Analytical tachyonic lump solutions in open superstring field theory

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Introduction

- String Field Theory (SFT) is a candidate for nonperturbative formulation of string theory.
- In bosonic SFT, some phenomena such as tachyon condensation have been investigated extensively using *level truncation numerically and exact solutions analytically*.
- In super SFT, similar works are done although concrete and detailed analysis is less developed than bosonic case.
- *We have constructed a class of exact classical solutions to super SFT and studied their properties.*

These are a generalization of marginal solutions in Witten's bosonic cubic SFT [Takahashi-Tanimoto(2001)] :

$$\Psi_0 = -V_L^a(F_a)I - \frac{1}{4}g^{ab}C_L(F_aF_b)I, \quad V_L^a(f) \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i}\frac{1}{\sqrt{2}}f(z)cJ^a(z), \cdots$$

to those in Berkovits' non-polynomial open super SFT.

A brief review of super SFT

- We use Berkovits' open super SFT.

The action for NS(+) sector is given by

WZW type

$$S[\Phi]$$

$$= \frac{1}{2g^2}\langle\langle (e^{-\Phi}Q_{\rm B}e^{\Phi})(e^{-\Phi}\eta_0 e^{\Phi}) - \int_0^1 dt (e^{-t\Phi}\partial_t e^{t\Phi})\{(e^{-t\Phi}Q_{\rm B}e^{t\Phi}),(e^{-t\Phi}\eta_0 e^{t\Phi})\}\rangle\rangle$$

$$= -\frac{1}{g^2}\int_0^1 dt \langle\langle (\eta_0\Phi)(e^{-t\Phi}Q_{\rm B}e^{t\Phi})\rangle\rangle$$

[Berkovits-Okawa-Zwiebach(2004)]

$$= -\frac{1}{g^2}\sum_{M,N=0}^{\infty}\frac{(-1)^M}{(M+N+2)(M+N+1)M!N!}\langle\langle(\eta_0\Phi)\Phi^M(Q_{\rm B}\Phi)\Phi^N\rangle\rangle\,.$$

String field Φ : ghost number 0, picture number 0, Grassmann even, represented by $X^\mu,\psi^\mu,b,c,\phi,\xi,\eta\quad(\beta=e^{-\phi}\partial\xi,\gamma=\eta e^{\phi})$

$$Q_{\rm B}=\oint\frac{dz}{2\pi i}(c(T^{\rm m}-\tfrac{1}{2}(\partial\phi)^2-\partial^2\phi+\partial\xi\eta)+bc\partial c+\eta e^{\phi}G^{\rm m}-\eta\partial\eta e^{2\phi}b)(z)$$

$$\eta_0=\oint\frac{dz}{2\pi i}\eta(z)$$

$Q_{\rm B}, \eta_0$ such as $Q_{\rm B}^2 = 0,\ \eta_0^2 = 0,\ \{Q_{\rm B}, \eta_0\} = 0$

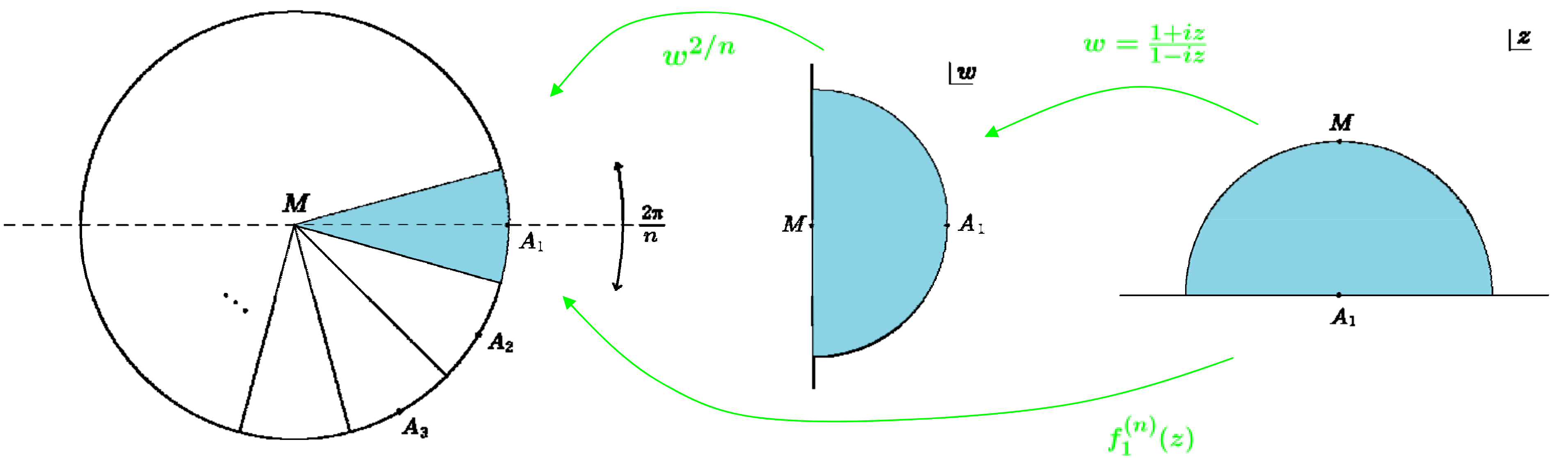
are derivations with respect to the star product:

$$Q_{\rm B}(A * B) = Q_{\rm B}A * B + (-1)^{|A|} A * Q_{\rm B}B, \quad \eta_0(A * B) = \eta_0 A * B + (-1)^{|A|} A * \eta_0 B$$

The star product is given by 3-string vertex: $\langle A * B| = \langle V_3|A\rangle|B\rangle$.

n-string vertex is defined using CFT correlator in the *large* Hilbert space:

$$\langle V_n|A_1\rangle \cdots |A_n\rangle = \langle\langle A_1 \cdots A_n \rangle\rangle := \left\langle f_1^{(n)}[\mathcal{O}_{A_1}] \cdots f_n^{(n)}[\mathcal{O}_{A_n}] \right\rangle$$
$$= \langle A_1|(\cdots(A_2 * A_3) * \cdots * A_{n-1}) * A_n\rangle = \langle A_1|A_2 * \cdots * A_n\rangle$$



- Variation of the action: $\delta S = \frac{1}{g^2}\langle\langle e^{-\Phi}\delta e^{\Phi}\,\eta_0(e^{-\Phi}Q_{\mathrm{B}}e^{\Phi})\rangle\rangle$
- Equation of motion: $\eta_0(e^{-\Phi}Q_{\mathrm{B}}e^{\Phi}) = 0$
- Gauge transformation: $\delta e^{\Phi} = Q_{\mathrm{B}}\Lambda_0 * e^{\Phi} + e^{\Phi} * \eta_0\Lambda_1$
- Re-expansion of the action around a classical solution Φ_0 :

$$S[\Phi] = S[\Phi_0] + S'[\Phi'] \qquad (\ e^{\Phi} = e^{\Phi_0}e^{\Phi'}\ )$$

where $S'[\Phi'] = S[\Phi']|_{Q_{\mathrm{B}}\to Q'_{\mathrm{B}}}$.

New BRST operator Q'_{B} is a derivation such as

$$Q'_{\mathrm{B}}A = Q_{\mathrm{B}}A + e^{-\Phi_0}Q_{\mathrm{B}}e^{\Phi_0} * A - (-1)^{|A|}A * e^{-\Phi_0}Q_{\mathrm{B}}e^{\Phi_0}$$

which satisfies $Q'^2_{\mathrm{B}} = 0,\ \{Q'_{\mathrm{B}}, \eta_0\} = 0$.

A class of classical solutions

- Suppose that there exists a supercurrent associated with G :

$\mathbf{J}^a(z,\theta) = \psi^a(z) + \theta J^a(z)$ $\quad (a = 1, \cdots, \mathrm{dim}G)$ such as

$$\psi^a(y)\psi^b(z) \sim \frac{1}{y-z}\frac{1}{2}\Omega^{ab},$$
$$J^a(y)\psi^b(z) \sim \frac{1}{y-z}f^{ab}{}_c\psi^c(z),$$
$$J^a(y)J^b(z) \sim \frac{1}{(y-z)^2}\frac{1}{2}\Omega^{ab} + \frac{1}{y-z}f^{ab}{}_cJ^c(z),$$

where

$$f^{ab}{}_c = -f^{ba}{}_c,$$
$$f^{ab}{}_d f^{cd}{}_e + f^{bc}{}_d f^{ad}{}_e + f^{ca}{}_d f^{bd}{}_e = 0,$$
$$\Omega^{ab} = \Omega^{ba},$$
$$f^{ab}{}_c\Omega^{cd} + f^{ad}{}_c\Omega^{cb} = 0.$$

If $\exists\, \Omega_{ab}$ such as $\Omega^{ac}\Omega_{cb} = \delta^a_b$, matter super Virasoro operators are given by Sugawara construction:

$$T^{\mathrm{m}}(z) = \Omega_{ab}:(J^aJ^b + \partial\psi^a\psi^b):(z) + \frac{2}{3}\Omega_{ad}\Omega_{be}f^{de}{}_c:(J^a:\psi^b\psi^c: + \psi^a:(\psi^bJ^c - J^b\psi^c):):(z),$$
$$G^{\mathrm{m}}(z) = 2\Omega_{ab}:J^a\psi^b:(z) + \frac{4}{3}\Omega_{ad}\Omega_{be}f^{de}{}_c:\psi^a:\psi^b\psi^c::(z),$$

where the central charge is $c^{\mathrm{m}} = \frac{3}{2}\mathrm{dim}G - f^{ac}{}_d f^{bd}{}_c\Omega_{ab}.$ [Mohammedi(1994)]

We use $c^{\mathrm{m}} = 15$ case for super SFT.

In this setup, we can construct a class of solution to equation of motion:

$$\Phi_0 = -\tilde{V}_L^a(F_a) I \,,$$

$$\tilde{V}_L^a(F_a) \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i} F_a(z)\tilde{v}^a(z), \quad F_a(-1/z) = z^2 F_a(z) \,,$$

$$\tilde{v}^a(z) \equiv \frac{1}{\sqrt{2}} c\xi e^{-\phi}\psi^a(z) \,,$$

where I is the identity string field given by $\langle I| \equiv \langle V_1|$.

In fact, we can compute as :

$$e^{-\Phi_0} Q_{\rm B} e^{\Phi_0} = e^{\tilde{V}_L^a(F_a)} Q_{\rm B} e^{-\tilde{V}^a(F_a)} I$$
$$= -V_L^a(F_a) I + \frac{1}{8}\Omega^{ab} C_L(F_a F_b) I$$

where

$$V_L^a(G) \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i} G(z) v^a(z),$$
$$v^a(z) = \frac{i}{2\sqrt{\alpha'}} cJ^a(z) + \frac{1}{\sqrt{2}}\eta e^{\phi}\psi^a(z),$$
$$C_L(G) \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i} G(z) c(z).$$

⇨ $\eta_0(e^{-\Phi_0} Q_{\rm B} e^{\Phi_0}) = 0$ due to $\eta_0|I\rangle = 0$

Note: the vacuum energy at this solution vanishes exactly,

$$S[\Phi_0] \;=\; \frac{1}{g^2}\int_0^1 dt \langle\langle \Phi_0\, \eta_0(e^{-t\Phi_0} Q_{\rm B} e^{t\Phi_0})\rangle\rangle = 0.$$

• Toward inclusion of GSO(−) sector

Super SFT on a non-BPS brane [Berkovits,Berkovits-Sen-Zwiebach(2000)]

$$S[\hat{\Phi}] = -\frac{1}{2g^2}\int_0^1 dt\, \mathrm{Tr}\langle\langle(\hat{\eta}_0\hat{\Phi})(e^{-t\hat{\Phi}}\hat{Q}_{\mathrm{B}}e^{t\hat{\Phi}})\rangle\rangle\,,$$

$$\hat{Q}_{\mathrm{B}} = Q_{\mathrm{B}}\otimes\sigma_3, \qquad \hat{\eta}_0 = \eta_0\otimes\sigma_3\,,$$

$$\hat{\Phi} = \Phi_+\otimes 1 + \Phi_-\otimes\sigma_1\,,$$

where $\Phi_+ : \mathrm{GSO}(+), \quad \Phi_- : \mathrm{GSO}(-).$

(※) Algebraic property is almost the same as that in GSO projected theory.

Equation of motion: $\hat{\eta}_0(e^{-\hat{\Phi}}\hat{Q}_{\mathrm{B}}e^{\hat{\Phi}}) = 0\,.$

The same form as that in super SFT on BPS (GSO-projected) D-brane.

⇓

We can also construct marginal solutions in the GSO(−) sector if there exists a supercurrent with GSO(−) components.

A class of GSO(−) solutions

Let us compactify X^9 direction to S^1 with the critical radius $R=\sqrt{2\alpha'}$.

Then, we find an SU(2) supercurrent $\mathbf{J}^a(z,\theta)=\psi^a(z)+\theta J^a(z)$ as

$$
\begin{aligned}
\mathrm{J}^1(z,\theta) &= \sqrt{2}\sin\left(\frac{X^9}{\sqrt{2\alpha'}}\right)(z)\otimes\sigma_2+\theta(-\sqrt{2})\psi^9\cos\left(\frac{X^9}{\sqrt{2\alpha'}}\right)(z)\otimes\sigma_1\,,\\
\mathrm{J}^2(z,\theta) &= \sqrt{2}\cos\left(\frac{X^9}{\sqrt{2\alpha'}}\right)(z)\otimes\sigma_2+\theta\sqrt{2}\psi^9\sin\left(\frac{X^9}{\sqrt{2\alpha'}}\right)(z)\otimes\sigma_1\,,\\
\mathrm{J}^3(z,\theta) &= \psi^9(z)\otimes\sigma_3+\theta\frac{i}{\sqrt{2\alpha'}}\partial X^9(z)\otimes 1\,.
\end{aligned}
$$

Note: $e^{in\frac{X^9}{\sqrt{2\alpha'}}}$ (n :odd) should be treated as "fermion."

We have assigned cocycle factors (Pauli matrices) to each component appropriately.

The above is an analogy with the SU(2) current in <u>bosonic</u> string theory:

$$
J^1=\sqrt{2}\cos\left(\frac{X^{25}}{\sqrt{\alpha'}}\right),\quad J^2=\sqrt{2}\sin\left(\frac{X^{25}}{\sqrt{\alpha'}}\right),\quad J^3=\frac{i}{\sqrt{2\alpha'}}\partial X^{25}\quad (R=\sqrt{\alpha'})
$$

Actually, we can check the SU(2) supercurrent algebra with

$\Omega^{ab} = 2\delta^{a,b}, \quad f^{ab}{}_{c} = -i\epsilon_{abc}$ and we have

$$T^9(z) = \left(-\frac{1}{4\alpha'}(\partial X^9)^2(z) - \frac{1}{2}\psi^9\partial\psi^9(z)\right)\otimes 1\,, \qquad G^9(z) = \frac{i}{\sqrt{2\alpha'}}\psi^9\partial X^9(z)\otimes\sigma_3\,.$$

which satisfies super Virasoro algebra with $c = \frac{3}{2}$.

Then, we can construct a solution to EOM $\hat{\eta}_0(e^{-\hat{\Phi}}\hat{Q}_{\mathrm{B}}e^{\hat{\Phi}}) = 0$:

$$\hat{\Phi}_0 = -\tilde{V}_L^a(F_a)I\,,$$

$$\tilde{V}_L^a(F_a) \equiv \int_{C_{\mathrm{left}}}\frac{dz}{2\pi i}F_a(z)\tilde{v}^a(z), \quad F_a(-1/z) = z^2F_a(z)\,,$$

$$\tilde{v}^a(z) \equiv \frac{1}{\sqrt{2}}(c\xi e^{-\phi}\otimes\sigma_3)\,\psi^a(z)\,, \qquad a = 1,2,3.$$

Around this solution, the new BRST operator is

$$\begin{aligned}
&\hat{Q}'_{\mathrm{B}}\hat{A} \\
&= \hat{Q}_{\mathrm{B}}\hat{A} + \left[\left(-V_L^a(F_a) + \frac{1}{4}C_L(F_aF_a)\right)I\right]*\hat{A} - (-1)^{\mathrm{gh}(\hat{A})}\hat{A}*\left[\left(-V_L^a(F_a) + \frac{1}{4}C_L(F_aF_a)\right)I\right] \\
&= \left((Q_{\mathrm{B}} + \frac{1}{4}C(F_aF_a))\sigma_3 - V^3(F_3) - V_L^1(F_1) - V_L^2(F_2) - (-1)^{\hat{F}+\hat{n}}(V_R^1(F_1) + V_R^2(F_2))\right)\hat{A}.
\end{aligned}$$

Here, $V^a_{L/R}(F) = \int_{C_{\rm left/right}} \frac{dz}{2\pi i} F(z) v^a(z),$

$$v^a(z) \equiv [\hat{Q}_{\rm B}, \tilde{v}^a(z)] = \frac{1}{\sqrt{2}} c\sigma_3 J^a(z) + \frac{1}{\sqrt{2}} \eta e^{\phi} \psi^a(z), \quad a = 1, 2, 3.$$

$\hat{n} = \oint \frac{dz}{2\pi i} \frac{i}{\sqrt{2\alpha'}} \partial X^9(z)$: momentum along the X^9 direction.

$(-1)^{\hat{F}}$: GSO($\pm$) which is given by $\hat{F} = \oint \frac{dz}{2\pi i} \left(\sum_{k=1}^{5} : \psi^k_+ \psi^k_- : (z) - \partial\phi(z) \right),$

$$\psi^1_\pm \equiv \frac{i}{\sqrt{2}}(\psi^0 \pm \psi^1), \quad \psi^k_\pm \equiv \frac{1}{\sqrt{2}}(\psi^{2k-2} \pm i\psi^{2k-1}),$$
$$k = 2, 3, 4, 5.$$

We can discuss physics around the solution $\hat{\Phi}_0$
by investigating the obtained new BRST operator: $\hat{Q}'_{\rm B}$.

In particular, let us consider a solution given by $F_a(z) = \delta^1_a F(z)$

and $\tilde{v}^1(z) = -ic\xi e^{-\phi} \sin\left(\frac{X^9}{\sqrt{2\alpha'}}\right)(z) \otimes \sigma_1$ in the following.

Technically, we use fermionization and rebosonization method after Sen's argument in the context of CFT. Namely,

$$e^{\pm\frac{i}{\sqrt{2\alpha'}}X^9(z)} = \frac{1}{\sqrt{2}}(\xi^9(z) \pm i\eta^9(z)) \otimes \tau_1 \quad : (\psi^9, X^9) \to (\psi^9, \xi^9, \eta^9)$$

$$\xi^9(z) \pm i\psi^9(z) = \sqrt{2}e^{\pm\frac{i}{\sqrt{2\alpha'}}\phi^9(z)} \otimes \tilde{\tau}_1 \quad : (\psi^9, \xi^9, \eta^9) \to (\phi^9, \eta^9)$$

where we introduce Pauli matrices $\tau_i, \tilde{\tau}_i \ (i = 1, 2, 3)$ as cocycle factors.

Then, the new BRST operator can be rewritten as

$$\hat{Q}'_{\rm B} = (Q_{\rm B} + \frac{1}{4}C(F^2))\sigma_3 - V_L^1(F) - (-1)^{\hat{F}+\hat{n}}V_R^1(F)$$

$$= \begin{cases} e^{-\frac{i}{2\sqrt{\alpha'}}(\phi_L^9(F)+\phi_R^9(F))\sigma_1\tau_2}\hat{Q}_{\rm B}\, e^{\frac{i}{2\sqrt{\alpha'}}(\phi_L^9(F)+\phi_R^9(F))\sigma_1\tau_2} \\ \qquad \text{for} \quad (-1)^{\hat{F}+\hat{n}} = +1 \\ e^{-\frac{i}{2\sqrt{\alpha'}}(\phi_L^9(F)-\phi_R^9(F))\sigma_1\tau_2}\hat{Q}_{\rm B}\, e^{\frac{i}{2\sqrt{\alpha'}}(\phi_L^9(F)-\phi_R^9(F))\sigma_1\tau_2} \\ \qquad \text{for} \quad (-1)^{\hat{F}+\hat{n}} = -1 \end{cases}$$

where $\phi^9_{L/R}(F) \equiv \int_{C_{\rm left/right}} \frac{dz}{2\pi i}F(z)\phi^9(z).$

This expression implies that our solution induces a string field redefinition:

$$\hat{\Phi}'' = e^{\frac{i}{2\sqrt{\alpha'}}\phi_L^9(F)I\sigma_1\tau_2} * \hat{\Phi}' * e^{-\frac{i}{2\sqrt{\alpha'}}\phi_L^9(F)I\sigma_1\tau_2}$$

$$= \begin{cases} e^{\frac{i}{2\sqrt{\alpha'}}(\phi_L^9(F)+\phi_R^9(F))\sigma_1\tau_2}\hat{\Phi}' & \text{for} \quad (-1)^{\hat{F}+\hat{n}} = +1 \\ e^{\frac{i}{2\sqrt{\alpha'}}(\phi_L^9(F)-\phi_R^9(F))\sigma_1\tau_2}\hat{\Phi}' & \text{for} \quad (-1)^{\hat{F}+\hat{n}} = -1 \end{cases}$$

in the sense that the action can be rewritten as

$$S[\hat{Q}_{\rm B};\hat{\Phi}] = S[\hat{Q}_{\rm B};\hat{\Phi}_0] + S[\hat{Q}'_{\rm B};\hat{\Phi}'] = S[\hat{Q}_{\rm B};\hat{\Phi}''].$$

$$S[\hat{Q}_{\rm B};\hat{\Phi}_0] = 0 \quad (e^{\hat{\Phi}} = e^{\hat{\Phi}_0}e^{\hat{\Phi}'})$$

Due to $\int_{C_{\rm left}}\frac{dz}{2\pi i}F(z) + \int_{C_{\rm right}}\frac{dz}{2\pi i}F(z) = 0$ by construction of the solution: $F(-1/z) = z^2F(z)$

$\phi_L^9(F) + \phi_R^9(F)$ does not include $\hat{\phi}_0^9$: zero mode of ϕ^9.

On the other hand, $\phi_L^9(F) - \phi_R^9(F) = 2f\hat{\phi}_0^9 + \cdots$ where $f \equiv \int_{C_{\rm left}}\frac{dz}{2\pi i}F(z)$.

⇓

ϕ^9-momentum changes by $\pm\frac{f}{\sqrt{\alpha'}}$ in $(-1)^{\hat{F}+\hat{n}} = -1$ sector by the field redefinition.

- Critical value of f

In the case of $f=\frac{2m+1}{\sqrt{2}},\ \ (m\in \mathbf{Z}),$ all states in $\boldsymbol{(-1)^{\hat{F}+\hat{n}}=-1}$ sector

change into $(-1)^{\hat{F}+\hat{n}}=+1$ and all states in $(-1)^{\hat{F}+\hat{n}}=+1$ remain

because $(-1)^{\hat{F}+\hat{n}}\partial\phi^9(z)(-1)^{-(\hat{F}+\hat{n})}=+\partial\phi^9(z)\,,$

$$(-1)^{\hat{F}+\hat{n}}e^{i\frac{2m+1}{\sqrt{2\alpha'}}\hat{\phi}_0^9}(-1)^{-(\hat{F}+\hat{n})}\ =\ -\,e^{i\frac{2m+1}{\sqrt{2\alpha'}}\hat{\phi}_0^9}.$$

Furthermore, the redefined string field has the following structure:

$$\hat{\Phi}''=\Psi_+^{\mathrm{e}}\otimes 1\otimes 1\otimes 1+\Psi_+^{\prime\mathrm{e}}\otimes 1\otimes \tau_1\otimes\tilde{\tau}_1+\Psi_-^{\mathrm{o}}\otimes\sigma_1\otimes\tau_2\otimes\tilde{\tau}_1+\Psi_-^{\prime\mathrm{o}}\otimes\sigma_1\otimes\tau_3\otimes 1$$

where superscript e/o denotes $\hat{n}$ and subscript ± denotes $(-1)^{\hat{F}}$.

And in this expression we should represent the derivations as

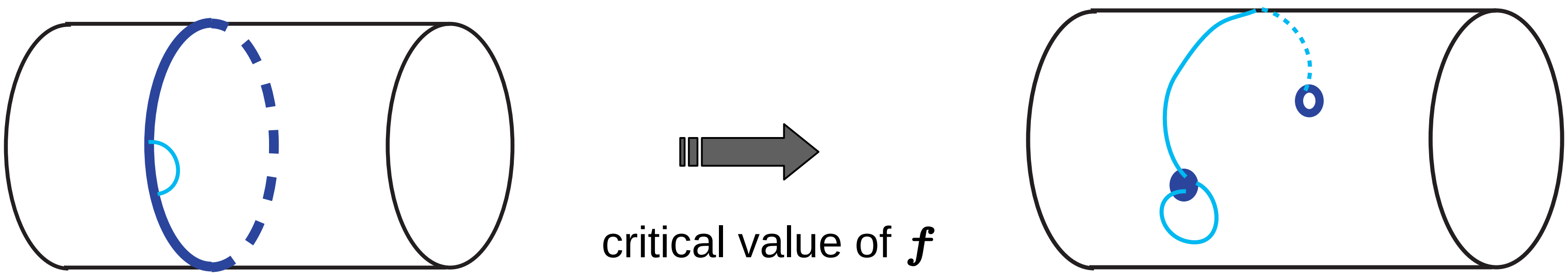
$$\hat{Q}_{\mathrm{B}}=Q_{\mathrm{B}}\otimes\sigma_3\otimes\tau_3\otimes\tilde{\tau}_3,\quad \hat{\eta}_0=\eta_0\otimes\sigma_3\otimes\tau_3\otimes\tilde{\tau}_3.$$

This redefined action $S[\hat{Q}_{\rm B};\hat{\Phi}'']$ has the same structure as

$$\hat{Q}_{\rm B}=Q_{\rm B}\otimes\sigma_3\otimes 1,\qquad \hat{\eta}_0=\eta_0\otimes\sigma_3\otimes 1,$$

$$\hat{\Phi}''=\Psi_+^{\rm e}\otimes 1\otimes 1+\Psi_+'^{\rm e}\otimes 1\otimes\tau_3+\Psi_-^{\rm o}\otimes\sigma_1\otimes\tau_1+\Psi_-'^{\rm o}\otimes\sigma_1\otimes\tau_2.$$

If we regard σ_i / τ_i as internal / external CP factor, and take T-dual picture (momentum ⟷ winding), this action represents super SFT on a D-brane-anti-D-brane system, in which a D-brane and an anti-D-brane are situated at antipodal points along the circle.



non-BPS D-brane D-brane-anti-D-brane

This picture is consistent with Sen's statement (1998) using boundary CFT!

Summary and Discussion

- We have constructed a class of exact classical solutions to Berkovits' super SFT, which have vanishing vacuum energy.
- These solutions are constructed by supercurrents and identity string field in general.
- We have obtained a GSO(−) solution at the critical radius using an SU(2) supercurrent. At the critical value of f, we find that it represents a process: non-BPS → D-anti-D.
- How about the Ramond sector around this solution?
- It is known that there exists a numerical solution in the Siegel gauge, whose vacuum energy cancels D-brane tension. Can we find an exact solution corresponding to it?
- It seems that we should regularize the identity string field appropriately, or use other methods to obtain a solution which has finite vacuum energy.