Identity-Based Solutions in Open String Field Theory Revisited

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References

- I. K. and T. Takahashi, "Open string field theory around universal solutions," Prog.Theor.Phys. 108 (2002) 591
- S. Inatomi, I.K. and T. Takahashi, "Homotopy Operators and One-Loop Vacuum Energy at the Tachyon Vacuum," arXiv:1106.5314; (+ to appear...)

Plan

- Introduction
- Brief review of open SFT and developments in the 21st century
- Review of a class of identity-based solution
- Cohomology around the tachyon vacuum (2002)
- Homotopy operator
- On one loop vacuum energy
- Concluding remarks
- (Extension to superstring field theory)

Introduction

- String Field Theory (SFT):
  - An old-fashioned non-perturbative definition of string theory
  - A string field includes infinite fields of particles in the space-time
  - Applicable to "D-brane physics"

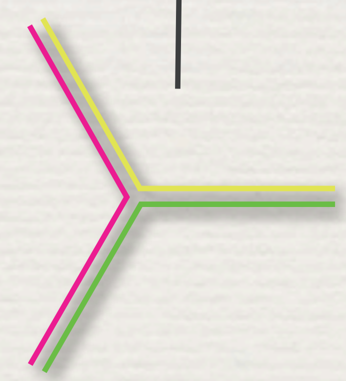
Cubic bosonic open SFT

- Action (Witten (1986)):

$$S[\Psi] = -\frac{1}{g^2}\left(\frac{1}{2}\langle\Psi, Q_{\rm B}\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right)$$

$$Q_{\rm B} = c_0 L_0 + b_0 M + \tilde{Q}$$

Kato-Ogawa's BRST operator (1983)



Midpoint interaction

- String field:

$$|\Psi\rangle = \phi(x) c_1|0\rangle + A_\mu(x)\alpha^\mu c_1|0\rangle + \chi(x) c_0|0\rangle + \cdots$$

tachyon

infinite component fields included

- Equation of motion:

$$Q_{\mathrm{B}}\Psi + \Psi * \Psi = 0$$

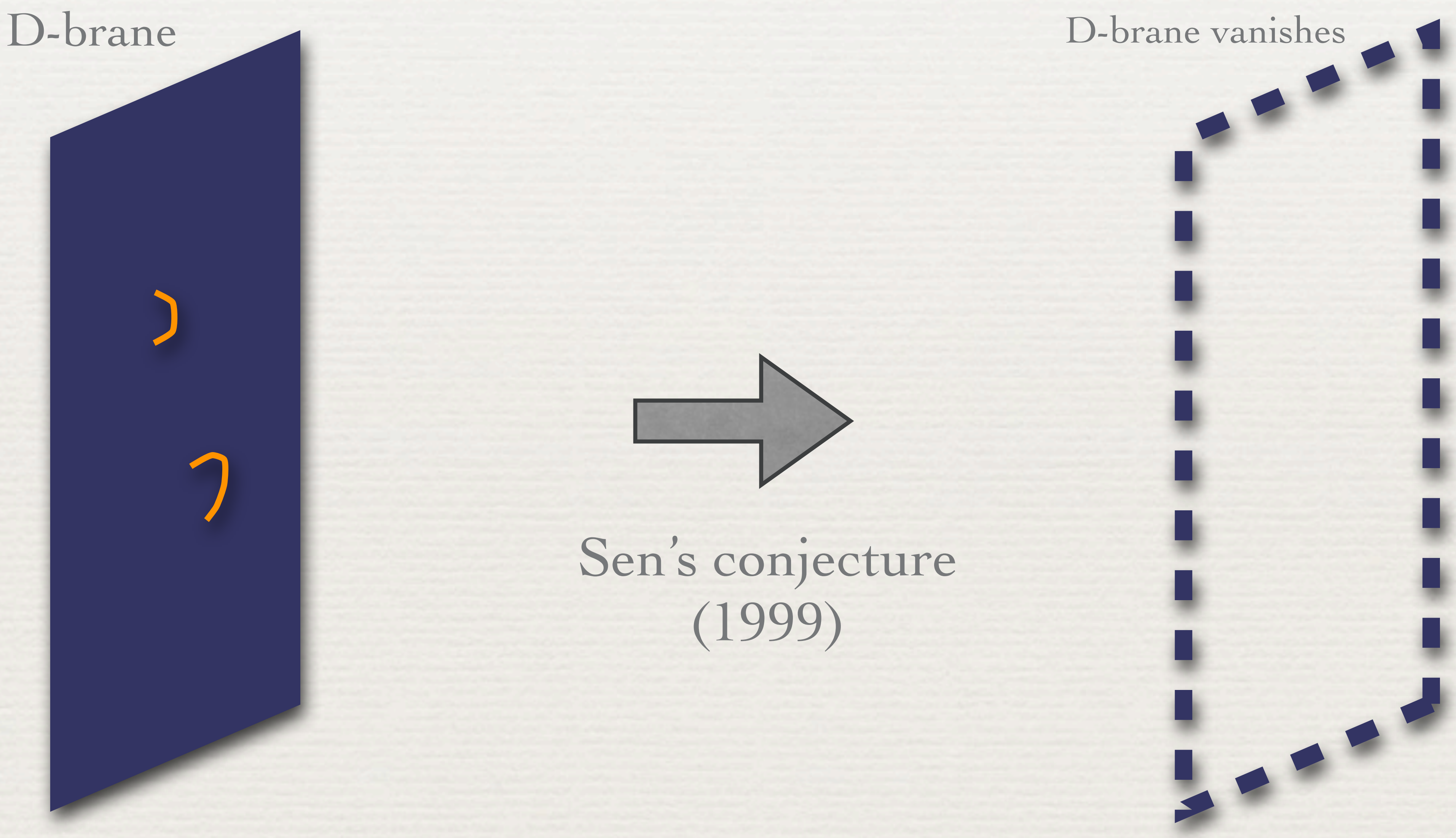
- Gauge transformation:

$$\delta\Psi = Q_{\mathrm{B}}\Lambda + \Psi * \Lambda - \Lambda * \Psi$$

Finite version:

$$\Psi' = e^{-\Lambda}\Psi e^{\Lambda} + e^{-\Lambda} Q_{\mathrm{B}} e^{\Lambda}$$

Non-perturbative vacuum



$\Psi = 0$

perturbative vacuum

$\Psi = \Psi_t$

$Q_{\mathrm{B}}\Psi_t + \Psi_t * \Psi_t = 0$

tachyon condensed vacuum

Solutions for tachyon condensation

- Numerical solution in Siegel gauge
  - Sen-Zwiebach (1999),Moeller-Taylor (2000), Gaiotto-Rastelli (2002), I.K.-Takahashi (2009, 2010) up to level 26
- Takahashi-Tanimoto's solution (2002-)
  - A class of identity-based solutions
- Schnabl's solution (2005-)
  - Based on "*KBc* algebra"

Brief review of TT solution

- Takahashi-Tanimoto's solution (2002)

$$\Psi_h = Q_L(e^h - 1)I - C_L((\partial h)^2 e^h)I$$

identity state (identity element of star product)

$$I * A = A * I = A, \quad \forall A$$

$$Q_L(f) = \int_{C_{\rm left}} \frac{dz}{2\pi i} f(z) j_{\rm B}(z), \quad C_L(f) = \int_{C_{\rm left}} \frac{dz}{2\pi i} f(z) c(z).$$

BRST current

$$h(z) = h(-1/z), \quad h(\pm i) = 0$$

- Theory around the solution:

$$S'[\Phi] = S[\Psi_h + \Phi] - S[\Psi_h]$$
$$= -\frac{1}{g^2}\left(\frac{1}{2}\langle\Phi, Q'\Phi\rangle + \frac{1}{3}\langle\Phi, \Phi * \Phi\rangle\right)$$

BRST operator at the TT solution:

$$Q' = Q(e^h) - C((\partial h)^2 e^h)$$

Explicit expression can be found!

$$Q(f) = \oint \frac{dz}{2\pi i} f(z) j_{\mathrm{B}}(z), \quad C(f) = \oint \frac{dz}{2\pi i} f(z) c(z).$$

- Examples of weighting function $\boldsymbol{h}$

$$h_a^l(z) = \log\left(1 - \frac{a}{2}(-1)^l\left(z^l - (-1)^l z^{-l}\right)^2\right)$$

$a \geq -1/2$: reality condition for the solution

$l = 1, 2, 3, \cdots$

- (Formal) similarity transformation:

$$Q' = e^{q(h_a^l)} Q_{\mathrm{B}} e^{-q(h_a^l)}$$

$$q(f) = \oint \frac{dz}{2\pi i} f(z) j_{\mathrm{gh}}(z)$$

ghost current

$$j_{\mathrm{gh}}(y) j_{\mathrm{gh}}(z) \sim \frac{1}{(y-z)^2}$$

$e^{\pm q(h_a^l)}$: ill-defined at $a = -1/2$

- Pure gauge in the case $a > -1/2$

$$Q' = e^{q(h_a^l)} Q_{\rm B} e^{-q(h_a^l)}$$ is well-defined

$$\simeq Q_{\rm B}$$ cohomologically the same as perturbative vacuum

The solution can be rewritten as a pure gauge form:

$$\Psi_{h_a^l} = \exp(q_L(h_a^l) I) Q_{\rm B} \exp(-q_L(h_a^l) I)$$

$$q_L(f) = \int_{C_{\rm left}} \frac{dz}{2\pi i} f(z) j_{\rm gh}(z)$$

Vanishing cohomology at $a = -1/2$

I.K.-Takahashi (2002)

- BRST operator at the solution:

$$Q' = \frac{1}{2}Q_{\mathrm{B}} + \frac{(-1)^l}{4}(Q_{2l} + Q_{-2l}) + 2l^2 c_0 - (-1)^l l^2 (c_{2l} + c_{-2l})$$

$$j_{\mathrm{B}}(z) = \sum_n Q_n z^{-n-1}$$

- Similarity transformation:

$$Q' = \frac{(-1)^l}{4} U_l Q_{\mathrm{B}}^{(2l)} U_l^{-1} \qquad U_l = \exp\left(-2\sum_{n=1}^{\infty}\frac{(-1)^{n(l+1)}}{n} q_{-2nl}\right)$$

$$j_{\mathrm{gh}}(z) = \sum_n q_n z^{-n-1}$$

$$Q_{\mathrm{B}}^{(2l)} = Q_{\mathrm{B}}|_{b_n \to b_{n-2l}, c_n \to c_{n+2l}} = Q_{2l} - 4l^2 c_{2l}$$

- Solving cohomology using Kato-Ogawa's result

$$Q'\psi = 0 \quad \Leftrightarrow$$

$$|\psi\rangle = |P\rangle \otimes U_l \, b_{-2l} b_{-2l+1} \cdots b_{-2} |0\rangle$$

$$+ |P'\rangle \otimes U_l \, b_{-2l+1} b_{-2l+2} \cdots b_{-2} |0\rangle + Q'|\phi\rangle$$

DDF states in the matter sector

Non-trivial states have ghost number $-2l+1, \; -2l+2$

⇕

In bosonic open SFT, string fields have ghost number 1.

No cohomology in the ghost number one sector.

No physical excitations around the solution.

D-brane vanishes at the solution.

Homotopy operator

Inatomi-I.K.-Takahashi(2011)

- OPE:

$$j_{\mathrm{B}}(y)b(z) \sim \frac{3}{(y-z)^3} + \frac{1}{(y-z)^2} j_{\mathrm{gh}}(z) + \frac{1}{y-z} T(z)$$

- Anti-commutation relation:

$$\{Q(f),\, b(z)\} = \frac{3}{2}\partial^2 f(z) + \partial f(z)\, j_{\mathrm{gh}}(z) + f(z)\, T(z)$$

These terms vanish at **second order zeros** of $f(z)$

⇒ The RHS becomes a c-number!

Note: $Q' = Q(e^h) - C((\partial h)^2 e^h)$ $\{C(f),\, b(z)\} = f(z)$

- Homotopy operator for $Q' \quad (a = -1/2)$

$$\{Q', \hat{A}\} = 1, \qquad \hat{A}^2 = 0$$

$$\hat{A} = \sum_{k=1}^{2l} a_k \, l^{-2} z_k^2 \, b(z_k), \qquad z_k^{2l} + (-1)^l = 0, \qquad \sum_{k=1}^{2l} a_k = 1$$

second order zeros of $\exp(h^l_{a=-\frac{1}{2}}(z))$

- No cohomology in ***all*** ghost number sectors:

$$Q'\psi = 0 \quad \Leftrightarrow \quad \psi = Q'(\hat{A}\psi)$$

Note: for Schnabl's solution, Ellwood-Schnabl (2006) found a homotopy operator.

Consistency ?

- What happens in the non-trivial sector found in 2002?

$$|\varphi\rangle = |P\rangle \otimes U_l\, b_{-2l} b_{-2l+1} \cdots b_{-2}|0\rangle + |P'\rangle \otimes U_l\, b_{-2l+1} b_{-2l+2} \cdots b_{-2}|0\rangle$$

$$b(z)\, U_l = \exp\left(-2 \sum_{n=1}^{\infty} \frac{(-1)^{n(l+1)}}{n} z^{-2nl}\right) U_l\, b(z)$$

$$\hat{A}\, U_l b_{-m} \cdots b_{-2}|0\rangle = \exp\left(-2 \sum_{n=1}^{\infty} \frac{1}{n}\right) U_l\, \hat{A} b_{-m} \cdots b_{-2}|0\rangle = 0$$

$$|\varphi\rangle = Q'(\hat{A}|\varphi\rangle)$$

zero (in a single Fock space)

Beyond a Fock state expression, the equality holds.
(Or associativity of operator product is broken.)

One-loop vacuum energy

- In the Siegel gauge in the theory around the solution, the inverse propagator: $L' = \{b_0, Q'\}$

$$V_{\text{1-loop}} = -\frac{1}{2}\log\det(L') = \int_0^\infty \frac{dt}{2t} Z(t) \qquad Z(t) = \mathrm{Tr}\left[(-1)^{N_{\mathrm{FP}}} e^{-tL'} b_0 c_0\right]$$

- Let us consider a variation of moduli such as interbrane distances:

$$Q' \to Q' + \delta Q'$$

Noting the relations:

$$\delta L' = \{\delta Q', b_0\} \qquad \{\delta Q', \hat{A}\} = 0 \qquad \{\hat{A}, b_0\} = 0$$

$$\Rightarrow \quad \delta Z(t) = 0$$

This does not hold for Schnabl's solution.

: consistent with vanishing D-branes

A proof of moduli-independence

$$
\begin{aligned}
\delta Z(t) =& -t\int_0^1 d\alpha\,\mathrm{Tr}\left[(-1)^{N_{\rm FP}}e^{-\alpha t L'}\{\delta Q',\,b_0\}e^{-(1-\alpha)tL'}b_0c_0\right]\\
=& -t\int_0^1 d\alpha\,\mathrm{Tr}\left[(-1)^{N_{\rm FP}}e^{-\alpha t L'}b_0\,\delta Q'\,e^{-(1-\alpha)tL'}b_0c_0\right]\\
=& t\int_0^1 d\alpha\,\mathrm{Tr}\left[(-1)^{N_{\rm FP}}e^{-\alpha t L'}\,\delta Q'\,e^{-(1-\alpha)tL'}b_0c_0b_0\right]\\
=& t\int_0^1 d\alpha\,\mathrm{Tr}\left[(-1)^{N_{\rm FP}}e^{-\alpha t L'}\,\delta Q'\,e^{-(1-\alpha)tL'}b_0\right]\\
=& t\int_0^1 d\alpha\,\mathrm{Tr}\left[(-1)^{N_{\rm FP}}e^{-\alpha t L'}\,\delta Q'\,e^{-(1-\alpha)tL'}\{Q',\,\hat{A}\}b_0\right]\\
=& t\int_0^1 d\alpha\left(\mathrm{Tr}\left[(-1)^{N_{\rm FP}}e^{-\alpha t L'}\,\delta Q'\,e^{-(1-\alpha)tL'}Q'\,\hat{A}\,b_0\right]+\mathrm{Tr}\left[(-1)^{N_{\rm FP}}e^{-\alpha t L'}\,\delta Q'\,e^{-(1-\alpha)tL'}\hat{A}\,Q'\,b_0\right]\right)\\
=& t\int_0^1 d\alpha\left(\mathrm{Tr}\left[(-1)^{N_{\rm FP}}e^{-\alpha t L'}\,\delta Q'\,e^{-(1-\alpha)tL'}Q'\,\hat{A}\,b_0\right]+\mathrm{Tr}\left[(-1)^{N_{\rm FP}}e^{-\alpha t L'}\,\delta Q'\,e^{-(1-\alpha)tL'}\,Q'\,b_0\hat{A}\right]\right)\\
=& 0
\end{aligned}
$$

Comments on higher order zeros

- Other solution associated with function with higher order zeros [Igarashi-Itoh-Katsumata-Takahashi-Zeze (2005)]

$$h_a^{(4)}(z) = \log\left(1 + 2a - \frac{a}{8}\left(z - z^{-1}\right)^4\right) \qquad (a \geq -1/2)$$

$$Q^{(4)} = Q(F_4) + C(G_4)$$
$$= \frac{3}{8}Q_{\mathrm{B}} - \frac{1}{4}(Q_2 + Q_{-2}) + \frac{1}{16}(Q_4 + Q_{-4}) + 2c_0 - c_4 - c_{-4},$$
$$F_4(z) = \frac{1}{16}\left(z - z^{-1}\right)^4, \quad G_4(z) = -z^{-2}(z^2 - z^{-2})^2$$
$$(a = -1/2)$$

- Homotopy operator for the solution with 4-th order zeros:

$$\{Q^{(4)}, \hat{A}^{(4)}\} = 1, \quad (\hat{A}^{(4)})^2 = 0$$

$$\hat{A}^{(4)} = \frac{1}{8}(\partial^2 b(1) + \partial^2 b(-1)) + \frac{5}{8}(\partial b(1) - \partial b(-1)) + \frac{1}{2}(b(1) + b(-1)) = \sum_{n=-\infty}^{\infty} n^2\, b_{2n}$$

On cohomology

- Non-trivial part exists in ghost number: $\mathbf{-3, -2}$

$$Q^{(4)}\psi = 0 \quad \Leftrightarrow$$

$$|\psi\rangle = |P\rangle \otimes U_{(4)}\, b_{-4}b_{-3}b_{-2}|0\rangle + |P'\rangle \otimes U_{(4)}\, b_{-3}b_{-2}|0\rangle + Q^{(4)}|\phi\rangle$$

$$U_{(4)} = \exp\left(-4\sum_{n=1}^{\infty}\frac{1}{n}q_{-2n}\right)$$

- Using the homotopy operator, cohomology vanishes, in a similar way to the case of $h = h^{l}_{a=-1/2}$

$$|\varphi_{(4)}\rangle = |P\rangle \otimes U_{(4)}\, b_{-4}b_{-3}b_{-2}|0\rangle + |P'\rangle \otimes U_{(4)}\, b_{-3}b_{-2}|0\rangle$$

$$|\varphi_{(4)}\rangle = Q^{(4)}(\hat{A}^{(4)}|\varphi_{(4)}\rangle) \qquad \hat{A}^{(4)}\, U_{(4)}b_{-m}\cdots b_{-2}|0\rangle = 0$$

zero (in a single Fock space)

Beyond a Fock state expression, the equality holds.
(Or associativity of operator product is broken.)

- Details of vanishing state in the Fock space:

$$
\begin{aligned}
b(z)U_{(4)} &= \exp\left(-4\sum_{n=1}^{\infty}\frac{z^{-2n}}{n}\right)U_{(4)}b(z) = (1-z^{-2})^4\,U_{(4)}b(z),\\
\partial b(z)U_{(4)} &= (1-z^{-2})^4\,U_{(4)}\partial b(z) + 8z^{-3}(1-z^{-2})^3\,U_{(4)}b(z),\\
\partial^2 b(z)U_{(4)} &= (1-z^{-2})^4\,U_{(4)}\partial^2 b(z) + 16z^{-3}(1-z^{-2})^3\,U_{(4)}\partial b(z)\\
&\quad - 24z^{-4}(1-3z^{-2})(1-z^{-2})^2\,U_{(4)}b(z)
\end{aligned}
$$

$\Downarrow$ $z \to \pm 1$

$$
\begin{aligned}
b(\pm 1)\,U_{(4)}b_{-m}\cdots b_{-2}|0\rangle &= 0,\\
\partial b(\pm 1)\,U_{(4)}b_{-m}\cdots b_{-2}|0\rangle &= 0,\\
\partial^2 b(\pm 1)\,U_{(4)}b_{-m}\cdots b_{-2}|0\rangle &= 0
\end{aligned}
$$

Concluding remarks

- We have found homotopy operators for a class of identity-base solutions.
- Moduli independence of 1-loop vacuum energy is shown by obtained homotopy operators.
- The existence of homotopy operator suggests that the non-trivial cohomology in the wrong ghost number sectors (2002) may be “trivial” beyond a single Fock space.
- Mathematically more rigorous definition of state space seems to be necessary to derive “physics” from SFT.

Comparison of analytic solutions in bosonic open SFT

	Takahashi-Tanimoto's identity-based solution	Schnabl's solution (and variants)
Equation of motion	2002	2005 (2009)
D-brane tension	? (indirectly, numerically 2009,2010)	O.K. 2005 (2009)
Gauge invariant overlap	? (indirectly, numerically 2009,2010)	O.K. 2008 (2009)
Cohomology	(ghost #1, 2002) all ghost #	all ghost #, 2006 (2009)
One-loop	? (moduli-independence)	?

Extension to superstring

Inatomi-I.K.-Takahashi (to appear)

- In the framework of modified cubic superstring field theory, a class of identity-based solution is constructed:

$$A_c = \left(Q_L \left(e^{\lambda} - 1 \right) - \frac{1}{2} C_L \left((\partial \lambda)^2 e^{\lambda} \right) - \frac{1}{4} \Theta_L \left((\partial \lambda) e^{\lambda} \right) \right) I$$

$$\lambda(-1/z) = \lambda(z), \quad \lambda(\pm i) = 0$$

$$\Theta_L(h) = \int_{C_{\mathrm{left}}} \frac{dz}{2\pi i} h(z) (c\beta\gamma(z) - \partial c(z))$$

$$Q_{\mathrm{B}} A_c + A_c * A_c = 0$$ ⇒ The EOM in the NS sector holds.

BRST operator at the solution

- Action expanded around the solution:

$$S'[A,\Psi] \equiv S[A+A_c,\Psi] - S[A_c,0]$$
$$= \frac{1}{2}\langle A, Y_{-2}Q'A\rangle + \frac{1}{3}\langle A, Y_{-2}A*A\rangle + \frac{1}{2}\langle \Psi, YQ'\Psi\rangle + \langle A, Y\Psi*\Psi\rangle$$

String field (A,Ψ)

NS sector (space-time boson) → A

R sector (space-time fermion) → Ψ

$$Q' = Q(e^{\lambda}) + C\left(-\frac{1}{2}(\partial\lambda)^2 e^{\lambda}\right) + \Theta\left(-\frac{1}{4}(\partial\lambda)e^{\lambda}\right)$$

Homotopy operators

- In the same way as the bosonic case, homotopy operators for the BRST operator at the solution with a particular type of associated function.

$$\begin{aligned}\{Q(f), b(z)\} =& \frac{3}{4}\partial^2 f(z) + \partial f(z)\left(-bc(z) - \frac{3}{4}\beta\gamma(z)\right) + f(z)T(z),\\ \{C(g), b(z)\} =& g(z),\\ \{\Theta(h), b(z)\} =& \partial h(z) + h(z)\beta\gamma(z)\end{aligned}$$

$$\{Q', \hat{A}\} = 1, \qquad \hat{A}^2 = 0$$

$$\hat{A} = \sum_{k=1}^{2l} a_k\, l^{-2} z_k^2\, b(z_k), \quad z_k^{2l} + (-1)^l = 0, \quad \sum_{k=1}^{2l} a_k = 1$$

the same as bosonic case

$$\lambda = h^l_{a=-1/2}$$

Comments on the solution

- Pure gauge form: $A_c = \exp(q_L(\lambda)I)Q_{\rm B}\exp(-q_L(\lambda)I)$

$$q_L(\lambda) = \int_{C_{\rm left}} \frac{dz}{2\pi i}\lambda(z)(-bc(z) - \beta\gamma(z))$$

↑

regular OPE

- "BPS" D-brane vanishes ??

Similar solutions have been constructed:
Erler's soltuion (2007) (extension of Schnabl's solution to superstring)