An Algebraic Model for the su(2|2) Light-Cone String Field Theory

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21世紀の弦の場の理論の進展

- 開弦の場の理論

  - 特にタキオン凝縮に応用され進展中...
    [QFT02,05,06,07,08,09, FPA-QFT02,05]

- 閉弦の場の理論

  - Non-polynomial型：Heterotic版...

  - cubic型：冪等方程式, OSp covariantized版...
    [QFT03,04]

  - light-cone gauge: BMN対応, LCSFT/MST対応...
    [FPA-QFT06]

LCSFT on pp-wave

- AdS/CFT対応の観点で応用された：

  - AdS$_5$×S^5 のPenrose limit: pp-wave時空

  - BMN(Berenstein-Maldacena-Nastase)対応

  4次元N=4 SU(N) SYMのalmost BPS operator

  pp-wave上の超弦理論のstring state

  弦の相互作用を含めて調べる

Spradlin-Volovich, Pankiewicz-Stefanski(2002), Pankiewicz(2003)による
光円錐ゲージの弦の場の理論(LCSFT)の振動子表示の定式化が使われた

Toward su(2|2) LCSFT

- flat, pp-wave以外のより一般の背景でのSFTの構成を直接、具体的に振動子レベルでやるのは難しい。

- 代わりにflat, pp-waveの例からSFTのbuilding blockを抜き出し超対称代数で形を決めよう。→「模型」の提案

- pp-wave背景の対称性はsu(2|2)を部分代数として含んでいる。

- spin chain模型でsu(2|2)代数が重要な役割を果たした。

↓

su(2|2)対称性をもつ背景上のLCSFTの模型

pp-wave上のSFTをより簡潔に再現、
一般化に向けた計算例

Contents

- Introduction ✔

- Review of flat LCSFT

- Algebraic model for su(2|2) LCSFT

  - Building blocks, ansatz, assumptions

  - Solutions

- Summary and discussion

Green-Schwarz形式の光円錐ゲージ

- Green-Schwarz形式光円錐ゲージの弦の座標、運動量

$$x^i(\sigma), \vartheta^a(\sigma) \qquad p^i(\sigma), \lambda^a(\sigma) \qquad :\text{モード展開される}$$
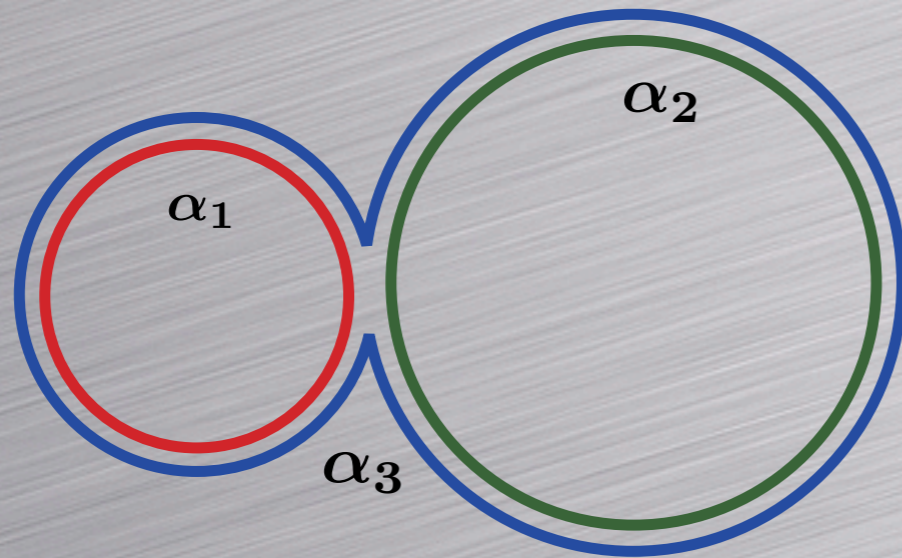
- 超電荷とハミルトニアン（free part）

$$q^{\dot{a}} = \frac{\sqrt{2}}{\alpha} \sum_{n=-\infty}^{\infty} \gamma_{a\dot{a}}^i Q_{-n}^a \alpha_n^i,$$

$$\widetilde{q}^{\dot{a}} = \frac{\sqrt{2}}{\alpha} \sum_{n=-\infty}^{\infty} \gamma_{a\dot{a}}^i \widetilde{Q}_{-n}^a \widetilde{\alpha}_n^i,$$

$$h = \frac{1}{\alpha} p^i p^i + \frac{1}{\alpha} \sum_{n=1}^{\infty} \left(\alpha_{-n}^i \alpha_n^i + \widetilde{\alpha}_{-n}^i \widetilde{\alpha}_n^i + \frac{n}{\alpha} (Q_{-n}^a Q_n^a + \widetilde{Q}_{-n}^a \widetilde{Q}_n^a) \right).$$

- $i, a, \dot{a}$ はSO(8)の**8ᵥ, 8ₛ, 8ᴄ**

Kinematical overlap

α_1 α_2 α_3

:光円錐型の３弦相互作用

３弦の接続条件を表すδ-汎関数:

$$\delta(\alpha_1 + \alpha_2 + \alpha_3)\delta^8(x^{i(3)} - \Theta_1 x^{i(1)} - \Theta_2 x^{i(2)})\delta^8(\vartheta^{i(3)} - \Theta_1 \vartheta^{i(1)} - \Theta_2 \vartheta^{i(2)})$$
$$= \langle \alpha_1, x^{(1)}, \vartheta^{(1)}| \langle \alpha_2, x^{(2)}, \vartheta^{(2)}| \langle \alpha_3, x^{(3)}, \vartheta^{(3)}|V \rangle$$

振動子を用いてケット $|V\rangle$ で表現できる

Algebra and prefactors

- SUSY代数： $\{Q^{\dot{a}}, Q^{\dot{b}}\} = \{\widetilde{Q}^{\dot{a}}, \widetilde{Q}^{\dot{b}}\} = 2H\delta^{\dot{a}\dot{b}}, \ \{Q^{\dot{a}}, \widetilde{Q}^{\dot{b}}\} = 0$

- 非自明な最低次（３弦相互作用項）の部分：

$$q^{\dot{a}}|Q^{\dot{b}}\rangle + q^{\dot{b}}|Q^{\dot{a}}\rangle = \widetilde{q}^{\dot{a}}|\widetilde{Q}^{\dot{b}}\rangle + \widetilde{q}^{\dot{b}}|\widetilde{Q}^{\dot{a}}\rangle = 2\delta^{\dot{a}\dot{b}}|H\rangle, \ q^{\dot{a}}|\widetilde{Q}^{\dot{b}}\rangle + \widetilde{q}^{\dot{b}}|Q^{\dot{a}}\rangle = 0$$

up to level matching conditionで満たす解

Green-Schwarz-Brinkの公式（の簡潔版 [I.K.-Moriyama(2006)]）

$$|H\rangle = X^i \widetilde{X}^j \left[\cosh \slashed{Y}\right]^{ij} |V\rangle,$$

$$|Q^{\dot{a}}\rangle = \sqrt{-\alpha_{123}} \widetilde{X}^i \left[\sinh \slashed{Y}\right]^{\dot{a}i} |V\rangle,$$

$$|\widetilde{Q}^{\dot{a}}\rangle = i\sqrt{-\alpha_{123}} X^i \left[\sinh \slashed{Y}\right]^{i\dot{a}} |V\rangle$$

$$\slashed{Y} = \sqrt{\tfrac{2}{-\alpha_{123}}} \eta^* Y^a \hat{\gamma}^a \qquad \hat{\gamma}^a = \begin{pmatrix} 0 & \hat{\gamma}^a_{i\dot{a}} \\ \hat{\gamma}^a_{\dot{a}i} & 0 \end{pmatrix} = \begin{pmatrix} 0 & \gamma^i_{a\dot{a}} \\ \gamma^i_{a\dot{a}} & 0 \end{pmatrix}$$

Building blocks (1)

- ３弦相互作用項の構成要素: $|V\rangle \quad Y^a, X^i, \widetilde{X}^i$

それぞれ振動子とノイマン係数を用いてあらわに表現されている：

$$|V\rangle = (2\pi)^9 \delta(\alpha_1 + \alpha_2 + \alpha_3)\delta^8(p_1^i + p_2^i + p_3^i)\delta^8(\lambda_1^a + \lambda_2^a + \lambda_3^a)$$

$$\times e^{\frac{1}{2}\sum \bar{N}_{nm}^{rs}(\alpha_{-n}^{(r)}\alpha_{-m}^{(s)} + \tilde{\alpha}_{-n}^{(r)}\tilde{\alpha}_{-m}^{(s)}) + \sum \bar{N}_n^r(\alpha_{-n}^{(r)} + \tilde{\alpha}_{-n}^{(r)})P - \frac{\tau_0}{\alpha_{123}}P^2}$$

$$\times e^{\sum Q_{-n}^{\mathrm{II}(r)}\alpha_r^{-1}n\bar{N}_{nm}^{rs}Q_{-m}^{\mathrm{I}(s)} - \sqrt{2}\Lambda \sum \alpha_r^{-1}n\bar{N}_n^r Q_{-n}^{\mathrm{II}(r)}}|0\rangle$$

$$P^i = \alpha_1 p_2^i - \alpha_2 p_1^i, \ \Lambda^a = \alpha_1 \lambda_2^a - \alpha_2 \lambda^a, \ Q_{-n}^{\mathrm{I/II}a} = \frac{1}{\sqrt{2}}(\eta^{\pm 1}Q_{-n}^a + \eta^{\mp 1}\widetilde{Q}_{-n}^a) \qquad \alpha_{123} \equiv \alpha_1\alpha_2\alpha_3$$

$$\bar{N}_{nm}^{rs} = -\frac{\alpha_{123}}{\alpha_r/n + \alpha_s/m}\bar{N}_n^r\bar{N}_m^s, \ \ \bar{N}_n^r = \frac{\Gamma(-n\alpha_{r+1}/\alpha_r)e^{n\tau_0/\alpha_r}}{\alpha_r n!\Gamma(1 - n(1 + \alpha_{r+1}/\alpha_r))}, \ (\alpha_4 \equiv \alpha_1), \ \ \tau_0 = \sum_{r=1}^{3}\alpha_r \log|\alpha_r|$$

$$Y^a = \Lambda^a - \frac{\alpha_{123}}{2}\sum_{r=1}^{3}\sum_{n=1}^{\infty}\alpha_r^{-1}n\bar{N}_n^r(\eta Q_{-n}^{(r)a} + \eta^*\widetilde{Q}_{-n}^{(r)a})$$

$$X^i = \mathrm{P}^i - \alpha_{123}\sum_{r=1}^{3}\sum_{n=1}^{\infty}\alpha_r^{-1}n\bar{N}_n^r\alpha_{-n}^{i(r)}, \quad \widetilde{X}^i = \mathrm{P}^i - \alpha_{123}\sum_{r=1}^{3}\sum_{n=1}^{\infty}\alpha_r^{-1}n\bar{N}_n^r\tilde{\alpha}_{-n}^{i(r)}.$$

Building blocks (2)

- ３弦相互作用項の構成要素: $|V\rangle \quad Y^a, X^i, \widetilde{X}^i$

実は... $\qquad \lambda^a(\sigma)|V\rangle \sim \frac{1}{4\pi|\alpha_{123}|^{1/2}} \frac{1}{|\sigma-\sigma_{\text{int}}|^{1/2}} Y^a|V\rangle$

$$\{q^{\dot{a}}, Y^a\} = \tfrac{1}{\sqrt{2}}\eta\gamma^i_{a\dot{a}}X^i, \qquad \{\widetilde{q}^{\dot{a}}, Y^a\} = \tfrac{1}{\sqrt{2}}\eta^*\gamma^i_{a\dot{a}}\widetilde{X}^i$$

またハミルトニアンには $\quad [q, X] \sim \partial Y, \ [\widetilde{q}, \widetilde{X}] \sim \overline{\partial}Y$

を用いた不定性も入りうる: [Lee-Russo(2004), Dobashi-I.K.-Moriyama(unpublished)]

$$|Q^{\dot{a}}\rangle = \left(\widetilde{f}_0\widetilde{X}^i[\sinh Y]^{i\dot{a}} + X^i\left[f_1 Y + \frac{1}{7!}f_7 Y^7\right]^{i\dot{a}}\right)|V\rangle = \left(\widetilde{f}_0\widetilde{X}^i[\sinh Y]^{i\dot{a}} + q^{\dot{a}}i\sqrt{-\alpha_{123}}(f_1 + f_7 y_0^8\delta^8(Y))\right)|V\rangle$$

$$|\widetilde{Q}^{\dot{a}}\rangle = \left(g_0[\sinh Y]_{\dot{a}i}X^i + \left[\widetilde{g}_1 Y + \frac{1}{7!}\widetilde{g}_7 Y^7\right]^{\dot{a}i}\widetilde{X}^i\right)|V\rangle = \left(g_0[\sinh Y]^{\dot{a}i}X^i + \widetilde{q}^{\dot{a}}\sqrt{-\alpha_{123}}(\widetilde{g}_1 + \widetilde{g}_7 y_0^8\delta^8(Y))\right)|V\rangle$$

$$|H\rangle = \left(\frac{1}{\sqrt{-\alpha_{123}}}g_0\widetilde{X}^i X^j[\cosh Y]^{ij} + h\sqrt{-\alpha_{123}}(\widetilde{g}_1 + y_0^8\widetilde{g}_7\delta^8(Y))\right)|V\rangle$$

Algebraic model

ここで扱う超代数： $\mathfrak{psu}(2|2) \ltimes \mathbb{R}^3$

$$
\begin{aligned}
\{\mathcal{Q}^\alpha{}_a, \mathcal{Q}^\beta{}_b\} &= \epsilon^{\alpha\beta}\epsilon_{ab}\mathcal{H}, \\
\{\widetilde{\mathcal{Q}}^\alpha{}_a, \widetilde{\mathcal{Q}}^\beta{}_b\} &= \epsilon^{\alpha\beta}\epsilon_{ab}\widetilde{\mathcal{H}}, \\
\{\mathcal{Q}^\alpha{}_a, \widetilde{\mathcal{Q}}^\beta{}_b\} &= \epsilon^{\alpha\beta}\epsilon_{ac}\mathcal{R}^c{}_b + \epsilon_{ab}\mathcal{L}^\alpha{}_\gamma\epsilon^{\gamma\beta} + \epsilon^{\alpha\beta}\epsilon_{ab}\mathcal{N}.
\end{aligned}
$$

$\mathcal{J} = j + g_s|J\rangle + \cdots$ のように展開しfree部分は知っているとし、

$$
\begin{aligned}
q^\alpha{}_a|Q^\beta{}_b\rangle + q^\beta{}_b|Q^\alpha{}_a\rangle &= \epsilon^{\alpha\beta}\epsilon_{ab}|H\rangle, \\
\widetilde{q}^\alpha{}_a|\widetilde{Q}^\beta{}_b\rangle + \widetilde{q}^\beta{}_b|\widetilde{Q}^\alpha{}_a\rangle &= \epsilon^{\alpha\beta}\epsilon_{ab}|\widetilde{H}\rangle, \\
q^\alpha{}_a|\widetilde{Q}^\beta{}_b\rangle + \widetilde{q}^\beta{}_b|Q^\alpha{}_a\rangle &= \epsilon^{\alpha\beta}\epsilon_{ab}|N\rangle.
\end{aligned}
$$

を満たす「３弦相互作用項」を構成しよう！

$\mathcal{L}^\alpha{}_\beta = l^\alpha{}_\beta, \quad \mathcal{R}^a{}_b = r^a{}_b$ はkinematicalだと仮定している。

Building blocks

- ３弦相互作用項　　$|H\rangle,\ |\widetilde{H}\rangle,\ |N\rangle,\ |Q^\alpha{}_a\rangle,\ |\widetilde{Q}^\alpha{}_a\rangle$

を構成するもの（仮定）

$$|V\rangle : \text{kinematical overlap} \sim \text{弦の接続条件を表す}\delta\text{-汎函数}$$

$$Y^\alpha{}_{\dot{b}},\, Y'^a{}_{\dot{\beta}},\, X^a{}_{\dot{b}},\, \widetilde{X}^a{}_{\dot{b}},\, X'^\alpha{}_{\dot{\beta}},\, \widetilde{X}'^\alpha{}_{\dot{\beta}},\, W^\alpha{}_{\dot{b}},\, \widetilde{W}^\alpha{}_{\dot{b}},\, W'^a{}_{\dot{\beta}},\, \widetilde{W}'^a{}_{\dot{\beta}}$$

$$: \text{prefactor}\text{の構成要素} \quad (\alpha, \dot{\alpha}, a, \dot{a}, \cdots = 1, 2)$$

$Y^\alpha{}_{\dot{b}},\, Y'^a{}_{\dot{\beta}}$　$\sim$fermion運動量を$|V\rangle$ にかけ相互作用点に近づけ発散を取り除いた部分
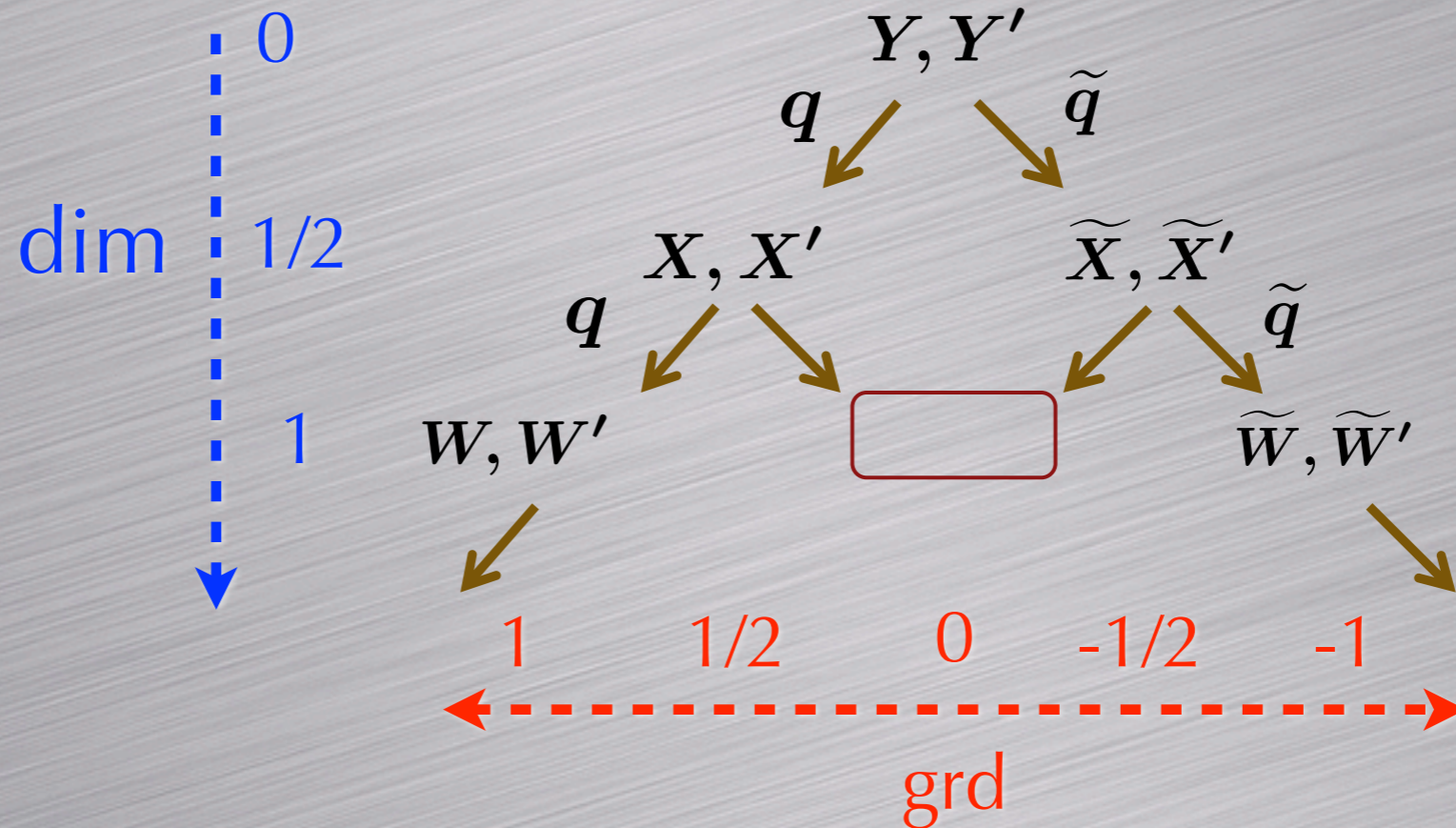
交換関係:

$$\{q^\alpha{}_a, Y^\beta{}_{\dot{b}}\} = -\epsilon^{\beta\alpha}(\epsilon X)_{a\dot{b}}, \qquad \{q^\alpha{}_a, Y'^b{}_{\dot{\beta}}\} = \delta^b_a X'^\alpha{}_{\dot{\beta}}, \qquad \sim X, X'\text{の定義}$$

$$[q^\alpha{}_a, X^b{}_{\dot{b}}] = \delta^b_a W^\alpha{}_{\dot{b}}, \qquad\qquad [q^\alpha{}_a, X'^\beta{}_{\dot{\beta}}] = -\epsilon^{\beta\alpha}(\epsilon W')_{a\dot{\beta}}. \qquad \sim W, W'\text{の定義}$$

$\widetilde{q}^\alpha{}_a$ との交換関係も同様

※さらに $q^\alpha{}_a|V\rangle,\ [q^\alpha{}_a, \widetilde{X}^b{}_{\dot{b}}],\ [q^\alpha{}_a, \widetilde{X}'^\beta{}_{\dot{\beta}}]$ の形を決めておく必要がある！

Dimension and grade

- dimとgrdをassignする:

- consistency

$$\{[q^{\alpha}{}_{a}, \widetilde{X}^{c}{}_{\dot{c}}], q^{\beta}{}_{b}\} + \{q^{\alpha}{}_{a}, [q^{\beta}{}_{b}, \widetilde{X}^{c}{}_{\dot{c}}]\} = \epsilon^{\alpha\beta}\epsilon_{ab}[h, \widetilde{X}^{c}{}_{\dot{c}}],$$

$$\{q^{\alpha}{}_{a}, q^{\beta}{}_{b}\}|V\rangle = \epsilon^{\alpha\beta}\epsilon_{ab}h|V\rangle, \quad \{q^{\alpha}{}_{a}, \widetilde{q}^{\beta}{}_{b}\}|V\rangle = \epsilon^{\alpha\beta}\epsilon_{ab}n|V\rangle, \quad$$ 等は満たすように決める

$$r^{a}{}_{b}|V\rangle = l^{\alpha}{}_{\beta}|V\rangle = 0 \quad$$ は仮定

Ansatz

$$q^\alpha{}_a|V\rangle \;=\; \sum_{n,m}\Big\{v_{nm}(Y^nXY'^m)^\alpha{}_a + v'_{mn}(Y^mX'Y'^n)^\alpha{}_a\Big\}|V\rangle$$

up to dim 1/2,
grd=1/2 (12項)

$[q^\alpha{}_a, \tilde{X}^b{}_{\dot b}] = v_1\delta_a^b Y^\alpha{}_{\dot b} + v_2\delta_a^b(Y^3)^\alpha{}_{\dot b} + v_3 Y^\alpha{}_{\dot b}(Y'^2)^b{}_a + v_4\delta_a^b Y^\alpha{}_{\dot b}(Y'^4)^c{}_c + v_5(Y^3)^\alpha{}_{\dot b}(Y'^2)^b{}_a + v_6\delta_a^b(Y^3)^\alpha{}_{\dot b}(Y'^4)^c{}_c + v_7 Y^\alpha{}_{\dot b}(\tilde{X}X)^b{}_a$

$+v_8 Y^\alpha{}_{\dot b}(X\tilde{X})^b{}_a + v_9(YX\epsilon)^{\alpha b}(\epsilon\tilde{X})_{a\dot b} + v_{10}(YX)^\alpha{}_a\tilde{X}^b{}_{\dot b} + v_{11}(Y\tilde{X}\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{12}(Y\tilde{X})^\alpha{}_a X^b{}_{\dot b} + v_{13}(Y^3)^\alpha{}_{\dot b}(\tilde{X}X)^b{}_a$

$+v_{14}(Y^3)^\alpha{}_{\dot b}(X\tilde{X})^b{}_a + v_{15}(Y^3X\epsilon)^{\alpha b}(\epsilon\tilde{X})_{a\dot b} + v_{16}(Y^3X)^\alpha{}_a\tilde{X}^b{}_{\dot b} + v_{17}(Y^3\tilde{X}\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{18}(Y^3\tilde{X})^\alpha{}_a X^b{}_{\dot b}$

$+v_{19}Y^\alpha{}_{\dot b}(Y'^2)^b{}_a(X\tilde{X})^c{}_c + v_{20}Y^\alpha{}_{\dot b}(\tilde{X}XY'^2)^b{}_a + v_{21}Y^\alpha{}_{\dot b}(Y'^2X\tilde{X})^b{}_a + v_{22}Y^\alpha{}_{\dot b}(X\tilde{X}Y'^2)^b{}_a + v_{23}Y^\alpha{}_{\dot b}(Y'^2\tilde{X}X)^b{}_a$

$+v_{24}(YX\tilde{X})^\alpha{}_{\dot b}(Y'^2)^b{}_a + v_{25}(YXY'^2)^\alpha{}_a\tilde{X}^b{}_{\dot b} + v_{26}(YXY'^2\epsilon)^{\alpha b}(\epsilon\tilde{X})_{a\dot b} + v_{27}(Y\tilde{X}X)^\alpha{}_{\dot b}(Y'^2)^b{}_a + v_{28}(Y\tilde{X}Y'^2)^\alpha{}_a X^b{}_{\dot b}$

$+v_{29}(Y\tilde{X}Y'^2\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{30}(YX)^\alpha{}_a(Y'^2\tilde{X})^b{}_{\dot b} + v_{31}(YX\epsilon)^{\alpha b}(\epsilon Y'^2\tilde{X})_{a\dot b} + v_{32}(Y\tilde{X})^\alpha{}_a(Y'^2X)^b{}_{\dot b} + v_{33}(Y\tilde{X}\epsilon)^{\alpha b}(\epsilon Y'^2 X)_{a\dot b}$

$+v_{34}Y^\alpha{}_a(\tilde{X}X)^b{}_a(Y'^4)^c{}_c + v_{35}Y^\alpha{}_{\dot b}(X\tilde{X})^b{}_a(Y'^4)^c{}_c + v_{36}(YX\epsilon)^{\alpha b}(\epsilon\tilde{X})_{a\dot b}(Y'^4)^c{}_c + v_{37}(YX)^\alpha{}_a\tilde{X}^b{}_{\dot b}(Y'^4)^c{}_c$

$+v_{38}(Y\tilde{X}\epsilon)^{\alpha b}(\epsilon X)_{a\dot b}(Y'^4)^c{}_c + v_{39}(Y\tilde{X})^\alpha{}_a X^b{}_{\dot b}(Y'^4)^c{}_c + v_{40}(Y^3)^\alpha{}_{\dot b}(Y'^2)^b{}_a(X\tilde{X})^c{}_c + v_{41}(Y^3)^\alpha{}_{\dot b}(\tilde{X}XY'^2)^b{}_a$

$+v_{42}(Y^3)^\alpha{}_{\dot b}(Y'^2X\tilde{X})^b{}_a + v_{43}(Y^3)^\alpha{}_{\dot b}(X\tilde{X}Y'^2)^b{}_a + v_{44}(Y^3)^\alpha{}_{\dot b}(Y'^2\tilde{X}X)^b{}_a + v_{45}(Y^3X\tilde{X})^\alpha{}_{\dot b}(Y'^2)^b{}_a + v_{46}(Y^3XY'^2)^\alpha{}_a\tilde{X}^b{}_{\dot b}$

$+v_{47}(Y^3XY'^2\epsilon)^{\alpha b}(\epsilon\tilde{X})_{a\dot b} + v_{48}(Y^3\tilde{X}X)^\alpha{}_{\dot b}(Y'^2)^b{}_a + v_{49}(Y^3\tilde{X}Y'^2)^\alpha{}_a X^b{}_{\dot b} + v_{50}(Y^3\tilde{X}Y'^2\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{51}(Y^3X)^\alpha{}_a(Y'^2\tilde{X})^b{}_{\dot b}$

$+v_{52}(Y^3X\epsilon)^{\alpha b}(\epsilon Y'^2\tilde{X})_{a\dot b} + v_{53}(Y^3\tilde{X})^\alpha{}_a(Y'^2X)^b{}_{\dot b} + v_{54}(Y^3\tilde{X}\epsilon)^{\alpha b}(\epsilon Y'^2 X)_{a\dot b} + v_{55}(Y^3)^\alpha{}_{\dot b}(\tilde{X}X)^b{}_a(Y'^4)^c{}_c$

$+v_{56}(Y^3)^\alpha{}_{\dot b}(X\tilde{X})^b{}_a(Y'^4)^c{}_c + v_{57}(Y^3X\epsilon)^{\alpha b}(\epsilon\tilde{X})_{a\dot b}(Y'^4)^c{}_c + v_{58}(Y^3X)^\alpha{}_a\tilde{X}^b{}_{\dot b}(Y'^4)^c{}_c + v_{59}(Y^3\tilde{X}\epsilon)^{\alpha b}(\epsilon X)_{a\dot b}(Y'^4)^c{}_c$

$+v_{60}(Y^3\tilde{X})^\alpha{}_a X^b{}_{\dot b}(Y'^4)^c{}_c + v_{61}Y^\alpha{}_{\dot b}\delta_a^b(X'\tilde{X}')^\beta{}_\beta + v_{62}(X'\tilde{X}'Y)^\alpha{}_{\dot b}\delta_a^b + v_{63}(\tilde{X}'X'Y)^\alpha{}_{\dot b}\delta_a^b + v_{64}(Y^3)^\alpha{}_{\dot b}\delta_a^b(X'\tilde{X}')^\beta{}_\beta$

$+v_{65}(X'\tilde{X}'Y^3)^\alpha{}_{\dot b}\delta_a^b + v_{66}(\tilde{X}'X'Y^3)^\alpha{}_{\dot b}\delta_a^b + v_{67}Y^\alpha{}_{\dot b}(Y'^2)^b{}_a(X'\tilde{X}')^\beta{}_\beta + v_{68}Y^\alpha{}_{\dot b}\delta_a^b(X'\tilde{X}')^\beta{}_\beta(Y'^4)^c{}_c + v_{69}(X'\tilde{X}'Y)^\alpha{}_{\dot b}\delta_a^b(Y'^4)^c{}_c$

$+v_{70}(\tilde{X}'X'Y)^\alpha{}_{\dot b}\delta_a^b(Y'^4)^c{}_c + v_{71}(Y^3)^\alpha{}_{\dot b}(Y'^2)^b{}_a(X'\tilde{X}')^\beta{}_\beta + v_{72}(Y^3)^\alpha{}_{\dot b}\delta_a^b(X'\tilde{X}')^\beta{}_\beta(Y'^4)^c{}_c + v_{73}(X'\tilde{X}'Y^3)^\alpha{}_{\dot b}\delta_a^b(Y'^4)^c{}_c$

$+v_{74}(\tilde{X}'X'Y^3)^\alpha{}_{\dot b}\delta_a^b(Y'^4)^c{}_c + v_{75}(\tilde{X}'Y'\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{76}(\tilde{X}'Y')^\alpha{}_a X^b{}_{\dot b} + v_{77}(X'Y'\epsilon)^{\alpha b}(\epsilon\tilde{X})_{a\dot b} + v_{78}(X'Y')^\alpha{}_a\tilde{X}^b{}_{\dot b}$

$+v_{79}(\tilde{X}'Y'^3\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{80}(\tilde{X}'Y'^3)^\alpha{}_a X^b{}_{\dot b} + v_{81}(X'Y'^3\epsilon)^{\alpha b}(\epsilon\tilde{X})_{a\dot b} + v_{82}(X'Y'^3)^\alpha{}_a\tilde{X}^b{}_{\dot b} + v_{83}(\tilde{X}'Y'\epsilon)^{\alpha b}(\epsilon X)_{a\dot b}(Y^4)^\beta{}_\beta$

$+v_{84}(\tilde{X}'Y')^\alpha{}_a X^b{}_{\dot b}(Y^4)^\beta{}_\beta + v_{85}(X'Y'\epsilon)^{\alpha b}(\epsilon\tilde{X})_{a\dot b}(Y^4)^\beta{}_\beta + v_{86}(X'Y')^\alpha{}_a\tilde{X}^b{}_{\dot b}(Y^4)^\beta{}_\beta + v_{87}(\tilde{X}'Y'^3\epsilon)^{\alpha b}(\epsilon X)_{a\dot b}(Y^4)^\beta{}_\beta$

$+v_{88}(\tilde{X}'Y'^3)^\alpha{}_a X^b{}_{\dot b}(Y^4)^\beta{}_\beta + v_{89}(X'Y'^3\epsilon)^{\alpha b}(\epsilon\tilde{X})_{a\dot b}(Y^4)^\beta{}_\beta + v_{90}(X'Y'^3)^\alpha{}_a\tilde{X}^b{}_{\dot b}(Y^4)^\beta{}_\beta + v_{91}(Y^2\tilde{X}'Y'\epsilon)^{\alpha b}(\epsilon X)_{a\dot b}$

$+v_{92}(Y^2\tilde{X}'Y')^\alpha{}_a X^b{}_{\dot b} + v_{93}(\tilde{X}'Y'\epsilon)^{\alpha b}(\epsilon XY^2)_{a\dot b} + v_{94}(\tilde{X}'Y')^\alpha{}_a(XY^2)^b{}_{\dot b} + v_{95}(Y^2X'Y'\epsilon)^{\alpha b}(\epsilon\tilde{X})_{a\dot b} + v_{96}(Y^2X'Y')^\alpha{}_a\tilde{X}^b{}_{\dot b}$

$+v_{97}(X'Y'\epsilon)^{\alpha b}(\epsilon\tilde{X}Y^2)_{a\dot b} + v_{98}(X'Y')^\alpha{}_a(\tilde{X}Y^2)^b{}_{\dot b} + v_{99}(Y^2\tilde{X}'Y'^3\epsilon)^{\alpha b}(\epsilon X)_{a\dot b} + v_{100}(Y^2\tilde{X}'Y'^3)^\alpha{}_a X^b{}_{\dot b}$

$+v_{101}(\tilde{X}'Y'^3\epsilon)^{\alpha b}(\epsilon XY^2)_{a\dot b} + v_{102}(\tilde{X}'Y'^3)^\alpha{}_a(XY^2)^b{}_{\dot b} + v_{103}(Y^2X'Y'^3\epsilon)^{\alpha b}(\epsilon\tilde{X})_{a\dot b} + v_{104}(Y^2X'Y'^3)^\alpha{}_a\tilde{X}^b{}_{\dot b}$

$+v_{105}(X'Y'^3\epsilon)^{\alpha b}(\epsilon\tilde{X}Y^2)_{a\dot b} + v_{106}(X'Y'^3)^\alpha{}_a(\tilde{X}Y^2)^b{}_{\dot b}$

up to dim 1,
grd=0 (106項)

$[q^\alpha{}_a, \tilde{X}'^\beta{}_{\dot\beta}]$ も同様に106項

※consistencyから係数に関係がつく

Linearized version

- Y,Y'について線形な場合： $q^{\alpha}{}_{a}|V\rangle = \dfrac{i}{2}[(YX)^{\alpha}{}_{a} + (X'Y')^{\alpha}{}_{a}]|V\rangle$

$$[q^{\alpha}{}_{a}, \widetilde{X}^{b}{}_{b}] = \dfrac{i}{2}\delta^{b}_{a}Y^{\alpha}{}_{b}, \quad [q^{\alpha}{}_{a}, \widetilde{X}'^{\beta}{}_{\dot{\beta}}] = \dfrac{i}{2}\epsilon^{\beta\alpha}(\epsilon Y')_{a\dot{\beta}}$$

- このとき$su(2|2)$代数を満たすもの：

$$|Q^{\alpha}{}_{a}\rangle = \dfrac{1}{2}q_{1}\Big\{\eta^{*}\big[(\sinh\overline{Y})\widetilde{X}(\cosh\overline{Y}')\big]^{\alpha}{}_{a} + \eta\big[(\cosh\overline{Y})\widetilde{X}'(\sinh\overline{Y}')\big]^{\alpha}{}_{a}\Big\}|V\rangle$$
$$+ q^{\alpha}{}_{a}|W\rangle$$

$$|H\rangle = \dfrac{1}{2}q_{1}\Big\{\dfrac{1}{12}\big[\mathrm{Tr}Y^{4} - \mathrm{Tr}Y'^{4}\big]$$
$$+ \mathrm{Tr}X\cosh\overline{Y}\widetilde{X}\cosh\overline{Y}' - i\mathrm{Tr}\sinh\overline{Y}\widetilde{X}\sinh\overline{Y}'X'$$
$$+ \mathrm{Tr}\cosh\overline{Y}\widetilde{X}'\cosh\overline{Y}'X' + i\mathrm{Tr}X\sinh\overline{Y}\widetilde{X}'\sinh\overline{Y}'\Big\}|V\rangle$$
$$+ h|W\rangle$$

$$\overline{Y} = Y\eta, \ \overline{Y}' = Y'\eta^{*}, \ (\eta \equiv e^{i\pi/4})$$

pp-waveの場合のLCSFTの３弦相互作用項

[Pankiewicz(2003)] + [Di Vecchia-Petersen-Petrini-Russo-Tanzini(2003)]
を再現している！

Toy model Ⅰ

仮定： $q^\alpha{}_a|V\rangle = \dfrac{i}{2}[(YX)^\alpha{}_a + (X'Y')^\alpha{}_a]|V\rangle + (v_{30}Y^3X + v_{34}Y^3XY'^4 + v'_{03}X'Y'^3 - v_{34}Y^4X'Y'^3)^\alpha{}_a|V\rangle$

$$[q^\alpha{}_a, \widetilde{X}^b{}_{\dot b}] = \frac{i}{2}\delta^b_a Y^\alpha{}_{\dot b}, \quad [q^\alpha{}_a, \widetilde{X}'^\beta{}_{\dot\beta}] = \frac{i}{2}\epsilon^{\beta\alpha}(\epsilon Y')_{a\dot\beta} \quad \longleftarrow \quad \text{pp-waveのときと同じ}$$

このときsu(2|2)代数を満たすもの：

$$|Q^\alpha{}_a\rangle = \frac{1}{2}q_1\Big\{\eta^*\big[(\sinh\overline{Y})\widetilde{X}(\cosh\overline{Y}' - (v'_{03}/2)Y'^4)\big]^\alpha{}_a + \eta\big[(\cosh\overline{Y} + (v_{30}/2)Y^4)\widetilde{X}'\sinh\overline{Y}'\big]^\alpha{}_a\Big\}|V\rangle$$
$$+q^\alpha{}_a|W\rangle$$

$$|H\rangle = \frac{1}{2}q_1\Big\{\frac{1}{12}\big[\text{Tr}\,Y^4(1 - (v'_{03}/4)\text{Tr}\,Y'^4) - \text{Tr}\,Y'^4(1 + (v_{30}/4)\text{Tr}\,Y^4)\big]$$
$$+\text{Tr}\,X(\cosh\overline{Y} - (v_{30}/2)Y^4)\widetilde{X}(\cosh\overline{Y}' - (v'_{03}/2)Y'^4) - i\text{Tr}\,\sinh\overline{Y}\widetilde{X}\sinh\overline{Y}'X'$$
$$+\text{Tr}\,(\cosh\overline{Y} + (v_{30}/2)Y^4)\widetilde{X}'(\cosh\overline{Y}' + (v'_{03}/2)Y'^4)X' + i\text{Tr}\,X\sinh\overline{Y}\widetilde{X}'\sinh\overline{Y}'$$
$$-(v_{34}/4)(\text{Tr}XY^4\widetilde{X}Y'^4 + \text{Tr}\,Y^4\widetilde{X}'Y'^4X')\Big\}|V\rangle$$
$$+h|W\rangle$$

Toy model II

仮定：　$q^{\alpha}{}_{a}|V\rangle = \dfrac{i}{2}[(YX)^{\alpha}{}_{a} + (X'Y')^{\alpha}{}_{a}]|V\rangle$　　⟵　pp-waveのときと同じ

$$[q^{\alpha}{}_{a}, \widetilde{X}^{\dot{b}}{}_{b}] = -\dfrac{i}{2}(Y\epsilon)^{\dot{b}\alpha}\epsilon_{ab} + y_{03}\big[(Y^{3}\epsilon)^{\dot{b}\alpha}\epsilon_{ab} + 2i(Y\epsilon)^{\dot{b}\alpha}(\epsilon X\widetilde{X})_{ab} + 4i(YX)^{\alpha}{}_{a}\widetilde{X}^{\dot{b}}{}_{b}\big],$$

$$[q^{\alpha}{}_{a}, \widetilde{X}'^{\beta}{}_{\dot{\beta}}] = \dfrac{i}{2}\epsilon^{\beta\alpha}(\epsilon Y')_{a\dot{\beta}} - y'_{30}\big[\epsilon^{\beta\alpha}(\epsilon Y'^{3})_{a\dot{\beta}} + 2i(\widetilde{X}'X'\epsilon)^{\beta\alpha}(\epsilon Y')_{a\dot{\beta}} + 4i(X'Y')^{\alpha}{}_{a}\widetilde{X}'^{\beta}{}_{\dot{\beta}}\big]$$

up to dim 1でconsistentに変形

このときup to dim 1でsu(2|2)代数を満たすものを求めた。

⟹　pp-waveからのハミルトニアンのズレ：

$$\dfrac{1}{2}q_{1}\Big[y_{03}\mathrm{Tr}Y^{4}\Big(1 - \dfrac{1}{48}\mathrm{Tr}Y'^{4}\Big) + y'_{30}\mathrm{Tr}Y'^{4}\Big(1 - \dfrac{1}{48}\mathrm{Tr}Y^{4}\Big)\Big]|V\rangle.$$

超電荷はpp-waveのときと同じ。

Summary and discussion

- su(2|2)対称性をもつ背景でのLCSFTの構成に向けた「模型」を提案。

- 従来知られているpp-waveの場合の形を簡潔に再現した。

- "SUGRA極限"は $\widetilde{X}^a{}_{\dot{a}} = X^a{}_{\dot{a}}, \ \widetilde{X}'^{\alpha}{}_{\dot{\alpha}} = X'^{\alpha}{}_{\dot{\alpha}} \quad W^{\alpha}{}_b = \widetilde{W}^{\alpha}{}_b = W'^a{}_{\dot{\beta}} = \widetilde{W}'^a{}_{\dot{\beta}} = 0$
  でとれてSUGRAの式と比較することにより係数を制限できる。

- LCSFTの3弦相互作用項をdim.に関する展開で系統的に構成できる。

- pp-waveから少し一般化した具体例(toy model I,II)の計算をした。

- pp-waveより一般の背景の場合の正当化？あるいは対応する背景は？
  (bubbling geometry?) また、高次の相互作用項は？

- AdS/CFT (BMN) 対応の一般化への応用？