Comments on Schnabl's marginal and scalar solutions in open string field theory

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（identity stateに基づいていない）開弦の場の理論の古典解

開弦の場の理論: $S[\Psi]=-\frac{1}{g^2}\left(\frac{1}{2}\langle\Psi,Q_B\Psi\rangle+\frac{1}{3}\langle\Psi,\Psi*\Psi\rangle\right)$ → EOM: $Q_B\Psi+\Psi*\Psi=0$

"sliver frame" $\tilde{z}=\arctan z$ （z : UHP) が便利:

$$U_r^\dagger U_r\tilde{\phi}_1(\tilde{x}_1)\cdots\tilde{\phi}_n(\tilde{x}_n)|0\rangle * U_s^\dagger U_s\tilde{\psi}_1(\tilde{y}_1)\cdots\tilde{\psi}_m(\tilde{y}_m)|0\rangle$$
$$=U_{r+s-1}^\dagger U_{r+s-1}\tilde{\phi}_1(\tilde{x}_1+\frac{\pi}{4}(s-1))\cdots\tilde{\phi}_n(\tilde{x}_n+\frac{\pi}{4}(s-1))\tilde{\psi}_1(\tilde{y}_1-\frac{\pi}{4}(r-1))\cdots\tilde{\psi}_m(\tilde{y}_m-\frac{\pi}{4}(r-1))|0\rangle$$

特にwedge state: $|r\rangle=U_r^\dagger|0\rangle=U_r^\dagger U_r|0\rangle$ $\quad U_r=\left(\frac{2}{r}\right)^{\mathcal{L}_0}=\left(\frac{2}{r}\right)^{L_0}e^{-\frac{r^2-4}{3r^2}L_2+\frac{r^4-16}{30r^4}L_4+\cdots}$

Schnablのmarginal解 [Schnabl,Nov.1 (2006) Hawaii,hep-th/0701248] [KORZ,hep-th/0701249]:

$$\Psi_\lambda^J=\lambda_a cJ^a(0)|0\rangle+\sum_{k=1}^\infty\left(\frac{-\pi}{2}\right)^k\int_0^1dr_1\cdots\int_0^1dr_k\psi_k^J(r_1,\cdots,r_k)=|3/2\rangle*\frac{1}{1+\hat{\phi}^J*A}*\hat{\phi}^J*|3/2\rangle,$$

$$\psi_k^J(r_1,\cdots,r_k)=U^\dagger_{2+\sum_{i=1}^k r_i}U_{2+\sum_{i=1}^k r_i}\prod_{m=0}^{k}\lambda_{a_m}\tilde{J}^{a_m}\Big(\frac{\pi}{4}(-\sum_{l=1}^m r_l+\sum_{l=m+1}^k r_l)\Big)\Big[-\frac{1}{\pi}\hat{\mathcal{B}}\tilde{c}(\frac{\pi}{4}\sum_{l=1}^k r_l)\tilde{c}(-\frac{\pi}{4}\sum_{l=1}^k r_l)+\frac{1}{2}\Big(\tilde{c}(\frac{\pi}{4}\sum_{l=1}^k r_l)+\tilde{c}(-\frac{\pi}{4}\sum_{l=1}^k r_l)\Big)\Big]|0\rangle.$$

$$A=\frac{\pi}{2}\int_1^2 dr B_1^L|r\rangle,\quad \hat{\phi}^J=U_1^\dagger U_1\lambda_a cJ^a(0)|0\rangle.\quad \mathcal{B}_0=b_0+\sum_{k=1}^\infty\frac{2(-1)^{k+1}}{4k^2-1}b_{2k},\quad B_1=b_1+b_{-1},\quad \hat{\mathcal{B}}=\mathcal{B}_0+\mathcal{B}_0^\dagger,\quad B_1^L=\frac{1}{2}B_1+\frac{1}{\pi}\hat{\mathcal{B}}.$$

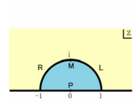
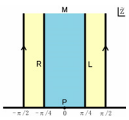
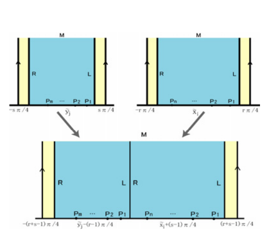
ただし "non-singularity"を仮定している: $\lambda_a\lambda_b g^{ab}=0,\quad J^a(y)J^b(z)\sim\frac{-g^{ab}}{(y-z)^2}+\frac{1}{y-z}f^{ab}{}_cJ^c(z)+\cdots$

Schnablのtachyon解 [Schnabl,hep-th/0511286]:

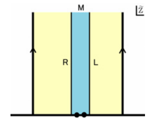
$$\Psi_\lambda=-\sum_{n=0}^\infty\lambda^{n+1}\partial_t\psi_{t+n}|_{t=0}=\lambda Q_B\Lambda_0*(1-\lambda\Lambda_0)^{-1},\quad \Lambda_0=B_1^Lc_1|0\rangle,\qquad S[\Psi_\lambda]/V_{26}=\begin{cases}\frac{1}{2\pi^2g^2} & (\lambda=1)\\ 0 & (|\lambda|<1)\end{cases},$$

$$\psi_n=\frac{2}{\pi}U_{n+2}^\dagger U_{n+2}\left[-\frac{1}{\pi}\hat{\mathcal{B}}\tilde{c}(\pi n/4)\tilde{c}(-\pi n/4)+\frac{1}{2}(\tilde{c}(\pi n/4)+\tilde{c}(-\pi n/4))\right]|0\rangle.\quad Q_{\Psi_{\lambda=1}}A=I=U_1^\dagger U_1|0\rangle.$$

[Ellwood-Schnabl,hep-th/0606142]

sliver frameでのスター積



$c\lambda_aJ^a(\epsilon)\,c\lambda_bJ^b(0)\sim 0$

Note: $Q_BA=I-|r=2\rangle=I-|0\rangle,\quad Q_B\hat{\phi}^J=0,\quad \hat{\phi}^J*\hat{\phi}^J=0.$

$$\hat{\phi}_s\equiv Q_BU_1^\dagger U_1\Lambda_0=Q_BU_1^\dagger U_1B_1^Lc_1|0\rangle,\quad Q_B\hat{\phi}_s=0,\quad \hat{\phi}_s*\hat{\phi}_s=0.$$

解の「生成」

$\hat{\phi}$ がBRST不変で冪零 $\quad Q_B\hat{\phi}=0,\quad \hat{\phi}*\hat{\phi}=0$ なら

$$\Psi^{(r,s)}=|r\rangle*\frac{1}{1+\hat{\phi}*A^{(r+s-1)}}*\hat{\phi}*|s\rangle,\quad A^{(r+s-1)}\equiv\frac{\pi}{2}\int_1^{r+s-1}drB_1^L|r\rangle$$

も解である

$$\begin{aligned}\because\ Q_B\Psi^{(r,s)}&=|r\rangle*Q_B\left(\frac{1}{1+\hat{\phi}*A^{(r+s-1)}}\right)*\hat{\phi}*|s\rangle\\&=-|r\rangle*\frac{1}{1+\hat{\phi}*A^{(r+s-1)}}*(Q_B(I+\hat{\phi}*A^{(r+s-1)}))*\frac{1}{1+\hat{\phi}*A^{(r+s-1)}}*\hat{\phi}*|s\rangle\\&=|r\rangle*\frac{1}{1+\hat{\phi}*A^{(r+s-1)}}*\hat{\phi}*(Q_BA^{(r+s-1)})*\frac{1}{1+\hat{\phi}*A^{(r+s-1)}}*\hat{\phi}*|s\rangle\\&=|r\rangle*\frac{1}{1+\hat{\phi}*A^{(r+s-1)}}*\hat{\phi}*(I-|r+s-1\rangle)*\frac{1}{1+\hat{\phi}*A^{(r+s-1)}}*\hat{\phi}*|s\rangle\\&=|r\rangle*\frac{1}{1+\hat{\phi}*A^{(r+s-1)}}*\hat{\phi}*\hat{\phi}*\frac{1}{1+A^{(r+s-1)}*\hat{\phi}}*|s\rangle-|r\rangle*\frac{1}{1+\hat{\phi}*A^{(r+s-1)}}*\hat{\phi}*|s\rangle*|r\rangle*\frac{1}{1+\hat{\phi}*A^{(r+s-1)}}*\hat{\phi}*|s\rangle\\&=-\Psi^{(r,s)}*\Psi^{(r,s)}.\end{aligned}$$

※もし、「負の角度をもつwedge state」を使うと形式的には

$$\Psi^{(r,s)}=V^{-1}*\hat{\phi}*V+V^{-1}*Q_BV,\quad V=(I+\hat{\phi}*A^{(r+s-1)})*|2-r\rangle,\quad V^{-1}=|r\rangle*\frac{1}{1+\hat{\phi}*A^{(r+s-1)}},$$

とかけるので互いにゲージ変換で移りあう。

- $r=s=3/2$ のとき, Schnabl, Kiermaier-Okawa-Rastelli-Zwiebach の解の場合が 再現される。
- $\hat{\phi}=\hat{\phi}_s+\hat{\phi}^J$ から生成される解など別の解は？
- 各解の物理的な意味は？各解まわりのBRST cohomologyは？
- singularityがある場合: $\lambda_a\lambda_bg^{ab}\neq 0$ の（identity stateを使わない）解の閉じた形は？
- 弦の場に関して何がregularで何がsingularなのか？
- BerkovitsのWZW型の超弦の場の理論への拡張は？

1. marginal解の一般化

$(\mathcal{B}_0-\mathcal{B}_0^\dagger)\hat{\phi}^J=0$ を満たすBRST不変で冪零な $\hat{\phi}^J=U_1^\dagger U_1\lambda_acJ^a(0)|0\rangle$ から

$$\Psi_\lambda^{J(r,s)}=|r\rangle*\hat{\phi}^J*|s\rangle+\sum_{k=1}^\infty(-1)^k|r\rangle*(\hat{\phi}^J*A^{(r+s-1)})^k*\hat{\phi}*|s\rangle=\sum_{n=1}^\infty\phi_n^J,$$

$$\phi_{k+1}^J=\left(-\frac{\pi}{2}\right)^k\int_0^{r+s-2}dr_1\cdots\int_0^{r+s-2}dr_kU^\dagger_{r+s-1+\sum_{i=1}^kr_i}U_{r+s-1+\sum_{i=1}^kr_i}\left[\prod_{m=0}^k\lambda_{a_m}\tilde{J}^{a_m}\Big(\frac{\pi}{4}(s-r-\sum_{l=1}^mr_l+\sum_{l=m+1}^kr_l)\Big)\right]$$
$$\times\left[-\frac{1}{\pi}\hat{\mathcal{B}}\tilde{c}(\frac{\pi}{4}(s-r+\sum_{l=1}^kr_l))\tilde{c}(\frac{\pi}{4}(s-r-\sum_{l=1}^kr_l))+\frac{1}{2}\Big(\tilde{c}(\frac{\pi}{4}(s-r+\sum_{l=1}^kr_l))+\tilde{c}(\frac{\pi}{4}(s-r-\sum_{l=1}^kr_l))\Big)\right]|0\rangle,$$

という解が得られる。実際、直接計算により、

$$Q_B\phi_1^J=0,\quad \mathcal{B}^{(r,s)}\phi_1^J=0,\quad \phi_{k+1}^J=-\frac{\mathcal{B}^{(r,s)}}{\mathcal{L}^{(r,s)}}\sum_{l=1}^k\phi_l^J*\phi_{k-l+1}^J,$$
$$\mathcal{B}^{(r,s)}=\frac{1}{2}(r+s-3)\hat{\mathcal{B}}+\mathcal{B}_0+\frac{\pi}{4}(r-s)B_1,\quad \mathcal{L}^{(r,s)}\equiv\{Q_B,\mathcal{B}^{(r,s)}\}.$$

がわかる。 → これは「generalized Schnablゲージ」: $\mathcal{B}^{(r,s)}\Psi_\lambda^{J(r,s)}=0$ の解である。

ex.1) Rolling tachyon $\lambda_aJ^a=\lambda{:}e^{X^0}{:}$

$$\Psi_\lambda^{J(r,r)}=\left[\lambda e^{X^0}-\frac{64\cot^3\frac{\pi(2r-1)}{2(4r-3)}}{3(4r-3)^3}\lambda^2e^{2X^0}+\cdots+(\sim r^{-k^2-2k}\text{ for }r\gg1)\lambda^{k+1}e^{kX^0}\right]c_1|0\rangle+\cdots$$

ex. 2) lightcone-like deformation $\lambda_aJ^a=\lambda i\partial X^+$

$$\Psi_\lambda^{J(r,r)}=\left[\lambda\alpha_{-1}^+-\frac{4\cot\frac{\pi(2r-1)}{2(4r-3)}}{4r-3}\lambda^2\alpha_{-1}^+\alpha_{-1}^++\cdots\right]c_1|0\rangle+\cdots$$

2. tachyon解の一般化

同様に $(\mathcal{B}_0-\mathcal{B}_0^\dagger)\hat{\phi}_s=0$ を満たすBRST不変で冪零な $\hat{\phi}_s=Q_BU_1^\dagger U_1B_1^Lc_1|0\rangle$ から

$$\hat{\Psi}_{\hat{\lambda}}^{(r,s)}=|r\rangle*\frac{\hat{\lambda}}{1+\hat{\lambda}\hat{\phi}_s*A^{(r+s-1)}}*\hat{\phi}_s*|s\rangle=\sum_{k=0}^\infty(-1)^k\hat{\lambda}^{k+1}|r\rangle*\hat{\phi}_s*(A^{(r+s-1)}*\hat{\phi}_s)^k*|s\rangle$$
$$=\sum_{k=0}^\infty\hat{\lambda}^{k+1}Q_B\Lambda_0^{(r,s)}*(\Lambda_0^{(r,s)}-I)^k=\frac{\hat{\lambda}}{1+\hat{\lambda}}Q_B\Lambda_0^{(r,s)}*\frac{1}{1-\frac{\hat{\lambda}}{1+\hat{\lambda}}\Lambda_0^{(r,s)}}=\Psi^{(r,s)}_{\lambda=\frac{\hat{\lambda}}{1+\hat{\lambda}}}$$

という解を得る。ここで、

$$\Psi_\lambda^{(r,s)}=-\sum_{n=0}^\infty\lambda^{n+1}\partial_t\psi_{t,n}^{(r,s)}|_{t=0}=-\sum_{n=0}^\infty\lambda^{n+1}|r-1/2\rangle*\partial_t\psi_{t+n(r+s-2)}|_{t=0}*|s-1/2\rangle$$

特に $\lambda=1\ (\leftrightarrow\hat{\lambda}=\infty)$ のときは次のように「正則化」する:

$$\Psi_{\lambda=1}^{(r,s)}=\frac{1}{r+s-2}\sum_{n=0}^\infty\frac{B_n}{n!}(r+s-2)^n\partial_t^n\psi_{t,n=0}^{(r,s)}|_{t=0}=\lim_{N\to\infty}\left(\frac{1}{r+s-2}\psi_{t=0,N}^{(r,s)}-\sum_{n=0}^N\partial_t\psi_{t,n}^{(r,s)}|_{t=0}\right).$$

恒等式: $\mathcal{B}^{(r,s)}e^{\frac{\pi}{4}(s-r)K_1}(r+s-2)^{\frac{D}{2}}=e^{\frac{\pi}{4}(s-r)K_1}(r+s-2)^{\frac{D}{2}}\mathcal{B}_0$

$(K_1=L_1+L_{-1},\ D=\mathcal{L}_0-\mathcal{L}_0^\dagger$: derivations, BPZ odd)

に注意すると $\Psi_\lambda^{(r,s)}=e^{\frac{\pi}{4}(s-r)K_1}(r+s-2)^{\frac{D}{2}}\Psi_\lambda^{(\frac{3}{2},\frac{3}{2})}$

⇒

$$Q_B\partial_t\psi_{t,0}^{(r,s)}|_{t=0}=0,\quad \mathcal{B}^{(r,s)}\partial_t\psi_{t,0}^{(r,s)}|_{t=0}=0,$$
$$\partial_t\psi_{t,n}^{(r,s)}|_{t=0}-\partial_t\psi_{t,0}^{(r,s)}|_{t=0}=\frac{\mathcal{B}^{(r,s)}}{\mathcal{L}^{(r,s)}}\sum_{m=0}^{n-1}\partial_t\psi_{t,m}^{(r,s)}|_{t=0}*\partial_t\psi_{t,n-1-m}^{(r,s)}|_{t=0}.$$
$$\mathcal{B}^{(r,s)}\Psi_\lambda^{(r,s)}=0,\qquad S[\Psi_\lambda^{(r,s)}]/V_{26}=\begin{cases}\frac{1}{2\pi^2g^2} & (\lambda=1)\\ 0 & (|\lambda|<1)\end{cases},$$
$$Q_{\Psi_{\lambda=1}^{(r,s)}}A^{(r+s-1)}=I.$$

超弦の場の理論への拡張 EOM: $\eta_0(e^{-\Phi}Q_Be^{\Phi})=0$

Schnabl解の教訓: *大雑把に* $\frac{\mathcal{B}_0}{\mathcal{L}_0}\to *A*$

⇒ 超弦の場合の「自然な」拡張: $\frac{\tilde{\mathcal{G}}_0^-}{\mathcal{L}_0}\frac{\mathcal{B}_0}{\mathcal{L}_0}\to *\hat{A}*$

ここで $\hat{A}\equiv-\frac{\pi}{2}\int_0^1du\int_0^1dvJ_1^{--L}|uv+1\rangle-\left(\frac{\pi}{2}\right)^2\int_0^1du\int_0^1dv\,uv\tilde{G}_1^{-L}B_1^L|uv+1\rangle$

$J^{--}=\xi b,\quad \tilde{G}^-=[Q_B,J^{--}],$

$\eta_0Q_B\hat{A}=I-|r=2\rangle,\quad \eta_0\hat{A}=-\frac{\pi}{2}\int_1^2drB_1^L|r\rangle,\quad Q_B\hat{A}=-\frac{\pi}{2}\int_1^2dr\tilde{G}_1^{-L}|r\rangle.$

$\hat{\Phi}_0\equiv U_1^\dagger U_1c\xi e^{-\phi}\lambda_a\psi^a(0)|0\rangle$ は超カレントがnon-singularなら

$\eta_0Q_B\hat{\Phi}_0=0,\ \hat{\Phi}_0*Q_B\hat{\Phi}_0=0,\ (Q_B\hat{\Phi}_0)*\hat{\Phi}_0=0,\ \hat{\Phi}_0*\hat{\Phi}_0=0,\ \hat{\Phi}_0*\eta_0\hat{\Phi}_0=0,\ (\eta_0\hat{\Phi}_0)*\hat{\Phi}_0=0$

bosonicのときと同様に、これから得られる $\Phi_0=|3/2\rangle*\hat{\Phi}_0*|3/2\rangle$
を解の最低次の項だとして次のカレントの2次の項を摂動的に求めると

$$\begin{aligned}\Phi_1&=\frac{1}{2}\frac{\tilde{\mathcal{G}}_0^-}{\mathcal{L}_0}\frac{\mathcal{B}_0}{\mathcal{L}_0}(\eta_0\Phi_0*Q_B\Phi_0+Q_B\Phi_0*\eta_0\Phi_0)\\&=-\frac{1}{2}|3/2\rangle*(\eta_0\hat{\Phi}_0*\hat{A}*Q_B\hat{\Phi}_0+Q_B\hat{\Phi}_0*\hat{A}*\eta_0\hat{\Phi}_0)*|3/2\rangle.\end{aligned}$$

次のオーダー（カレントの3次）の項まで求めた。
一般項はきれいにまとまるか？

⇒ work in progress