Some Properties of Classical Solutions in CSFT

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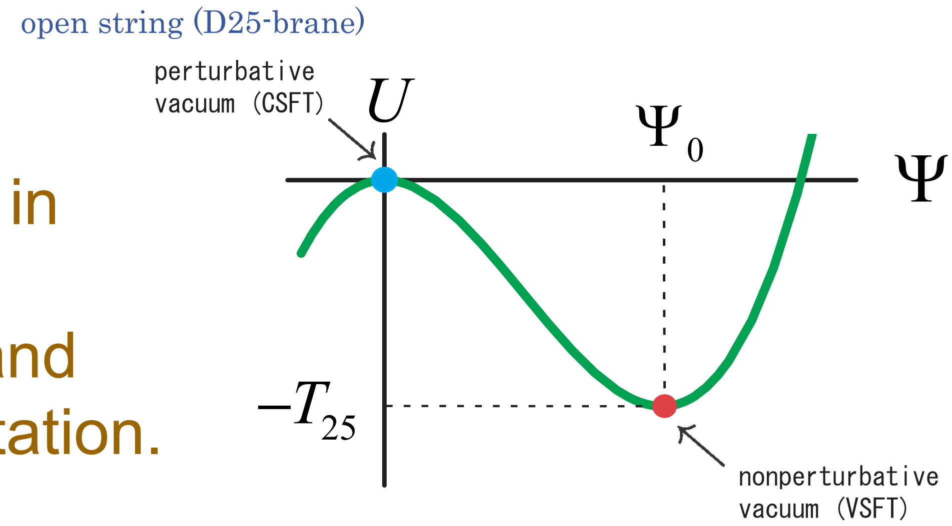
Introduction

- Sen's conjecture

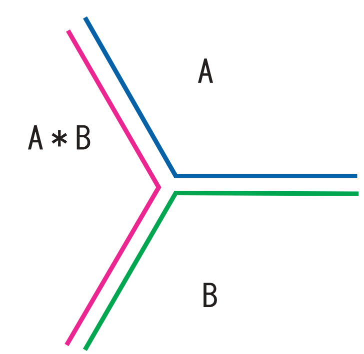
There is a tachyon vacuum in bosonic open string theory.
D25-brane vanishes on it and there is no open string excitation.



- Cubic String Field Theory (CSFT) [Witten(1986)]

Here we consider CSFT
to investigate Sen's conjecture.

$$S = -\frac{1}{g^2}\left[\frac{1}{2}\langle\Psi, Q_B\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right]$$



Sen's conjecture can be rephrased in CSFT :

- There is a solution Ψ_0 of equation of motion:

$$Q_B\Psi + \Psi * \Psi = 0.$$

- The potential height equals to D25-brane tension: $$-S|_{\Psi_0} / V_{26} = T_{25}.$$

- The cohomology of new BRST Q'_B operator around it is trivial: $$Q'_B\psi = 0 \Rightarrow \psi = Q'_B\phi, \quad \exists\phi.$$

There are some numerical evidences, but an exact solution is necessary to prove them.

Another approach: Vacuum String Field Theory (VSFT) [(G)RSZ(2001)]

(Gaiotto-)Rastelli-Sen-Zwiebach proposed SFT around tachyon vacuum:

$$S_V = -\kappa\left[\frac{1}{2}\langle\Psi, Q_{\mathrm{GRSZ}}\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right],$$

$$Q_{\mathrm{GRSZ}} = \frac{1}{2i}\big(c(i) - c(-i)\big) = c_0 + \sum_{n\geq 1}(-1)^n (c_{2n} + c_{-2n}).$$

This Q_{GRSZ} has trivial cohomology.

There are some solutions of equation of motion which are constructed by projectors with respect to $*$.

Can VSFT reconstruct CSFT?

Recently, Okawa proved D25-brane tension can be reproduced.

Contents

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Some Solutions in CSFT

Horowitz et al.(1988) discussed rather formal solutions in the context of purely CSFT.

$$|\Psi_0\rangle = -Q_L|I\rangle + C_L(f)|I\rangle, \qquad f(\pi-\sigma)=f(\sigma), f(\pi/2)=0.$$

In particular it can be used to derive VSFT action from CSFT:

$$f_{\mathrm{GRSZ}}(\sigma) = \lim_{\varepsilon\to 0}\Big(\delta\big(\sigma-(\pi/2-\varepsilon)\big)+\delta\big((\pi/2+\varepsilon)-\sigma\big)\Big).$$

The corresponding solution is singular:

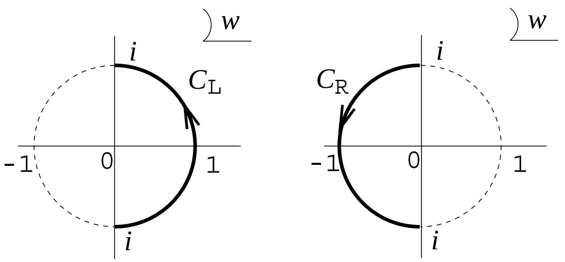
$$|\Psi_0^{\mathrm{VSFT}}\rangle = -Q_L|I\rangle + \lim_{\varepsilon\to 0}\frac{(g^2\kappa)^{1/3}}{4i}\left(e^{-i\varepsilon}c(ie^{i\varepsilon})-e^{i\varepsilon}c(-ie^{-i\varepsilon})\right)|I\rangle$$

$$=\frac{2}{\pi}\sum_{m\geq 0}\frac{(-1)^m}{2m+1}Q_{-(2m+1)}|I\rangle + \frac{(g^2\kappa)^{1/3}}{2}\lim_{\varepsilon\to 0}\left(\left(1+2\sum_{k\geq 1}\cos 2k\varepsilon\right)c_0-\left(\sum_{k\geq 0}\sin(2k+1)\varepsilon\right)(c_1-c_{-1})\right)|I\rangle$$

- There are some subtleties about identity string field, but here we treat $|I\rangle$ as the identity with respect to $*$:

$$A * I = I * A = A, \quad Q_R I * A = -I * Q_L A = -Q_L A, \dots$$

Notation:



$$C(f) = \oint \frac{dw}{2\pi i} f(w) c(w), \quad C_L(f) = \int_{C_L} \frac{dw}{2\pi i} f(w) c(w),$$

$$Q(f) = \oint \frac{dw}{2\pi i} f(w) j_{\mathrm{BRST}}(w), \quad Q_L(f) = \int_{C_L} \frac{dw}{2\pi i} f(w) j_{\mathrm{BRST}}(w),$$

$$q(f) = \oint \frac{dw}{2\pi i} f(w) j_{\mathrm{gh}}(w), \quad q_L(f) = \int_{C_L} \frac{dw}{2\pi i} f(w) j_{\mathrm{gh}}(w), \dots$$

$$j_{\mathrm{BRST}}(w) = cT^X + :bc\partial c: + \frac{3}{2}\partial^2 c = \sum_n Q_n w^{-n-1}, \quad j_{\mathrm{gh}}(w) = -:bc: = \sum_n q_n w^{-n-1}, \dots$$

- Takahashi and Tanimoto proposed a new regular solution of CSFT which might represent tachyon vacuum.(2002)

First, they found solutions of CSFT:

$$|\Psi_0\rangle = Q_L\left(e^h - 1\right)|I\rangle - C_L\left((\partial h)^2 e^h\right)|I\rangle,$$

where $h(w)$ is some function such that

$$h(w) = \sum h_n\left(w^n + (-w^{-1})^n\right), \quad h(\pm i) = 0, \partial h\left(\pm i\right) = 0.$$

But these solutions can be rewritten as pure gauge ones at least formally.

$$\Psi_0 = e^{q_L(h)I} * Q_B e^{-q_L(h)I}$$

In fact, new BRST operator is formally rewritten as

$$Q'_B = Q(e^h) - C((\partial h)^2 e^h)$$

$$Q'_B = e^{q(h)} Q_B e^{-q(h)}.$$

However they might become nontrivial solutions at some limits.

Example

$$h_a(w) = \log\left(1 + \frac{a}{2}\left(w + w^{-1}\right)^2\right)$$

Noting

$$q(h_a) = -q_0 \log(1 - Z(a))^2 + q^{(+)}(h_a) + q^{(-)}(h_a), \quad Z(a) = \frac{1 + a - \sqrt{1 + 2a}}{a},$$

and $\left[q^{(+)}(h_a), q^{(-)}(h_a)\right] = -2\log(1 - Z(a)^2),$

$$\exp\left(\pm q(h_a)\right) = \left(1 - Z(a)^2\right)^{-1} \exp\left(\mp q_0 \log(1 - Z(a))^2\right) e^{\pm q^{(-)}(h_a)} e^{\pm q^{(+)}(h_a)}$$

is well-defined for $a > -1/2.$

Because $Z(a = -1/2) = -1,$ $a = -1/2$ case:

$h_{a=-1/2}(w) = \log\left(-\frac{1}{4}(w - w^{-1})^2\right)$ might be nontrivial.

More Examples

$$h_a(w^{2k-1}), \qquad h_a(w^{2k}) - \log(2a)$$

Noting $\quad Z(a=-1/2)=-1, \quad Z(a=\infty)=1,$

in the same sense, we have nontrivial limit at $a=-1/2, \quad a=\infty.$ respectively:

$$h^{(l)}(w) = \log\left(\frac{(-1)^l}{4}\left(w^l + (-w)^{-l}\right)^2\right).$$

There is well-defined oscillator representation:

$$\left|\Psi_0^{(l)}\right\rangle = Q_L\left(-\frac{1}{2}+\frac{(-1)^l}{4}(w^{2l}+w^{-2l})\right)\left|I\right\rangle + C_L\left(l^2 w^{-2}\left(2-(-1)^l(w^{2l}+w^{-2l})\right)\right)\left|I\right\rangle$$

$$= \frac{1}{\pi}\sum_{m\geq 0}\frac{-(-1)^m 4l^2}{(2m+1)(2m+1-2l)(2m+1+2l)}\left(-Q_{-(2m+1)}+4l^2 c_{-(2m+1)}\right)\left|I\right\rangle$$

$$+\frac{l^2}{\pi}\left(\gamma+2\log 2+\frac{1}{2}\psi\left(\frac{1}{2}-l\right)+\frac{1}{2}\psi\left(\frac{1}{2}+l\right)\right)(c_1-c_{-1})\left|I\right\rangle.$$

Potential Height

When we naively compute the value of the action at the solutions constructed on identity string field:

$$|I\rangle = \frac{1}{4i} b\left(\frac{\pi}{2}\right) b\left(-\frac{\pi}{2}\right) \exp\left(\sum_{n\geq 1}\left(\frac{-(-1)^n}{2n}\alpha_{-n}\cdot\alpha_{-n} + (-1)^n c_{-n} b_{-n}\right)\right) c_0 c_1 |0\rangle,$$

we encounter following divergence:

$$\langle I|(\cdots)|I\rangle \sim \left(\det_{n,m\geq 1}\left(\delta_{n,m} - \delta_{n,m}\right)\right)^{-26/2+1} = \infty.$$

It is necessary to regularize $|I\rangle$ appropriately.

On the other hand, potential height is zero if we use e.o.m. naively in the action:

$$Q_{\mathrm{B}}\Psi_0 + \Psi_0 * \Psi_0 = 0,$$

$$Q_{\mathrm{L}} I * Q_{\mathrm{L}} I = 0, \qquad C_L(f) I * C_L(f) I = 0,$$

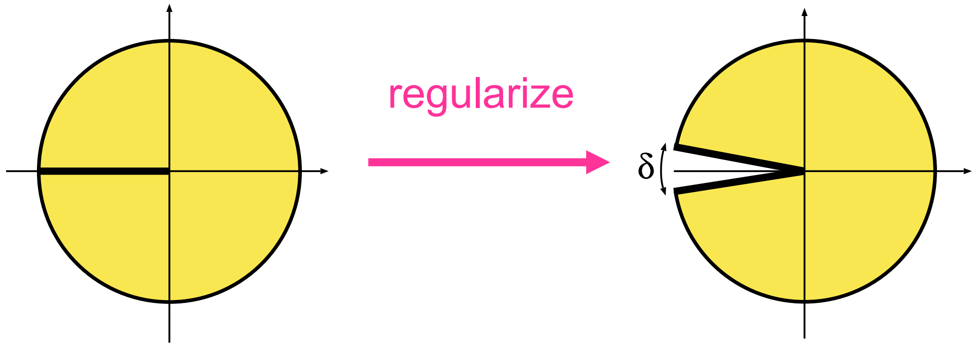
$$\Rightarrow \quad S|_{\Psi_0^{\mathrm{VSFT}}} = -\frac{1}{6g^2}\langle \Psi_0, Q_B \Psi_0 \rangle = \frac{1}{6g^2}\langle \Psi_0, \Psi_0 * \Psi_0 \rangle = 0.$$

Takahashi-Tanimoto solutions give also zero naively, because they are pure gauge formally.

There is a possibility of $\infty \times 0 = \text{finite}$ (!?)

- Pacman regularization

It is technically easy to use pacman regularization in the computation using LPP+GGRT. [I.K.-K.Ohmori]



But we could not get the definite value at $\delta = 0$, for example,

$$\left\langle \frac{1}{2i}\left(e^{-i\varepsilon}c(ie^{i\varepsilon}) - e^{i\varepsilon}c(-ie^{-i\varepsilon})\right) I_\delta, Q_B \frac{1}{2i}\left(e^{-i\varepsilon}c(ie^{i\varepsilon}) - e^{i\varepsilon}c(-ie^{-i\varepsilon})\right) I_\delta \right\rangle$$

$$= -\delta^2 \sin^2\varepsilon \left[\frac{1}{2}\left\{\left(\tan\frac{\varepsilon}{2}\right)^{\frac{2}{\delta}} + \left(\tan\frac{\varepsilon}{2}\right)^{-\frac{2}{\delta}}\right\} + 3\right] V_{26}, \cdots.$$

- Regularization in oscillator language

If we multiply damping factor $\exp(-\pi t L_0)$ on $|I\rangle$,

we get a formula for oscillator modes:

$$\langle I|q^{L_0} c_{m_1} c_{m_2} c_{m_3} q^{L_0}|I\rangle = \frac{i^{m_1+m_2+m_3} q^{-2} f(q^4)^{-12} V_{26}}{\sinh(2m_1\pi t)\sinh(2m_2\pi t)\sinh(2m_3\pi t)} \det\begin{pmatrix} 1 & 1 & 1 & 1 \\ (-q)^{m_1} & q^{m_1} & q^{-m_1} & (-q)^{-m_1} \\ (-q)^{m_2} & q^{m_2} & q^{-m_2} & (-q)^{-m_2} \\ (-q)^{m_3} & q^{m_3} & q^{-m_3} & (-q)^{-m_3} \end{pmatrix},$$

$$q := e^{-\pi t}, \qquad f(q^2) = \prod_{n=1}^{\infty}(1-q^{2n}) = (2\pi)^{-1/3} q^{-1/12} \vartheta'_1(0,it)^{1/3}.$$

Using some relations

$$Q_B|I\rangle = 0, \quad \{Q_B, Q_n\} = 0, \quad \{Q_B, c_n\} = -\sum_{j=-\infty}^{\infty}\left(j+\frac{n}{2}\right) c_{-j} c_{n+j}, \ldots,$$

we have the following expression:

$$S|_{\Psi_0^{(1)}} / V_{26} = \lim_{t\to 0+} \frac{-1}{6g^2 V_{26}} \langle I | q^{L_0} C_L \left(w^{-2}(w+w^{-1})^2 \right) Q_B C_L \left(w^{-2}(w+w^{-1})^2 \right) q^{L_0} | I \rangle$$

$$= \lim_{t\to 0+} \frac{-1}{6g^2} \int_{-\pi/2}^{\pi/2} dx \int_{-\pi/2}^{\pi/2} dy \cos^2 x \cos^2 y \frac{i\pi^2}{\left(\vartheta_1{}'(0,2it)\right)^4}$$

$$\cdot \det \begin{pmatrix} 1 & 1 & 1 & 1 \\ \frac{\vartheta_4{}'(x/2+\pi/2-i\pi t/4,2it)}{\vartheta_4(x/2+\pi/2-i\pi t/4,2it)} & \frac{\vartheta_3{}'}{\vartheta_3}(\cdots) & \frac{\vartheta_2{}'}{\vartheta_2}(\cdots) & \frac{\vartheta_1{}'}{\vartheta_1}(\cdots) \\ \frac{\vartheta_4{}'(y/2+\pi/2-i\pi t/4,2it)}{\vartheta_4(y/2+\pi/2-i\pi t/4,2it)} & \frac{\vartheta_3{}'}{\vartheta_3}(\cdots) & \frac{\vartheta_2{}'}{\vartheta_2}(\cdots) & \frac{\vartheta_1{}'}{\vartheta_1}(\cdots) \\ \partial_y\left(\frac{\vartheta_4{}'(y/2+\pi/2-i\pi t/4,2it)}{\vartheta_4(y/2+\pi/2-i\pi t/4,2it)}\right) & \partial_y\left(\frac{\vartheta_3{}'}{\vartheta_3}\right)(\cdots) & \partial_y\left(\frac{\vartheta_2{}'}{\vartheta_2}\right)(\cdots) & \partial_y\left(\frac{\vartheta_1{}'}{\vartheta_1}\right)(\cdots) \end{pmatrix}.$$

We do not have definite (or exact) value of the potential height for our solutions yet.

Numerically, it tends to be divergent.(?)

Cohomology of new BRST operator

- Kinetic term around a solution

$$S|_{\Psi_0+\psi} = -\frac{1}{g}\left[\frac{1}{2}\langle\psi, Q'_B\psi\rangle + \frac{1}{3}\langle\psi, \psi*\psi\rangle\right] + S|_{\Psi_0},$$

$$Q'_B\psi = Q_B\psi + \Psi_0*\psi - (-1)^{|\psi|}\psi*\Psi_0.$$

The new BRST operator around our solution $\Psi_0^{(1)}$ is

$$Q_B' = \frac{1}{2}Q_0 - \frac{1}{4}Q_{-2} - \frac{1}{4}Q_2 + 2c_0 + c_{-2} + c_2 = R_2 + R_0 + R_{-2}.$$

$$R_{\pm2} := -\frac{1}{4}Q_{\pm2} + c_{\pm2}, \quad R_0 := \frac{1}{2}Q_0 + 2c_0$$ satisfy following relations:

$$R_{\pm2}^2 = 0, \quad R_{\pm2}R_0 + R_0R_{\pm2} = 0, \quad R_0^2 + R_2R_{-2} + R_{-2}R_2 = 0.$$

Claim

Q'_B cohomology is trivial in ghost number 1 states.

Proof We consider the equation $Q'_B \psi = 0$

for the ghost number 1 state $\psi = \sum_{N \geq h} \psi_{-N}$.

In other words, we solve the following equations:

$$R_2 \psi_{-h-k} = 0, \qquad k = 0, 1,$$

$$R_2 \psi_{-h-k-2} = -R_0 \psi_{-h-k},$$

$$R_2 \psi_{-h-k-2l} = -R_0 \psi_{-h-k-2(l-1)} - R_{-2} \psi_{-h-k-2(l-2)}, \qquad l \geq 2.$$

R_2 is rewritten as $R_2 = -\frac{1}{4}Q_2 + c_2 = -\frac{1}{4}\widetilde{Q}_0$.

$\widetilde{Q}_0$ is given by replacing c_n, b_n with $\widetilde{c}_n := c_{n+2}, \widetilde{b}_n := b_{n-2}$ in $Q_0 = Q_B$.

Solutions of $Q_B\psi = 0$ are [Kato-Ogawa,M.Henneaux,…]

$$|\psi\rangle = A_{\mathrm{DDF}}|0, p_0\rangle + B_{\mathrm{DDF}}c_0|0, p_0\rangle + Q_B|\phi\rangle$$

where $|0, p_0\rangle = e^{ip_0x}|\Omega\rangle$, $\quad p_0{}^{\pm} = \pm\frac{1}{\sqrt{2\alpha'}}, \quad p_0{}^{i} = 0,$

$$|\Omega\rangle := c_1|0\rangle, \qquad c_n|\Omega\rangle = 0, n \geq 1, \qquad b_n|\Omega\rangle = 0, n \geq 0,$$

and $A_{\mathrm{DDF}}, B_{\mathrm{DDF}}$ are generated by DDF operators A_n^i :

$$\left[A_m^i, A_n^j\right] = m\delta_{m+n,0}\delta^{i,j}, \qquad \left[L_{m,}^X A_n^i\right] = 0.$$

Similarly, solutions of $\widetilde{Q}_0 \psi = 0$ are given by

$$|\psi\rangle = A_{\rm DDF} \left|\tilde{0}, p_0\right\rangle + B_{\rm DDF} \widetilde{c}_0 \left|\tilde{0}, p_0\right\rangle + \widetilde{Q}_0 |\phi\rangle$$

$$\left|\tilde{0}, p_0\right\rangle := b_{-2} b_{-1} |0, p_0\rangle, \quad \widetilde{c}_n \left|\tilde{0}, p_0\right\rangle = 0, n \geq 1, \quad \widetilde{b}_n \left|\tilde{0}, p_0\right\rangle = 0, n \geq 0.$$

Here $|\psi_{-h-k}\rangle$ is ghost number 1,

$$R_2 |\psi_{-h-k}\rangle = 0 \quad \Rightarrow \quad |\psi_{-h-k}\rangle = R_2 |\phi_{-h-k-2}\rangle, \qquad \exists |\phi_{-h-k-2}\rangle.$$

Then $$R_2 |\psi_{-h-k-2}\rangle = -R_0 |\psi_{-h-k}\rangle = -R_0 R_2 |\phi_{-h-k-2}\rangle = R_2 R_0 |\phi_{-h-k-2}\rangle,$$

$$\Rightarrow \quad |\psi_{-h-k-2}\rangle = R_0 |\phi_{-h-k-2}\rangle + R_2 |\phi_{-h-k-4}\rangle, \quad \exists |\phi_{-h-k-4}\rangle.$$

Suppose, $\left|\psi_{-h-k-2l}\right\rangle = R_{-2}\left|\phi_{-h-k-2(l-1)}\right\rangle + R_0\left|\phi_{-h-k-2l}\right\rangle + R_2\left|\phi_{-h-k-2(l+1)}\right\rangle, \quad l = m, m-1$

then

$$R_2\left|\psi_{-h-k-2(m+1)}\right\rangle = -R_0\left|\psi_{-h-k-2m}\right\rangle - R_{-2}\left|\psi_{-h-k-2(m-1)}\right\rangle$$
$$= -R_0\left(R_{-2}\left|\phi_{-h-k-2(m-1)}\right\rangle + R_0\left|\phi_{-h-k-2m}\right\rangle + R_2\left|\phi_{-h-k-2(m+1)}\right\rangle\right)$$
$$- R_{-2}\left(R_{-2}\left|\phi_{-h-k-2(m-2)}\right\rangle + R_0\left|\phi_{-h-k-2(m-1)}\right\rangle + R_2\left|\phi_{-h-k-2m}\right\rangle\right)$$
$$= R_2\left(R_{-2}\left|\phi_{-h-k-2m}\right\rangle + R_0\left|\phi_{-h-k-2(m+1)}\right\rangle\right),$$

$\Rightarrow$

$$\left|\psi_{-h-k-2(m+1)}\right\rangle = R_{-2}\left|\phi_{-h-k-2m}\right\rangle + R_0\left|\phi_{-h-k-2(m+1)}\right\rangle + R_2\left|\phi_{-h-k-2(m+2)}\right\rangle, \quad \exists\left|\phi_{-h-k-2(m+2)}\right\rangle$$

By induction, we conclude

$$\left|\psi_{-h-k-2l}\right\rangle = R_{-2}\left|\phi_{-h-k-2(l-1)}\right\rangle + R_0\left|\phi_{-h-k-2l}\right\rangle + R_2\left|\phi_{-h-k-2(l+1)}\right\rangle, \quad l \geq 0,$$

namely, $\left|\psi\right\rangle = Q'_B\left|\phi\right\rangle, \quad \exists\left|\phi\right\rangle = \sum_{k=0,1,l\geq 1}\left|\phi_{-h-k-2l}\right\rangle.$

- We have similar arguments for other solution $\Psi_0^{(l)}$ by using R_{2l} instead of R_2.

New BRST charge around it is

$$Q_B^{(l)} = \frac{1}{2}Q_0 + 2l^2 c_0 - (-1)^l \left(R_{2l} + R_{-2l}\right),$$

where $R_{\pm 2l} = -\frac{1}{4}Q_{\pm 2l} + c_{\pm 2l} = -\frac{1}{4}\widetilde{Q}_0^{(\pm l)}.$

$\widetilde{Q}_0^{(l)}$ is given by replacing c_n, b_n with $\widetilde{c_n} := c_{n+2l}, \widetilde{b_n} := b_{n-2l}$ in Q_B.

We can prove

$Q_B^{(l)}$ cohomology is trivial in ghost number 1 states.

- There are nontrivial solutions for $Q_B^{(l)}\left|\psi^{(l)}\right\rangle = 0$ of the form:

$$\left|\psi^{(l)}\right\rangle = \exp\left(-q(f^{(l)})\right) \cdot\left(A_{\mathrm{DDF}} b_{-2l} b_{-2l+1} \cdots b_{-2} b_{-1}\left|0, p_0\right\rangle + B_{\mathrm{DDF}} b_{-2l+1} \cdots b_{-2} b_{-1}\left|0, p_0\right\rangle\right),$$

where $q(f^{(l)}) = 2\sum_{n\geq 1}\frac{(-1)^{n(1+l)}}{n} q_{-2ln}$

in other ghost number sector.

This follows from the identity:

$$\exp\left(q(f^{(l)})\right) Q_B^{(l)} \exp\left(-q(f^{(l)})\right) = -(-1)^l R_{2l}.$$

Summary and Discussion

- We have investigated whether some solutions of CSFT using identity string field can be tachyon vacuum or not.

- Evaluation of potential height at the solutions expressed by identity string field is rather difficult by our two regularization methods. It is divergent at least naively.

- The new BRST charge cohomology around Takahashi-Tanimoto limit solutions is trivial in ghost number 1 states. This suggests they are nontrivial solutions of CSFT although we do not know yet that they are the tachyon vacuum in Sen's conjecture.

- In the context of Takahashi-Tanimoto solutions, the solution which plugs CSFT into VSFT directly is very singular limit (?):

$$|\Psi_0\rangle = Q_L\left(e^h - 1\right)|I\rangle - C_L\left((\partial h)^2 e^h\right)|I\rangle$$

$$h(w) \to -\infty \qquad |\Psi_0\rangle = -Q_L|I\rangle + C_L(f)|I\rangle.$$

There is subtlety about $Q_L I$:

By CFT calculation, we have $\langle \phi, Q_L I * Q_L I \rangle = \langle \phi, Q_L^2 I \rangle = 0,$

but by oscillator calculation, we have

$$Q_L^2|I\rangle = -\frac{2\zeta(0)}{\pi^2}\left(1 + 2\zeta(0)\right) Q_B c_0 |I\rangle = 0. \quad (?)$$

- Is the identity string field well-defined?

Mystery on $c_0 I$ [Rastelli-Zwiebach(2000)]

$$c_0 A = c_0 (I * A) = (c_0 I) * A + I * (c_0 A) = (c_0 I) * A + c_0 A,$$

$$\therefore (c_0 I) * A = 0, \forall A.$$

If we take $A = I$, $0 \neq c_0 I = (c_0 I) * I = 0 \quad (??)$

- Is there another exact solution which do not use identity string field in CSFT?

Solutions in CSFT

CSFT → VSFT

Q_B — $Q_B^{(l)}$ — Q_{GRSZ}

pure gauge — $\Psi_0^{(l)}$ — singular — Ψ_0^{VSFT}

Where is the tachyon vacuum solution?

numerical solution
in the Siegel gauge

Brief review of CSFT

- ket representation of string field:

$$|\Psi\rangle = \left(\varphi(x) + A_\mu(x)\alpha_{-1}^{\mu} + \cdots + B(x) b_{-1} c_0 + \cdots\right) c_1 |0\rangle.$$

Classically, these are ghost number 1 states.

- The $*$ product is defined as

$$|A*B\rangle_1 := {}_2\langle A|\,{}_3\langle B||V_3\rangle_{123}, \qquad {}_1\langle A| := {}_{12}\langle R||A\rangle_2 .$$

- There are some relations:

$$\langle A|B*C\rangle = \langle A, B*C\rangle = \langle B, C*A\rangle = \langle C, A*B\rangle,$$

$$(A*B)*C = A*(B*C) = A*B*C,$$

$$Q_B{}^2 = 0, \qquad \langle Q_B A, B\rangle = -(-1)^{|A|}\langle A, Q_B B\rangle,$$

$$Q_B(A*B) = (Q_B A)*B + (-1)^{|A|} A*(Q_B B).$$

- Equation of motion: $$Q_B\left|\Psi\right\rangle+\left|\Psi*\Psi\right\rangle=0.$$

- Gauge invariance of CSFT

Under the gauge transformation

$$\delta_\Lambda\left|\Psi\right\rangle=Q_B\left|\Lambda\right\rangle+\left|\Psi*\Lambda\right\rangle-\left|\Lambda*\Psi\right\rangle,$$

we can show the action is invariant: $\delta_\Lambda S=0$

by using previous relations.

In the context of first quantization, physical states are

$$Q_B\left|\mathrm{phys}\right\rangle=0,\qquad \left|\mathrm{phys}\right\rangle\equiv\left|\mathrm{phys}\right\rangle+Q_B\left|\Lambda\right\rangle.$$