On identity based solutions in open string field theory

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based on collaboration with

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Refs.

I.K. and T.T. : PTP108(2002)591[hep-th/0205275],
PTP122(2009)385[arXiv:0904.1095]

(and recent numerical results)

Dec. 11 (2009) APCTP Focus Program on Current Trends in SFT

CONTENTS

NON-PERTURBATIVE VACUUM IN OPEN BOSONIC STRING FIELD THEORY (1)

- Schnabl's solution $\Psi_{\rm Sch}$

Gauge invariants

(1) Action: D-brane tension

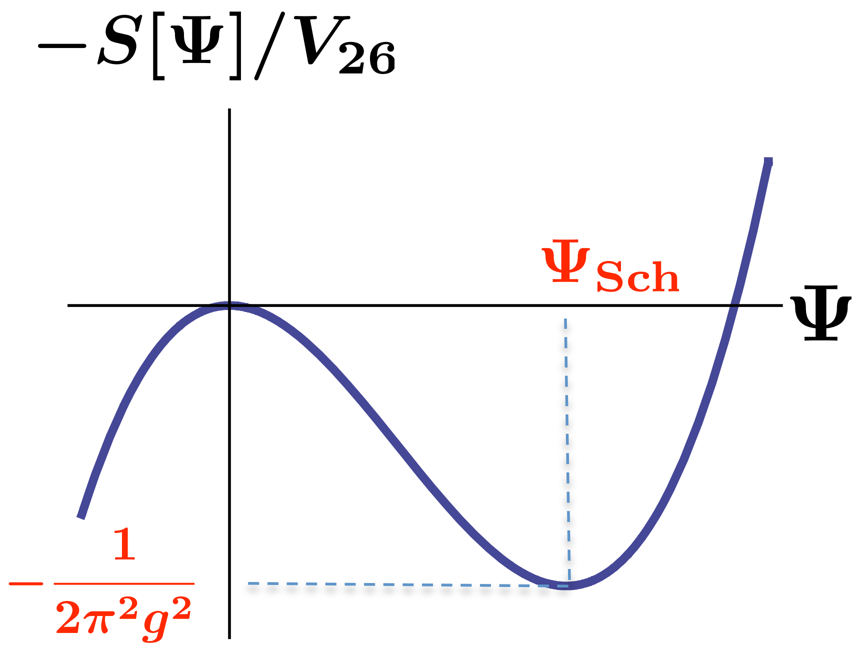
$$S[\Psi_{\rm Sch}]/V_{26} = \frac{1}{2\pi^2 g^2}$$

[Schnabl(2005),Okawa,Fuchs-Kroyter(2006)]

(2) Gauge invariant overlap:

$$\mathcal{O}_V(\Psi_{\rm Sch}) = \frac{1}{2\pi}\langle B|c_0^-|\Phi_V\rangle$$

[Ellwood, Kawano-Kishimoto-Takahashi(2008)]



NON-PERTURBATIVE VACUUM IN OPEN BOSONIC STRING FIELD THEORY (2)

$$Q' \equiv Q_{\rm B} + [\Psi_{\rm Sch}, \ \cdot \ \}_*$$

no cohomology (in all ghost number sectors) [Ellwood-Schnabl(2006)]

On the other hand,

in 2002, Takahashi and Tanimoto constructed a class of analytic solutions
("identity based solutions") and conjectured that
it represents a non-perturbative vacuum for a particular value of the parameter.

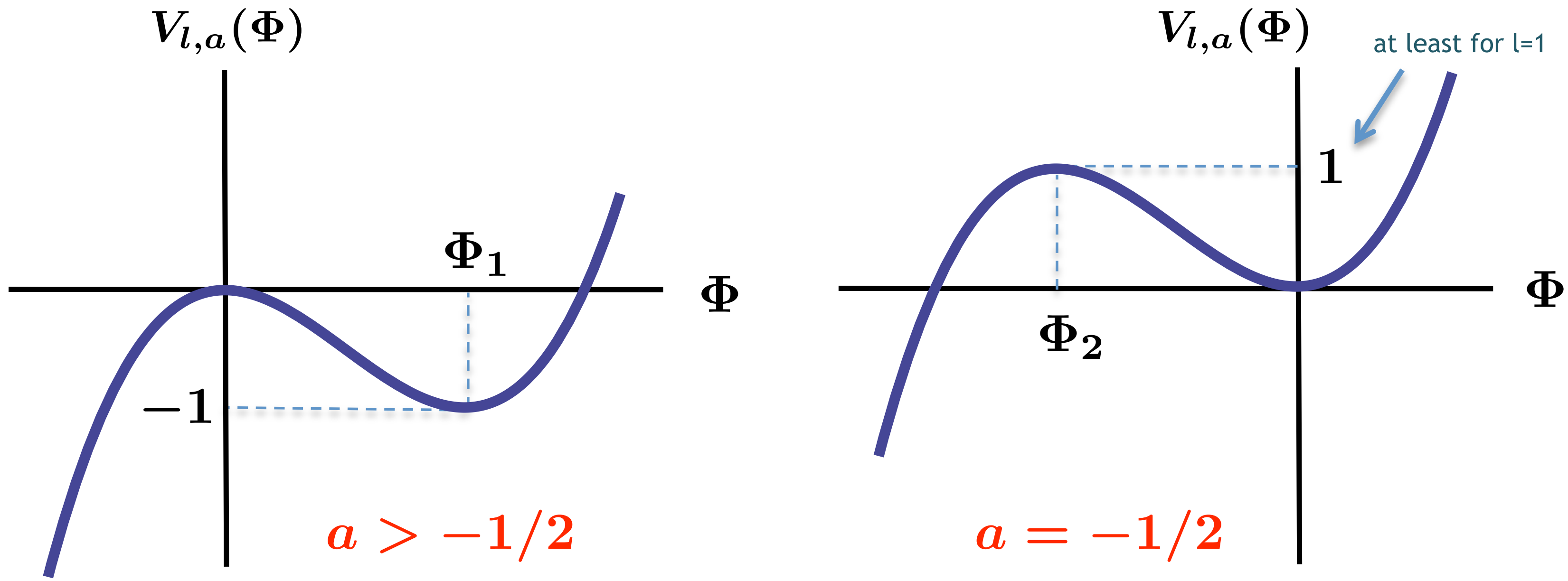
$(a = -1/2)$

Actually, around the solution $\Psi_{l,a=-1/2}$ there is no cohomology
in the ghost number 1 sector. [Kishimoto-Takahashi (2002)]

We have obtained further quantitative evidences. [Kishimoto-Takahashi (2009)]

NON-TRIVIAL SOLUTION IN SFT AROUND $\Psi_{l,a}$

- We numerically construct stable solution and unstable solution (up to level (24,72)) in the theory around $\Psi_{l,a}$ $(l = 1, 2, 3; a \geq -1/2)$ and evaluate gauge invariants.
- The results suggest the vacuum structure is like this:



- It is consistent with our previous result

$\Psi_{l,a>-1/2}$: pure gauge $\Psi_{l,a=-1/2}$: tachyon vacuum

OPEN BOSONIC STRING FIELD THEORY

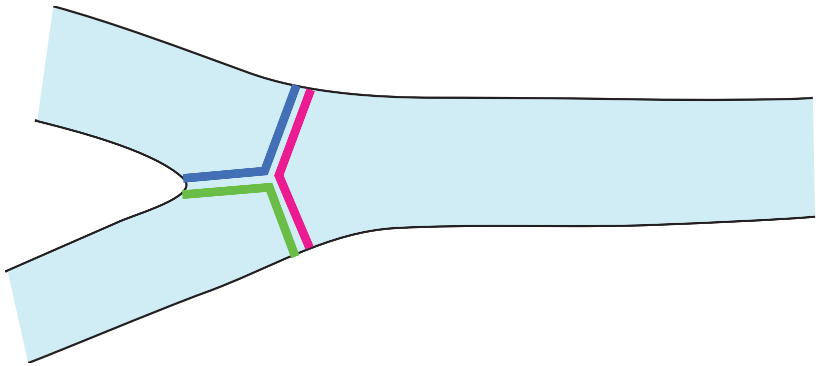
Action: $$S[\Psi] = -\frac{1}{g^2}\left(\frac{1}{2}\langle\Psi, Q_{\mathrm{B}}\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right)$$

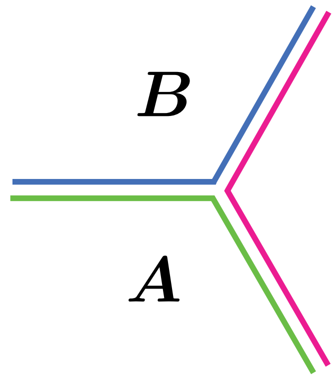
Equation of motion: $$Q_{\mathrm{B}}\Psi + \Psi * \Psi = 0$$

Gauge transformation: $$\delta_\Lambda\Psi = Q_{\mathrm{B}}\Lambda + \Psi * \Lambda - \Lambda * \Psi$$

$$\rightarrow \quad \delta_\Lambda S[\Psi] = 0$$

WITTEN TYPE INTERACTION





$$|A * B\rangle = \sum_i |\phi^i\rangle\langle f_{(1)}[\phi_i] f_{(2)}[A] f_{(3)}[B]\rangle_{\mathrm{UHP}}$$

$$= \sum_i |\phi^i\rangle\langle V_3(1,2,3)|\phi_i\rangle_1 |A\rangle_2 |B\rangle_3$$

ϕ^i :basis of worldsheet fields

$$f_{(r)}(z) = h^{-1}(e^{(1-r)\frac{2\pi}{3}i} h(z)^{\frac{2}{3}})$$

$$\langle \phi_i, \phi^j \rangle = \delta_i^j$$

$$h(z) = \frac{1 + iz}{1 - iz}$$

Gauge invariant overlap

Gauge invariant for on-shell closed string state

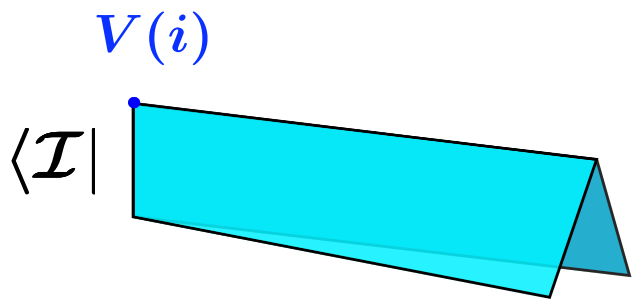
$$\mathcal{O}_V(\Psi) = \langle \mathcal{I}|V(i)|\Psi\rangle = \langle \hat{\gamma}(1_c, 2)|\Phi_V\rangle_{1_c}|\Psi\rangle_2$$

$$|\Phi_V\rangle = c_1\bar{c}_1|V_m\rangle$$

V_m :matter primary with (1,1)-dim.

$$\mathcal{O}_V(Q_B\Lambda) = 0$$

$$\mathcal{O}_V(\Psi * \Lambda) = \mathcal{O}_V(\Lambda * \Psi)$$



⇒ $$\delta_\Lambda \mathcal{O}_V(\Psi) = 0$$

In particular, it vanishes for pure gauge solutions:

$$\mathcal{O}_V(e^{-\Lambda}Q_B e^{\Lambda}) = 0$$

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TAKAHASHI-TANIMOTO'S SOLUTION

- a type of "identity based solutions"

$$\Psi_h = Q_L(e^h - 1)\mathcal{I} - C_L((\partial h)^2 e^h)\mathcal{I}$$

$\mathcal{I}$: identity state

$$Q_L(f) \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i} f(z) j_{\rm B}(z) \qquad C_L(f) \equiv \int_{C_{\rm left}} \frac{dz}{2\pi i} f(z) c(z)$$

$$h(-1/z) = h(z), \quad h(\pm i) = 0$$

Half-integration and identity state

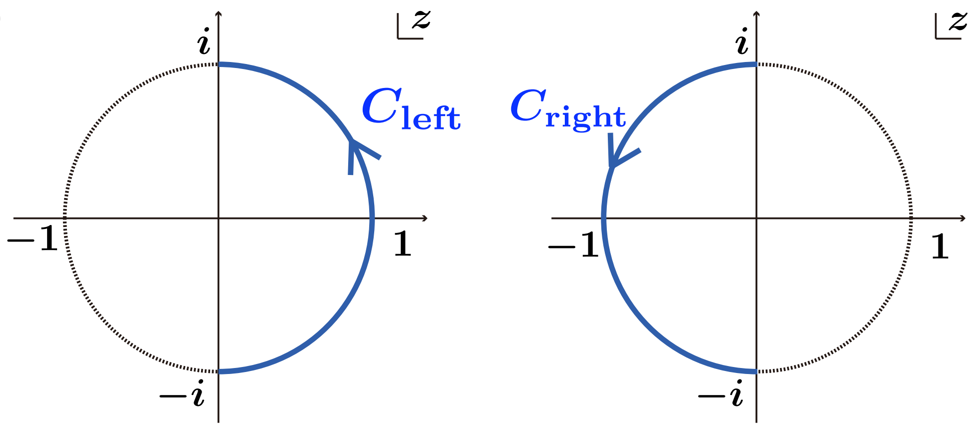
$$(\sigma(z)A) * B = (-1)^{|\sigma||A|} * (z'^{2h}\sigma(z')B) \quad (zz' = -1,\ |z| = 1,\ \mathrm{Re}\, z \leq 0)$$

primary field (dim *h*)

$$(\Sigma_R(F)A) * B = -(-1)^{|\sigma||A|} A * (\Sigma_L(F)B)$$

$$\Sigma_{L(R)}(F) = \int_{C_{\mathrm{left(right)}}} \frac{dz}{2\pi i} F(z)\,\sigma(z)$$

$$F(-1/z) = (z^2)^{1-h} F(z)$$

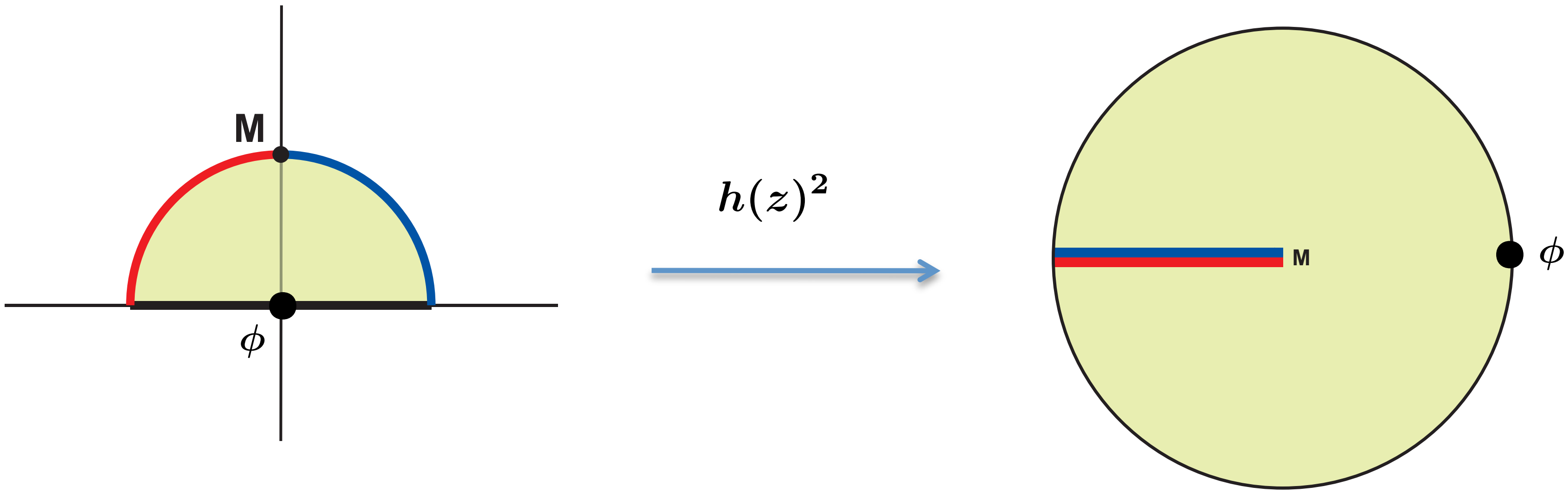


$$\Sigma_L(F)\mathcal{I} = -\Sigma_R(F)\mathcal{I}$$

$$\mathcal{I} * A = A * \mathcal{I} = A$$

: identity element with respect to the star product

IDENTITY STATE



$$\langle \mathcal{I}|\phi\rangle = \langle h_{\mathcal{I}}[\phi(0)]\rangle_{\text{UHP}}$$

$$h_{\mathcal{I}}(z) = h^{-1}(h(z)^2) = \frac{2z}{1-z^2}$$

$$\begin{aligned} |\mathcal{I}\rangle &= |r=1\rangle = 2^{\mathcal{L}_0^\dagger}|0\rangle \\ &= \cdots e^{-\frac{1}{2^{k-1}}L_{-2^k}} \cdots e^{-\frac{1}{8}L_{-16}} e^{-\frac{1}{4}L_{-8}} e^{-\frac{1}{2}L_{-4}} e^{L_{-2}}|0\rangle \end{aligned}$$

$$\mathcal{L}_0^\dagger = L_0 + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{4k^2-1} L_{-2k}$$

On the equation of motion

- Ansatz: $$\Psi = Q_L(F)\mathcal{I} + C_L(G)\mathcal{I}$$

  Note: $j_{\rm B} j_{\rm B} \sim cc, \quad j_{\rm B} c \sim cc$

$$Q_{\rm B}\Psi = \{Q_{\rm B}, C_L(G)\}\mathcal{I}$$

$$\Psi * \Psi = \{Q_{\rm B}, C_L((\partial F)^2 + FG)\}\mathcal{I}$$

$$j_{\rm B}(z) = cT^{\rm mat}(z) + bc\partial c(z) + \frac{3}{2}\partial^2 c(z) = \sum_n Q_n z^{-n-1}$$

$$\{Q_m, Q_n\} = 2mn\{Q_{\rm B}, c_{m+n}\}, \quad \{Q_m, c_n\} = \{Q_{\rm B}, c_{m+n}\} \qquad Q_{\rm B}|\mathcal{I}\rangle = 0$$

assumptions: $F(-1/z) = F(z), \quad G(-1/z) = z^4 G(z)$

$$F(\pm i) = 0, \quad G(\pm i) = 0$$

$$Q_{\rm B}\Psi + \Psi * \Psi = 0 \quad \longleftrightarrow \quad G = -\frac{(\partial F)^2}{1+F} \qquad e^h = 1 + F$$

PURE GAUGE FORM OF TT SOLUTION

- Formal pure gauge form:

$$\Psi_h = \exp(q_L(h)\mathcal{I}) Q_{\rm B} \exp(-q_L(h)\mathcal{I})$$

$$q_L(h) = \int_{C_{\rm left}} \frac{dz}{2\pi i} h(z) \left(j_{\rm gh}(z) - \frac{3}{2z} \right)$$

$$j_{\rm gh}(z) = cb(z) = \sum_n q_n z^{-n-1}$$

$$[Q_m, q_n] = -Q_{m+n} + 2mnc_{m+n}$$

gauge parameter string field: $\exp(\pm q_L(h)\mathcal{I}) = e^{\pm q_L(h)}\mathcal{I}$

"well-defined" ?

On gauge invariants for TT solution

Action

$$S[\Psi_h] = -\frac{1}{6g^2}\langle\mathcal{I}|C_L((\partial h)^2 e^h)Q_{\mathrm{B}}C_L((\partial h)^2 e^h)|\mathcal{I}\rangle$$

Gauge invariant overlap

$$\mathcal{O}_V(\Psi_h) = \langle\mathcal{I}|V(i)(Q_L(e^h-1) - C_L((\partial h)^2 e^h))|\mathcal{I}\rangle$$

At least naively, $\langle\mathcal{I}|(\cdots)|\mathcal{I}\rangle$ causes divergence.
Appropriate regularization is necessary to evaluate such quantities.

→ Identity based solutions may be "singular."

↓

Alternatively, we investigate SFT *around* Ψ_h .

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Around TT solution

- expansion of the action around TT solution:

$$S[\Psi_h + \Phi] = S[\Psi_h] - \frac{1}{g^2}\left(\frac{1}{2}\langle\Phi, Q'\Phi\rangle + \frac{1}{3}\langle\Phi, \Phi * \Phi\rangle\right)$$

$$\begin{aligned} Q'\Phi &= Q_{\mathrm{B}}\Phi + \Psi_h * \Phi + \Phi * \Psi_h \\ &= Q_{\mathrm{B}}\Phi + (Q_L(e^h - 1) - C_L((\partial h)^2 e^h))\mathcal{I} * \Phi \\ &\quad + \Phi * (Q_L(e^h - 1) - C_L((\partial h)^2 e^h))\mathcal{I} \\ &= Q_{\mathrm{B}}\Phi + (Q_L(e^h - 1) - C_L((\partial h)^2 e^h))\Phi \\ &\quad + (Q_R(e^h - 1) - C_R((\partial h)^2 e^h))\Phi \\ &= (Q(e^h) - C((\partial h)^2 e^h))\Phi \end{aligned}$$

$$Q(f) \equiv \oint \frac{dz}{2\pi i} f(z) j_{\mathrm{B}}(z), \quad C(f) \equiv \oint \frac{dz}{2\pi i} f(z) c(z)$$

$$q(h) \equiv \oint \frac{dz}{2\pi i} h(z) j_{\mathrm{gh}}(z)$$

Formally, $\quad Q' = e^{q(h)} Q_{\mathrm{B}} e^{-q(h)}$

A CLASS OF IDENTITY BASED SOLUTIONS

- a particular choice of function with 1-parameter

$$h_a^l(z) = \log\left(1 - \frac{a}{2}(-1)^l\left(z^l - (-1)^l\frac{1}{z^l}\right)^2\right)$$

$$= -\log((1-Z(a))^2) - \sum_{n=1}^{\infty}\frac{(-1)^{ln}}{n}Z(a)^n(z^{2ln} + z^{-2ln})$$

$$l = 1, 2, 3, \cdots$$

$$Z(a) = \frac{1 + a - \sqrt{1+2a}}{a} \qquad a \geq -1/2$$

$$-1 \leq Z(a) < 1$$

MODE EXPANSION OF THE SOLUTION

- Explicit form of the solution

$$\Psi_{l,a} = Q_L(e^{h^l_a} - 1)\mathcal{I} - C_L((\partial h^l_a)^2 e^{h^l_a})\mathcal{I}$$

$$= \frac{al^2}{\pi}\left(\sum_{k=1}^{\infty}\frac{(-1)^k 8}{(2k-1)((2k-1)^2-4l^2)}Q_{1-2k} + \sum_{k=0}^{\infty} g_k(a) c_{1-2k}\right)|\mathcal{I}\rangle$$

↑

$$(Q_n + (-1)^n Q_{-n})|\mathcal{I}\rangle = 0$$

$$c(z)|\mathcal{I}\rangle = \left(c_0\frac{z^3-z}{z^2+1} + c_1\frac{1}{z^2+1} + c_{-1}\frac{z^4+z^2+1}{z^2+1} + \sum_{n=2}^{\infty} c_{-n}(z^n - (-1)^n z^{-n})z\right)|\mathcal{I}\rangle$$

$$g_0(a) = \int_0^{\frac{\pi}{2}} d\theta \frac{-\cos 2\theta}{\sin\theta}\frac{\sin^2 2l\theta}{1 + a^{-1} - \cos 2l\theta}$$

$$g_1(a) = \int_0^{\frac{\pi}{2}} d\theta \frac{1 - 2\cos 2\theta}{\sin\theta}\frac{\sin^2 2l\theta}{1 + a^{-1} - \cos 2l\theta}$$

$$g_k(a) = \int_0^{\frac{\pi}{2}} d\theta (4\sin(2k-1)\theta)\frac{\sin^2 2l\theta}{1 + a^{-1} - \cos 2l\theta} \qquad (k \geq 2)$$

Finite coefficients for

$$a \geq -1/2$$

BRST OPERATOR AROUND THE SOLUTION

- Mode expansion

$$\begin{aligned} Q' &= Q(e^{h^l_a}) - C((\partial h^l_a)^2 e^{h^l_a}) \\ &= (1+a)Q_{\rm B} - (-1)^l \frac{a}{2}(Q_{2l} + Q_{-2l}) + 4al^2 c_0 + (-1)^l 2al^2 Z(a)^2 (c_{2l} + c_{-2l}) \\ & \quad -2al^2(1 - Z(a)^2) \sum_{n=2}^{\infty} (-1)^{nl} Z(a)^{n-1} (c_{2ln} + c_{-2ln}) \end{aligned}$$

Formally, $Q' = e^{q(h^l_a)} Q_{\rm B} e^{-q(h^l_a)}$

Normal ordered form:

$$e^{\pm q(h^l_a)} = e^{\frac{1}{2}[q^{(+)}(h^l_a), q^{(-)}(h^l_a)]} e^{\pm q^{(-)}(h^l_a)} e^{\pm q^{(0)}(h^l_a)} e^{\pm q^{(+)}(h^l_a)}$$

$$q(h^l_a) = q^{(+)}(h^l_a) + q^{(0)}(h^l_a) + q^{(-)}(h^l_a)$$

$$[q_m, q_n] = m\delta_{m+n,0}$$

On the exponential factor

$$q^{(0)}(h^l_a) = -q_0 \log((1 - Z(a))^2)$$

$$q^{(+)}(h^l_a) = -\sum_{n=1}^{\infty} \frac{(-1)^{nl}}{n}(Z(a))^n q_{nl} \qquad q^{(-)}(h^l_a) = -\sum_{n=1}^{\infty} \frac{(-1)^{nl}}{n}(Z(a))^n q_{-nl}$$

$$[q^{(+)}(h^l_a), q^{(-)}(h^l_a)] = \sum_{n=1}^{\infty} \frac{2l}{n}(Z(a))^{2n} = -2l \log(1 - (Z(a))^2)$$

$$e^{\pm q(h^l_a)} = (1 - (Z(a))^2)^{-l}(1 - Z(a))^{\mp 2q_0} e^{\pm q^{(-)}(h^l_a)} e^{\pm q^{(+)}(h^l_a)}$$

$$Z(a = -1/2) = -1$$

Divergent for $a = -1/2$

Finite for $a > -1/2$

$$Q' = e^{q(h^l_a)} Q_{\mathrm{B}} e^{-q(h^l_a)} \sim Q_{\mathrm{B}}$$ the same cohomology

$\Psi_{l,a>-1/2}$: pure gauge solution

Q' FOR A=-1/2

- Similarity transformation

$$Q'|_{a=-\frac{1}{2}} = \frac{1}{2}Q_{\mathrm{B}} + \frac{(-1)^l}{4}(Q_{2l}+Q_{-2l}) + 2l^2\left(c_0 - \frac{(-1)^l}{2}(c_{2l}+c_{-2l})\right)$$
$$= \frac{(-1)^l}{4}e^{-q(\lambda^l)}Q_{\mathrm{B}}^{(2l)}e^{q(\lambda^l)}$$

$$q(\lambda^l) = 2\sum_{n=1}^{\infty}\frac{(-1)^{n(l+1)}}{n}q_{-2nl} \qquad \lambda^l(z) = -2\log(1+(-1)^l z^{2l})$$

$$Q_{\mathrm{B}}^{(2l)} = Q_{\mathrm{B}}|_{b_n\to b_{n-2l},c_n\to c_{n+2l}} = Q_{2l} - 4l^2 c_{2l}$$

↑

Replacement of *bc* ghosts in the Kato-Ogawa BRST operator

Cohomology of Q' for a=-1/2

Solution of $Q'\psi = 0$

$$\psi = |P\rangle \otimes e^{-q(\lambda^l)} \prod_{m=2}^{2l} b_{-m}|0\rangle + |P'\rangle \otimes e^{-q(\lambda^l)} c_{2l} \prod_{m=2}^{2l} b_{-m}|0\rangle + Q'|\phi\rangle$$

ghost number $-2l+1$ ghost number $-2l+2$

$|P\rangle, |P'\rangle$: DDF states in the matter sector

In the ghost number 1 sector,

$$Q'\psi_{(1)} = 0 \quad \rightarrow \quad \exists \phi_{(0)}, \quad \psi_{(1)} = Q'\phi_{(0)}$$

$\Psi_{l,a=-1/2}$: tachyon vacuum (!?)

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- Introduction
- Takahashi-Tanimoto's identity based solution
- String field theory around TT solutions
- Construction of numerical solutions
- Comments on numerical solution in Siegel gauge
- Numerical stable solution around TT solution (l=1)
- Numerical unstable solution around TT solution (l=1)
- Numerical solutions around TT solution (l=2,3)
- Summary and discussion

Classical solutions for Q'

- Action of SFT around $\Psi_{l,a\geq -1/2}$

$$\begin{aligned} S_{l,a}[\Phi] &\equiv S[\Psi_{l,a}+\Phi]-S[\Psi_{l,a}] \\ &= -\frac{1}{g^2}\left(\frac{1}{2}\langle\Phi,Q'\Phi\rangle+\frac{1}{3}\langle\Phi,\Phi*\Phi\rangle\right) \end{aligned}$$

- Equation of motion: $Q'\Phi+\Phi*\Phi=0$
- We solve the above in the Siegel gauge *numerically*:

$$b_0\Phi=0$$

$$L_{l,a}\Phi+b_0(\Phi*\Phi)=0$$

$$\begin{aligned} L_{l,a} &= \{b_0,Q'\} \\ &= (1+a)L_0-(-1)^l\frac{a}{2}(L_{2l}+L_{-2l})-(-1)^l al(q_{2l}-q_{-2l})+4l^2aZ(a) \end{aligned}$$

CONSTRUCTING SOLUTION

- Iterative approach (Newton's method)

$$L_{l,a}\Phi^{(n)} + b_0(\Phi^{(n)} * \Phi^{(n)})$$
$$+L_{l,a}(\Phi^{(n+1)} - \Phi^{(n)}) + b_0(\Phi^{(n)} * (\Phi^{(n+1)} - \Phi^{(n)}) + (\Phi^{(n+1)} - \Phi^{(n)}) * \Phi^{(n)}) = 0$$

↓

$$L_{l,a}\Phi^{(n+1)} + b_0(\Phi^{(n)} * \Phi^{(n+1)} + \Phi^{(n+1)} * \Phi^{(n)}) = b_0(\Phi^{(n)} * \Phi^{(n)})$$

Linear equation with respect to $\Phi^{(n+1)}$

$$n \longrightarrow \infty$$

$$L_{l,a}\Phi^{(\infty)} + b_0(\Phi^{(\infty)} * \Phi^{(\infty)}) = 0$$

if iteration converges

We should choose an appropriate initial configuration $\Phi^{(0)}$.

TRUNCATION OF STRING FIELD

- Level (L,3L) truncation $L = L_0 + 1$

  Φ : up to level L

  $$|A * B\rangle = \sum_i |\phi^i\rangle \langle V_3(1,2,3)|\phi_i\rangle_1 |A\rangle_2 |B\rangle_3$$

  up to total level 3L

  $b_0 \Phi = 0$: Siegel gauge in the ghost number 1 sector

  $(-1)^{L_0+1} \Phi = \Phi$: twist even

  $$\Phi \sim L^{\rm mat}_{-n_1} L^{\rm mat}_{-n_2} \cdots b_{-m_1} b_{-m_2} \cdots c_{-k_1} c_{-k_2} \cdots |0\rangle$$

  : universal space

SU(1,1) SINGLET BASIS

[Zwiebach (2001)]

- Further truncation to SU(1,1) singlet sector

$$\Phi \sim L^{\rm mat}_{-n_1} L^{\rm mat}_{-n_2} \cdots L^{\rm gh\prime}_{-n_1} L^{\rm gh\prime}_{-n_2} \cdots c_1|0\rangle$$

$$L^{\rm gh\prime}_n \equiv L^{\rm gh}_n + nq_n + \delta_{n,0}$$

$$\mathcal{G}\Phi = X\Phi = Y\Phi = 0$$

$$\mathcal{G} = \sum_{n=1}^{\infty}(c_{-n}b_n - b_{-n}c_n) \quad X = -\sum_{n=1}^{\infty} nc_{-n}c_n \quad Y = \sum_{n=1}^{\infty}\frac{1}{n}b_{-n}b_n$$

$$\begin{aligned} L_{l,a} &= (1+a)(L^{\rm mat}_0 + L^{\rm gh\prime}_0 - 1) \\ &\quad -(-1)^l\frac{a}{2}(L^{\rm mat}_{2l} + L^{\rm gh\prime}_{2l} + L^{\rm mat}_{-2l} + L^{\rm gh\prime}_{-2l}) + 4l^2 aZ(a) \end{aligned}$$

Note: $$[L^{\rm gh\prime}_m, L^{\rm gh\prime}_n] = (m-n)L^{\rm gh\prime}_{m+n} - \frac{1}{6}(m^3-m)\delta_{m+n,0}$$

$$[b_0, L^{\rm gh\prime}_n] = 0 \quad [X, L^{\rm gh\prime}_n] = 0 \quad [Y, L^{\rm gh\prime}_n] = 0 \quad [\mathcal{G}, L^{\rm gh\prime}_n] = 0$$

Dimension of truncated space

L	dim H^+_{univ}	dim H^+_{singl}
0	1	1
2	3	3
4	9	8
6	26	21
8	69	51
10	171	117
12	402	259
14	898	549
16	1925	1124
18	3985	2236
20	7995	4328
22	15606	8176
24	29736	15121

$\mathcal{H}^+_{\mathbf{univ}}$

twist even, universal space,
Siegel gauge, ghost number 1

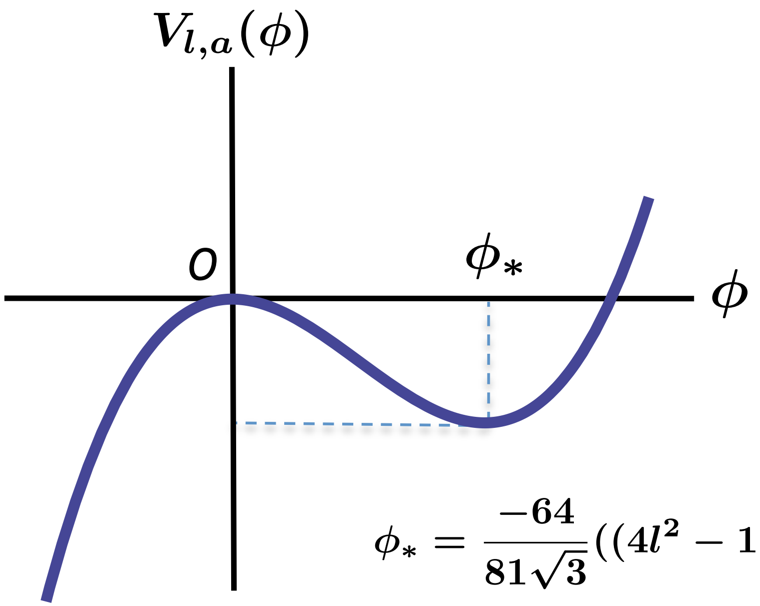
$\mathcal{H}^+_{\mathbf{singl}}$

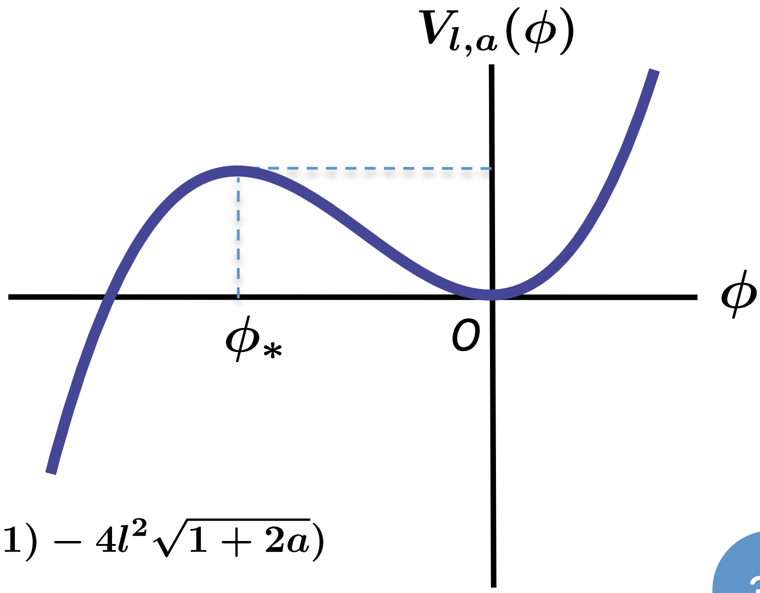
twist even, universal space,
Siegel gauge, ghost number 1,
SU(1,1) singlet

LEVEL 0 ANALYSIS

- Ansatz: $\Phi_{L=0} = \phi\, c_1 |0\rangle$

$$
\begin{aligned}
V_{l,a}(\phi) &= -2\pi^2 g^2 S_{l,a}[\Phi_{L=0}]/V_{26} \\
&= 2\pi^2 \left(\frac{1}{2}((4l^2-1)(a+1) - 4l^2\sqrt{1+2a})\phi^2 + \frac{27\sqrt{3}}{64}\phi^3 \right)
\end{aligned}
$$





$$
\phi_* = \frac{-64}{81\sqrt{3}}((4l^2-1)(a+1) - 4l^2\sqrt{1+2a})
$$

stable solution $(a \sim 0)$

unstable solution $(a \gtrsim -1/2)$

Construction of "stable solutions"

- The initial configuration for $a=0\ (Q'=Q_{\mathrm{B}})$

$$\Phi^{(0)}=\frac{64}{81\sqrt{3}}c_1\left|0\right\rangle \quad \xrightarrow{\text{iteration}} \quad \Phi_1\big|_{a=0}$$

"conventional" tachyon vacuum solution

the nontrivial solution for (0,0) truncation

- The initial configuration for $a=\epsilon\ (0<|\epsilon|\ll 1)$

$$\Phi^{(0)}=\Phi_1\big|_{a=0} \quad \xrightarrow{\text{iteration}} \quad \Phi_1\big|_{a=\epsilon}$$

- The initial configuration for $a=2\epsilon$

$$\Phi^{(0)}=\Phi_1\big|_{a=\epsilon} \quad \xrightarrow{\text{iteration}} \quad \Phi_1\big|_{a=2\epsilon}$$

$$\vdots$$

CONSTRUCTION OF "UNSTABLE SOLUTIONS" (1)

- The initial configuration for $a=-1/2 \quad l=1$

$$\Phi^{(0)} = -\frac{32}{27\sqrt{3}} c_1|0\rangle \xrightarrow{\text{iteration}} \Phi_2|_{a=-1/2}$$

the nontrivial solution for (0,0) truncation

- The initial configuration for $a=-1/2+\epsilon \ (0<\epsilon\ll 1)$

$$\Phi^{(0)} = \Phi_2|_{a=-1/2} \xrightarrow{\text{iteration}} \Phi_2|_{a=-1/2+\epsilon}$$

- The initial configuration for $a=-1/2+2\epsilon$

$$\Phi^{(0)} = \Phi_2|_{a=-1/2+\epsilon} \xrightarrow{\text{iteration}} \Phi_2|_{a=-1/2+2\epsilon}$$

$$\vdots$$

CONSTRUCTION OF "UNSTABLE SOLUTIONS" (2)

$$a = -1/2 \qquad l = 2, 3$$

To get the converged solution uniquely, we take the following strategy:

- Level (2,6) truncation:

$$\Phi^{(0)} = -\frac{32(4l^2-1)}{81\sqrt{3}} c_1|0\rangle \quad \xrightarrow{\text{iteration}} \quad \Phi_2\big|_{a=-1/2}^{(2,6)}$$

the nontrivial solution for (0,0) truncation

- Level (4,12) truncation:

$$\Phi^{(0)} = \Phi_2\big|_{a=-1/2}^{(2,6)} \quad \xrightarrow{\text{iteration}} \quad \Phi_2\big|_{a=-1/2}^{(4,12)}$$

- Level (6,18) truncation:

$$\Phi^{(0)} = \Phi_2\big|_{a=-1/2}^{(4,12)} \quad \xrightarrow{\text{iteration}} \quad \Phi_2\big|_{a=-1/2}^{(6,18)}$$

$$\vdots$$

Evaluation of gauge invariants

- We evaluate potential height and gauge invariant overlap for obtained solutions:

$$V_{l,a}(\Phi) = -2\pi^2 g^2 S_{l,a}[\Phi]/V_{26}$$

$$\mathcal{O}_V(\Phi) = 2\pi\langle\hat{\gamma}(1_c, 2)|\phi_V\rangle_{1_c}|\Phi\rangle_2/V_{26}$$

invariants under the gauge transformation: $\delta_\Lambda\Phi = Q'\Lambda + \Phi * \Lambda - \Lambda * \Phi$

$\langle\hat{\gamma}(1_c, 2)|$: Shapiro-Thorn's open-closed vertex

$\langle\hat{\gamma}(1_c, 2)|\phi_V\rangle_{1_c}(c_n^{(2)} + (-1)^n c_{-n}^{(2)}) = 0$ $\langle\hat{\gamma}(1_c, 2)|\phi_V\rangle_{1_c}(Q_n^{(2)} + (-1)^n Q_{-n}^{(2)}) = 0$

Normalization: $V_{l,a=0}(\Psi_{\rm Sch}) = -1$ $\mathcal{O}_V(\Psi_{\rm Sch}) = 1$

for Schnabl's analytic solution for tachyon vacuum

CONTENTS

- Introduction
- Takahashi-Tanimoto's identity based solution
- String field theory around TT solutions
- Construction of numerical solutions
- Comments on numerical solution in Siegel gauge
- Numerical stable solution around TT solution (l=1)
- Numerical unstable solution around TT solution (l=1)
- Numerical solutions around TT solution (l=2,3)
- Summary and discussion

Numerical values for a=0 ($Q'=Q_B$)

(L,3L)	$V_{l,a=0}(\Phi_1\|_{a=0})$	$\mathcal{O}_V(\Phi_1\|_{a=0})$
(0,0)	-0.6846162	0.7165627
(2,6)	-0.9593766	0.8898618
(4,12)	-0.9878218	0.9319524
(6,18)	-0.9951771	0.9510789
(8,24)	-0.9979301	0.9611748
(10,30)	-0.9991825	0.9681148
(12,24)	-0.9998223	0.9725595
(14,42)	-1.0001737	0.9761715
(16,48)	-1.0003755	0.9786768
(18,54)	-1.0004937	0.9809045
(20,60)	-1.0005630	0.9825168
(22,66)	-1.0006023	0.9840334
(24,72)	-1.0006227	0.9851603

[Sen-Zwiebach(1999), Moeller-Taylor(2000), Gaitto-Rastelli(2002)]

[Kawano-Kishimoto-Takahashi(2008)]

[Kishimoto-Takahashi,0910.3025]

Comments on results for a=0 ($Q' = Q_B$)

- Straightforward extrapolation of potential height

fitting function: $$F_N(L) = \sum_{n=0}^{N} \frac{a_n}{(L+1)^n}$$ [Gaiotto-Rastelli (2002)]

Using data for *L*=0,2,4,6,8,10,12,14,16 and *N*=9, we have

$$F_{N=9}(L=18) = -1.0004937$$

$$F_{N=9}(L=20) = -1.0005630$$

$$F_{N=9}(L=22) = -1.0006023$$

$$F_{N=9}(L=24) = -1.0006229$$

$$F_{N=9}(L=\infty) = -1.0000293$$

Good coincidence with our direct computation!

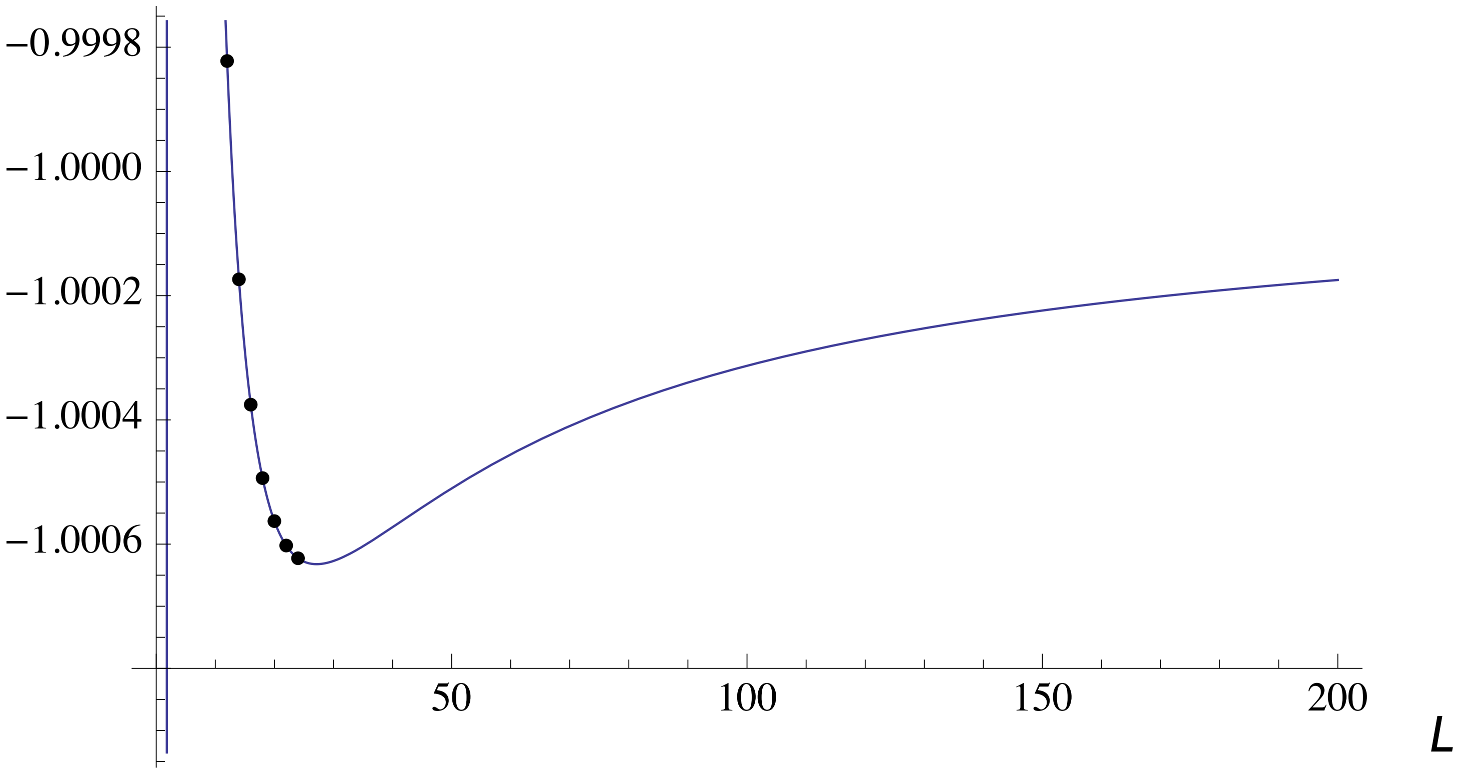
Extrapolation of potential height for a=0 ($Q' = Q_B$)

Using data for *L*=0,2,4,6,8,10,12,14,16,18,20,22,24 and *N*=13, we have

$$F_{N=13}(L=\infty) = -1.0000075$$

The extrapolated value further approaches -1.

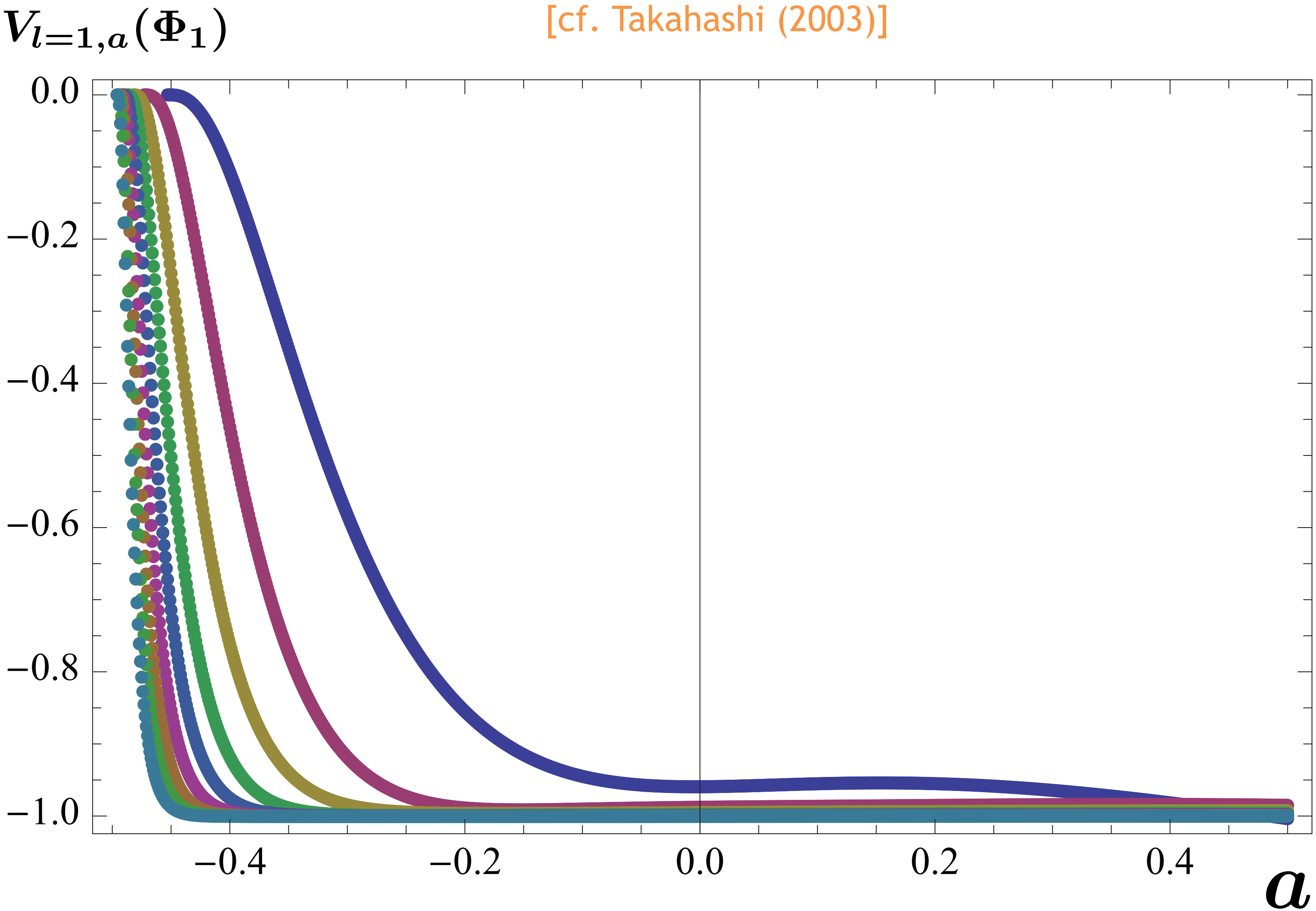
$V_{l,a=0}(\Phi_1|_{a=0})$



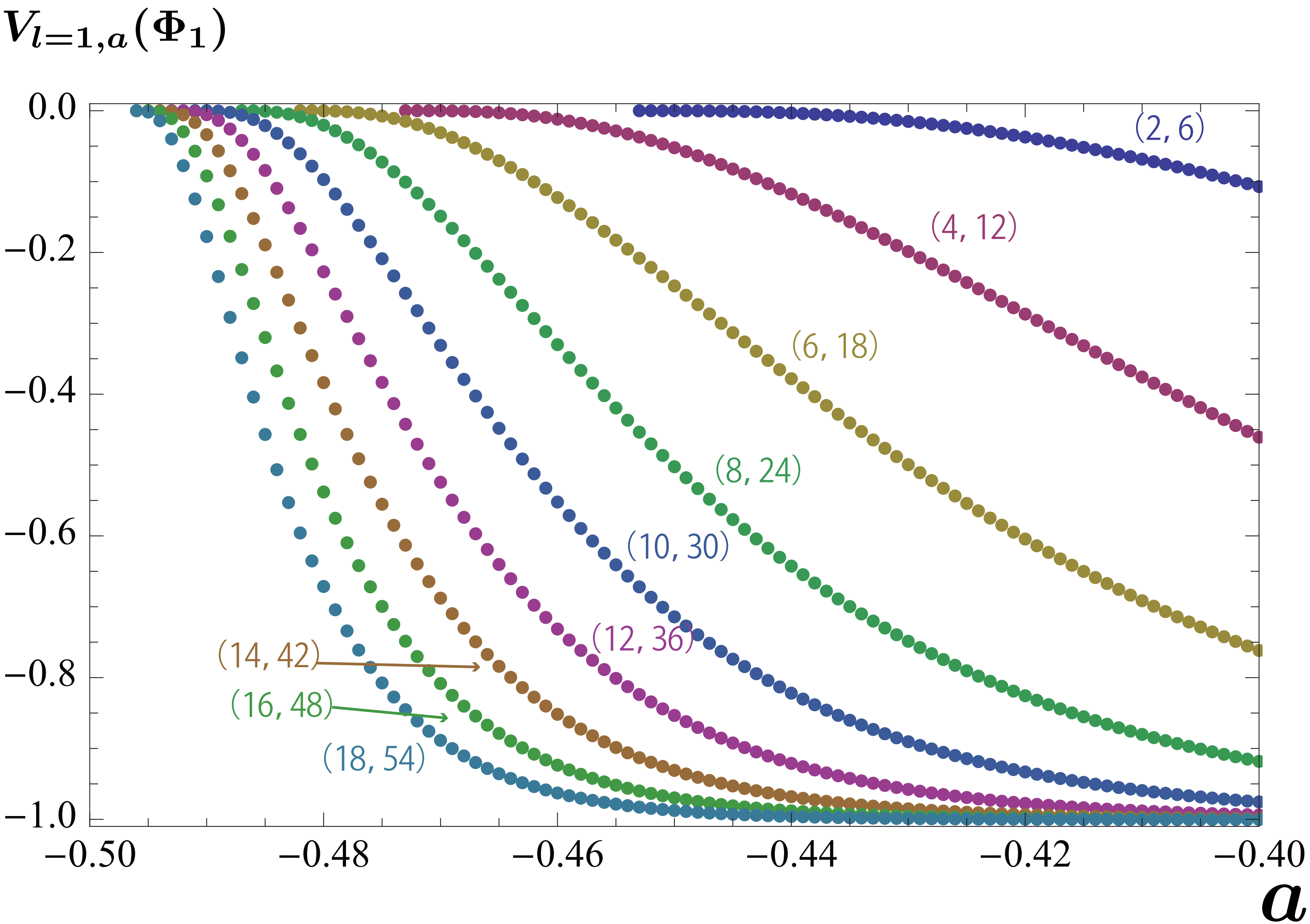
Contents

POTENTIAL HEIGHT FOR $\Phi_1 \ (l = 1)$

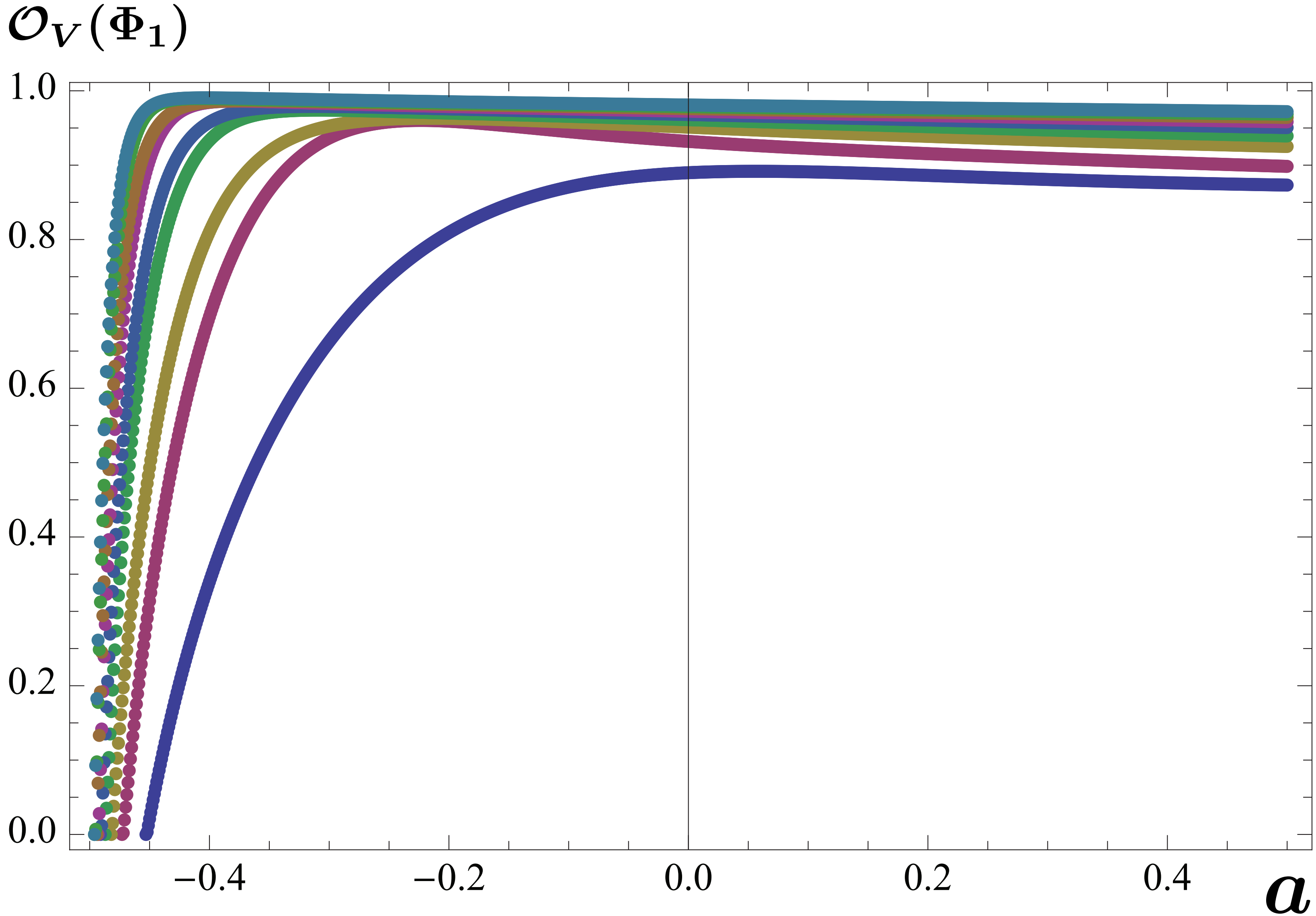
[cf. Takahashi (2003)]



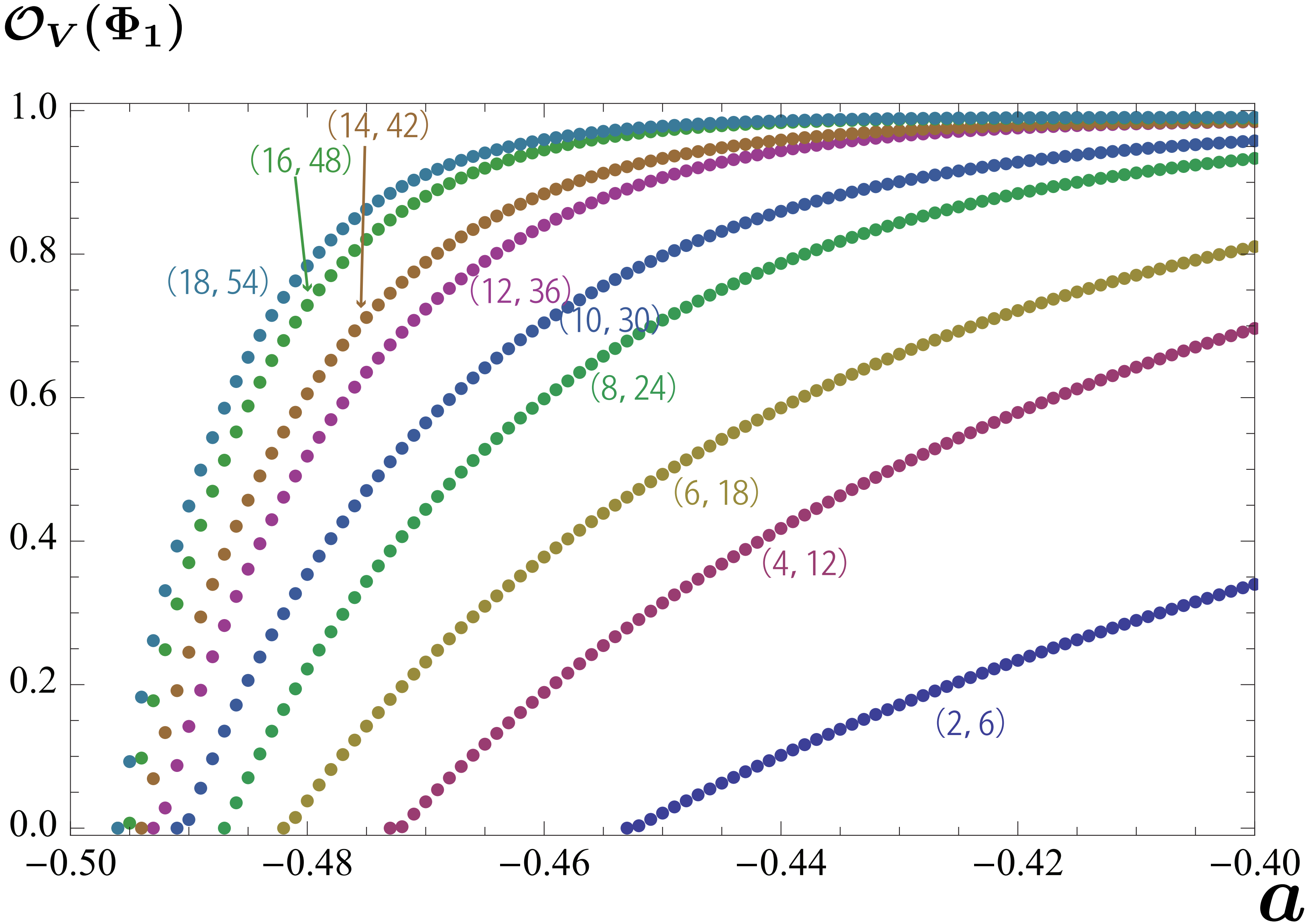
Potential height for Φ_1 ($l = 1$)



Gauge invariant overlap for Φ_1 $(l = 1)$

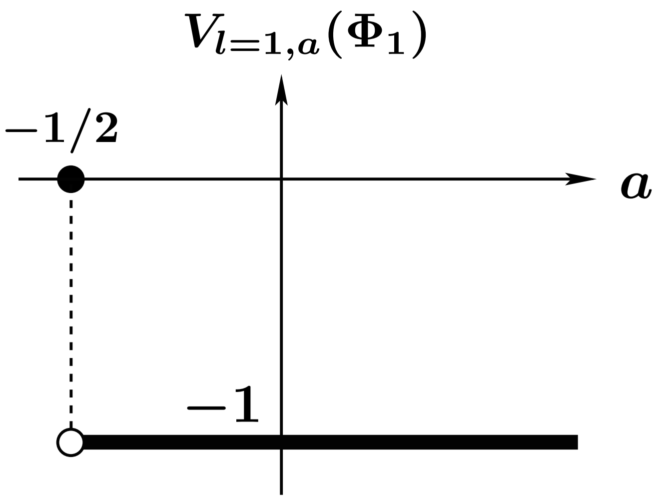


GAUGE INVARIANT OVERLAP FOR Φ_1 $(l = 1)$



Stable vacuum solution

- For $L \longrightarrow \infty$, numerical results suggest



$$a > -1/2$$

Φ_1 :nontrivial tachyon vacuum

$$a = -1/2$$

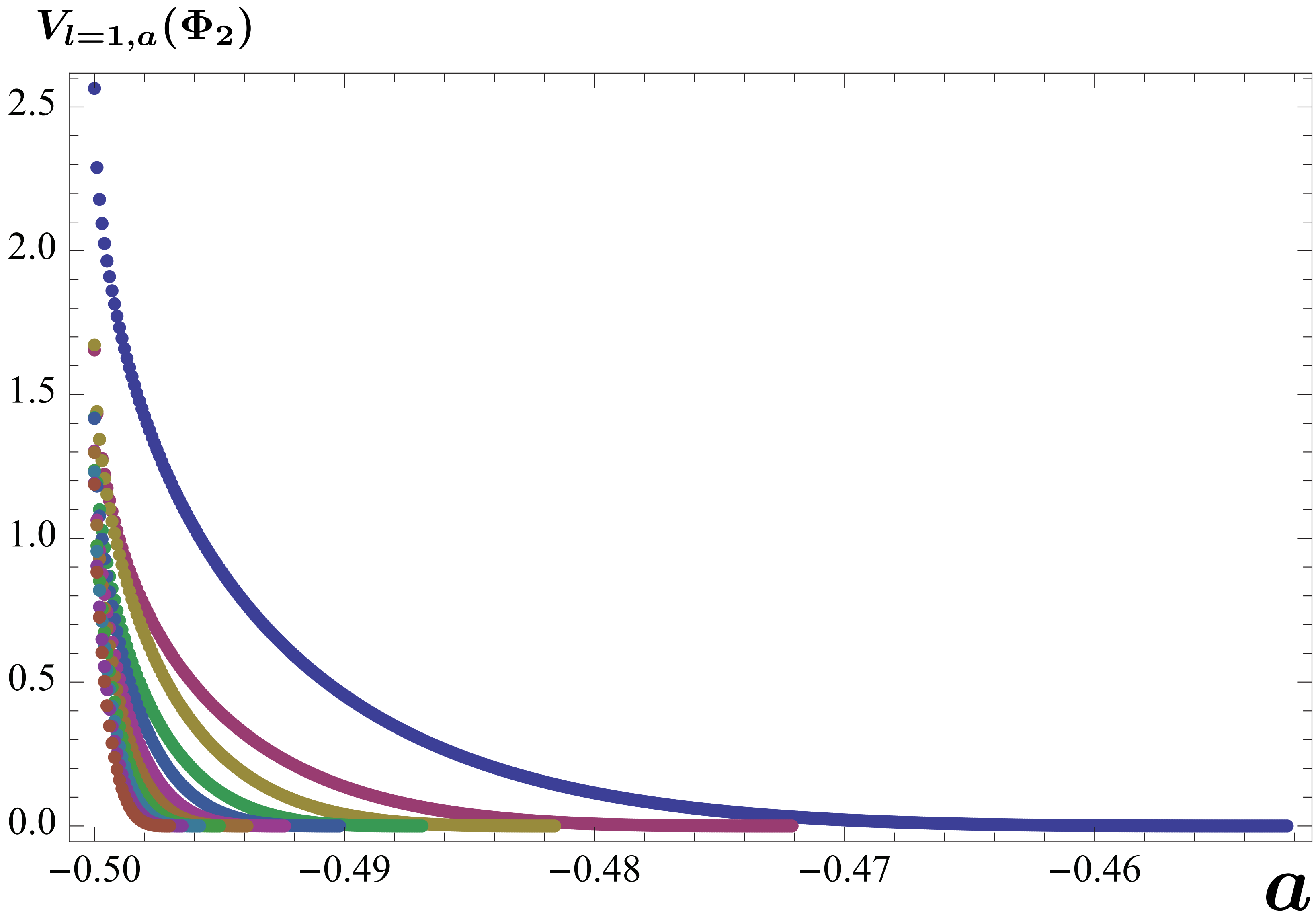
$$\Phi_1 = 0$$

$a > -1/2$ ⟶ $\Psi_{l=1,a}$: pure gauge

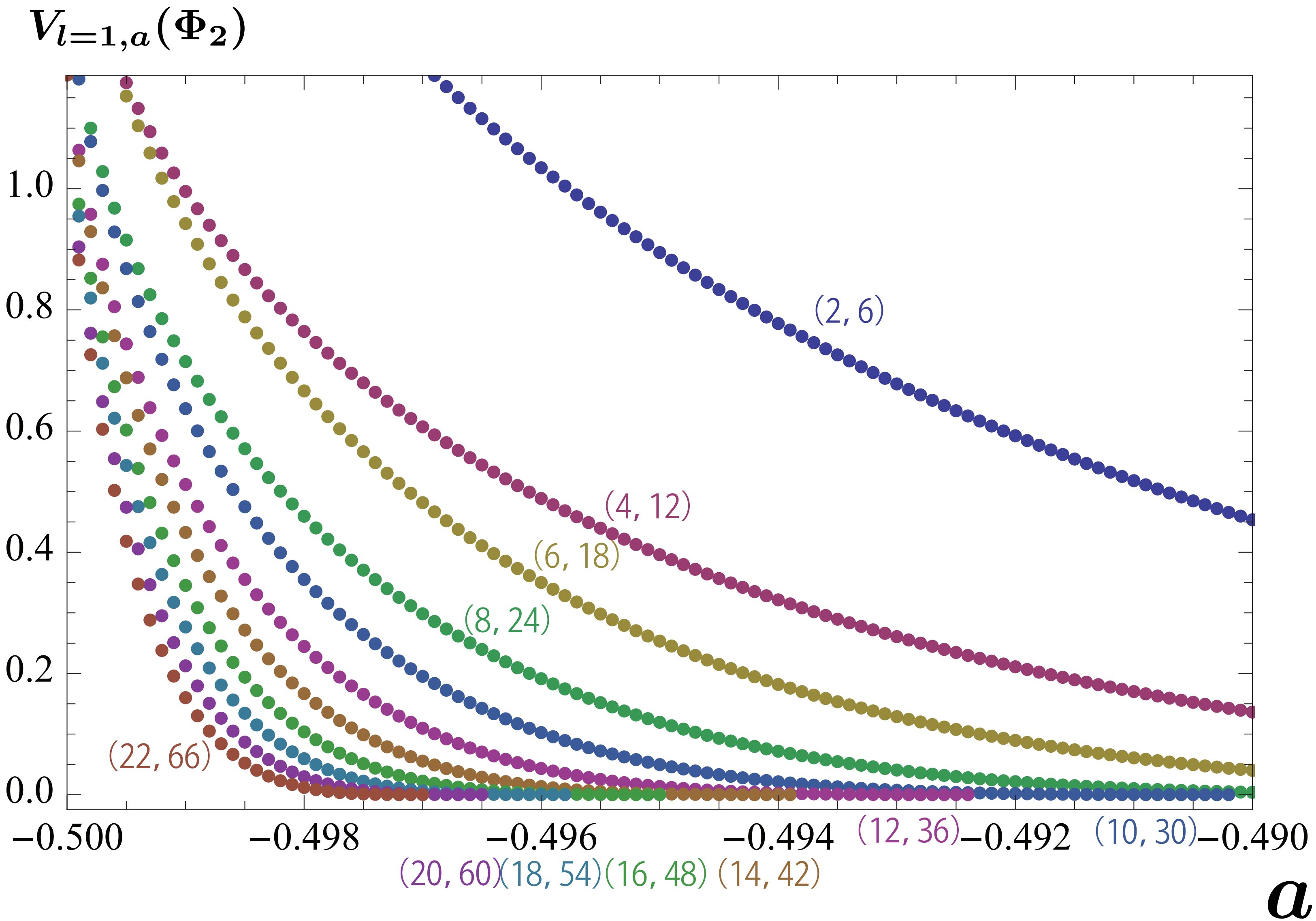
$a = -1/2$ ⟶ $\Psi_{l=1,a=-\frac{1}{2}}$: tachyon vacuum (!?)

CONTENTS

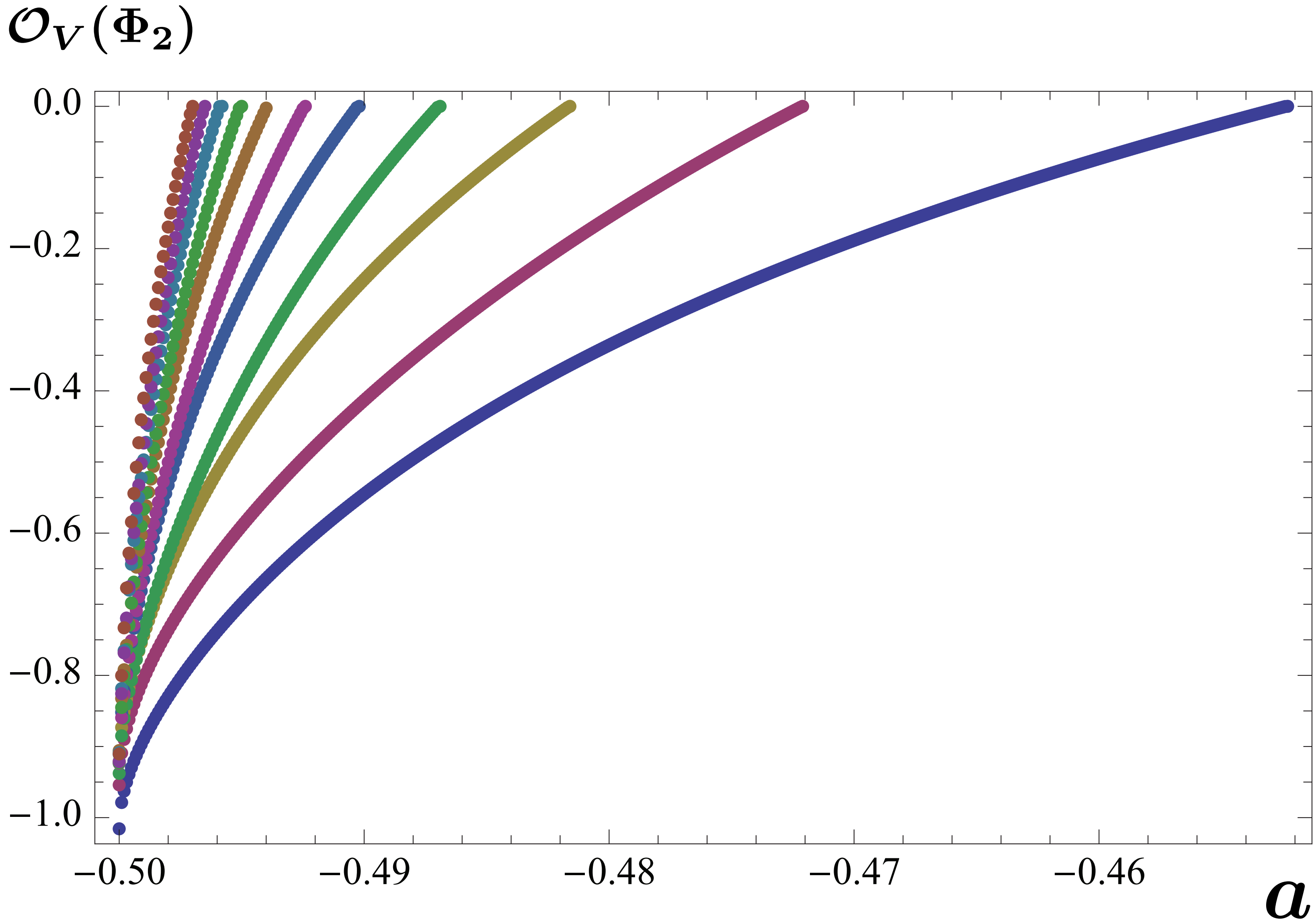
POTENTIAL HEIGHT FOR Φ_2 $(l = 1)$



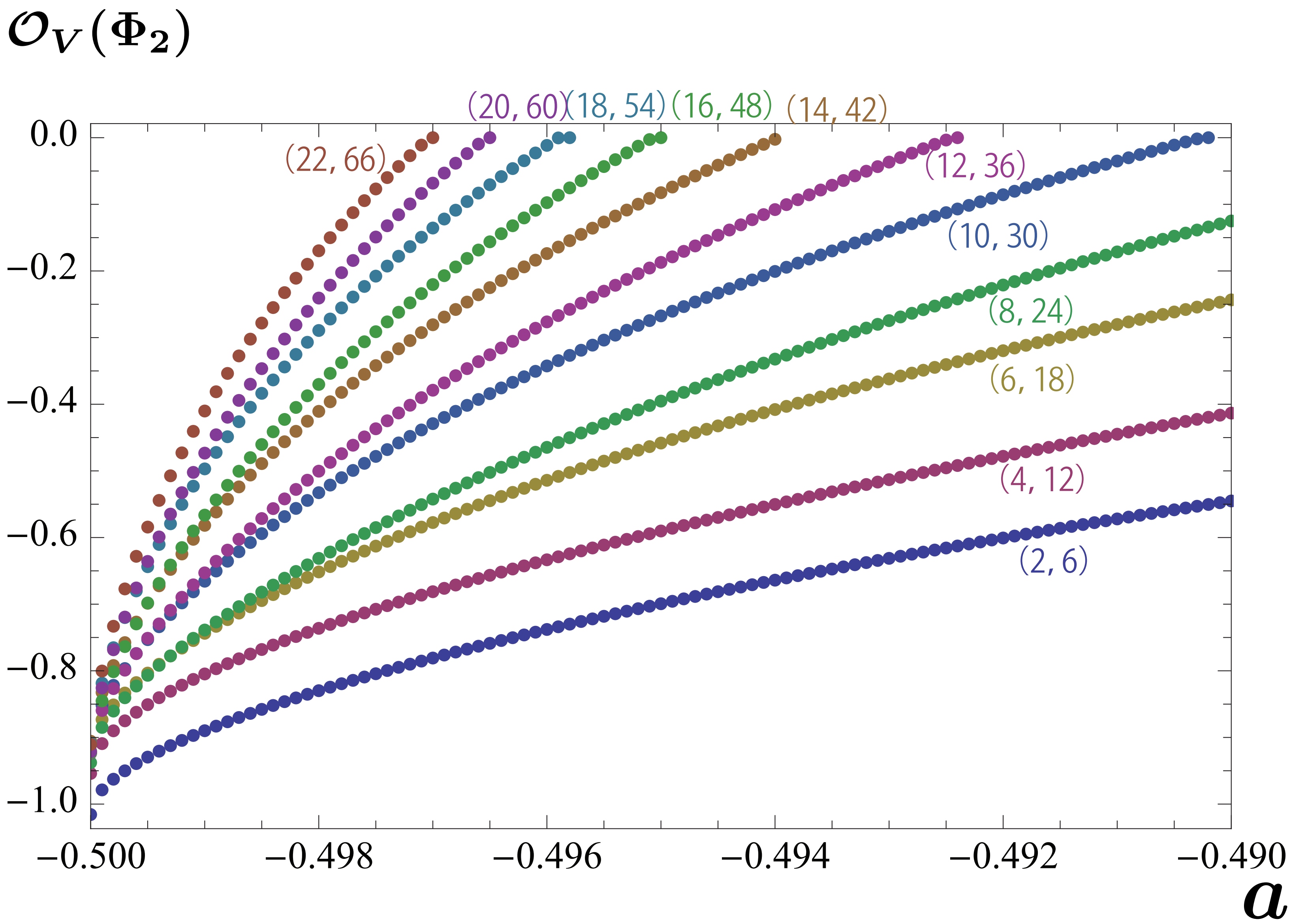
POTENTIAL HEIGHT FOR Φ_2 $(l = 1)$



Gauge invariant overlap for Φ_2 $(l = 1)$



Gauge invariant overlap for Φ_2 $(l = 1)$



NUMERICAL VALUES FOR $a = -1/2 \ (l = 1)$

[cf. Zeze(2003), Drukker-Okawa(2005)]

(L,3L)	$V_{l=1,a=-\frac{1}{2}}(\Phi_2^{l=1}\|_{a=-\frac{1}{2}})$	$\mathcal{O}_V(\Phi_2^{l=1}\|_{a=-\frac{1}{2}})$
(0,0)	2.3105795	-1.0748441
(2,6)	2.5641847	-1.0156983
(4,12)	1.6550774	-0.9539832
(6,18)	1.6727496	-0.9207572
(8,24)	1.4193393	-0.9377548
(10,30)	1.4168893	-0.9110994
(12,24)	1.3035715	-0.9237917
(14,42)	1.2986472	-0.9056729
(16,48)	1.2357748	-0.9229035
(18,54)	1.2310583	-0.9086563
(20,60)	1.1915648	-0.9212376
(22,66)	1.1874828	-0.9103838
(24,72)	1.1605884	-0.9231608

[Kishimoto-Takahashi, 0910.3026]

EXTRAPOLATION OF POTENTIAL HEIGHT $a=-1/2\ \ (l=1)$

(L,3L)	Extrapolation of $V_{l=1,a=-\frac{1}{2}}(\Phi_2^{l=1}\vert_{a=-\frac{1}{2}})$
$(4\infty, 12\infty)$	0.9893181
$(4\infty+2, 12\infty+6)$	0.9891240

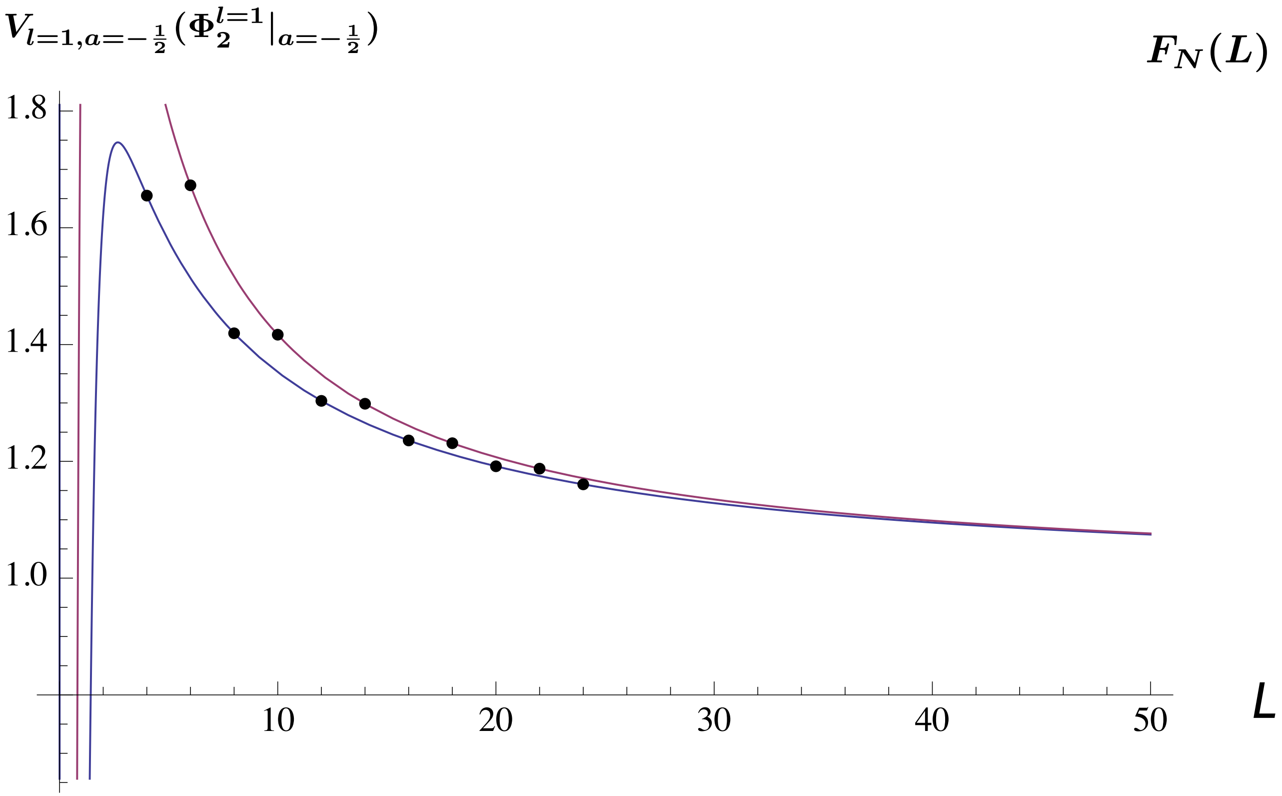
← (L=0,4,8,12,16,20,24;N=7)

← (L=2,6,10,14,18,22;N=6)

The value approaches +1 (!?)

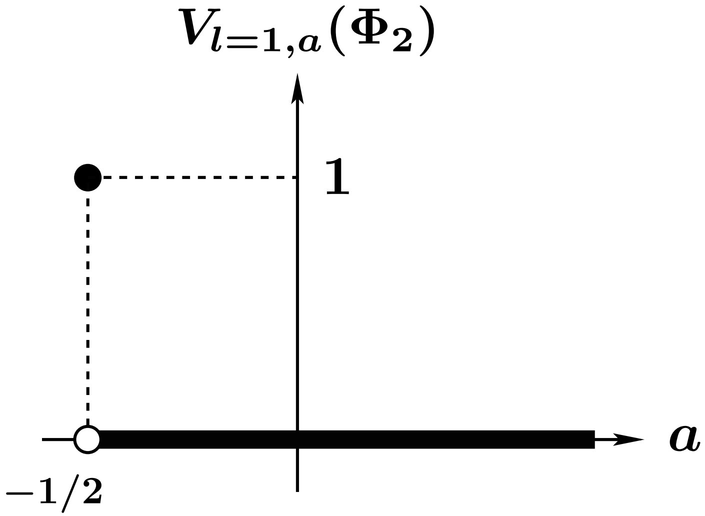
fitting function:

$$F_N(L)=\sum_{n=0}^{N}\frac{a_n}{(L+1)^n}$$



Unstable vacuum solution

- For $L \longrightarrow \infty$, numerical results suggest



$$a > -1/2$$

$$\Phi_2 = 0$$

$$a = -1/2$$

Φ_2 :nontrivial vacuum
(perturbative vacuum!?)

$a > -1/2$ ⟹ $\Psi_{l=1,a}$: pure gauge

$a = -1/2$ ⟹ $\Psi_{l=1,a=-\frac{1}{2}}$: tachyon vacuum (!?)

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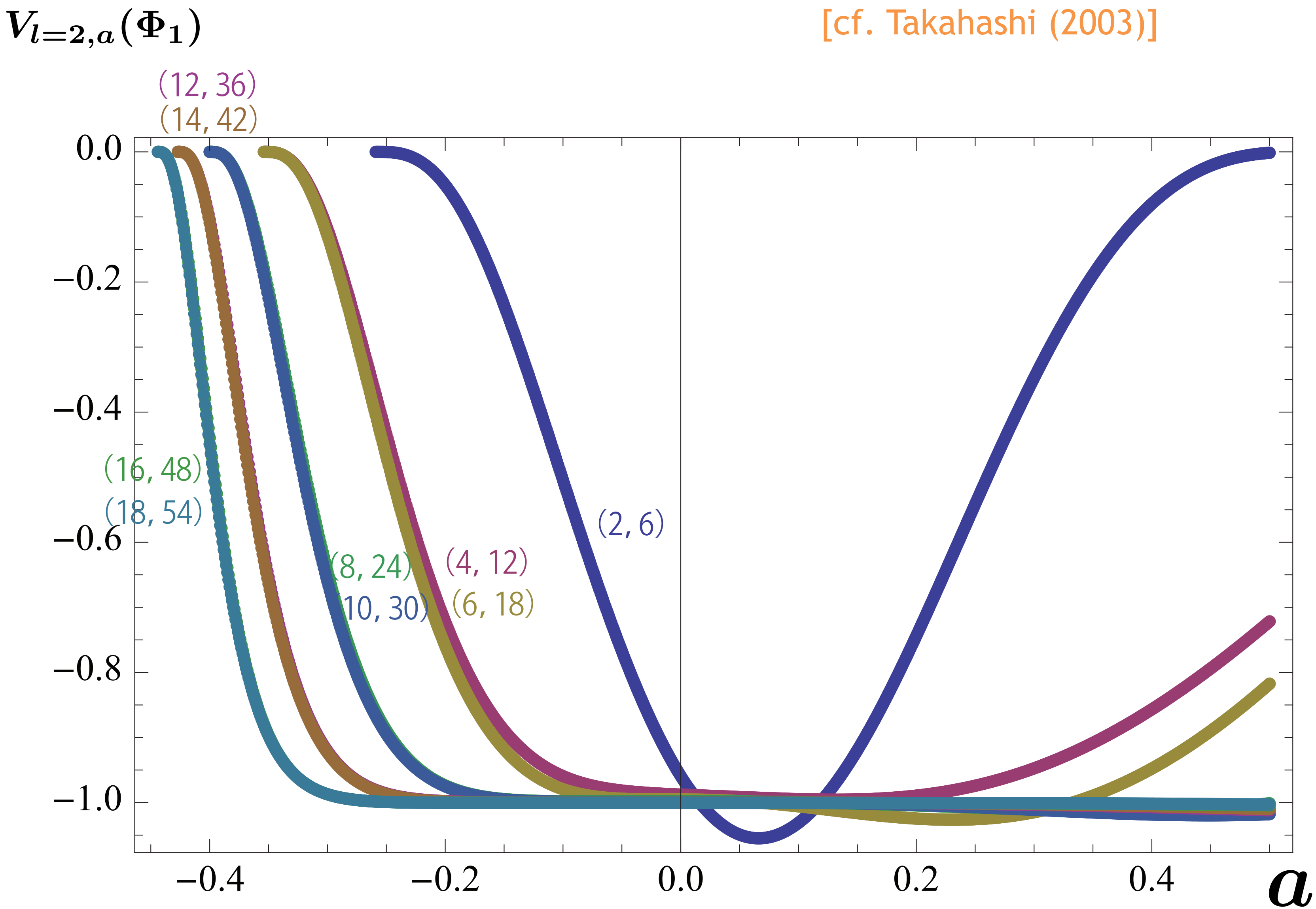
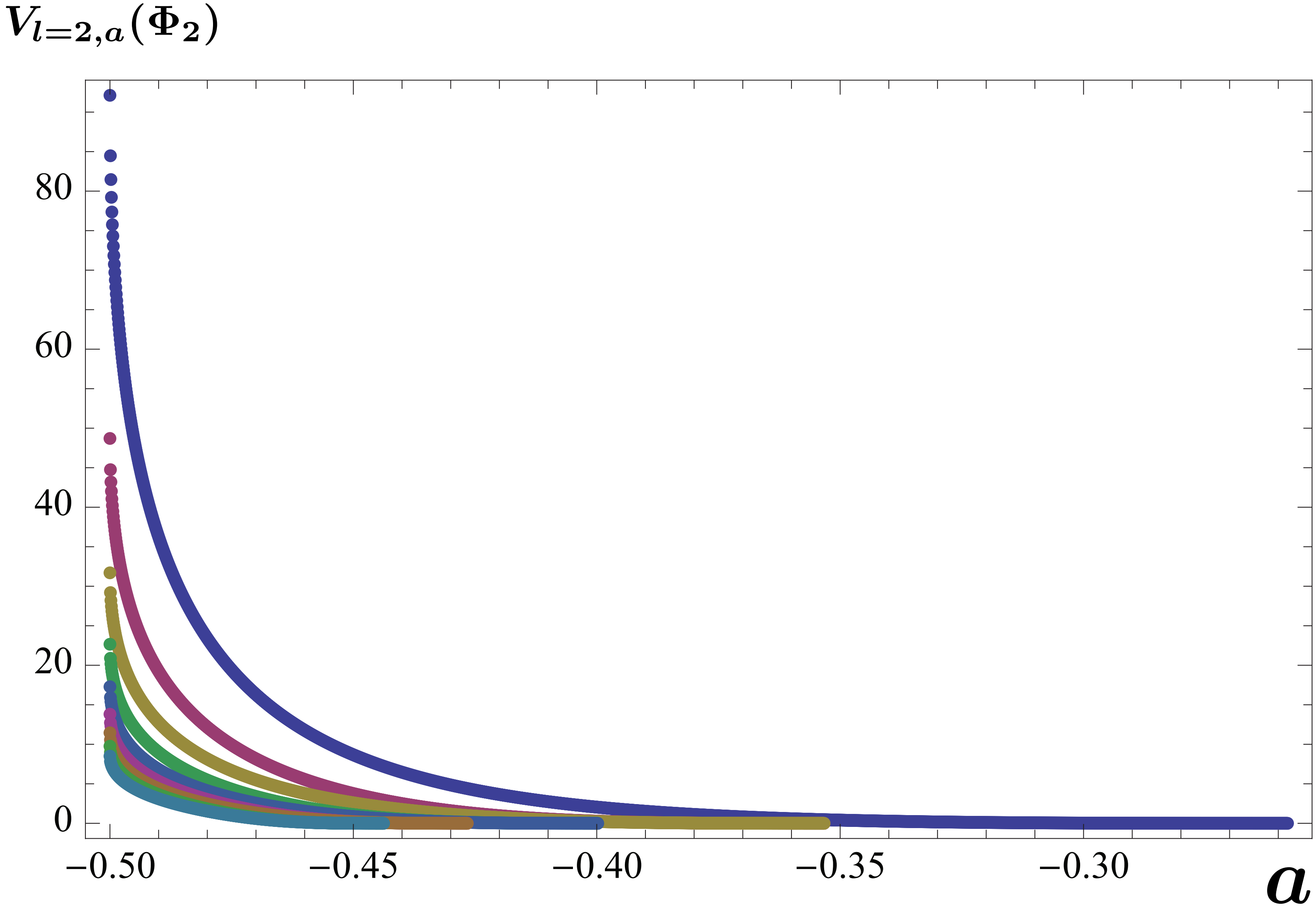
Potential height for Φ_1 $(l = 2)$

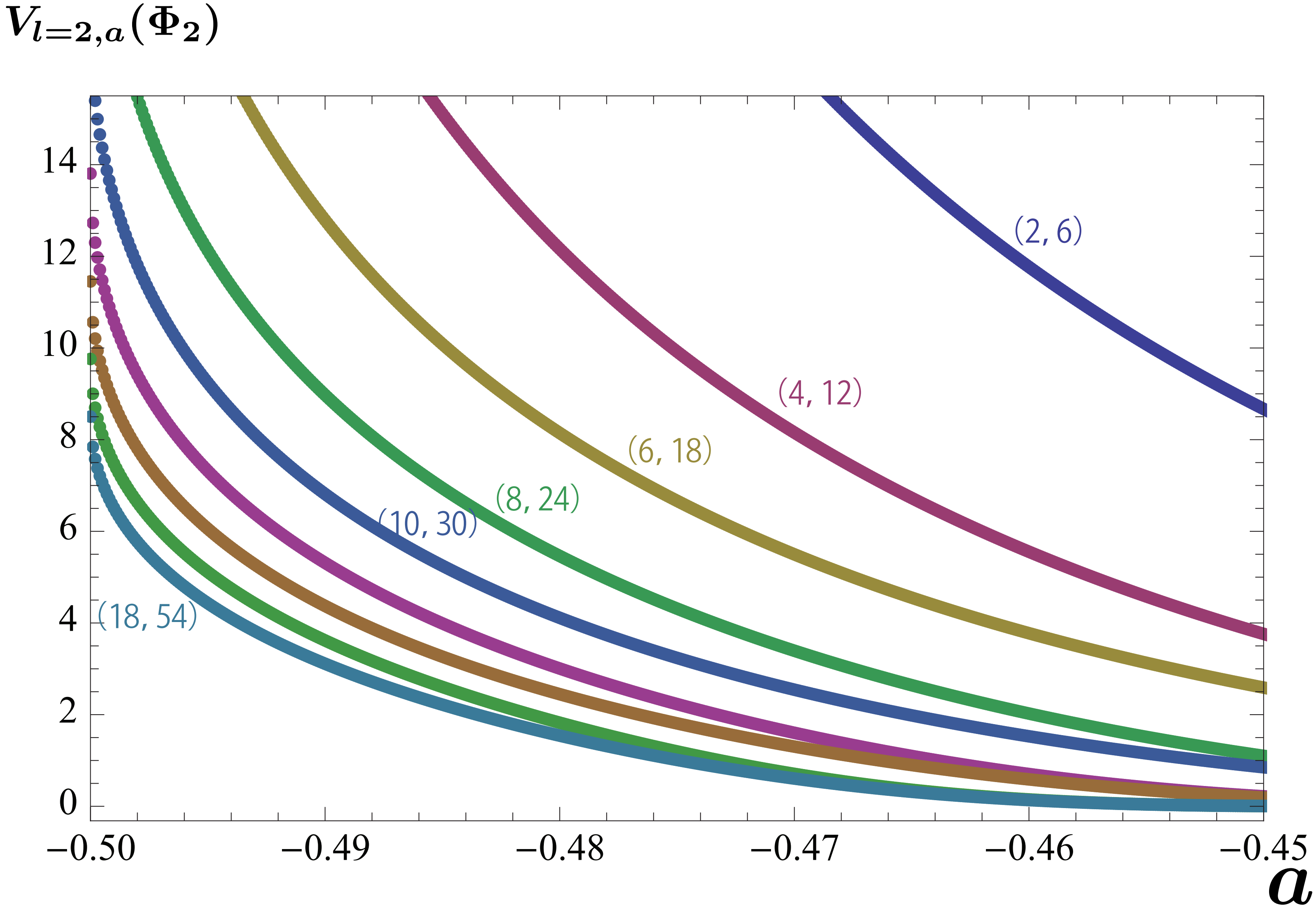
[cf. Takahashi (2003)]



POTENTIAL HEIGHT FOR Φ_2 $(l = 2)$



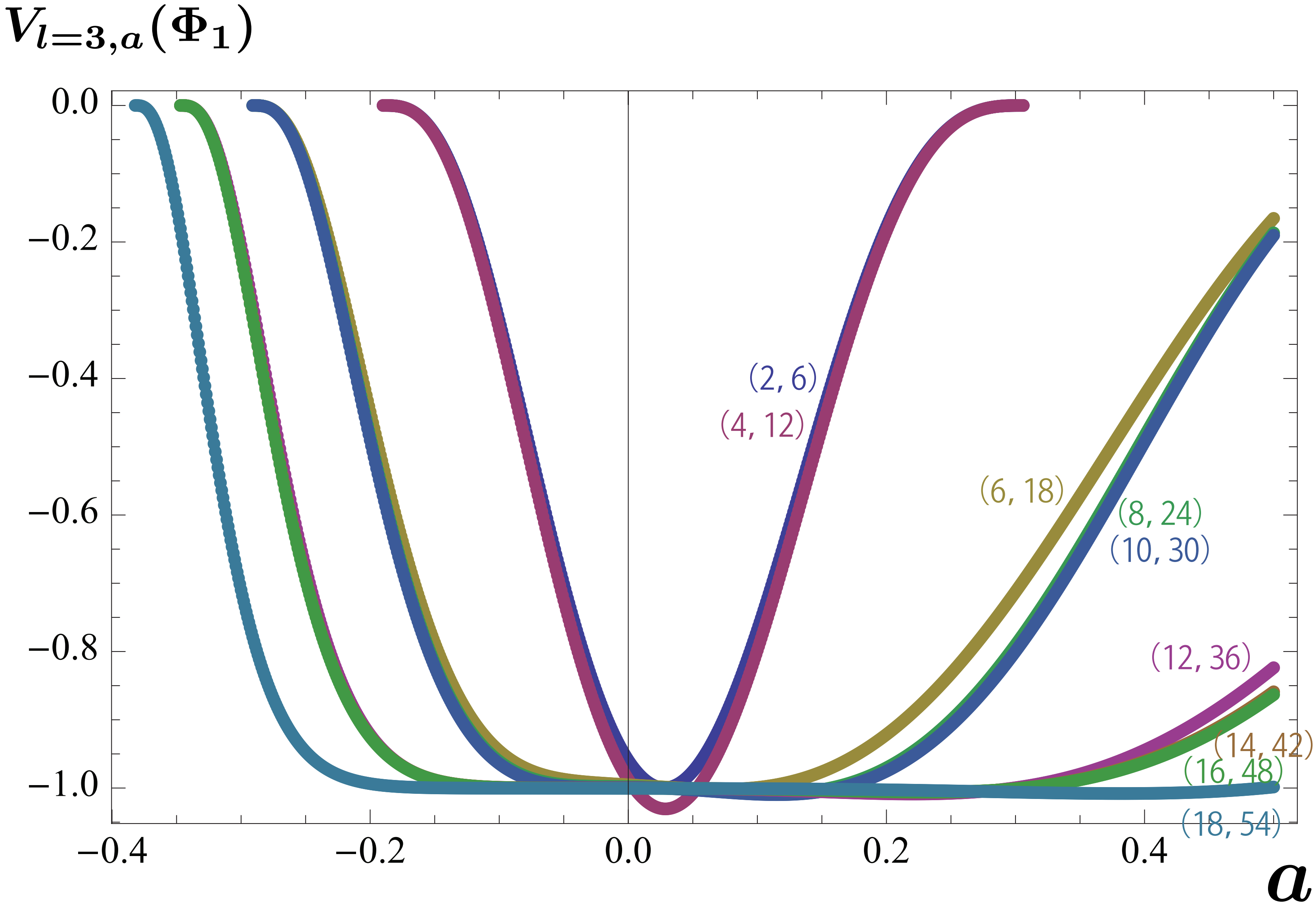
Potential height for $\Phi_2\ (l=2)$



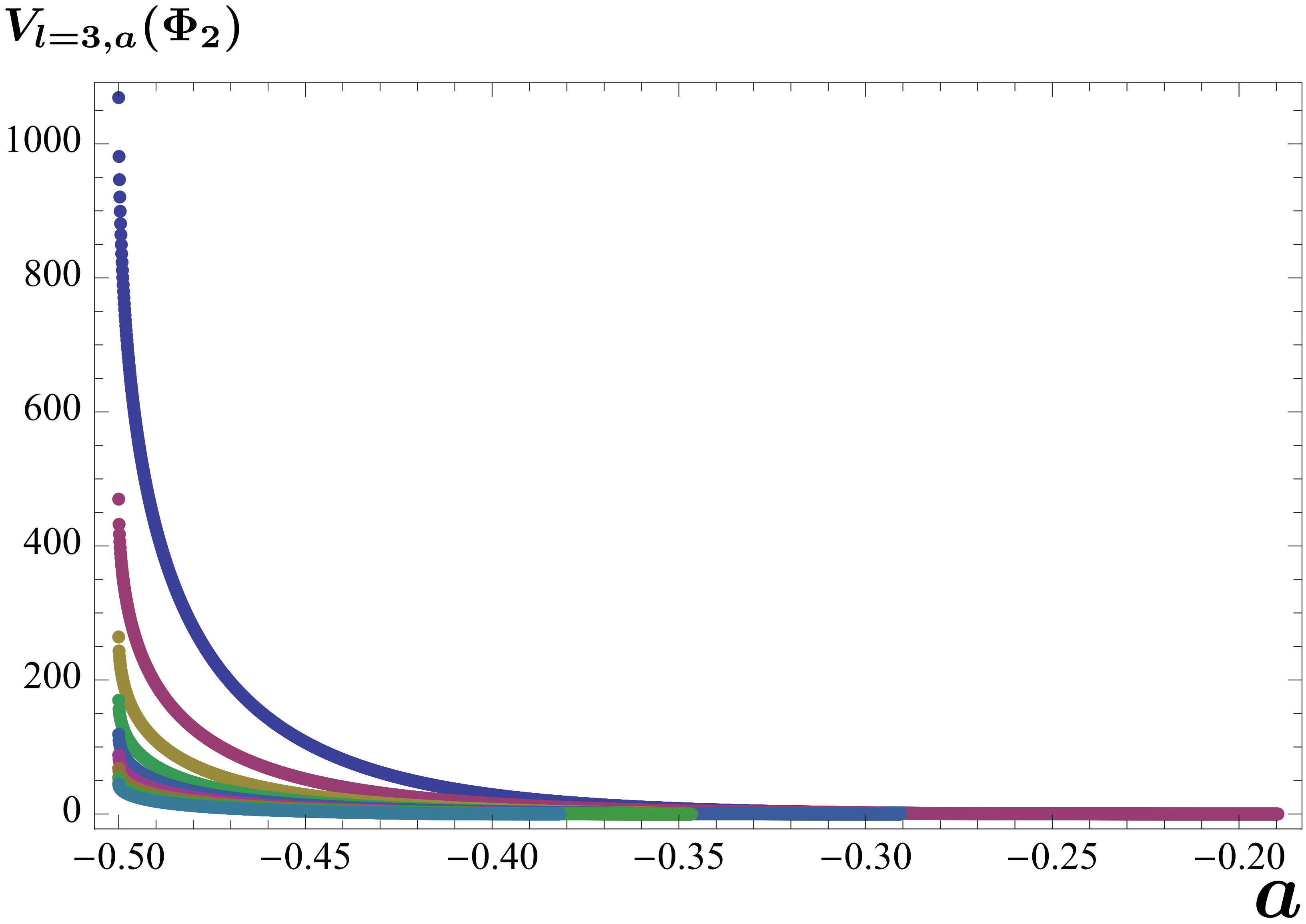
NUMERICAL VALUES FOR $a = -1/2 \ (l = 2)$

(L,3L)	$V_{l=2,a=-\frac{1}{2}}(\Phi_2^{l=2}\|_{a=-\frac{1}{2}})$	$\mathcal{O}_V(\Phi_2^{l=2}\|_{a=-\frac{1}{2}})$
(0,0)	288.8224425	-5.3742203
(2,6)	92.1238442	-3.1048971
(4,12)	48.7033363	-2.6003723
(6,18)	31.6992499	-2.2279366
(8,24)	22.6595219	-2.0575256
(10,30)	17.2812044	-1.8761349
(12,24)	13.8061273	-1.7589056
(14,42)	11.4523287	-1.6466427
(16,48)	9.7610020	-1.5812240
(18,54)	8.5029788	-1.5129234
(20,60)	7.5338958	-1.4632552
(22,66)	6.7726232	-1.4139118
(24,72)	6.1591160	-1.3803795

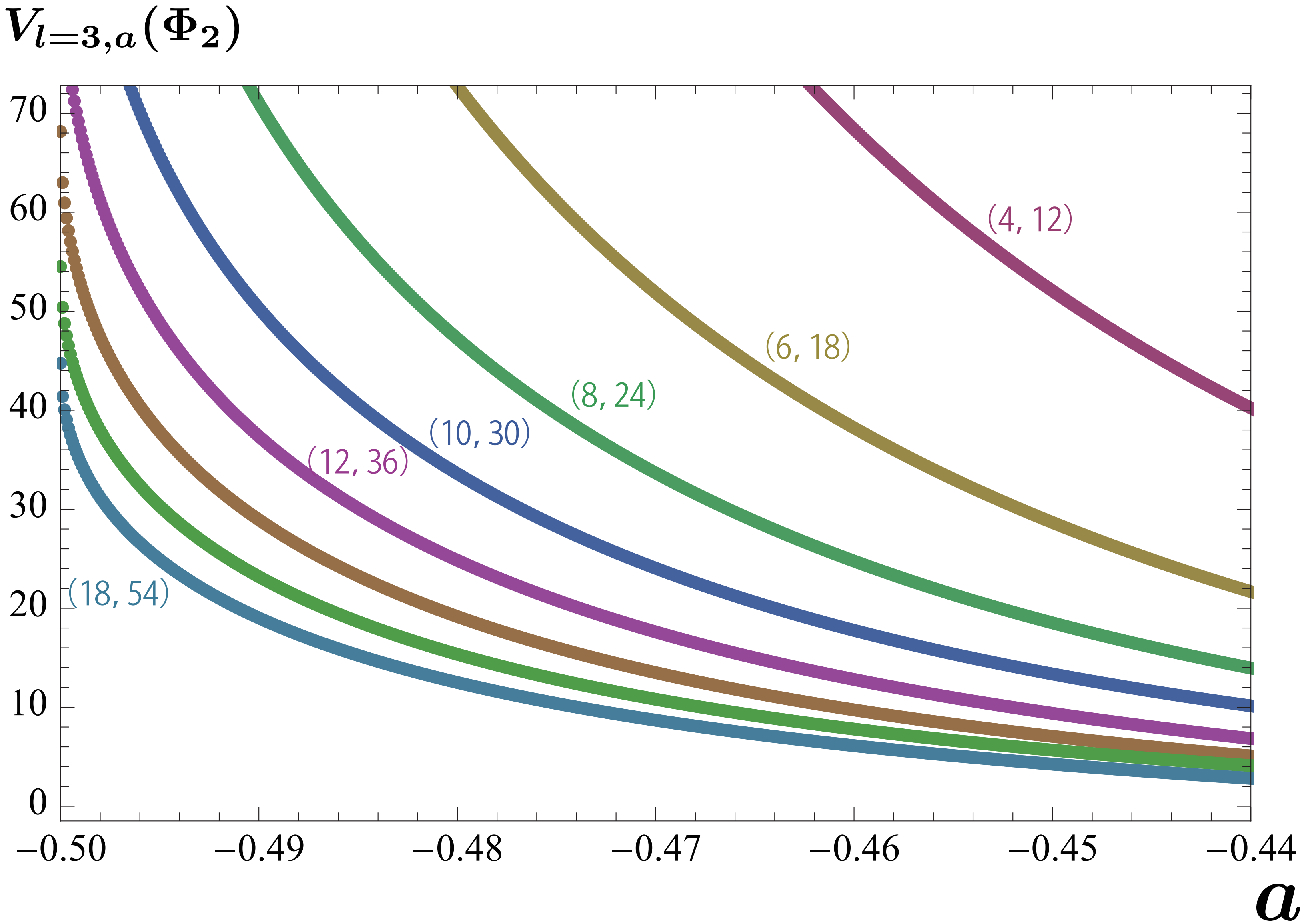
POTENTIAL HEIGHT FOR Φ_1 $(l = 3)$



POTENTIAL HEIGHT FOR $\Phi_2\ (l = 3)$



POTENTIAL HEIGHT FOR $\Phi_2\ (l=3)$



NUMERICAL VALUES FOR $a = -1/2 \;\; (l = 3)$

(L,3L)	$V_{l=3,a=-\frac{1}{2}}(\Phi_2^{l=3}\vert_{a=-\frac{1}{2}})$	$\mathcal{O}_V(\Phi_2^{l=3}\vert_{a=-\frac{1}{2}})$
(0,0)	3669.1147320	-12.5398475
(2,6)	1069.0267362	-6.9447829
(4,12)	469.8576394	-5.6412136
(6,18)	264.1631512	-4.4016913
(8,24)	169.6466508	-3.9168243
(10,30)	118.8569322	-3.4132963
(12,24)	88.1014995	-3.1705974
(14,42)	68.1558589	-2.8864809
(16,48)	54.5068784	-2.7221662
(18,54)	44.7485588	-2.5342661
(20,60)	37.5395492	-2.4137280
(22,66)	32.0687967	-2.2791648
(24,72)	27.8108828	-2.1939288

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SUMMARY (1)

- We investigated cohomology of $Q' \equiv Q_{\mathrm{B}} + [\Psi_{l,a},\ \cdot\ \}_*$ around a class of identity-based solutions:

$$\Psi_{l,a}\ (l = 1, 2, \cdots ; a \geq -1/2)$$

$a > -1/2$ $\quad Q' = e^{q(h_a^l)} Q_{\mathrm{B}} e^{-q(h_a^l)}$

$\longrightarrow$ the same cohomology as Q_{B}

$a = -1/2$ $\quad Q' = e^{-q(\lambda^l)} (Q_{2l} - 4l^2 c_{2l}) e^{q(\lambda^l)}$

$\longrightarrow$ no cohomology in the ghost number 1 sector

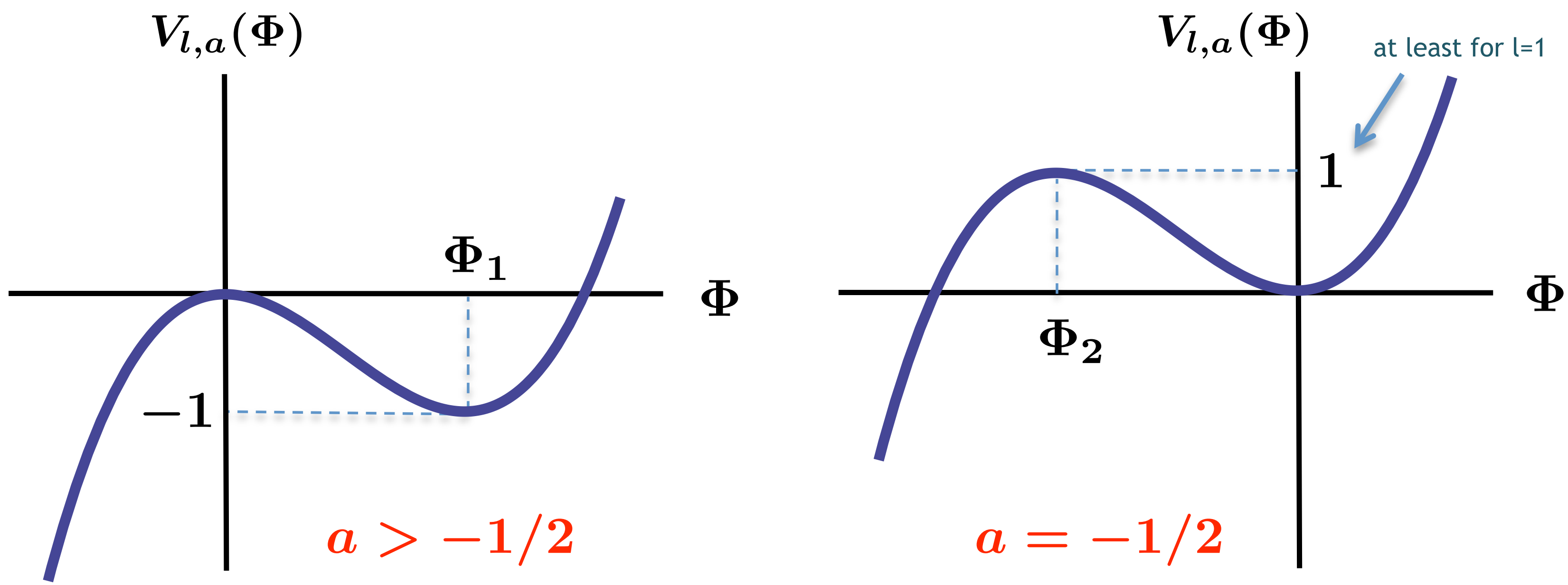
- The result suggests

$\Psi_{l,a>-1/2}$: pure gauge

$\Psi_{l,a=-1/2}$: nontrivial solution, no open string

SUMMARY (2)

- We numerically constructed stable solution and unstable solution (up to level (24,72)) in the theory around $\Psi_{l,a}\ (l=1,2,3; a \geq -1/2)$ and evaluated gauge invariants.
- The results suggest the vacuum structure is like this:



- It is consistent with our previous interpretation.

DISCUSSION

- Our result on TT's solution suggests that the TT solution (*a*=−1/2) may be "gauge equivalent" to the Schnabl solution and give an alternative approach to investigating the nonperturbative vacuum.

[cf. Drukker(2003), Zeze(2004), Igarashi-Itoh-Katsumata-Takahashi-Zeze(2005)]

- Values of gauge invariant overlap slowly approach +1 (or -1(?)) for nontrivial solutions. Appropriate extrapolation with respect to the truncation level ?

- *Regular* solutions? *Definition* of the space of string fields?

- Extension to superstring field theory?