HOMOTOPY OPERATORS AND IDENTITY-BASED SOLUTIONS IN SUPERSTRING FIELD THEORY

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REFERENCES

- S. Inatomi, I.K. and T. Takahashi, "Homotopy Operators and Identity-Based Solutions in Cubic Superstring Field Theory," arXiv: 1109.2406

  See, also,

  - S. Inatomi, I.K. and T. Takahashi, "Homotopy Operators and One-Loop Vacuum Energy at the Tachyon Vacuum," arXiv:1106.5314

CONTENTS

A CLASS OF IDENTITY-BASED SOLUTIONS IN SFT

- Takahashi-Tanimoto's identity-based solutions (2002)

- Marginal solution $\Psi = -V_L^a(F_a)I - \frac{1}{4}g^{ab}C_L(F_aF_b)I$

  $\sim c\partial X^\mu$

  - extension to marginal solutions in super SFT [Kishimoto-Takahashi (2005)] $\Phi = -\tilde{V}_L^a(F_a)I$

    $\sim c\xi e^{-\phi}\psi^\mu$

- Scalar solution $\Psi_h = Q_L(e^h - 1)I - C_L((\partial h)^2 e^h)I$

  - a candidate for tachyon vacuum

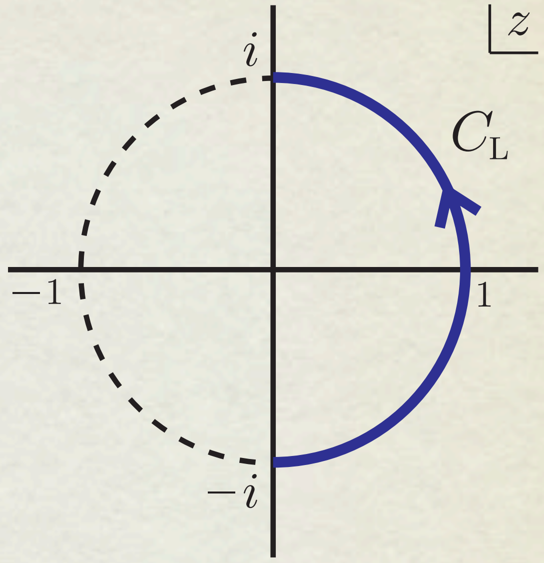
  - extension to solution in super SFT ? [This talk]

THE SCALAR SOLUTION IN BOSONIC SFT

- Structure of the solution:

$$\Psi_h = Q_L(e^h - 1)I - C_L((\partial h)^2 e^h)I$$

identity state



$$h(z) = h(-1/z), \quad h(\pm i) = 0$$

$$Q_L(f) = \int_{C_{\rm L}} \frac{dz}{2\pi i} f(z) j_{\rm B}(z), \quad C_L(f) = \int_{C_{\rm L}} \frac{dz}{2\pi i} f(z) c(z).$$

$$j_{\rm B} = cT^{\rm m} + bc\partial c + \frac{3}{2}\partial^2 c$$:BRST current (primary)

OPERATOR PRODUCT EXPANSION IN BOSONIC STRING

- The following OPEs were essential to prove the equation of motion: $Q_{\rm B}\Psi_h + \Psi_h * \Psi_h = 0$

$$j_{\rm B}(y) j_{\rm B}(z) \sim \frac{-4}{(y-z)^3} c\partial^3 c(z) + \frac{-2}{(y-z)^2} c\partial^2 c(z)$$

$$j_{\rm B}(y) c(z) \sim \frac{1}{y-z} c\partial c(z)$$

$j_{\rm B}, c$ form a closed algebra.

- The identity state I is an identity element of the star product.

OPERATOR PRODUCT EXPANSIONS IN RNS SUPERSTRING

- BRST current and c-ghost and...

$$j_{\mathrm{B}} = cT^{\mathrm{m}} + \gamma G^{\mathrm{m}} + bc\partial c + \frac{1}{4}c\partial\beta\gamma - \frac{3}{4}c\beta\partial\gamma + \frac{3}{4}\partial c\beta\gamma - b\gamma^2 + \frac{3}{4}\partial^2 c$$

:primary, dim. 1, s.t., $\{Q_{\mathrm{B}}, b(z)\} = T(z)$

$\theta \equiv c\beta\gamma - \partial c$:primary, dim. 0

$$j_{\mathrm{B}}(y)\, j_{\mathrm{B}}(z) \sim \frac{1}{(y-z)^3}\left(-\frac{17}{8}c\partial c(z) + 3\gamma^2(z)\right) + \frac{1}{(y-z)^2}\frac{1}{2}\partial\left(-\frac{17}{8}c\partial c(z) + 3\gamma^2(z)\right)$$
$$+ \frac{1}{y-z}\partial\left(\frac{1}{4}c\gamma G^{\mathrm{m}}(z) + \frac{1}{2}bc\gamma^2(z) + \frac{1}{4}\beta\gamma^3(z)\right)$$

$$j_{\mathrm{B}}(y)\, \theta(z) \sim \frac{1}{(y-z)^2}\left(\frac{1}{4}c\partial c(z) - \gamma^2(z)\right) + \frac{1}{y-z}\left(-c\gamma G^{\mathrm{m}}(z) - 2bc\gamma^2(z) - \beta\gamma^3(z)\right)$$

$$j_{\mathrm{B}}(y)\, c(z) \sim \frac{1}{y-z}(c\partial c(z) - \gamma^2(z)) \qquad \theta(y)\,\theta(z) \sim \frac{1}{y-z}c\partial c(z)$$

ANTI-COMMUTATION RELATIONS FROM OPE

- Mode expansions:

$$j_{\mathrm{B}}(z) = \sum_n Q_n z^{-n-1}, \quad c(z) = \sum_n c_n z^{-n+1}, \quad \theta(z) = \sum_n \theta_n z^{-n}$$

- Anti-commutation relations can be derived from OPE:

$$\{Q_n, Q_m\} = nm\left(-\frac{7}{16}\{\theta_n, \theta_m\} + \frac{3}{2}\{Q_{\mathrm{B}}, c_{n+m}\}\right) + \frac{n+m}{4}\{Q_{\mathrm{B}}, \theta_{n+m}\},$$

$$\{Q_n, c_m\} = \{Q_{\mathrm{B}}, c_{n+m}\},$$

$$\{Q_n, \theta_m\} = \{Q_{\mathrm{B}}, \theta_{n+m}\} + n\left(-\frac{3}{4}\{\theta_n, \theta_m\} + \{Q_{\mathrm{B}}, c_{n+m}\}\right).$$

Note: $Q_0 = Q_{\mathrm{B}}$

ANTI-COMMUTATION RELATIONS INCLUDING HALF INTEGRATION

- Half-integration with a weighting function:

$$Q_L(f) = \int_{C_\mathrm{L}} \frac{dz}{2\pi i} f(z) j_\mathrm{B}(z), \quad C_L(g) = \int_{C_\mathrm{L}} \frac{dz}{2\pi i} g(z) c(z), \quad \Theta_L(h) = \int_{C_\mathrm{L}} \frac{dz}{2\pi i} h(z) \theta(z)$$

- Anti-commutation relations from mode expansions:

$$\{Q_\mathrm{B}, Q_L(f)\} = \frac{1}{4}\{Q_\mathrm{B}, \Theta_L(\partial f)\},$$

$$\{Q_L(f), Q_L(f)\} = -\frac{7}{16}\{\Theta_L(\partial f), \Theta_L(\partial f)\} + \frac{3}{2}\{Q_\mathrm{B}, C_L((\partial f)^2)\} + \frac{1}{2}\{Q_\mathrm{B}, \Theta_L(f\partial f)\},$$

$$\{Q_L(f), C_L(g)\} = \{Q_\mathrm{B}, C_L(fg)\},$$

$$\{Q_L(f), \Theta_L(h)\} = \{Q_\mathrm{B}, \Theta_L(fh)\} - \frac{3}{4}\{\Theta_L(\partial f), \Theta_L(h)\} + \{Q_\mathrm{B}, C_L((\partial f)h)\}$$

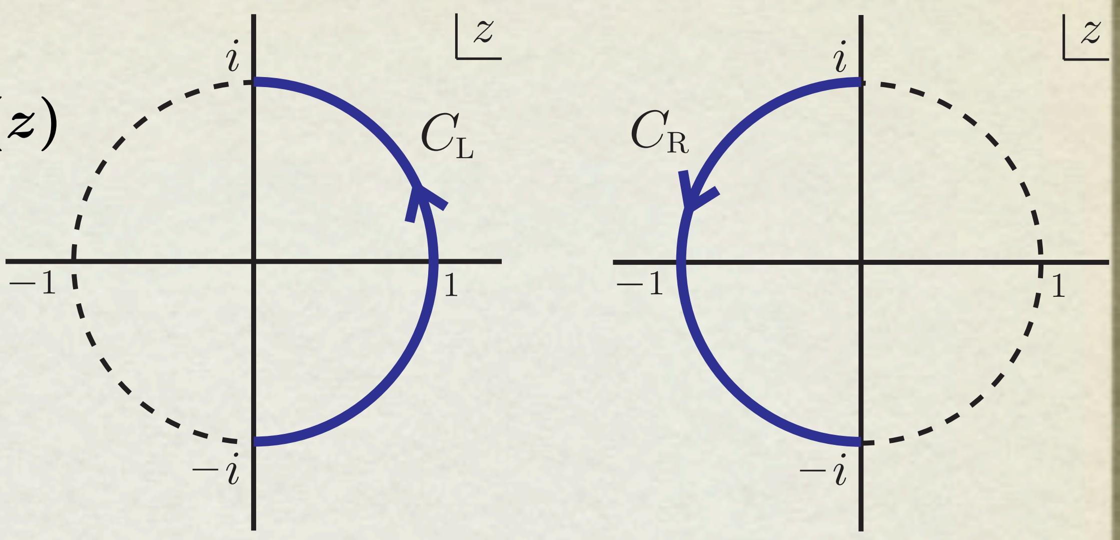
$f(\pm i) = 0$ for partial integrations

HALF INTEGRATIONS AND THE STAR PRODUCT

- For a primary field $\boldsymbol{\sigma}$ with dim. h, half integrations:

$$\Sigma_L(F) = \int_{C_{\rm L}} \frac{dz}{2\pi i} F(z)\sigma(z), \quad \Sigma_R(F) = \int_{C_{\rm R}} \frac{dz}{2\pi i} F(z)\sigma(z)$$

$$F(-1/z) = (z^2)^{1-h} F(z)$$



$$(\Sigma_R(F)B_1) * B_2 = -(-1)^{|\sigma||B_1|} B_1 * (\Sigma_L(F)B_2)$$

- For identity state, we have simple formulas:

$$\Sigma_R(F)I = -\Sigma_L(F)I,$$
$$(\Sigma_L(F)I) * B = \Sigma_L(F)B,$$
$$B * (\Sigma_L(F)I) = -(-1)^{|\sigma||B|} \Sigma_R(F)B.$$

ANSATZ FOR STRING FIELD

- Identity-based string field (ghost number 1, picture number 0)

$$A_c = Q_L(f)I + C_L(g)I + \Theta_L(h)I$$

$$f(-1/z) = f(z),\ \ g(-1/z) = z^4\, g(z),\ \ h(-1/z) = z^2\, h(z),\ \ f(\pm i) = 0$$

- Calculation using the previous formulas:

$$\begin{aligned} & Q_{\mathrm{B}} A_c + A_c * A_c \\ & = \Big[\Big\{Q_{\mathrm{B}}, C_L\Big((1+f)g + \frac{3}{4}(\partial f)^2 + h\partial f\Big)\Big\} + \Big\{Q_{\mathrm{B}}, \Theta_L\Big((1+f)\Big(h + \frac{1}{4}\partial f\Big)\Big)\Big\} \\ & \quad - \frac{7}{32}\{\Theta_L(\partial f), \Theta_L(\partial f)\} + \frac{1}{2}\{\Theta_L(h), \Theta_L(h)\} - \frac{3}{4}\{\Theta_L(\partial f), \Theta_L(h)\}\Big] I. \end{aligned}$$

- The above vanishes if we choose the functions as follows:

$$f = e^{\lambda} - 1,\ \ g = -\frac{1}{2}(\partial\lambda)^2 e^{\lambda},\ \ h = -\frac{1}{4}(\partial\lambda)e^{\lambda} \qquad \lambda(-1/z) = \lambda(z),\ \lambda(\pm i) = 0.$$

IDENTITY-BASED SOLUTION TO MODIFIED CUBIC SSFT

- Equations of motion of modified cubic SSFT:

$$Y_{-2}(Q_{\mathrm{B}}A + A * A) + Y\Psi * \Psi = 0,$$
$$Y(Q_{\mathrm{B}}\Psi + A * \Psi + \Psi * A) = 0.$$

- A class of identity-based solution in the NS sector (as an extension of Takahashi-Tanimoto's scalar solution to SSFT):

$$A_c = Q_L(e^{\lambda} - 1)I + C_L\Big(-\frac{1}{2}(\partial\lambda)^2 e^{\lambda}\Big)I + \Theta_L\Big(-\frac{1}{4}\partial e^{\lambda}\Big)I$$

$$\lambda(-1/z) = \lambda(z), \ \lambda(\pm i) = 0.$$

$$\Rightarrow \quad Q_{\mathrm{B}}A_c + A_c * A_c = 0$$

BRST OPERATOR AT THE SOLUTION

- Re-expansion of the action of SSFT around the solution:

$$S'[A,\Psi] \equiv S[A+A_c,\Psi] - S[A_c,0]$$
$$= \frac{1}{2}\langle A, Y_{-2}Q'A\rangle + \frac{1}{3}\langle A, Y_{-2}A*A\rangle + \frac{1}{2}\langle \Psi, YQ'\Psi\rangle + \langle A, Y\Psi*\Psi\rangle$$

- BRST operator at the solution can be expressed as:

$$Q' = Q_{\rm B} + [A_c, \ \cdot \ \}_*$$
$$= Q_{\rm B} + (Q_L(f) + C_L(g) + \Theta_L(h)) + (Q_R(f) + C_R(g) + \Theta_R(h))$$
$$= Q(e^{\lambda}) + C\left(-\frac{1}{2}(\partial\lambda)^2 e^{\lambda}\right) + \Theta\left(-\frac{1}{4}\partial e^{\lambda}\right)$$

$$Q(f) = \oint\frac{dz}{2\pi i}f(z)j_{\rm B}(z),\quad C(g) = \oint\frac{dz}{2\pi i}g(z)c(z),\qquad \Theta(h) = \oint\frac{dz}{2\pi i}h(z)\theta(z)$$

TOWARD HOMOTOPY OPERATOR FOR *Q'*

- OPE with *b*-ghost:

$$j_{\rm B}(y)b(z) \sim \frac{3/2}{(y-z)^3} + \frac{1}{(y-z)^2}\left(-bc(z) - \frac{3}{4}\beta\gamma(z)\right) + \frac{1}{y-z}T(z),$$

$$c(y)b(z) \sim \frac{1}{y-z}, \qquad \theta(y)b(z) \sim \frac{1}{(y-z)^2} + \frac{1}{y-z}\beta\gamma(z)$$

- Anti-commutation relation from the OPEs:

$$\{Q', b(z)\} = \frac{1}{2}(\partial^2\lambda(z))e^{\lambda(z)} + (\partial e^{\lambda(z)})j_{\rm gh}(z) + e^{\lambda(z)}T(z).$$

$$j_{\rm gh} = -bc - \beta\gamma$$

- It becomes a c-number at a <u>second order zero</u> $z = z_0$ of $e^{\lambda(z)}$

$$\{Q', b(z_0)\} = \frac{1}{2}(\partial^2\lambda(z))e^{\lambda(z)}|_{z=z_0}$$

HOMOTOPY OPERATOR FOR *Q'* AT A PARTICULAR FUNCTION

- Example of the function: $\lambda(z) = h_a^l(z)$

$$h_a^l(z) = \log\left(1 - \frac{a}{2}(-1)^l(z^l - (-1)^l z^{-l})^2\right), \quad (a \geq -1/2;\ l = 1, 2, 3, \cdots).$$

:the same functions with the bosonic case

- $e^{h_a^l(z)}$ has second order zeros z_k $(z_k^{2l} = -(-1)^l)$ only for $a = -\frac{1}{2}$

- Homotopy operator $\hat{A}$ for $\lambda(z) = h^l_{a=-1/2}(z)$

$$\{Q', \hat{A}\} = 1, \quad \hat{A}^2 = 0.$$

$$\hat{A} = \sum_{k=1}^{2l} a_k l^{-2} z_k^2\, b(z_k), \quad \sum_{k=1}^{2l} a_k = 1.$$

:the same form with the bosonic case

SIMILARITY TRANSFORMATION OF BRST OPERATOR

- It turns out that *Q'* can be rewritten as a similarity transform using the ghost number current: $j_{\rm gh} = -bc - \beta\gamma$

$$Q' = e^{q(\lambda)} Q_{\rm B} e^{-q(\lambda)}$$
$$= Q(e^{\lambda}) + C\left(-\frac{1}{2}(\partial\lambda)^2 e^{\lambda}\right) + \Theta\left(-\frac{1}{4}\partial e^{\lambda}\right).$$

$$q(\lambda) = \oint \frac{dz}{2\pi i} \lambda(z) j_{\rm gh}(z)$$

- Unlike the bosonic case, $e^{\pm q(\lambda)}$ is <u>not</u> singular even for $\lambda = h^l_{a=-\frac{1}{2}}$

$j_{\rm gh}(y) j_{\rm gh}(z) \quad (y \to z)$:regular for superstring

- Nevertheless, there exists a homotopy operator for $\lambda = h^l_{a=-\frac{1}{2}}$
It implies:

$$Q'\psi = 0 \quad \Leftrightarrow \quad \psi = Q'(\hat{A}\psi)$$

Vanishing cohomology for all ghost number sectors!

COMMENTS ON COHOMOLOGY OF THE BRST OPERATOR

- At least formally, we have

$$Q_{\mathrm{B}}\phi = 0 \quad \Leftrightarrow \quad Q'(e^{q(h^l_{-1/2})}\phi) = 0$$

$$\Leftrightarrow \quad e^{q(h^l_{-1/2})}\phi = Q'(\hat{A}e^{q(h^l_{-1/2})}\phi)$$

- Using the explicit form of the <u>nontrivial part</u> φ of Q_{B}-cohomology in the NS and R sector, we find:

$$\phi = \varphi + Q_{\mathrm{B}}\chi$$

⬇

$$e^{q(h^l_{-1/2})}\phi = U2^{-2g}\varphi + Q'(e^{q(h^l_{-1/2})}\chi)$$

g: ghost number

$$U = \exp\left(-\sum_{n=1}^{\infty}\frac{(-1)^{n(l+1)}}{n}q_{-2nl}\right) \qquad j_{\mathrm{gh}}(z) = \sum_n q_n z^{-n-1}$$

ZERO IN THE FOCK SPACE BUT NONZERO IN A LARGER SPACE

- In a similar way to the bosonic case,

$$[q_n, b_m] = -b_{n+m} \qquad U^{-1} b(z) U = e^{-\sum_{n=1}^{\infty} \frac{(-1)^{n(l+1)}}{n} z^{-2nl}} b(z)$$

$$\Rightarrow \quad \hat{A} U 2^{-2g} |\varphi\rangle = \exp\left(-\sum_{n=1}^{\infty} \frac{1}{n}\right) U \hat{A} 2^{-2g} |\varphi\rangle = 0$$

- It implies that all coefficients of $\hat{A} U 2^{-2g} \varphi$ vanish in the Fock space.

- However, we should have $e^{q(h^l_{-1/2})} \varphi = Q'(\hat{A} U 2^{-2g} \varphi)$
$\neq 0$

- Nontrivial part of Q_{B} -cohomology becomes *Q'*-exact <u>outside</u> the Fock space by $e^{q(h^l_{-1/2})}$ *as far as we respect the homotopy relation:*

$$\{Q', \hat{A}\} = 1$$

PURE GAUGE FORM OF THE SOLUTION

- In a similar way to the bosonic case, we have

$$A_c = \exp(q_L(\lambda)I)Q_{\mathrm{B}} \exp(-q_L(\lambda)I)$$

$$q_L(\lambda) = \int_{C_{\mathrm{L}}} \frac{dz}{2\pi i} \lambda(z) j_{\mathrm{gh}}(z)$$

- Unlike the bosonic case, $\exp(\pm q_L(\lambda)I)$ is not singular even for $\lambda = h^l_{a=-1/2}$ (?)

- Formally, it corresponds to a pure gauge solution to Berkovits' WZW-type SSFT: $\Phi_c = -q_L(\lambda)I = \eta_0(-\xi_0 q_L(\lambda)I)$

$$\eta_0(e^{-\Phi_c} Q_{\mathrm{B}} e^{\Phi_c}) = 0$$

SUMMARY

- A type of identity-based solution to modified cubic SSFT was constructed, which is a straightforward extension of Takahashi-Tanimoto's scalar solution in bosonic SFT.
- A homotopy operator for the BRST operator *Q'* at the solution was constructed in the same way as the bosonic case.
- *Q'* can be expressed as a similarity transformation from the conventional BRST operator, even in the case that there exists a homotopy operator.
- Nontrivial part of the conventional BRST cohomology is mapped to *Q'*-exact form outside the Fock space by a similarity transformation.

DISCUSSION

- Vanishing cohomology at the solution might imply that the D-brane vanishes although it is in the GSO(+) sector.(?)

  [cf. Erler(2007)]

- Evaluation of vacuum energy or gauge invariant overlap with appropriate regularization is desired.

- Why does there exist a homotopy operator for (at least apparently) pure gauge solution ?

- More rigorous statement of $\hat{A}U2^{-2g}\varphi \simeq 0$ and $e^{q(h^{l}_{-1/2})}\varphi = Q'(\hat{A}U2^{-2g}\varphi)$
  $\neq 0$

- Or the relation $\{Q', \hat{A}\} = 1$ might be broken?

  $(Q'\hat{A} + \hat{A}Q')U|\cdot\rangle \neq Q'(\hat{A}U|\cdot\rangle) + \hat{A}(Q'U|\cdot\rangle)$ (?)

CALCULATION OF THE STAR PRODUCT FOR THE ANSATZ

- Using star product formulas and anti-commutation relations,

$$A_c * A_c = \Big[\frac{1}{2}\{Q_L(f), Q_L(f)\} + \frac{1}{2}\{C_L(g), C_L(g)\} + \frac{1}{2}\{\Theta_L(h), \Theta_L(h)\} + \{Q_L(f), C_L(g)\} + \{Q_L(f), \Theta_L(h)\} + \{C_L(g), \Theta_L(h)\}\Big] I$$

$$= \Big[-\frac{7}{32}\{\Theta_L(\partial f), \Theta_L(\partial f)\} + \frac{3}{4}\{Q_{\rm B}, C_L((\partial f)^2)\} + \frac{1}{4}\{Q_{\rm B}, \Theta_L(f\partial f)\} + \frac{1}{2}\{\Theta_L(h), \Theta_L(h)\} + \{Q_{\rm B}, C_L(fg)\} + \{Q_{\rm B}, \Theta_L(fh)\} - \frac{3}{4}\{\Theta_L(\partial f), \Theta_L(h)\} + \{Q_{\rm B}, C_L((\partial f)h)\}\Big] I$$

- BRST multiplication noting $Q_{\rm B} I = 0$:

$$Q_{\rm B} A_c = [\{Q_{\rm B}, Q_L(f)\} + \{Q_{\rm B}, C_L(g)\} + \{Q_{\rm B}, \Theta_L(h)\}]\, I$$

$$= \Big[\frac{1}{4}\{Q_{\rm B}, \Theta_L(\partial f)\} + \{Q_{\rm B}, C_L(g)\} + \{Q_{\rm B}, \Theta_L(h)\}\Big] I$$

MODIFIED CUBIC SUPER STRING FIELD THEORY

[Preitschopf-Thorn-Yost, Arefeva-Medvedev-Zubarev(1990)]

- Action:

$$S[A,\Psi] = \frac{1}{2}\langle A, Y_{-2}Q_{\mathrm{B}}A\rangle + \frac{1}{3}\langle A, Y_{-2}A * A\rangle + \frac{1}{2}\langle \Psi, YQ_{\mathrm{B}}\Psi\rangle + \langle A, Y\Psi * \Psi\rangle$$

String field

A :NS sector, 0-picture, ghost number 1, Grassmann odd

Ψ :R sector, (-1/2)-picture, ghost number 1/2, Grassmann odd

Inverse picture changing operators

$Y = Y(i) = c(i)\delta'(\gamma(i))$:picture number -1

$Y_{-2} = Y(i)Y(-i)$:picture number -2

- A part of (finite) gauge transformations:

$$A' = e^{-\Lambda}A\,e^{\Lambda} + e^{-\Lambda}Q_{\mathrm{B}}e^{\Lambda}, \qquad \Psi' = e^{-\Lambda}\Psi\,e^{\Lambda}$$

BRST COHOMOLOGY IN 0-PICTURE

- BRST cohomology for $p^+ \neq 0$ [cf. Kohriki-Kunitomo-Murata(2010)]

$$Q_{\rm B}\phi = 0 \quad \Leftrightarrow$$

$$\phi = \mathcal{P}|\text{tach}\rangle_0 + \mathcal{P}'\left(c_0|\text{tach}\rangle_0 + \frac{\sqrt{2}}{\sqrt{\alpha'}k^+}\gamma_{-\frac{1}{2}}|0,k_1\rangle_0\right) + Q_{\rm B}\chi$$

$$|\text{tach}\rangle_0 = \left(\psi^-_{-\frac{1}{2}} - \frac{1}{\sqrt{2\alpha'}k^+}b_{-1}\gamma_{\frac{1}{2}} + \frac{1}{4\alpha'(k^+)^2}\psi^+_{-\frac{1}{2}}\right)|0,k_1\rangle_0,$$:onshell tachyon in 0-picture

$$|0,k_1\rangle_0 = |k_1\rangle_{\rm mat}\otimes c_1|0\rangle_{bc}\otimes|P=0\rangle_{\beta\gamma}, \quad k_1^+ = k^+,\ k_1^- = \frac{1}{4\alpha' k^+},\ k^i = 0.$$

$\mathcal{P}, \mathcal{P}'$:DDF operators in the matter sector

Note: $|0,k_1\rangle_{-1} = \delta(\gamma_{\frac{1}{2}})|0,k_1\rangle_0$:onshell tachyon in (-1)-picture

$$|\text{tach}\rangle_0 = X(0)\frac{1}{\sqrt{2\alpha'}k^+}|0,k_1\rangle_{-1}$$

BRST COHOMOLOGY IN 0-PICTURE WITH ZERO MOMENTUM

- BRST cohomology for $p^\mu = 0$ can be obtained by expanding the BRST operator with respect to $\gamma_{\frac{1}{2}}$ and $\beta_{-\frac{1}{2}}$.

$$Q_{\rm B}\phi = 0 \quad \Leftrightarrow$$

$$\phi = \mathcal{C}^{(0)} b_{-1}\,|{\downarrow}\rangle_0 + \mathcal{C}^{(1)}_\mu (\alpha^\mu_{-1} + \psi^\mu_{-\frac{1}{2}} b_{-1}\gamma_{\frac{1}{2}})\,|{\downarrow}\rangle_0 + \mathcal{C}^{(2)}_\mu \left(\alpha^\mu_{-1} c_0 + 2\psi^\mu_{-\frac{1}{2}}\gamma_{-\frac{1}{2}} + \psi^\mu_{-\frac{1}{2}} b_{-1} c_0 \gamma_{\frac{1}{2}}\right)|{\downarrow}\rangle_0$$
$$+\, \mathcal{C}^{(3)}\left(2\gamma^2_{-\frac{1}{2}} + c_{-1}c_0 + \gamma_{-\frac{1}{2}}\gamma_{\frac{1}{2}} b_{-1} c_0\right)|{\downarrow}\rangle_0 + Q_{\rm B}\chi$$

$$|{\downarrow}\rangle_0 = |0\rangle_{\rm mat} \otimes c_1|0\rangle_{bc} \otimes |P=0\rangle_{\beta\gamma} \qquad \mathcal{C}^{(0)}, \mathcal{C}^{(1)}_\mu, \mathcal{C}^{(2)}_\mu, \mathcal{C}^{(3)}$$:constants

- It turned out that they can be also obtained by *X(0)* from the result of (-1)-picture, up to BRST-exact term.

$$Q_{\rm B}\phi = 0 \Leftrightarrow$$

$$\phi = \mathcal{C}^{(0)}\beta_{-\frac{1}{2}}\,|{\downarrow}\rangle_{-1} + \mathcal{C}^{(1)}_\mu \psi^\mu_{-\frac{1}{2}}\,|{\downarrow}\rangle_{-1} + \mathcal{C}^{(2)}_\mu \psi^\mu_{-\frac{1}{2}} c_0\,|{\downarrow}\rangle_{-1} + \mathcal{C}^{(3)}\gamma_{-\frac{1}{2}} c_0\,|{\downarrow}\rangle_{-1} + Q_{\rm B}\chi$$

BRST COHOMOLOGY IN (-1/2)-PICTURE

[Henneaux(1987),Lian-Zuckerman(1989)]

- BRST cohomology for $p^+ \neq 0$

$$Q_{\rm B}\phi = 0 \quad \Leftrightarrow \quad \phi = |\mathcal{P}\rangle_{-\frac{1}{2}} + \left(c_0 + \gamma_0 \frac{\psi_0^+}{\sqrt{2\alpha'}p^+}\right)|\mathcal{P}'\rangle_{-\frac{1}{2}} + Q_{\rm B}\chi$$

$|\mathcal{P}\rangle_{-\frac{1}{2}}, |\mathcal{P}'\rangle_{-\frac{1}{2}}$:DDF states in the matter sector

- BRST cohomology for $p^\mu = 0$

$$Q_{\rm B}\phi = 0 \quad \Leftrightarrow \quad \phi = A_0^a\,|S_a\rangle_{-\frac{1}{2}} + A_1^a\,\gamma_0|S_a\rangle_{-\frac{1}{2}} + Q_{\rm B}\chi$$

$|S_a\rangle_{-\frac{1}{2}}$:ground states

A_0^a, A_1^a :constants with spacetime spinor index

ON THE NONTRIVIAL PART OF THE BRST COHOMOLOGY

- Both NS and R sectors, nontrivial part φ of $Q_{\mathbf{B}}$-cohomology satisfies: $q_n|\varphi\rangle = 0 \quad (n > 1)$

$$q(h^l_{-1/2}) = q^{(+)}(h^l_{-1/2}) + q^{(0)}(h^l_{-1/2}) + q^{(-)}(h^l_{-1/2})$$

$$q^{(0)}(h^l_{-1/2}) = -q_0 \log 4, \quad q^{(\pm)}(h^l_{-1/2}) = -\sum_{n=1}^{\infty} \frac{(-1)^{n(l+1)}}{n} q_{\pm 2nl}$$

$$[q_n, q_m] = 0$$

⇓

$$e^{q(h^l_{-1/2})}|\varphi\rangle = e^{q^{(-)}(h^l_{-1/2})} 2^{-2q_0}|\varphi\rangle = U 2^{-2g}|\varphi\rangle$$