \sum_{N\geq 2} \Phi_N \{\eta_0\Phi_0, \eta_0\xi_0 Q\Phi_{-N}\} \\ & + O(\Phi^3_{N\neq 0}) + O(\Phi^4_N) \end{aligned}$$

However,

As long as S_1 reproduces 1st quantized amplitudes,

S_1 and S_0 give the same on-shell tree level amplitudes.

1st quantized on-shell amplitudes reproduced?:

4-point amplitudes

Berkovits-Echevarria(1999), Berkovits-Schnabl(2003),

Y.M. (2004), Fuji-Nakayama-Suzuki(2006)

$\Rightarrow$ reproduced !

Further check: calculation of 5-point amplitudes

Calculation of on-shell amplitudes: Generalities

- Bosonic string

Expression expected to be reproduced

$$\int d\alpha_4 \ldots \int d\alpha_N \langle \int d^2w_4 \mu_{\alpha_4} b(w_4) \ldots \int d^2w_N \mu_{\alpha_N} b(w_N) \times \Phi_1(z_1) \ldots \Phi_N(z_N) \rangle$$

Propagator = b_0/L_0

$$1/L_0 = \int_0^\infty d\tau \exp(-\tau L_0)$$

: inserting a strip of the length τ

$$b_0 \to \int d^2w \mu_\tau b(w)$$

:Beltrami differential insertion

⇒Integrand: reproduced

The region of integration is also reproduced.

- Superstring case

Expression expected to be reproduced:

  - Picture numbers adjusted by Q
  - superfluous ξ_0 removed by η_0

Propagator: $\xi_0 b_0 / L_0$

Infinitely many interaction vertices with Q and η_0 insertions

Q and η_0 should be relocated to adjust picture numbers.

Those may hit $\xi_0 b_0 / L_0$ and remove $1/L_0$

Number of propagators reduced. (zero length propagators)
$\Rightarrow$should be canceled by diagrams with higher order vertices.

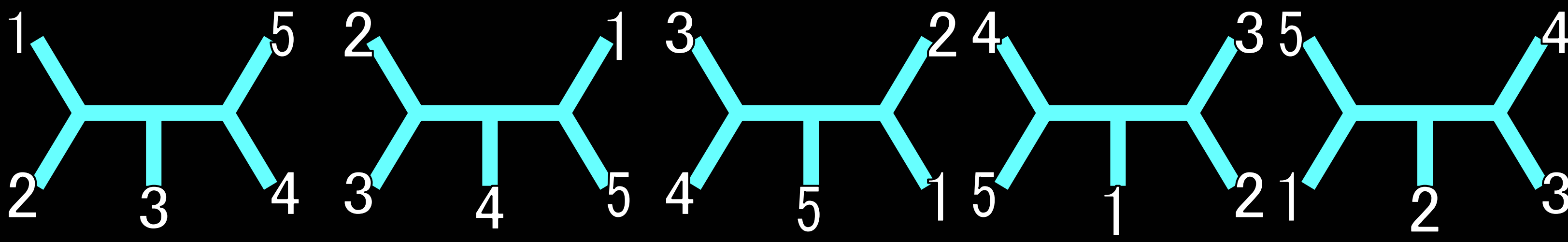
5-point amplitude: 5 bosons

- 1st quantized amplitude

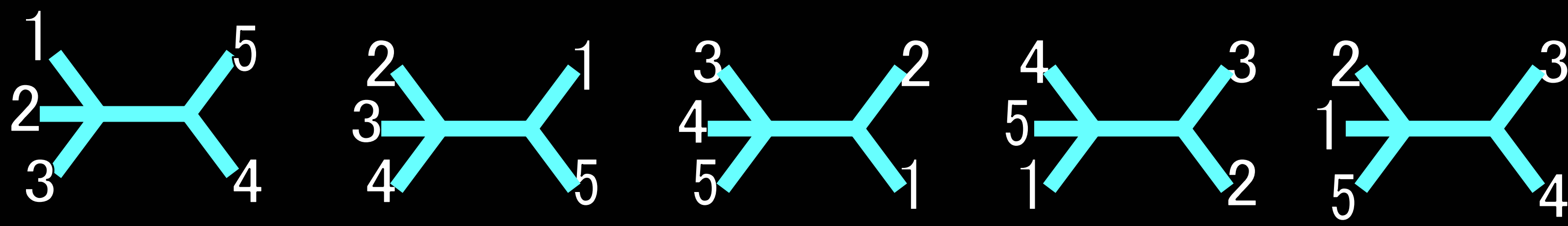
$$\begin{aligned} A_{5B} \;=\;& \Phi_1 Q \Phi_2 \frac{b_0}{L_0} Q \Phi_3 \frac{b_0}{L_0} Q \Phi_4 \eta_0 \Phi_5 \\ & + Q \Phi_2 Q \Phi_3 \frac{b_0}{L_0} Q \Phi_4 \frac{b_0}{L_0} \eta_0 \Phi_5 \Phi_1 \\ & - Q \Phi_3 Q \Phi_4 \frac{b_0}{L_0} \eta_0 \Phi_5 \frac{b_0}{L_0} \Phi_1 Q \Phi_2 \\ & - Q \Phi_4 \eta_0 \Phi_5 \frac{b_0}{L_0} \Phi_1 \frac{b_0}{L_0} Q \Phi_2 Q \Phi_3 \\ & - \eta_0 \Phi_5 \Phi_1 \frac{b_0}{L_0} Q \Phi_2 \frac{b_0}{L_0} Q \Phi_3 Q \Phi_4 \end{aligned}$$

Let us see if the superstring field theory reproduces this.

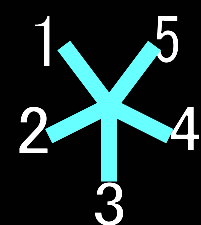
- Diagrams with 2 propagators



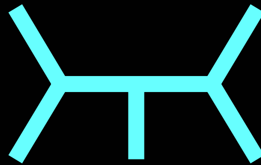
- Diagrams with 1 propagator

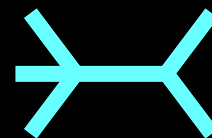


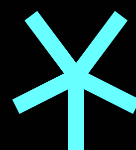
- Diagrams with no propagator

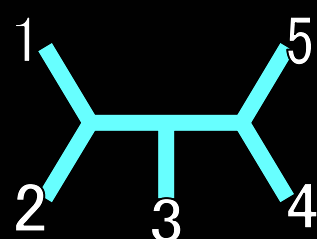


We expect that

1. Each  gives one of terms in A_{5B} and extra terms with 1 propagator through relocating Q and η_0 .

2. Those extra terms and  are combined into terms with no propagator.

3. Those terms cancel 

Let us compute 

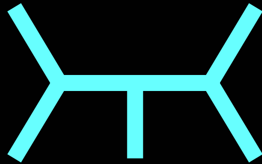
first.

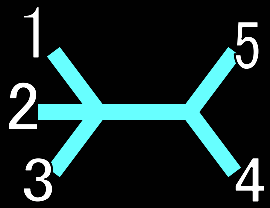
$$\left(-\frac{1}{6}\right)^3 \cdot 3[(Q\Phi_1\eta_0\Phi_2+\eta_0\Phi_1 Q\Phi_2)\Phi_{p1}] \times 3[(Q\Phi_{p1}\eta_0\Phi_3+\eta_0\Phi_{p1}Q\Phi_3)\Phi_{p2}]$$
$$\times 3[\Phi_{p2}(Q\Phi_4\eta_0\Phi_5+\eta_0\Phi_4Q\Phi_5)]$$
$$= \frac{1}{8}(Q\Phi_1\eta_0\Phi_2+\eta_0\Phi_1Q\Phi_2)\xi_0\frac{b_0}{L_0}\times\left[Q\left\{\eta_0\Phi_3\xi_0\frac{b_0}{L_0}(Q\Phi_4\eta_0\Phi_5+\eta_0\Phi_4Q\Phi_5)\right\}\right.$$
$$\left.+\eta_0\left\{Q\Phi_3\xi_0\frac{b_0}{L_0}(Q\Phi_4\eta_0\Phi_5+\eta_0\Phi_4Q\Phi_5)\right\}\right]$$

expected term

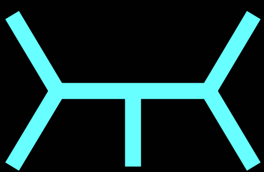
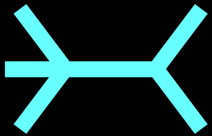
terms with 1 propagator

$$= -\frac{1}{8}\left[8\Phi_1Q\Phi_2\frac{b_0}{L_0}Q\Phi_3\frac{b_0}{L_0}Q\Phi_4\eta_0\Phi_5\right.$$
$$-4\eta_0\Phi_1Q\Phi_2\frac{b_0}{L_0}Q\Phi_3\Phi_4\Phi_5-4\eta_0\Phi_1Q\Phi_2Q\Phi_3\frac{b_0}{L_0}\Phi_4\Phi_5$$
$$+2\Phi_1\Phi_2Q\Phi_3\frac{b_0}{L_0}(Q\Phi_4\eta_0\Phi_5+\eta_0\Phi_4Q\Phi_5)+2\Phi_1\Phi_2\frac{b_0}{L_0}Q\Phi_3(Q\Phi_4\eta_0\Phi_5+\eta_0\Phi_4Q\Phi_5)$$
$$-(Q\Phi_1\eta_0\Phi_2+\eta_0\Phi_1Q\Phi_2)\Phi_3\xi_0\frac{b_0}{L_0}(Q\Phi_4\eta_0\Phi_5+\eta_0\Phi_4Q\Phi_5)$$
$$\left.+(Q\Phi_1\eta_0\Phi_2+\eta_0\Phi_1Q\Phi_2)\xi_0\frac{b_0}{L_0}\Phi_3(Q\Phi_4\eta_0\Phi_5+\eta_0\Phi_4Q\Phi_5)\right]$$

- Other  : cyclic permutations



$$= \frac{1}{24}(2Q\Phi_1\Phi_2\eta_0\Phi_3 - 2\eta_0\Phi_1\Phi_2Q\Phi_3 \\ +\eta_0\Phi_1Q\Phi_2\Phi_3 - \Phi_1Q\Phi_2\eta_0\Phi_3 \\ +\Phi_1\eta_0\Phi_2Q\Phi_3 - Q\Phi_1\eta_0\Phi_2\Phi_3) \\ \times\xi_0\frac{b_0}{L_0}(Q\Phi_4\eta_0\Phi_5 + \eta_0\Phi_4Q\Phi_5)$$

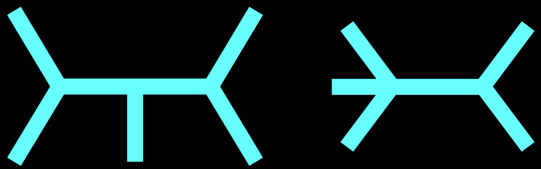
$\Phi_1\Phi_2\Phi_2(\mathrm{prop})\Phi_4\Phi_5$ in  

should sum up to terms with no propagator.

The sum is

$$
\begin{aligned}
= \ & \frac{1}{24}[(2Q\Phi_1\Phi_2\eta_0\Phi_3 - 2\eta_0\Phi_1\Phi_2Q\Phi_3 \\
& +\eta_0\Phi_1Q\Phi_2\Phi_3 - \Phi_1Q\Phi_2\eta_0\Phi_3 + \Phi_1\eta_0\Phi_2Q\Phi_3 - Q\Phi_1\eta_0\Phi_2\Phi_3) \\
& \times\xi_0\frac{b_0}{L_0}(Q\Phi_4\eta_0\Phi_5 + \eta_0\Phi_4Q\Phi_5) \\
& +12\eta_0\Phi_1Q\Phi_2Q\Phi_3\frac{b_0}{L_0}\Phi_4\Phi_5 - 6\Phi_1\Phi_2Q\Phi_3\frac{b_0}{L_0}(Q\Phi_4\eta_0\Phi_5 + \eta_0\Phi_4Q\Phi_5) \\
& +3(Q\Phi_1\eta_0\Phi_2 + \eta_0\Phi_1Q\Phi_2)\Phi_3\xi_0\frac{b_0}{L_0}(Q\Phi_4\eta_0\Phi_5 + \eta_0\Phi_4Q\Phi_5) \\
& -12Q\Phi_1\Phi_2\Phi_3\frac{b_0}{L_0}\eta_0\Phi_4Q\Phi_5 + 6Q\Phi_1(Q\Phi_2\eta_0\Phi_3 + \eta_0\Phi_2Q\Phi_3)\frac{b_0}{L_0}\Phi_4\Phi_5 \\
& -3\Phi_1(Q\Phi_2\eta_0\Phi_3 + \eta_0\Phi_2Q\Phi_3)\xi_0\frac{b_0}{L_0}(Q\Phi_4\eta_0\Phi_5 + \eta_0\Phi_4Q\Phi_5) \\
& +24\Phi_1Q\Phi_2\Phi_3\frac{b_0}{L_0}Q\Phi_4\eta_0\Phi_5 + 24Q\Phi_1Q\Phi_2\eta_0\Phi_3\frac{b_0}{L_0}\Phi_4\Phi_5] \\
= \ & \frac{1}{24}[2\Phi_1\Phi_2\Phi_3(Q\Phi_4\eta_0\Phi_5 + \eta_0\Phi_4Q\Phi_5) \\
& +6\eta_0(\Phi_1\Phi_2Q\Phi_3 - \Phi_1Q\Phi_2\Phi_3 + 2Q\Phi_1\Phi_2\Phi_3)\Phi_4\Phi_5 \\
& +12\Phi_1\Phi_2\Phi_3Q\Phi_4\eta_0\Phi_5]
\end{aligned}
$$

Then



$$= A_{5B} - \frac{1}{12}\Phi_1[Q\Phi_2\Phi_3\eta_0\Phi_4\Phi_5 - \eta_0\Phi_2 Q\Phi_3\Phi_4\Phi_5 + \Phi_2 Q\Phi_3\eta_0\Phi_4\Phi_5 + \Phi_2 Q\Phi_3\Phi_4\eta_0\Phi_5 - \eta_0\Phi_2\Phi_3\Phi_4 Q\Phi_5 - \Phi_2\Phi_3\eta_0\Phi_4 Q\Phi_5]$$

$$= A_{5B} \quad -$$

1st quantized amplitude reproduced !

How to describe R-sector

- Classical description (Y.M. 2004)

R-sector string field $\Psi \sim \xi_0 V_{-1/2}$

Naïve kinetic term $\Psi Q \eta_0 \Psi$

vanishes due to picture # conservation!

This is a situation similar to field theories with self dual fields.

⇒introduction of an additional field Ξ corresponding to anti self dual part

Action $$S_F = \frac{1}{2}(Q_B \Xi) e^{\Phi} (\eta_0 \Psi) e^{-\Phi}$$

+ constraint $$Q_B \Xi = e^{\Phi} (\eta_0 \Psi) e^{-\Phi}$$

- Naïve inference of Feynman Rules
  1. Propagator $\Xi_p \Psi_p = -2\xi_0 \frac{b_0}{L_0}$
  2. External on-shell $Q\Xi$ replaced by $\eta_0 \Psi$
  3. Interaction vertices: higher terms in the action

We have to impose the constraint on the interaction vertices.

Here we naively impose the linearized constraint $Q_B \Xi = \eta_0 \Psi$

i.e. $Q\Xi$ and $\eta_0 \Psi$ is replaced by $\omega = \frac{1}{2}(Q\Xi + \eta_0 \Psi)$

Too naïve?

Then…

$$v_3^{FFB} = \omega\omega\Phi \qquad v_5^{FFBBB} = \frac{1}{6}\omega\omega\Phi\Phi\Phi$$

$$v_4^{FFBB} = 0 \qquad v_5^{FBFBB} = -\frac{1}{2}\omega\Phi\omega\Phi\Phi$$

$$v_4^{FBFB} = 0 \qquad v_5^{FFFFB} = 0$$

$$v_4^{FFFF} = 0$$

Correctness of cubic and quartic vertices has been confirmed by calculating 4-point amplitudes.

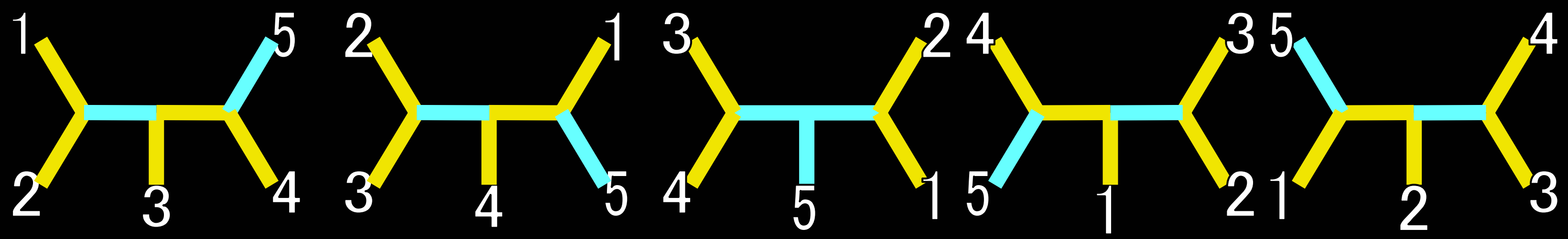
5-point vertices correct?

5-point amplitude:
4 fermions and 1 boson

- 1st quantized amplitude

$$
\begin{aligned}
-A_{FFFFB} = \ & \Psi_1 \eta_0 \Psi_2 \frac{b_0}{L_0} \eta_0 \Psi_3 \frac{b_0}{L_0} \eta_0 \Psi_4 Q \Phi_5 \\
& + \eta_0 \Psi_2 \eta_0 \Psi_3 \frac{b_0}{L_0} \eta_0 \Psi_4 \frac{b_0}{L_0} Q \Phi_5 \Psi_1 \\
& - \eta_0 \Psi_3 \eta_0 \Psi_4 \frac{b_0}{L_0} Q \Phi_5 \frac{b_0}{L_0} \Psi_1 \eta_0 \Psi_2 \\
& - \eta_0 \Psi_4 Q \Phi_5 \frac{b_0}{L_0} \Psi_1 \frac{b_0}{L_0} \eta_0 \Psi_2 \eta_0 \Psi_3 \\
& - Q \Phi_5 \Psi_1 \frac{b_0}{L_0} \eta_0 \Psi_2 \frac{b_0}{L_0} \eta_0 \Psi_3 \eta_0 \Psi_4
\end{aligned}
$$

- Diagrams with 2 propagators



- Diagrams with 1 propagator

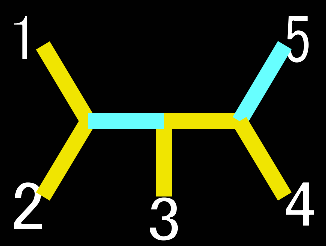
None

- Diagrams with no propagator

None

Used vertices: $v_3^{BBB}, v_3^{FFB}, v_4^{FFBB} = 0, v_4^{FBFB} = 0,$
$v_4^{FFFF} = 0, v_5^{FFFFB} = 0$

- Expectation
  1. Each diagram gives one of terms in the 1st quantized amplitude and extra terms with 1 propagator.
  2. Those extra terms cancel each other.



expected term

Terms with 1 propagator

$$= [\eta_0\Psi_1\eta_0\Psi_2\Phi_p \cdots \Phi_p\eta_0\Psi_3\omega_p][\omega_p\eta_0\Psi_4\Phi_5]$$

$$= -\frac{1}{2}\eta_0\Psi_1\eta_0\Psi_2 \cdots \xi_0\frac{b_0}{L_0}\eta_0\Psi_3\left[Q\left(\xi_0\frac{b_0}{L_0}\eta_0\Psi_4\eta_0\Phi_5\right) \cdots \eta_0\left(\xi_0\frac{b_0}{L_0}\eta_0\Psi_4 Q\Phi_5\right)\right]$$

$$= -\frac{1}{2}\Big[2\Psi_1\eta_0\Psi_2\frac{b_0}{L_0}\eta_0\Psi_3\frac{b_0}{L_0}\eta_0\Psi_4 Q\Phi_5$$

$$+\eta_0\Psi_1\eta_0\Psi_2\xi_0\frac{b_0}{L_0}\eta_0\Psi_3\eta_0\Psi_4\Phi_5 - \eta_0\Psi_1\eta_0\Psi_2\eta_0\Psi_3\xi_0\frac{b_0}{L_0}\eta_0\Psi_4\Phi_5\Big]$$

As is expected, terms with 1 propagator cancel, and remaining terms reproduce the 1st quantized amplitude !

Therefore

$$v_3^{BBB}, v_3^{FFB}, v_4^{FFBB} = 0, v_4^{FBFB} = 0,$$
$$v_4^{FFFF} = 0, v_5^{FFFFB} = 0$$

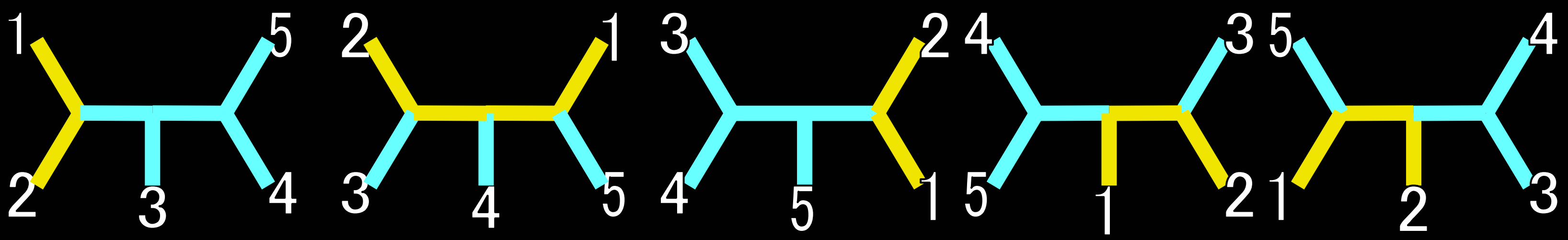
seems correct.

5-point amplitude:
2 fermions and 3 bosons
in the order FFBBB

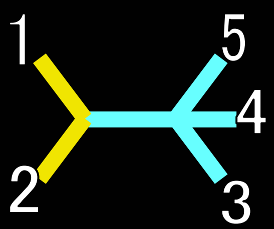
- 1st quantized amplitude

$$
\begin{aligned}
A_{FFBBB} \;=\;& \eta_0\Psi_1\eta_0\Psi_2\frac{b_0}{L_0}Q\Phi_3\frac{b_0}{L_0}Q\Phi_4\Phi_5 \\
& -\eta_0\Psi_2 Q\Phi_3\frac{b_0}{L_0}Q\Phi_4\frac{b_0}{L_0}\Phi_5\eta_0\Psi_1 \\
& -Q\Phi_3 Q\Phi_4\frac{b_0}{L_0}\Phi_5\frac{b_0}{L_0}\eta_0\Psi_1\eta_0\Psi_2 \\
& -Q\Phi_4\Phi_5\frac{b_0}{L_0}\eta_0\Psi_1\frac{b_0}{L_0}\eta_0\Psi_2 Q\Phi_3 \\
& +\Phi_5\eta_0\Psi_1\frac{b_0}{L_0}\eta_0\Psi_2\frac{b_0}{L_0}Q\Phi_3 Q\Phi_4
\end{aligned}
$$

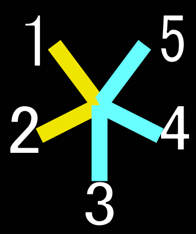
- Diagrams with 2 propagators



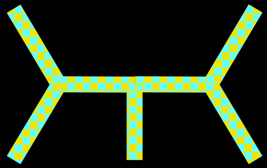
- Diagram with 1 propagator

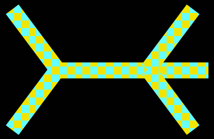


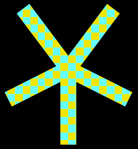
- Diagram with no propagator

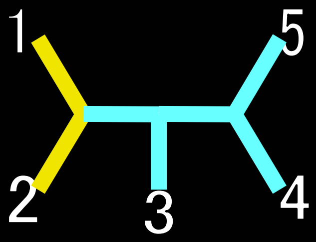


- Expectation

1. Each  gives one of terms in A_{FFBBB} and extra terms with 1 propagator through relocating Q and η_0 .

2. Those extra terms and  are combined into terms with no propagator.

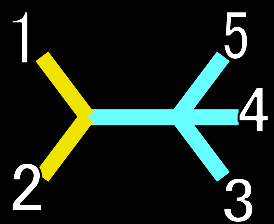
3. Those terms cancel 



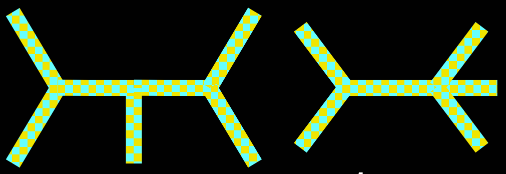
$$
\begin{aligned}
&= \left(-\frac{1}{2}\right)^2 [\eta_0\Psi_1\eta_0\Psi_2\Phi_{p1}][\Phi_{p1}(Q\Phi_3\eta_0\Phi_{p2} + \eta_0\Phi_3 Q\Phi_{p2})] \\
&\quad \times [\Phi_{p2}(Q\Phi_4\eta_0\Phi_5 + \eta_0\Phi_4 Q\Phi_5)] \\
&= \frac{1}{4}\eta_0\Psi_1\eta_0\Psi_2\xi_0\frac{b_0}{L_0}\left[Q\Phi_3\eta_0\left\{\xi_0\frac{b_0}{L_0}(Q\Phi_4\eta_0\Phi_5 + \eta_0\Phi_4 Q\Phi_5)\right\}\right. \\
&\quad \left. +\eta_0\Phi_3 Q\left\{\xi_0\frac{b_0}{L_0}(Q\Phi_4\eta_0\Phi_5 + \eta_0\Phi_4 Q\Phi_5)\right\}\right] \\
&= \frac{1}{4}\left[4\eta_0\Psi_1\eta_0\Psi_2\frac{b_0}{L_0}Q\Phi_3\frac{b_0}{L_0}Q\Phi_4\Phi_5\right. \\
&\quad -2\eta_0\Psi_1\eta_0\Psi_2\frac{b_0}{L_0}Q\Phi_3\Phi_4\Phi_5 - 2\eta_0\Psi_1\eta_0\Psi_2 Q\Phi_3\frac{b_0}{L_0}\Phi_4\Phi_5 \\
&\quad -\eta_0\Psi_1\eta_0\Psi_2\Phi_3\xi_0\frac{b_0}{L_0}(Q\Phi_4\eta_0\Phi_5 + \eta_0\Phi_4 Q\Phi_5) \\
&\quad \left.+\eta_0\Psi_1\eta_0\Psi_2\xi_0\frac{b_0}{L_0}\Phi_3(Q\Phi_4\eta_0\Phi_5 + \eta_0\Phi_4 Q\Phi_5)\right]
\end{aligned}
$$

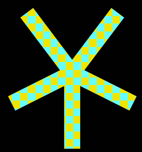
expected term

Terms with 1 propagator



$$
\begin{aligned}
&= \frac{1}{12}[\eta_0\Psi_1\eta_0\Psi_2\Phi_p][\Phi_p(2\eta_0\Phi_3\Phi_4Q\Phi_5 - 2Q\Phi_3\Phi_4\eta_0\Phi_5 \\
&\quad -\eta_0\Phi_3Q\Phi_4\Phi_5 + Q\Phi_3\eta_0\Phi_4\Phi_5 - \Phi_3\eta_0\Phi_4Q\Phi_5 - \Phi_3Q\Phi_4\eta_0\Phi_5)] \\
&= \frac{1}{12}[\eta_0\Psi_1\eta_0\Psi_2\xi_0\frac{b_0}{L_0}(2\eta_0\Phi_3\Phi_4Q\Phi_5 - 2Q\Phi_3\Phi_4\eta_0\Phi_5 \\
&\quad -\eta_0\Phi_3Q\Phi_4\Phi_5 + Q\Phi_3\eta_0\Phi_4\Phi_5 - \Phi_3\eta_0\Phi_4Q\Phi_5 - \Phi_3Q\Phi_4\eta_0\Phi_5)]
\end{aligned}
$$

Terms with 1 propagator in 
should sum up to terms with no propagator.
The sum is $+\frac{1}{12}\eta_0\Psi_1\eta_0\Psi_2\Phi_3\Phi_4\Phi_5$ as expected.

However,  $= +\frac{1}{6}\eta_0\Psi_1\eta_0\Psi_2\Phi_3\Phi_4\Phi_5$

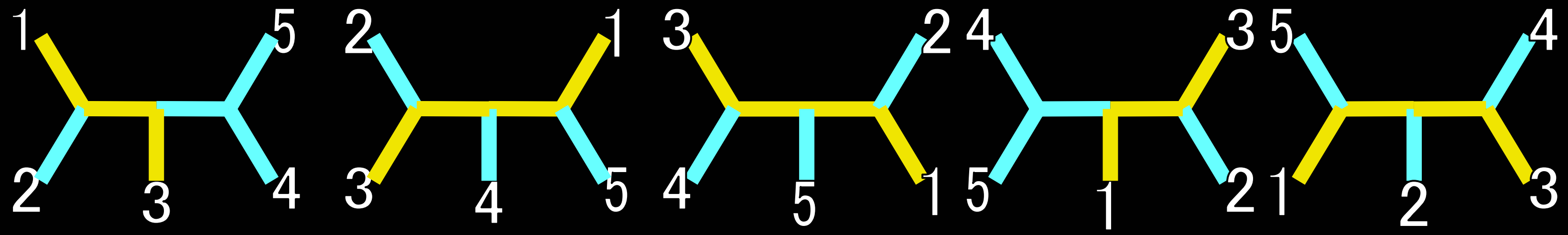
Should be modified to $v_5^{FFBBB} = -\frac{1}{12}\eta_0\Psi_1\eta_0\Psi_2\Phi_3\Phi_4\Phi_5$???

5-point amplitude: 2 fermions and 3 bosons in the order FBFBB

- 1st quantized amplitude

$$
\begin{aligned}
A_{FBFBB} \;=\;& \eta_0\Psi_1 Q\Phi_2 \frac{b_0}{L_0}\eta_0\Psi_3\frac{b_0}{L_0}Q\Phi_4\Phi_5 \\
& -Q\Phi_2\eta_0\Psi_3\frac{b_0}{L_0}Q\Phi_4\frac{b_0}{L_0}\Phi_5\eta_0\Psi_1 \\
& -\eta_0\Psi_3 Q\Phi_4\frac{b_0}{L_0}\Phi_5\frac{b_0}{L_0}\eta_0\Psi_1 Q\Phi_2 \\
& -Q\Phi_4\Phi_5\frac{b_0}{L_0}\eta_0\Psi_1\frac{b_0}{L_0}Q\Phi_2\eta_0\Psi_3 \\
& +\Phi_5\eta_0\Psi_1\frac{b_0}{L_0}Q\Phi_2\frac{b_0}{L_0}\eta_0\Psi_3 Q\Phi_4
\end{aligned}
$$

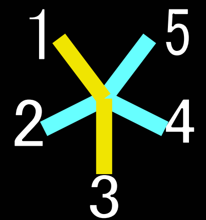
- Diagrams with 2 propagators



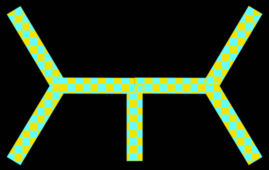
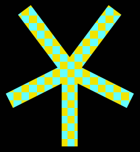
- Diagrams with 1 propagator

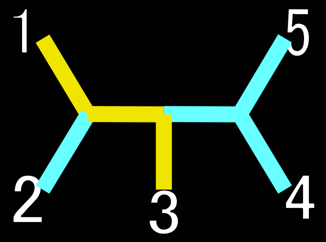
None

- Diagrams with no propagator



- Expectation

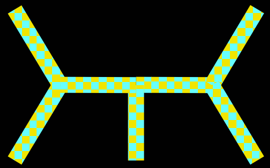
1. Each  gives one of terms in the 1st quantized amplitude and extra terms with 1 propagator through relocating Q and η_0 .
2. Those extra terms sum up to terms with no propagator.
3. Those cancel  .



$$
\begin{aligned}
&= \frac{1}{2}[\eta_0\Psi_1\Phi_2\omega_p][\omega_p\eta_0\Psi_3\Phi_p][\Phi_p(Q\Phi_4\eta_0\Phi_5+\eta_0\Phi_4Q\Phi_5)] \\
&= \frac{1}{4}\eta_0\Psi_1\Phi_2\left[Q\left\{\xi_0\frac{b_0}{L_0}\eta_0\left\{\eta_0\Psi_3\xi_0\frac{b_0}{L_0}(Q\Phi_4\eta_0\Phi_5+\eta_0\Phi_4Q\Phi_5)\right\}\right\}\right. \\
&\quad \left.+\eta_0\left\{\xi_0\frac{b_0}{L_0}Q\left\{\eta_0\Psi_3\xi_0\frac{b_0}{L_0}(Q\Phi_4\eta_0\Phi_5+\eta_0\Phi_4Q\Phi_5)\right\}\right\}\right] \\
&= -\frac{1}{4}\left[-4\eta_0\Psi_1Q\Phi_2\frac{b_0}{L_0}\eta_0\Psi_3\frac{b_0}{L_0}Q\Phi_4\Phi_5\right. \\
&\quad +2\eta_0\Psi_1Q\Phi_2\frac{b_0}{L_0}\eta_0\Psi_3\Phi_4\Phi_5+2\eta_0\Psi_1Q\Phi_2\eta_0\Psi_3\frac{b_0}{L_0}\Phi_4\Phi_5 \\
&\quad -\eta_0\Psi_1\Phi_2\eta_0\Psi_3\xi_0\frac{b_0}{L_0}(Q\Phi_4\eta_0\Phi_5+\eta_0\Phi_4Q\Phi_5) \\
&\quad \left.+\eta_0\Psi_1\Phi_2\xi_0\frac{b_0}{L_0}\eta_0\Psi_3(Q\Phi_4\eta_0\Phi_5+\eta_0\Phi_4Q\Phi_5)\right]
\end{aligned}
$$

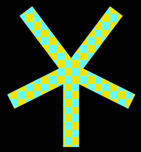
expected term

Terms with 1 propagator

Terms with 1 propagator in 

should sum up to terms with no propagator.

The sum is $+\frac{1}{4}\eta_0\Psi_1\Phi_2\eta_0\Psi_3\Phi_4\Phi_5$ as expected.

However,  $= +\frac{1}{2}\eta_0\Psi_1\Phi_2\eta_0\Psi_3\Phi_4\Phi_5$

Should be modified to $v_5^{FBFBB} = -\frac{1}{4}\eta_0\Psi_1\Phi_2\eta_0\Psi_3\Phi_4\Phi_5$???

Summary

- Naïve gauge fixing procedure using the unfixed action with ghost number restriction removed, does not fit BV formalism. However as long as it reproduces on-shell 1st quantized amplitudes, tree level amplitudes have no problem.

- It reproduces the 1st quantized amplitude with 5 bosons through many nontrivial cancellations.

- For 5-point amplitudes with fermions, we have found some discrepancies, and those are remedied by changing coefficients of 5-point vertices. (subtlety of imposing constraints to the vertices?)